Quasilinear relaxation of energetic particles interacting with Alfvénic modes

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With invaluable contributions from
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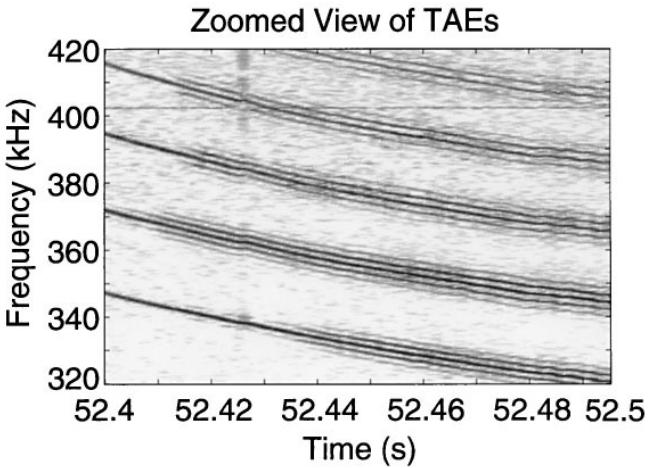
Transport Task Force, Williamsburg, VA, April 27th 2017

Alfvén waves can exhibit a range of bifurcations upon their interaction with fast ions

Typical scenarios:

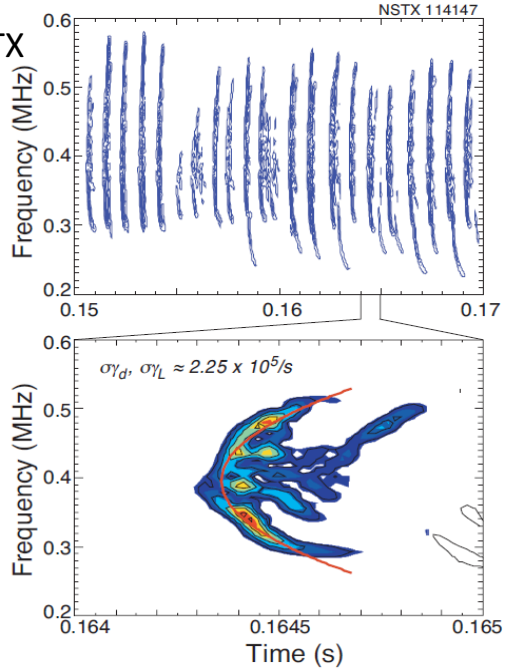
- fixed frequency and frequency splitting-> frequency is determined by the equilibrium
- chirping and avalanches -> frequency is highly affected by the fast ions response

JET



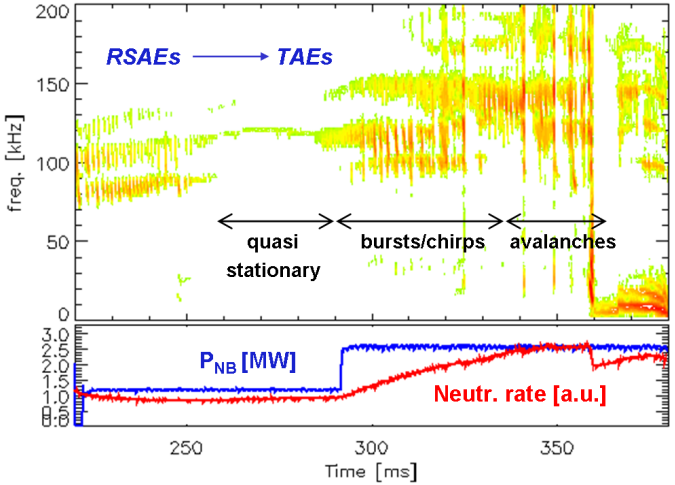
Fasoli, PRL 1998

NSTX



Fredrickson, PoP 2006

NSTX



Podestà, NF 2011

Prediction of character of energetic-particle-driven transport in tokamaks

What tools can be used to model each type of transport?

