Magnetic self-organization in Tokamaks

Co authors:
N. Ferraro
I. Krebs

Acknowledgments:
A. Bhattacharjee
J. Breslau
J. Callen
G. Fu
S. Günter
S. Hudson
D. Meshcheriakov
R. Nazikian
C. Petty
C. Sovinec

S.C. Jardin

Princeton Plasma Physics Laboratory

Presented at

American Physical Society
Division of Plasma Physics
November 17, 2015
Savannah, GA

PRINCETON UNIVERSITY · PPPL PRINCETON PLASMA PHYSICS LABORATORY · U.S. DEPARTMENT OF ENERGY Office of Science · SciDAC Scientific Discovery through Advanced Computing · CEMM · MPPC

Motivation

- Some high-β operating modes of existing tokamaks do not exhibit sawteeth and maintain central safety factor values near $q_0 = 1$
  - hybrid modes in DIII-D, ASDEX-U, JET
  - long-lived mode in MAST, NSTX

- However, applying 1 ½ D transport codes (i.e. TRANSP) to many of these discharges indicate q_0 should fall below 1

- Mechanism responsible for keeping $q_0=1$ not previously understood, but often referred to as “flux-pumping”

- We offer an explanation for this as a “self-organized” state which we found using 3D MHD code M3D-C1

3D resistive MHD in torus with source terms

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = S_n$$

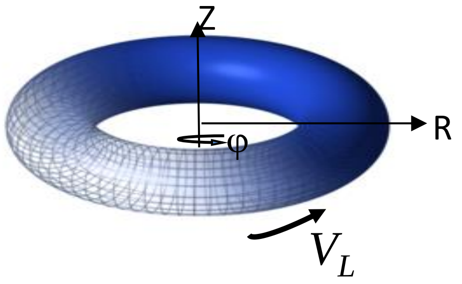
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{B}$$

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} \qquad \mu = \mu_0$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\frac{3}{2} p\mathbf{V}\right) = -p\nabla \bullet \mathbf{V} + \underline{\nabla \bullet n\kappa_\perp}\, \mathbf{bb} \bullet \nabla T + \nabla \bullet n\kappa_\| + \eta J^2 + \underline{S_e}$$

These balance in axisymmetric equilibrium

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \qquad T = p/n \qquad \eta = \eta_0 (T/T_0)^{-3/2}$$



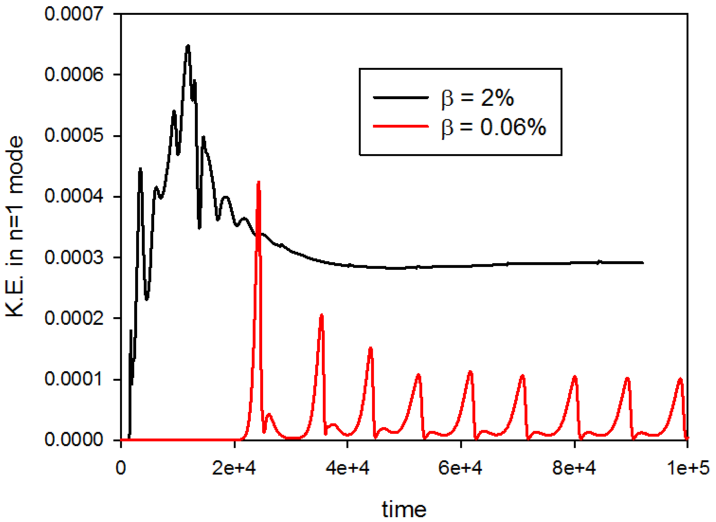
S_n(t) and S_e(t) volume sources adjusted to keep β and total # of particles constant
V_L(t) applied at boundary in control loop to keep total plasma toroidal current constant

Series of runs with same $\beta = \mu_0 p/B^2 \sim 2\%$ but differing $\kappa_{\perp 0}$ and S_e (proportional)

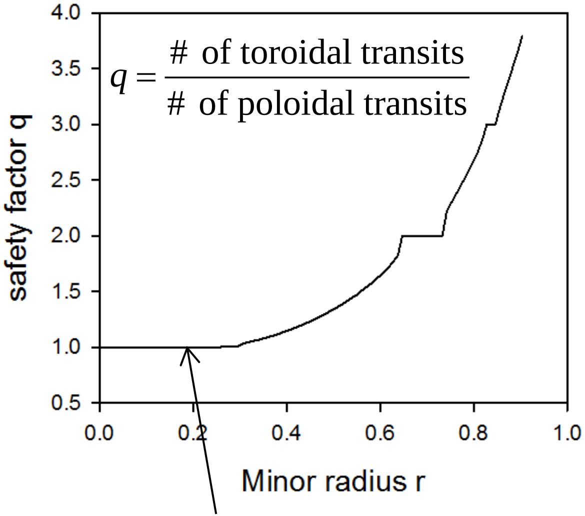
$\kappa_\perp \sim \kappa_{\perp 0} (T/T_0)^{-1/2}$ $\qquad \kappa_\| \sim 10^5 \times \kappa_\perp$ $\qquad \eta_0 \sim 10^{-6}$ $\qquad \kappa_{\perp 0} = 18\eta_0, 36\eta_0, 72\eta_0, 144\eta_0$

