



# *Things Fall Apart:*

Waves and Turbulence in a Weakly Collisional ICM

Matthew Kunz



with Jono Squire, Eliot Quataert, and Alex Schekochihin

an excerpt from my talk at Snowcluster 2015:

What I'd love to see at ~~Snowcluster 2017~~<sup>8</sup>

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- Someone will provide a model for the parallel conductivity.
- Simulations of ICM gas dynamics will be using (at least) Braginskii-MHD (with appropriate limits imposed on parallel viscosity)
- I might have a kinetic simulation of fluctuation dynamo in a collisionless plasma.

Komarov, Churazov, Kunz & Schekochihin (2016, *MNRAS*)

- ▶ mirrors on ion-Larmor scales reduce  $\kappa$  by  $\approx 1/5$

Roberg-Clark, Drake, Reynolds & Swisdak (2016, *ApJ*; 2018, *PRL*)

- ▶ see next talk
- ▶ also Komarov, Schekochihin, Churazov & Spitkovsky (2018)

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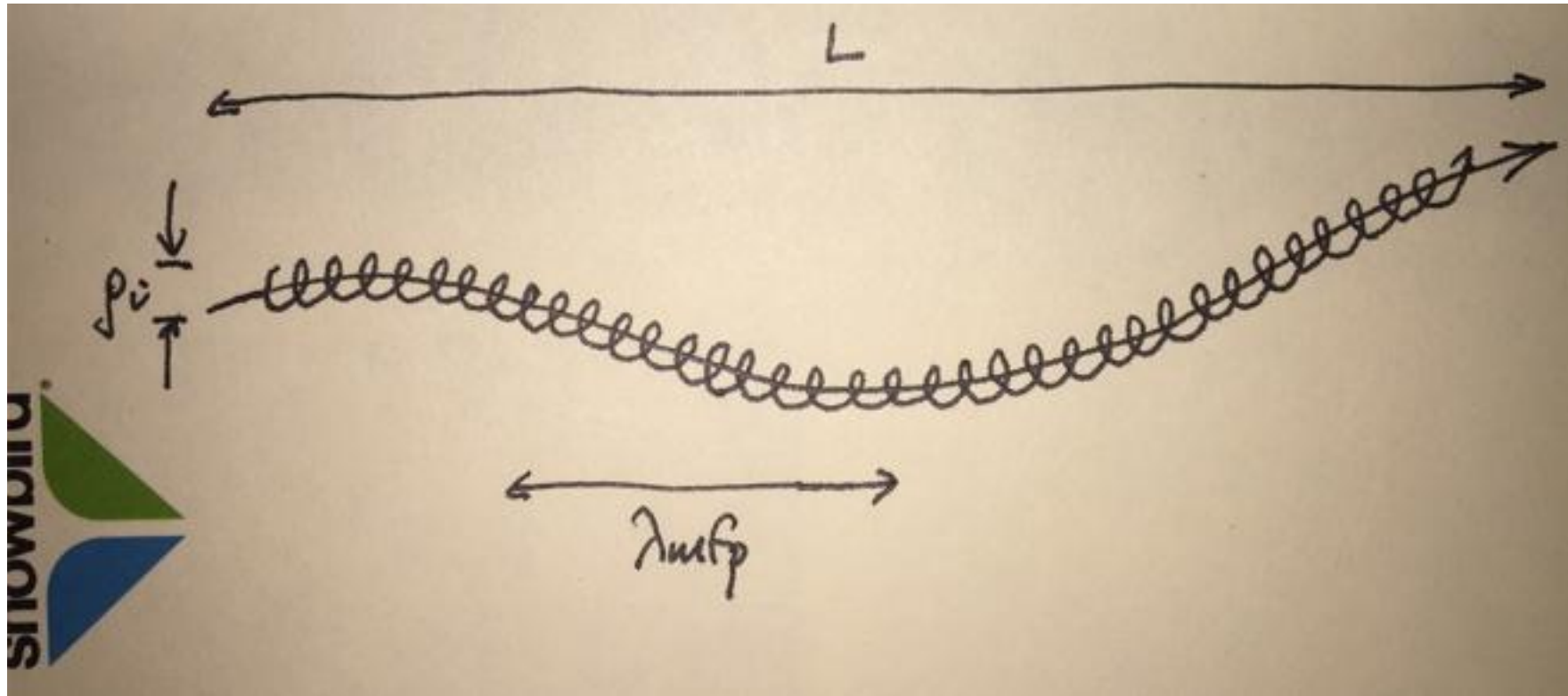