Diffusive transport (typical for fixed-frequency modes)

- can be modelled using reduced theories, such as quasilinear
- typical in conventional tokamaks

Convective transport (typical for chirping frequency modes)

- needs to retain full nonlinear features of the wave, is sustained by nonlinear phase-space structures
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Both can lead to similar fast ion loss levels, up to 40%

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- **development of a criterion for the likelihood of each nonlinear scenario and its comparison with experiments**
- **quasilinear diffusion approach and perspectives for whole device modeling**

Weak nonlinear dynamics of driven kinetic systems can be used to distinguish between fixed-frequency and chirping responses

Starting point: **kinetic equation** plus **wave power balance**

Assumptions:

- Perturbative procedure for $\omega_b \ll \gamma$
- Truncation at third order due to closeness to marginal stability
- Bump-on-tail modal problem, uniform mode structure

Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude A:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^3 \tau^2 (2\tau/3+\tau_1) + i\nu_{drag}^2 \tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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stabilizing

destabilizing (makes integral sign flip)

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stabilizing

destabilizing (makes integral sign flip)

- If nonlinearity is weak: linear stability, solution saturates at a low level and f merely flattens (system not allowed to further evolve nonlinearly).
- If solution of cubic equation explodes: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (precursor of chirping modes).

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

A criterion for chirping onset in tokamaks

Using an action and angle formulation, the previous weak nonlinear theory leads to

$$\sum_{l,\sigma_\parallel} \int dP_\varphi \int d\mu \frac{\tau_b}{\nu_{drag}^4} |V_l|^4 \left|\frac{\partial \Omega_l}{\partial I}\right| \frac{\partial F}{\partial I} Int > 0$$

>0: fixed-frequency solution likely

<0: chirping likely to occur

$$Int \equiv Re \int_0^\infty dz \frac{z}{\frac{\nu_{stoch}^3}{\nu_{drag}^3} z - i} exp\left[-\frac{2}{3}\frac{\nu_{stoch}^3}{\nu_{drag}^3} z^3 + iz^2\right]$$

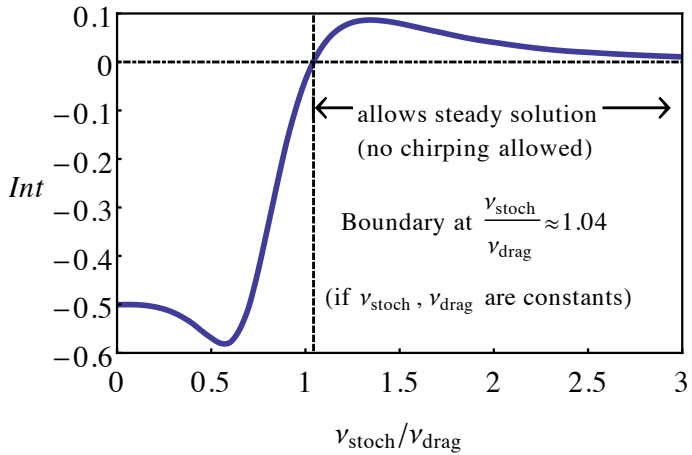
Phase space integration

Eigenstructure information:

$$q \int dt \mathbf{v}_{dr} \cdot \delta \mathbf{E} e^{i\omega t}$$

Resonance surfaces:

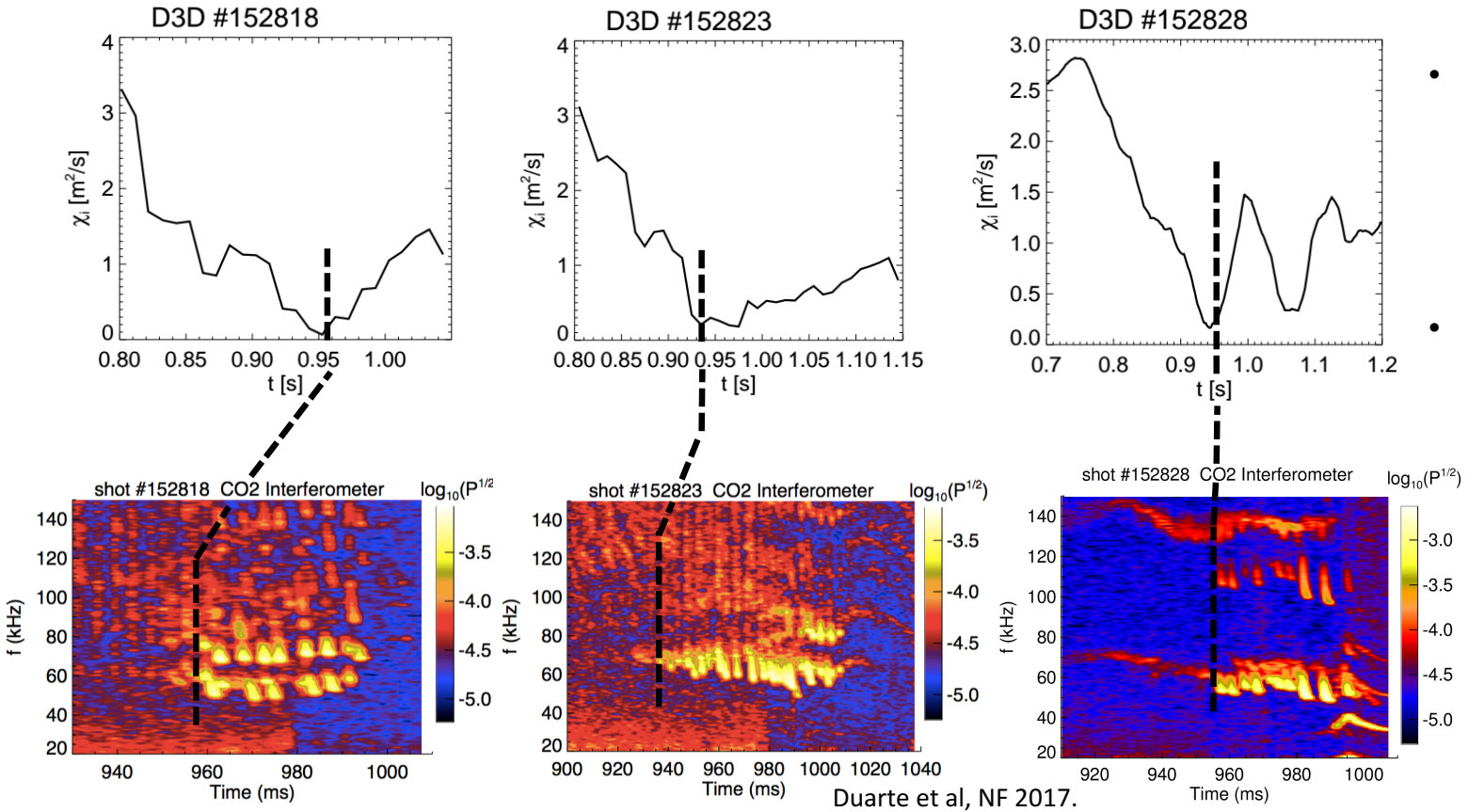
$$\Omega_l\left(\mathcal{E}' + \omega P_\varphi / n, P_\varphi, \mu\right) \equiv$$
$$\equiv n \frac{d\varphi}{dt} - l \frac{d\theta}{dt} - \omega_0$$



Criterion was incorporated into NOVA-K code: nonlinear prediction from linear physics elements

The criterion predicts that micro-turbulence should be key in determining the likely nonlinear character of a mode, e.g., fixed-frequency or chirping