(4 3D cases will be presented + 1 2D)

$\beta \equiv \mu_0 \, p/B^2$ =2% behavior much different from low β



2% β



Large region in center with q = 1

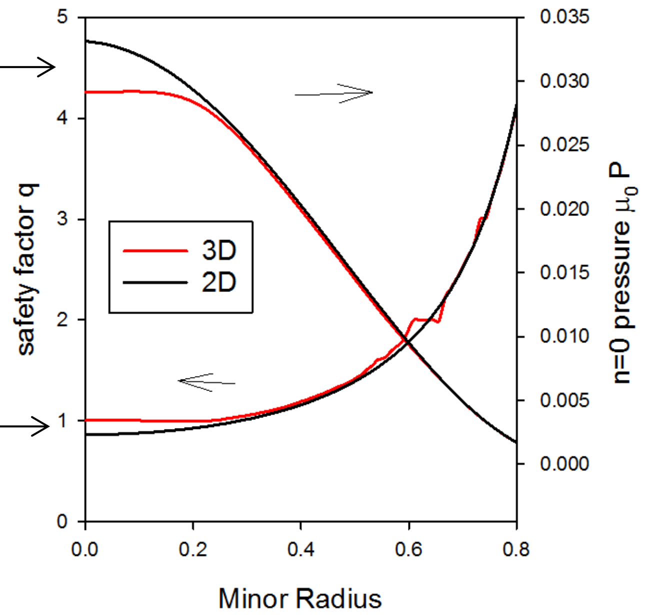
- At low-β, plasma kinetic energy (and T_{e0} and q_0) undergo periodic oscillations where current peaks, reconnection occurs and process repeats (sawteeth)
- At 2% β, plasma goes into a stationary state with large stationary kinetic energy and ultra-low magnetic shear with q=1 in center region

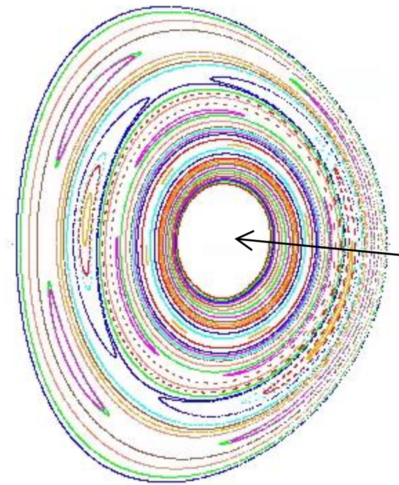
This self-organized stationary state with q=1 occurs only in 3D simulations (not in 2D)

Comparison of profiles from 2D and 3D stationary states (both with same β=2% and same κ) shows 2 differences

(1)

(2)





Poincare "field line mapping" plot of 3D final configuration

Volume in center with q=1 and no flux surfaces

Next slide focuses on central region with q=1

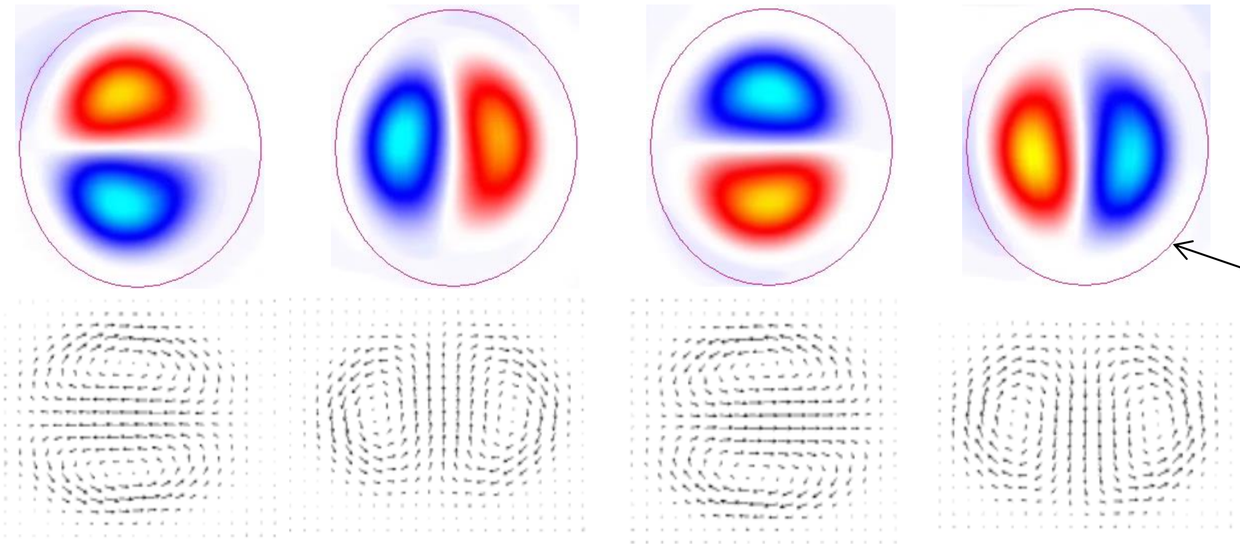
(1) Central pressure is flattened in 3D calculation compared to 2D

(2) Central q-profiles is less than 1 in 2D, equal to 1 and <u>ultra-flat</u> in 3D

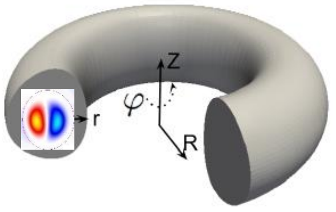
3D simulations show stationary (1,1) helical flow in q=1 region

U

$\mathbf{V}_{1,1} = R^2 \nabla U \times \nabla \varphi$



$\varphi = \varphi_0$ $\varphi = \varphi_0 + 90^0$ $\varphi = \varphi_0 + 180^0$ $\varphi = \varphi_0 + 270^0$



q =1.01

NOTE:
Only the central shear-free region with q=1 is plotted.