more on this  
in this talk

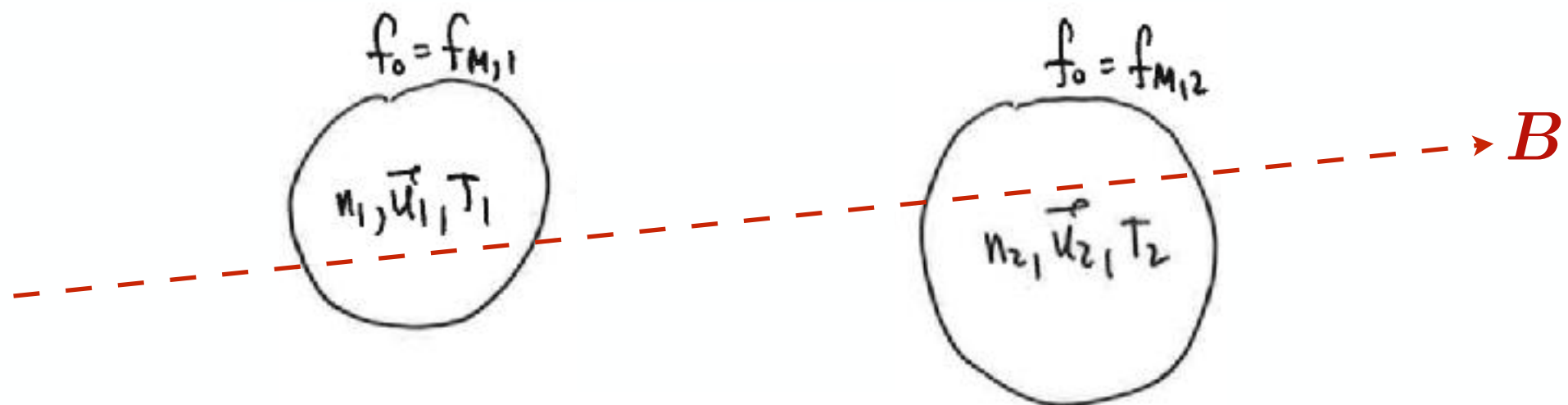


A student of mine,  
Denis St-Onge,  
and I now do.  
(but likely won't have  
time to talk about)

Consider a plasma with  $\rho_i \ll \lambda_{\text{mfp}} \ll L$  (ICM at large scales)



imagine two magnetically connected fluid elements in LTE  
with different flow velocities and temperatures