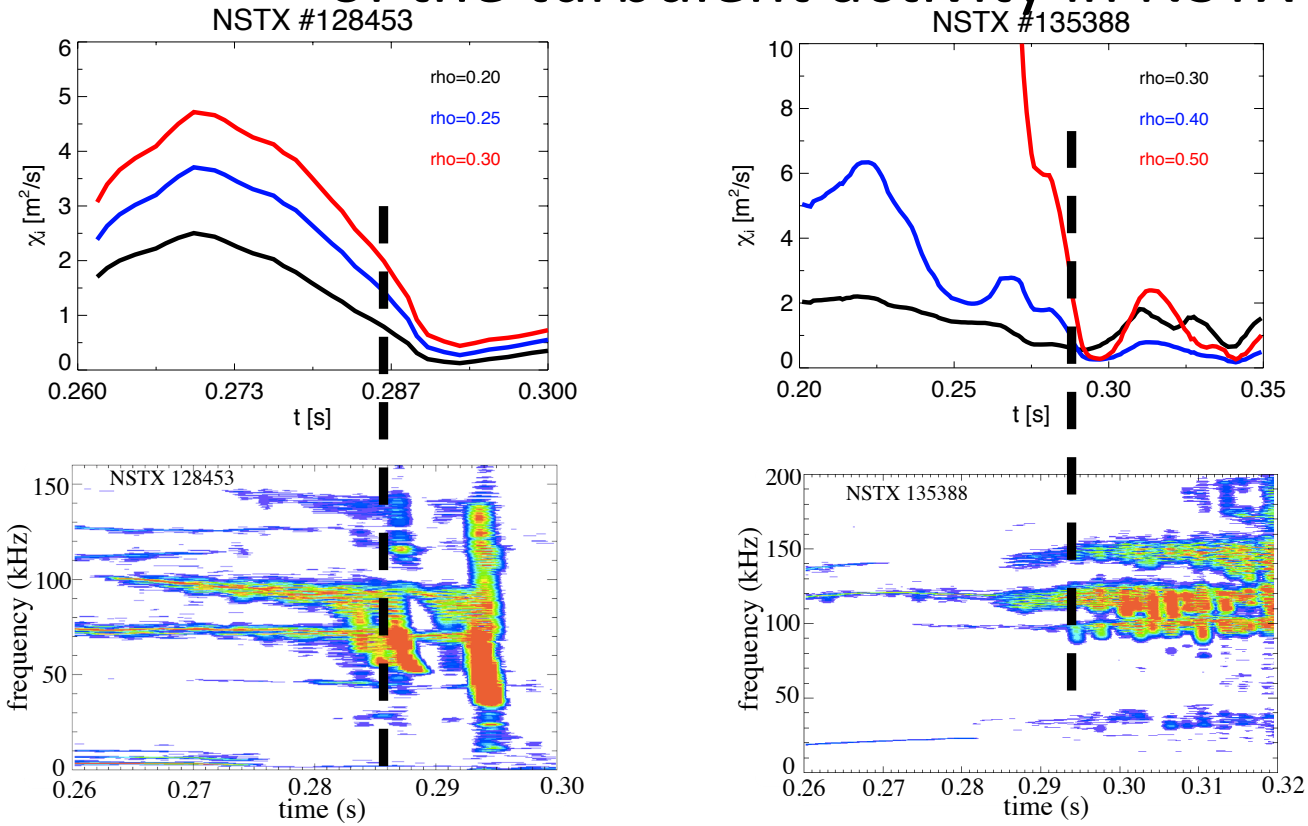
Correlation between chirping onset and a marked reduction of the turbulent activity in DIII-D



- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport
- experiments in DIII-D are scheduled to further test the proposed criterion

Duarte et al, NF 2017.

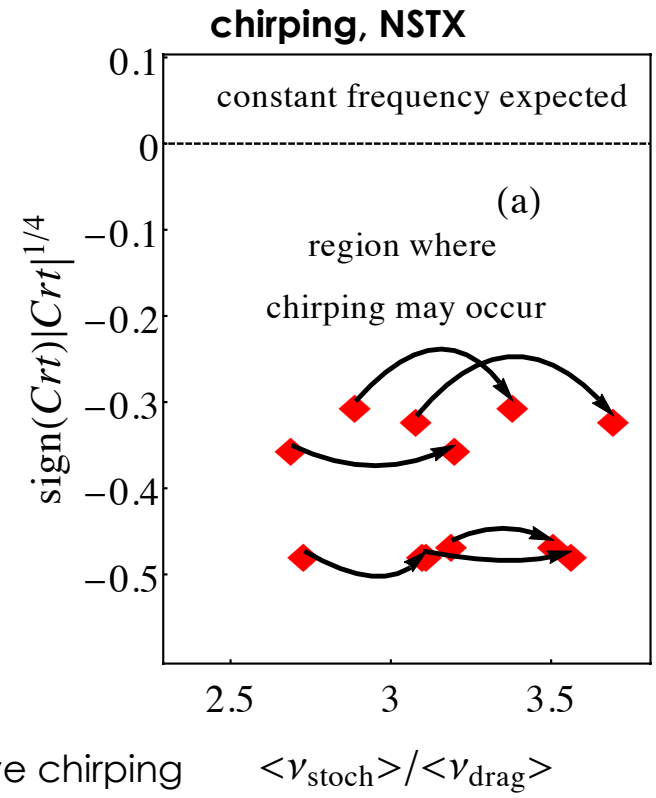
Correlation between chirping onset and a marked reduction of the turbulent activity in NSTX