Plotted on top is poloidal velocity stream function U where $\mathbf{V}_{1,1} = R^2 \nabla U \times \nabla \varphi$

On bottom are vectors of poloidal velocity $\mathbf{V}_{1,1}$. Hill's vortex like structure.

What causes this flow and how does it enforce q_0=1?

Why doesn't q_0 continue to decrease in 3D run?

In a stationary state, $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = 0 \quad \Rightarrow \quad \mathbf{E} = -\nabla\Phi + \frac{V_L}{2\pi}\nabla\varphi \qquad (1)$$

Most general form for a curl-free vector field in a torus

Generalized Ohm's law: $$\mathbf{E} + \mathbf{V}\times\mathbf{B} = \eta\mathbf{J} \qquad (2)$$

In the stationary state, (1)+(2) becomes:

$$\nabla\Phi - \mathbf{V}\times\mathbf{B} = -\eta\mathbf{J} + \frac{V_L}{2\pi}\nabla\varphi \qquad (3)$$

$\hat{\varphi}\bullet$

$\mathbf{B}\bullet$

If we dot $\mathbf{B}$ into Eq. (3):

$$\mathbf{B}\bullet\nabla\Phi = \nabla\bullet(\mathbf{B}\Phi) = -\eta\mathbf{B}\bullet\mathbf{J} + \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi \qquad (4)$$

If magnetic surfaces exist, and we surface average (4), we get the well-known condition that the surface averaged current is completely determined by the resistivity profile:

$$\eta\langle\mathbf{J}\bullet\mathbf{B}\rangle = \frac{V_L}{2\pi}\langle\mathbf{B}\bullet\nabla\varphi\rangle$$

This is satisfied exactly in 2D stationary states ➔ $q_0 < 1$

V_L is a constant (applied loop voltage)

φ is the toroidal angle

But, it is not satisfied in 3D in the central region (if 2D surfaces are used!) Why not?

Basic Physics of self-organized stationary discharge with q_0=1

(1) Central ultra-low shear (ULS) region with q=1 (and no flux surfaces)

(2) Drives stationary interchange instability producing $\mathbf{V}_{1,1}$

(3) Produces electric potential

$$\nabla\Phi_{1,1} \simeq \mathbf{V}_{1,1} \times \mathbf{B}$$

(4) Also perturbs temperature:

$$(2/3)\nabla\bullet\left[n\kappa_\perp \nabla T_{1,1}\right] \simeq n\mathbf{V}_{1,1}\bullet\nabla T$$

(5) Also perturbs magnetic field:

$$\eta \mathbf{J}_{1,1} = \mathbf{V}_{1,1} \times \mathbf{B} - \eta_{1,1}\mathbf{J}$$

$$\nabla \times \mathbf{B}_{1,1} = \mu_0 \mathbf{J}_{1,1}$$

(6) 3+5 produce effective enhanced loop voltage in central region:

$$V_{n=0} = \mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1} + \cdots$$

(7) 2+4 produce velocity field that tends to flatten temperature profile

$$\delta T_{n=0} \simeq -\mathbf{V}_{1,1}\bullet\nabla T_{1,1}$$

(8)Axisymmetric (n=0) parallel Ohm's law is modified in two ways that broaden current

$$\eta \mathbf{J}\bullet\mathbf{B}_{n=0} + V_{n=0} = \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

resistivity profile is flattened ($\eta \mathbf{J}\bullet\mathbf{B}_{n=0}$)

positive in center, flattening current ($V_{n=0}$)

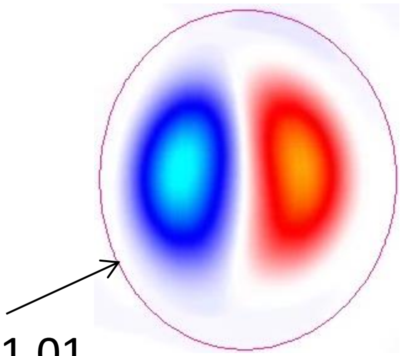
(9) No longer constrained to $\eta\langle \mathbf{J}\bullet\mathbf{B}\rangle = \text{const.}$ central region approaches minimum energy Taylor State $\mathbf{J} = \lambda\mathbf{B}$ → ULS

- These perturbations will also produce other islands via mode coupling, but we believe they do not play a central role

(2) Ultra-flat q profile drives interchange instability[1]