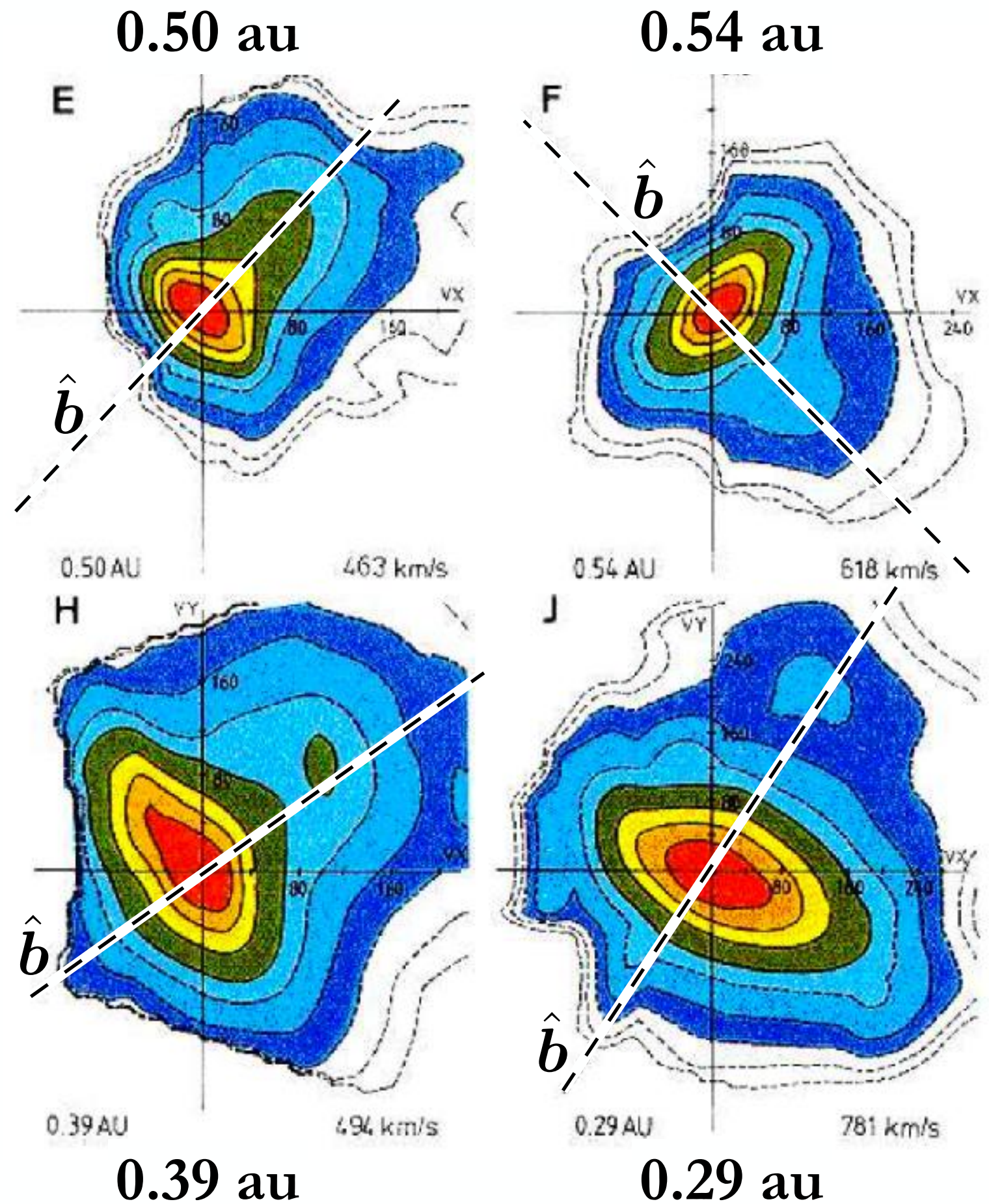
These gradients will distort the distribution function in each fluid element:

$$f_i = f_{Mi} \left[ \underbrace{1 - \nu_i^{-1} \frac{3}{2} \left( \frac{v_{\parallel}^2 - v_{\perp}^2/2}{v_{thi}^2} \right) \left( \hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3} \right) : \mathbf{W}_i}_{\text{distortion in } f \text{ caused by differences in flow velocity between different fluid elements}} \underbrace{g\left(\frac{v}{v_{thi}}\right) + \nu_i^{-1} \frac{5}{4} v_{\parallel} \hat{\mathbf{b}} \cdot \nabla \ln T_i}_{\text{distortion in } f \text{ caused by differences in temperature between different fluid elements}} h\left(\frac{v}{v_{thi}}\right) \right]$$

These distortions are  $\sim \mathcal{O}(\lambda_{mfp}/L)$ , which is  $\sim 0.01 - 0.1$  in ICM.

They drive collisional exchange of momentum and heat along field lines.  
(Braginskii 1965)

You can easily  
see these  
distortions in  
the collisionless  
solar wind.





The momentum exchange is proportional to the *pressure anisotropy* :

$$\frac{P_{\perp}}{P_{\parallel}} - 1 = \frac{3}{\nu_{ii}} \left( b_i b_j - \frac{1}{3} \delta_{ij} \right) \frac{\partial u_i}{\partial x_j}$$

$$\sim \mathcal{O}(M \lambda_{\text{mfp}} / L) \sim 0.01 \text{ in ICM}$$

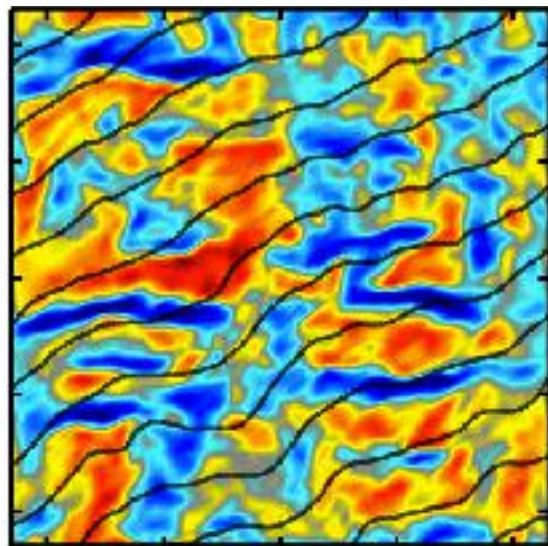
which might sound small

...but, then again, so is  $\frac{1}{\beta}$

This renders the Braginskii equations potentially unstable to ion-Larmor-scale instabilities:

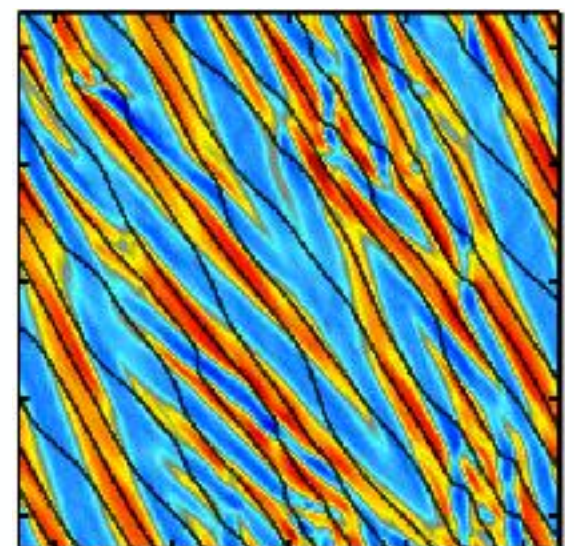
firehose  

$$\frac{P_{\perp}}{P_{\parallel}} - 1 \lesssim -\frac{2}{\beta}$$
 scatters



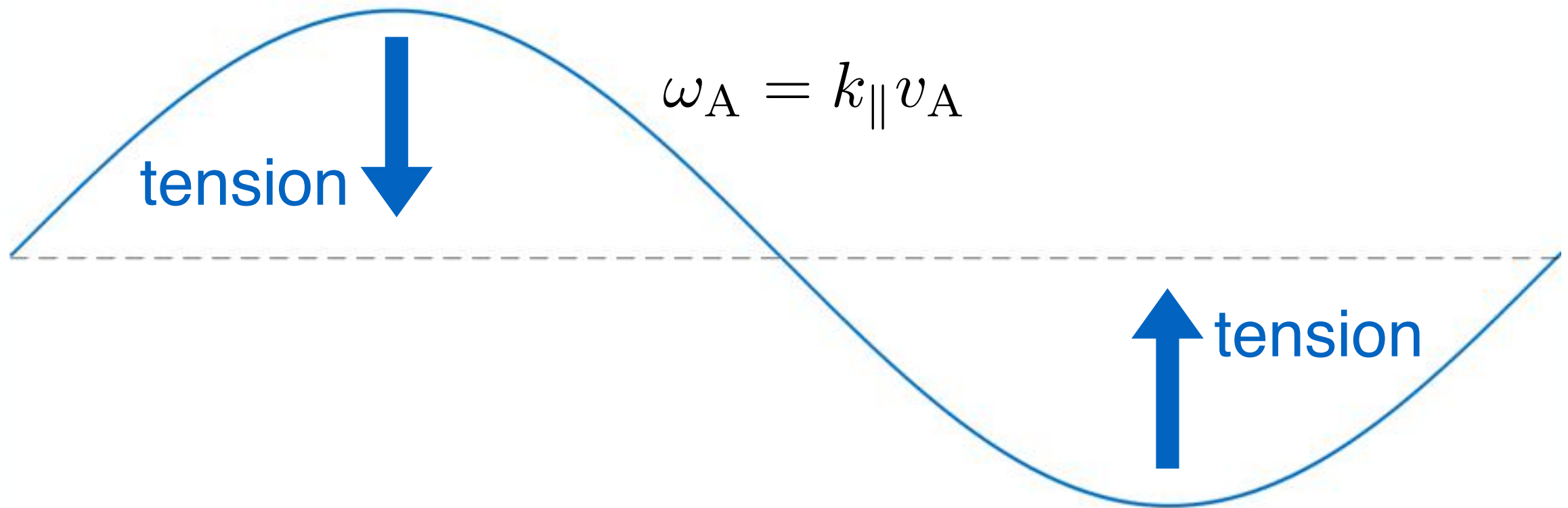
mirror  

$$\frac{P_{\perp}}{P_{\parallel}} - 1 \gtrsim \frac{1}{\beta}$$
 traps



Here's a simple example  
(following Squire *et al.* 2016, ApJL)