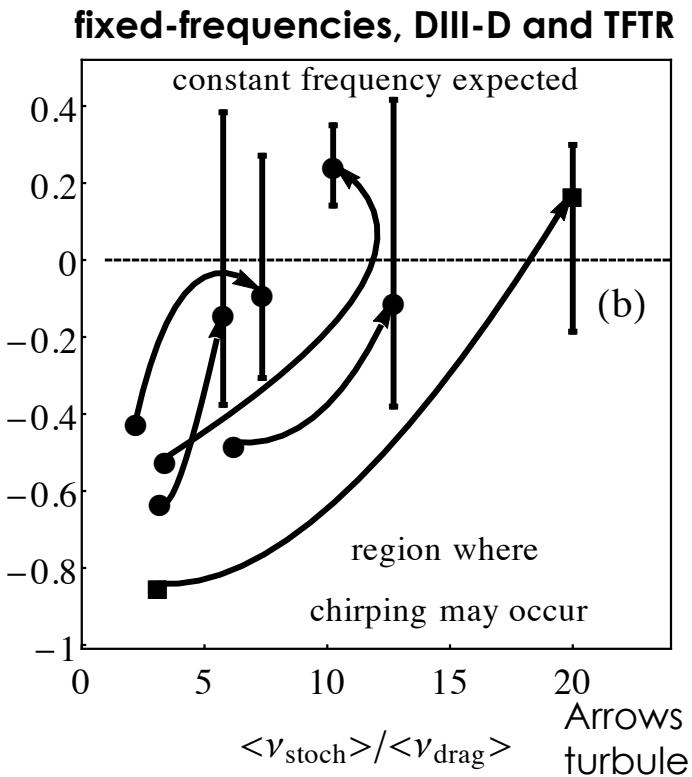


- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport
- GTS code is being used as na independent calculatation of fast ion diffusivity

Chirping is ubiquitous in NSTX but rare in DIII-D, which is consistent with the inferred fast ion micro-turbulent levels

Criterion evaluation





Alfvén wave chirping quantitatively agrees with the criterion

Arrows represent the turbulent diffusion that adds up to pitch-angle scattering

Nonlinear chirping vs Quasilinear approach

- Requires particles to remember their phases from one trapping bounce to another;
- Full kinetic approach necessary;
- Entropy is conserved in the absence of collisions.

- Requires particles to forget their phase (via collisions, turbulence or mode overlap);
- Assumes that the **modes remain linear** (therefore NOVA is suited) while the distribution function is allowed to slowly evolve nonlinearly in time;
- Entropy increases due to particle memory loss.

The chirping criterion is a useful tool to make sure the quasilinear approach is applicable for a given mode

Resonance-broadened quasilinear (RBQ) diffusion model[1]

Formulation in action and angle variables[2,3]

- Diffusion equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial I}\left(\sum_{n,l,m,m'} D(I;t)\right)\frac{\partial}{\partial I} f + C[f]$$

$$D(I;t) = \pi n^2 C_n^2(t)\,\mathcal{E}^2 \frac{\mathcal{F}(I - I_r)}{\left|\frac{\partial \Omega_l}{\partial I}\right|} G^*_{m'l} G_{ml}$$