Plotted is U on one toroidal plane (φ=0) from a 3D simulation where:

$$\mathbf{V}_{1,1} = R^2 \nabla U \times \nabla \varphi$$

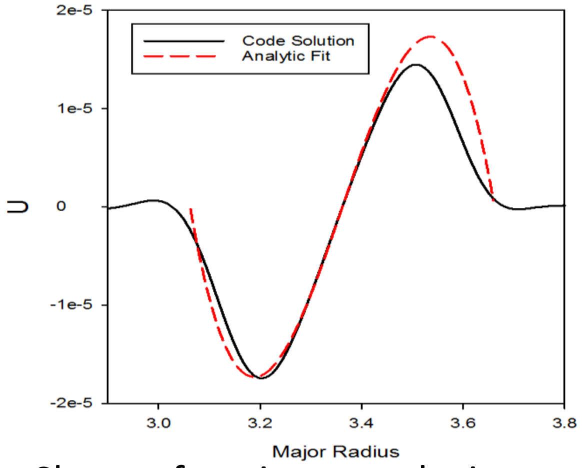


q =1.01

Compare with the unstable eigenfunction found in [1]

$$U(r,\theta,\varphi) = U_0 r[1-(r/r_1)^2]\sin(\theta-\varphi)$$

Mid-plane values



Shape of stationary velocity stream function from 3D nonlinear code agrees well with analytic linear eigenfunction.

Almost the same for all values of κ_0 -- but amplitude depends on β

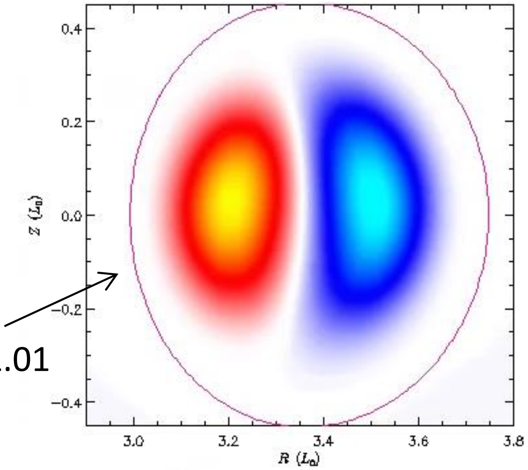
[1] Hastie and Hender, NF **28** (1988) p. 585 "Toroidal internal kink stability in tokamaks with ultra flat q profiles"

(3) Driven flow from interchange produces electric potential

$$\nabla\Phi - \mathbf{V}\times\mathbf{B} = -\eta\mathbf{J} + \frac{V_L}{2\pi}\nabla\varphi$$

These 2 large terms must almost cancel

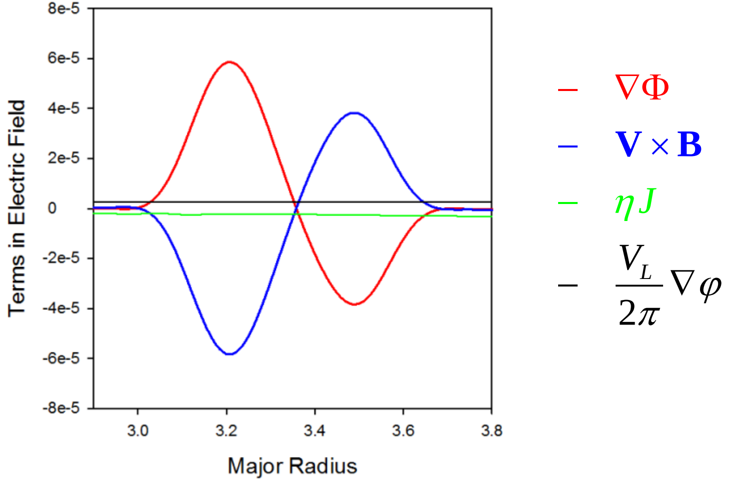
potential Φ at one toroidal plane



q =1.01

$$\varphi = 0$$

Mid-plane values of individual terms making up toroidal electric field (color coded) at one toroidal location



Terms on either side of equal sign mostly cancel (but not exactly)

(4) Velocity field also perturbs temperature and pressure.

$$\frac{\partial T}{\partial t} + n\mathbf{V}\bullet\nabla T + \frac{2}{3}nT\nabla\bullet\mathbf{V} - \frac{2}{3}\nabla\bullet n\kappa_\perp\bullet\nabla T + \frac{2}{3}\nabla\bullet\mathbf{q}_\parallel = \eta J^2 + S_e$$

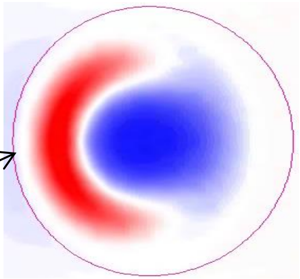
(1,1) components balance in 3D

These balance in 2D

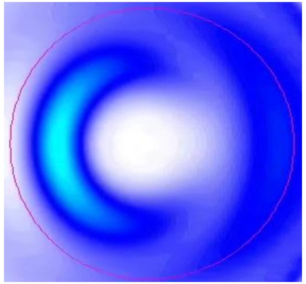
$n\mathbf{V}\bullet\nabla T + \frac{2}{3}nT\nabla\bullet\mathbf{V}$