Consider a standing, shear-Alfvén wave:



$$\left| \frac{P_{\perp}}{P_{\parallel}} - 1 \right| \simeq \frac{3}{2} \frac{\omega_A}{\nu_i} \left| \frac{\delta B_{\perp}^2}{B_0^2} \right|$$



$$\text{If } \frac{\delta B_{\perp}}{B_0} \gtrsim \sqrt{\frac{\nu_i / \omega_A}{\beta}} = \frac{\beta^{-1/4}}{\sqrt{k_{\parallel} \lambda_{\text{mfp}}}} ,$$

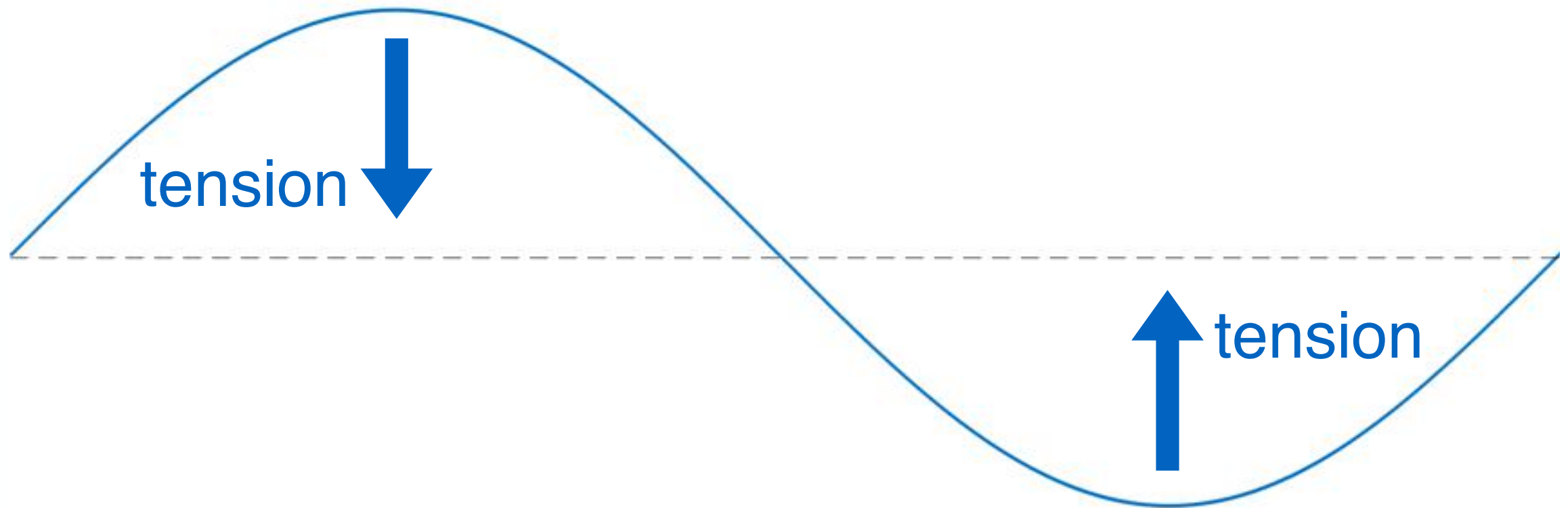
$$\text{then } \left| \frac{P_{\perp}}{P_{\parallel}} - 1 \right| \simeq \frac{3}{2} \frac{\omega_A}{\nu_i} \left| \frac{\delta B_{\perp}^2}{B_0^2} \right| \gtrsim \frac{1}{\beta} .$$

Thus, a nonlinear shear-Alfvén wave could produce enough pressure anisotropy to drive a weakly collisional plasma unstable.

At collisionless scales,  
pressure anisotropy is produced by adiabatic invariance:  
(Chew, Goldberger & Low 1956)

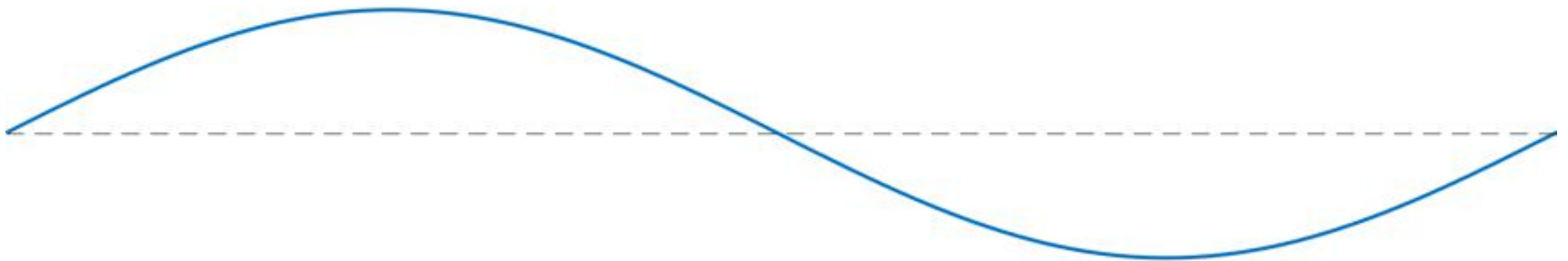
$$\frac{P_{\perp}}{nB} \simeq \text{const}$$