Broadened delta function

eigenstucture information

- Mode amplitude evolution:

$$\frac{dC_n^2(t)}{dt} = 2\left(\gamma_{L,n} - \gamma_{d,n}\right) C_n^2(t)$$

$$\frac{\partial}{\partial I} = \omega \frac{\partial}{\partial \mathcal{E}} - n \frac{\partial}{\partial P_\varphi}$$

$\mathcal{E}$: unperturbed (kinetic) energy; P_φ : canonical toroidal momentum

Broadening is the platform that allows for momentum and energy exchange between particles and waves:

$$\triangle\Omega = a\omega_{b,n} + b\left|\gamma_n\right| + c\nu_{scatt}$$

[1]Berk, Breizman, Fitzpatrick, NF 1995.
[2]Kaufman PoF 1972.
[3]Duarte, Gorelenkov and Berk, 2017 (unpublished).

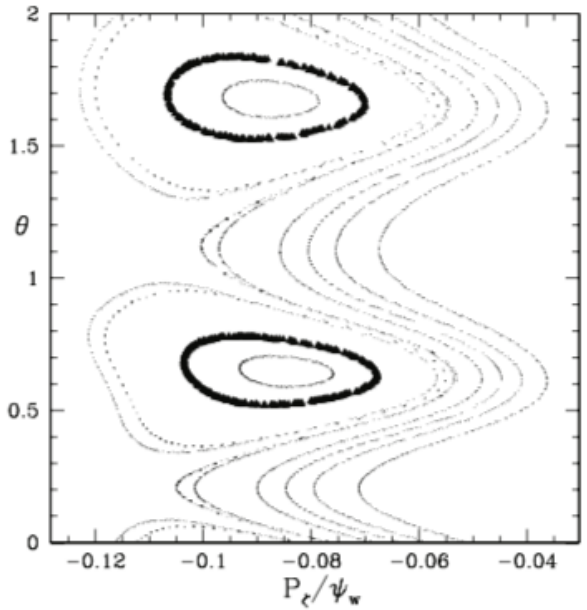
Parametric dependencies of a broadened resonance

- old idea (Dupree PoF 1966): the turbulent spectrum contributes to diffuse particles away from their original unperturbed trajectories

- broadening specification: $\triangle\Omega = a\omega_{b,n} + b\left|\gamma_n\right| + c\nu_{scatt}$

- analytical predictions for the simplified driven, bump-on-tail system close to and far from marginal stability lead to particular choices of coefficients a, b and c.

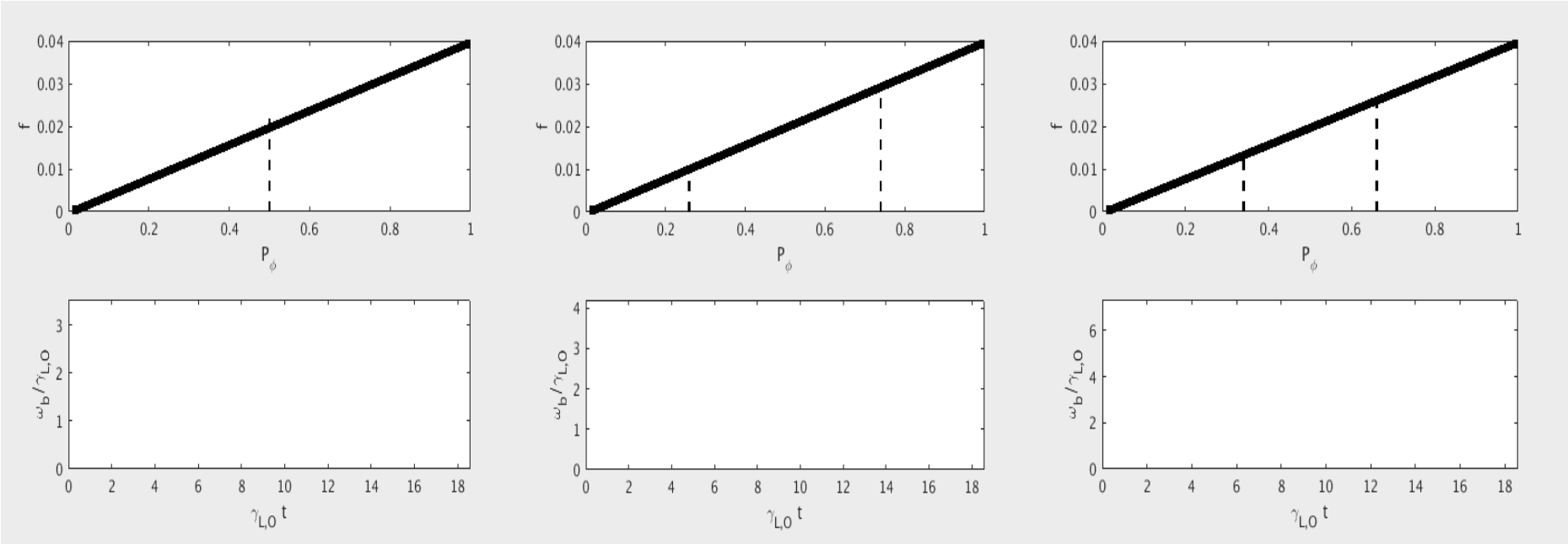
- guiding-center code ORBIT is being used to verify the resonance width scaling for realistic mode structure and resonances calculated by NOVA.