$-\frac{2}{3}\nabla\bullet n\kappa_\perp\nabla T$

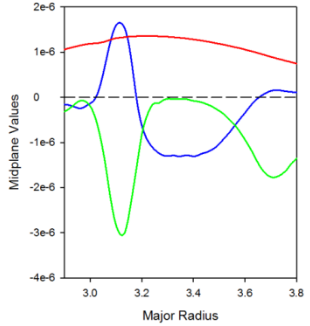
low κ →



q =1.01



Mid-plane values at φ=0



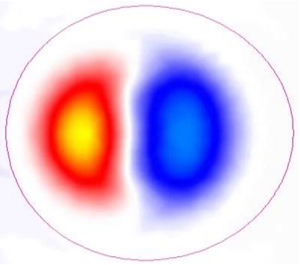
←

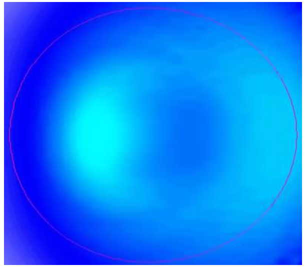
In low κ cases velocity terms dominate energy balance near center

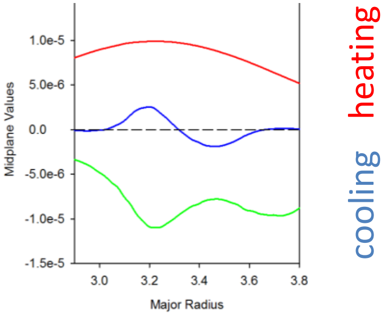
➔ large $T_{1,1}$

$\varphi = 0$

high κ →







←

In high κ cases thermal conductivity terms still dominate energy balance

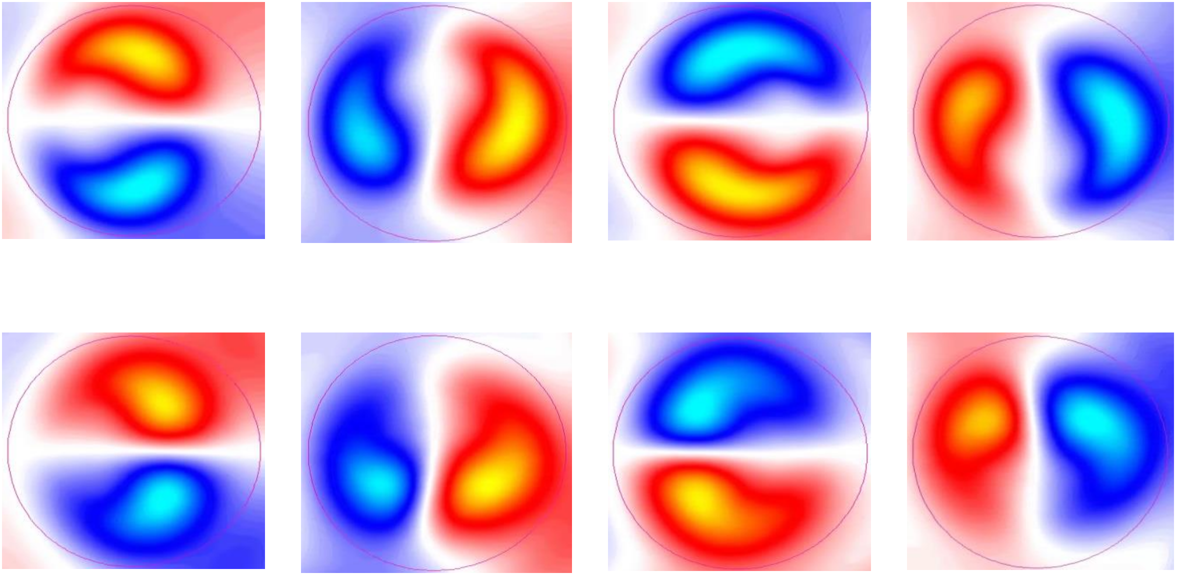
Toroidal derivative of pressure profile at 4 locations

Pressure develops a (1,1) component from $V_{1,1}$. It is of a similar form but about twice as large for the lowest κ case as for the highest κ case

0° 090° 180° 270°

Low $\kappa = 18\eta_0$
$\delta p \sim \pm 0.0020$

High $\kappa = 144\eta_0$
$\delta p \sim \pm 0.0009$

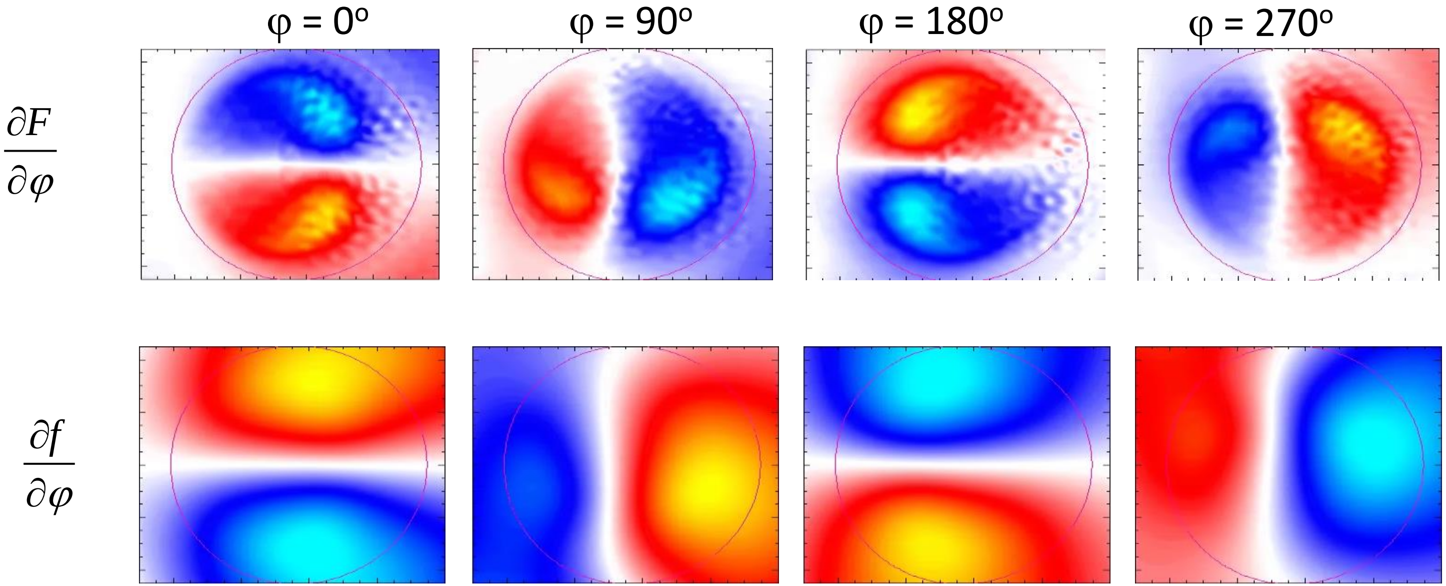


(5) The toroidal magnetic field is perturbed by the perturbed pressure

$$\delta\left(p+\frac{1}{2}B_T^2\right)=\delta p+\frac{1}{R^2}F\,\delta F\sim 0 \quad\Rightarrow\quad \delta F\sim -\delta p\frac{R^2}{F}$$

$$\mathbf{B}=\nabla\psi\times\nabla\varphi-\nabla_\perp\frac{\partial f}{\partial\varphi}+F\nabla\varphi$$

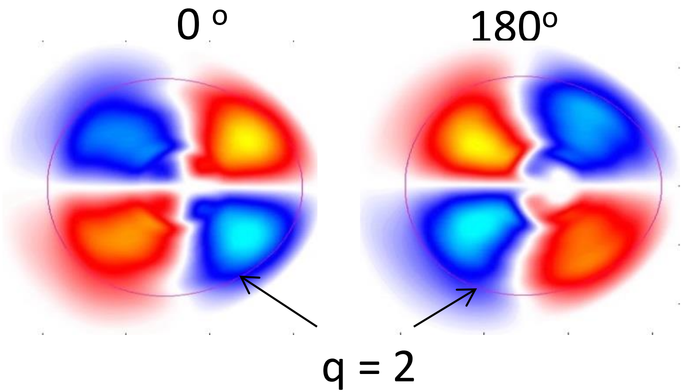
$$F=F_0+R^2\nabla_\perp^2 f$$



(1,1) perturbation in the toroidal field agrees with simple estimate

Because div **B**=0 this causes a (1,1) poloidal field component

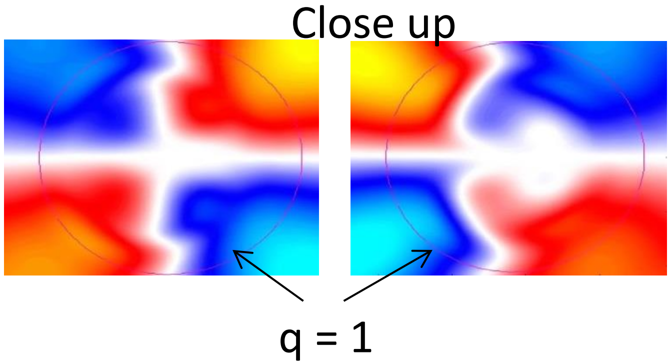
The poloidal flux ψ is also perturbed by the velocity field.



$$\mathbf{B} = \nabla\psi \times \nabla\varphi - \nabla_{\perp}\frac{\partial f}{\partial\varphi} + F\nabla\varphi$$

- Perturbed ψ is dominantly (m,n) = (2,1) due to the (2,1) island

- However, it also has a (1,1) component that plays an essential role.



$$\boxed{\frac{\partial\psi}{\partial t} = \left[-\mathbf{V}\bullet\nabla\psi - \frac{\partial\Phi}{\partial\varphi}\right] + \eta\Delta^{*}\psi = V_L / 2\pi} \qquad (*)$$