Resonance Broadened Quasilinear (RBQ) code computation of fast ion relaxation

Upper plot: distribution function as a function of the canonical toroidal momentum
Lower plot: evolution of the nonlinear bounce frequency (~square root of mode amplitude)

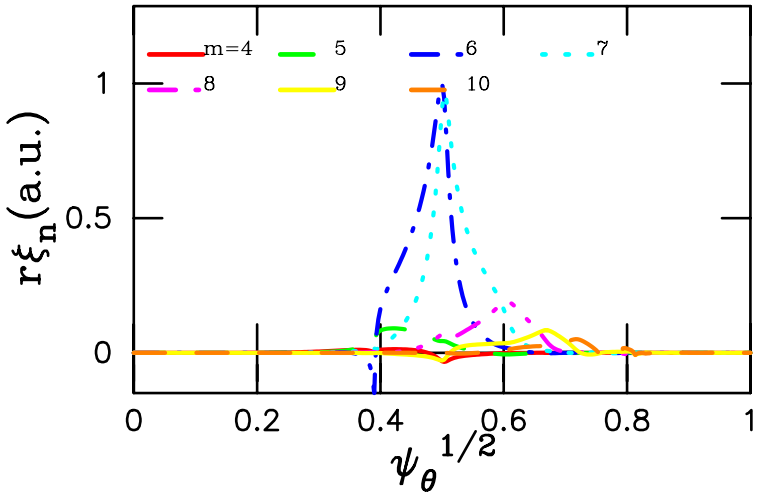
Single mode saturation | Two isolated modes | Two overlapping modes



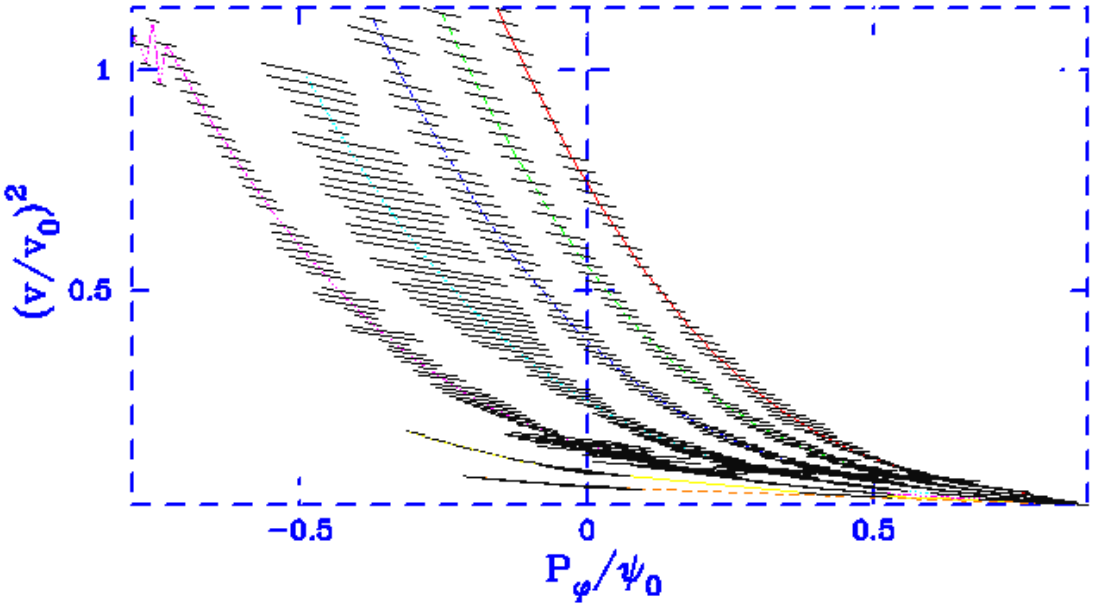
When modes overlap, there is a sudden release of stored fast ion energy, which lead to substantial mode growth