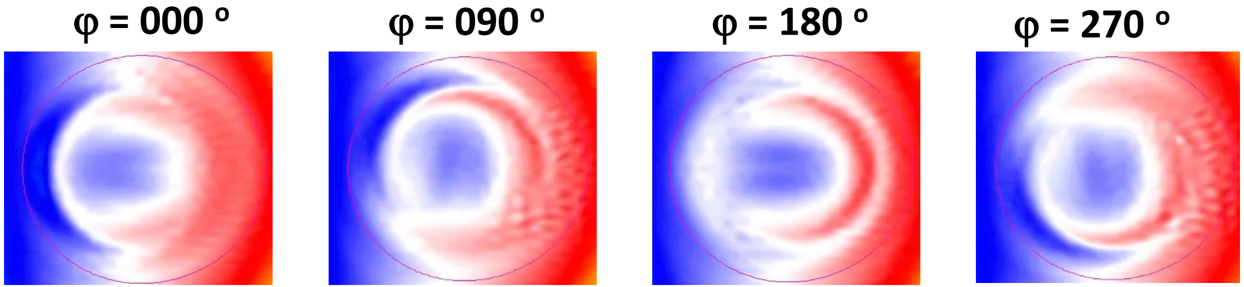
$$(*)_{1,1} \Longrightarrow \quad \eta_{0,0}\Delta^{*}\psi_{1,1} = \left[\mathbf{V}\bullet\nabla\psi + \frac{\partial\Phi}{\partial\varphi}\right]_{1,1} - \eta_{1,1}\Delta^{*}\psi_{0,0}$$

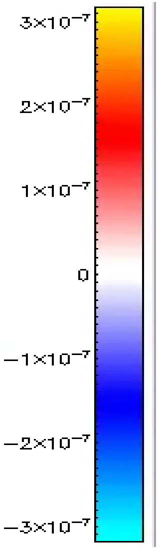
(6) Terms in parallel Ohm's law

$$\eta \mathbf{J}\bullet\mathbf{B} = -\mathbf{B}\bullet\nabla\Phi + \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

In 3D, the $\mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1}$ term leads to an effective voltage along the field in center

$$-\mathbf{B}\bullet\nabla\Phi$$





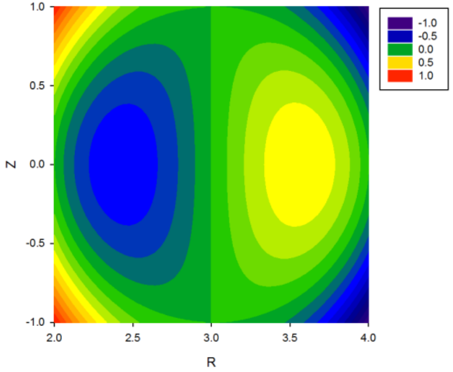
- Contours at 4 planes around torus.
- White is zero, blue is negative, red is positive.
- Center region is blue (negative) at all toroidal locations
- Implies a (0,0) voltage generated non-linearly

(6b) How can $\mathbf{B}\bullet\nabla\Phi$ *have a non-zero toroidal average in a volume?*

Suppose $\tilde{\mathbf{B}}$ is a small (1,1) field component resonant with Φ:

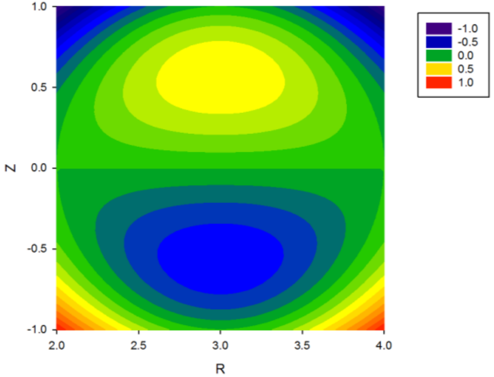
$$\mathbf{B} = \mathbf{B}^0 + \hat{\varphi}\times\nabla\psi_{1,1} \qquad \psi_{1,1} = \varepsilon r\left(1-r^2\right)\sin\left(\theta-\varphi\right)$$

Potential Φ



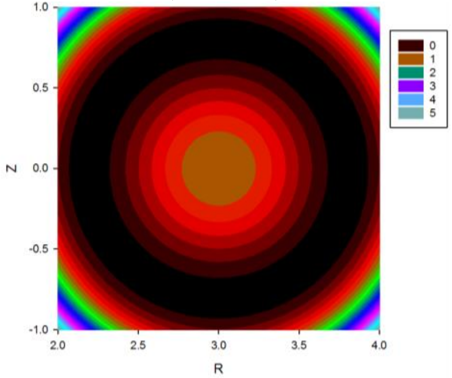
$$\Phi = \Phi_0 r\left(1-r^2\right)\cos\left(\theta-\varphi\right)$$

Perturbed flux $\psi_{1,1}$



$$\psi_{1,1} = \varepsilon r\left(1-r^2\right)\sin\left(\theta-\varphi\right)$$

$\left\langle \tilde{\mathbf{B}}\bullet\nabla\Phi \right\rangle$



$$\left\langle \mathbf{B}\bullet\nabla\Phi \right\rangle = \varepsilon\Phi_0\left\langle \left(1-3r^2\right)\left(1-r^2\right)\right\rangle$$

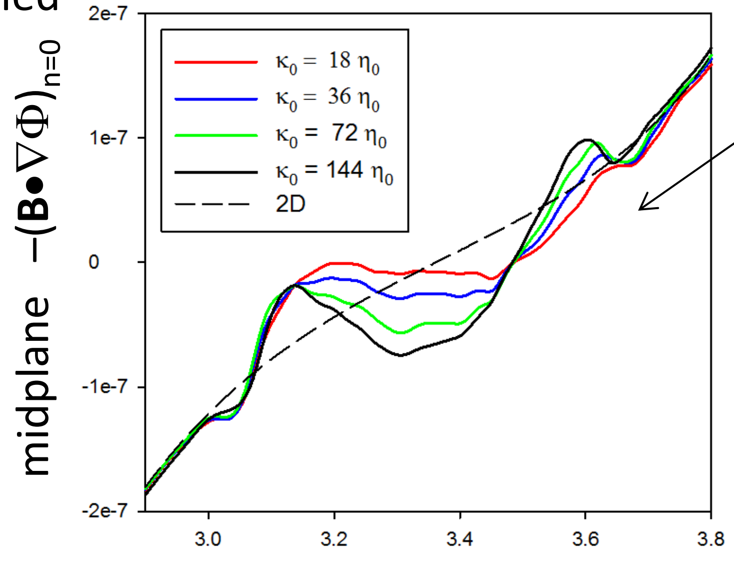
Resonant field perturbation produces an effective voltage along perturbed field!

positive definite for r sufficiently small!

(8) Parallel Ohm's law is modified in two ways that broaden current

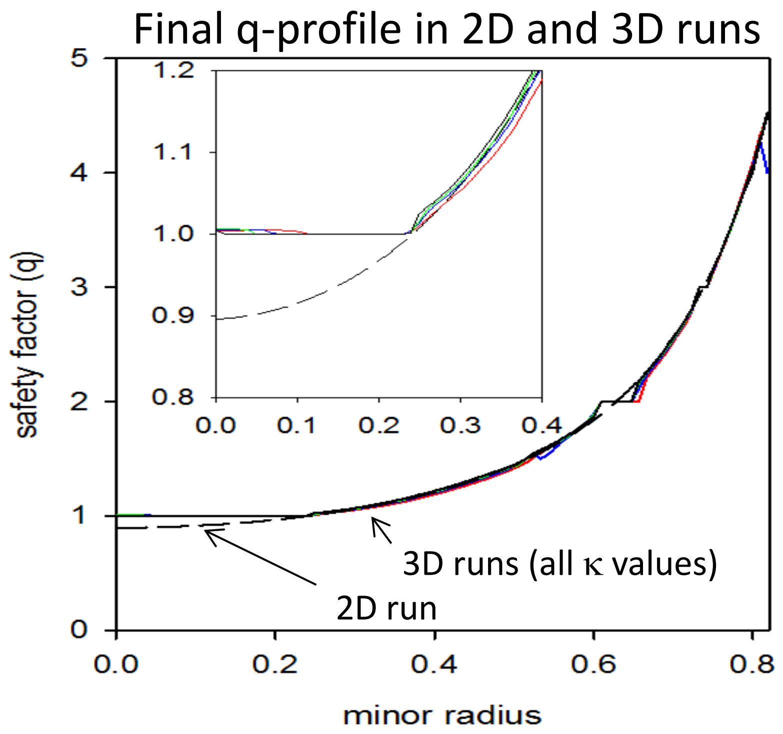
(1) Resistivity is flattened because temperature is partially flattened

$$\left(\eta \mathbf{J}\bullet\mathbf{B}\right)_{n=0} = -\left(\mathbf{B}\bullet\nabla\Phi\right)_{n=0} + \frac{V_L}{2\pi}\left(\mathbf{B}\bullet\nabla\varphi\right)_{n=0}$$



- (2) Nonlinear processes from $\mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1}$ produce an effective n=0 toroidal voltage in the center (as needed) to keep q=1 in central volume.
- This voltage is different for each of the 4 runs that have a different degree of temperature flattening
- Also different from 2D result (dashed line)

(9) No longer constrained to $\eta\langle \mathbf{J}\bullet\mathbf{B}\rangle = \mathrm{const}$, central regions in all 3D runs approach minimum energy Taylor State with q=1



The nonlinear drive that keeps the current from peaking gets stronger as q→1 from above

This feedback mechanism results in an ultra-flat q-profile in center with $q_0 = 1 + \varepsilon$ (where $\varepsilon << 1$)

Summary

- Some operating modes of existing tokamaks (i.e. hybrid modes in DIII-D, ASDEX-U, JET; long-lived mode in MAST, NSTX) do not exhibit sawteeth. However, (2D) transport codes indicate q_0 should fall below 1.

- In our long-time 3D MHD simulations, we similarly find that for some parameters, the discharge does not sawtooth but remains in a *stationary state* with $q_0 \cong 1$ and *ultra-low shear* in center

- This configuration is *unstable to a pressure driven interchange mode* which creates a stationary flow $\mathbf{V}$, which leads to a (1,1) electrical potential : $\nabla\Phi_{1,1} \cong \mathbf{V} \times \mathbf{B}$

- Flow from interchange mode also perturbs temperature and magnetic fields, and leads to a (1,1) component of the magnetic field $\mathbf{B}_{1,1}$

- The (0,0) dynamo voltage from $\mathbf{B}_{1,1} \bullet \nabla\Phi_{1,1}$ is just that needed to maintain the current that keeps $q_0 \cong 1$

S.C. Jardin, N. Ferraro, I. Krebs, PRL (2015) online Nov. 17!