Example: DIII-D discharge 153072

Mode structure

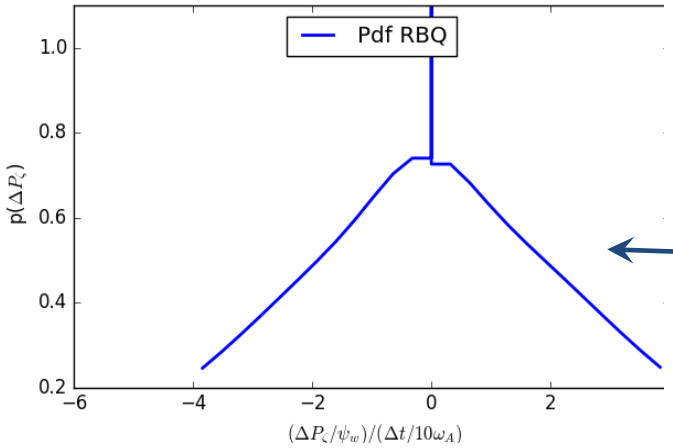


Resonance broadening for a given value of μ

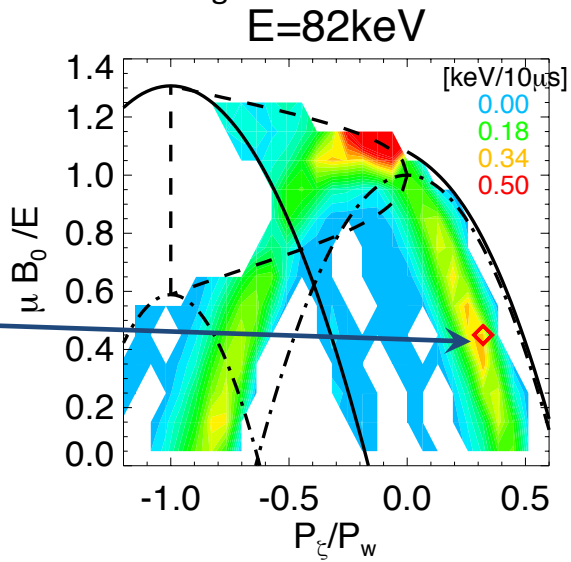


Resonance Broadened Quasilinear (RBQ) code is being interfaced with TRANSP

Whole device modeling using TRANSP is interfaced with RBQ via a Probability Distribution Function (PDF)



PDFs are used actively in Kick modeling[1]



RBQ can self-consistently provide:

- mode amplitude evolution
- diffusion coefficient at any given phase space location
- Intermittency and domino behaviors

[1] Podestà, PPCF 2014

Outcome

- Criterion gives confidence in the application of quasilinear modeling;
- Refinement of the fast ion diffusivity that enters the chirping criterion is being done with the gyrokinetic code GTS;
- Experiments on DIII-D to further test the proposed chirping criterion predictions;
- Although a reduced model, RBQ provides a platform for rich dynamics seen in experiments: it resolves velocity space and account for losses and intermittency;
- Whole device modeling, RBQ interfacing with TRANSP.

Thank you