$$\frac{P_{\parallel} B^2}{n^3} \simeq \text{const}$$



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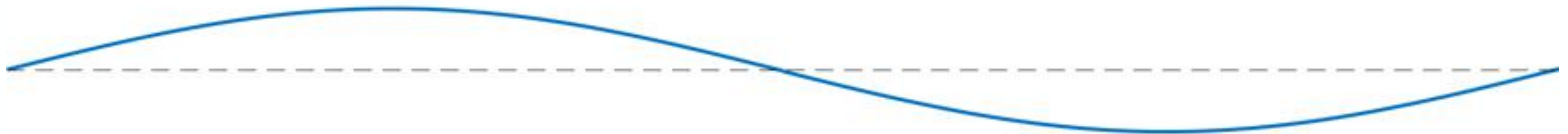
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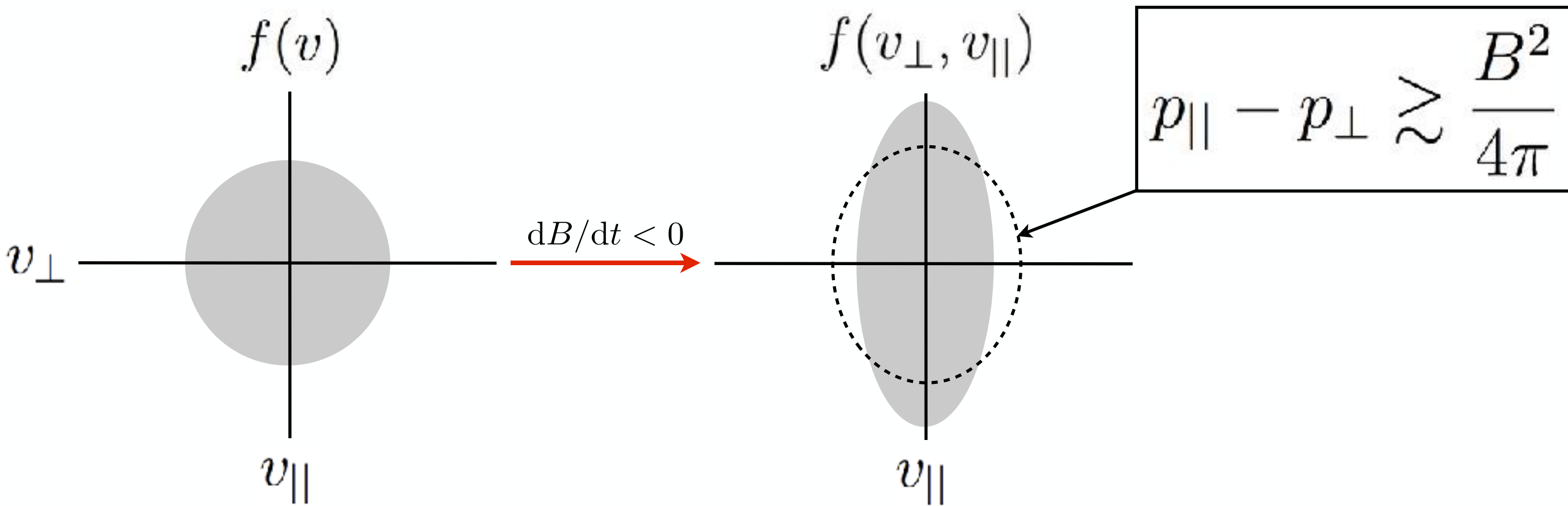


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---

Now, how much pressure anisotropy was driven  
by this decrease in field strength?



Answer: 
$$\frac{P_{\perp}}{P_{\parallel}} - 1 \simeq -\frac{3}{4} \left| \frac{\delta B_{\perp}}{B_0} \right|^2$$

If  $\left| \frac{\delta B_{\perp}}{B_0} \right| \gtrsim \frac{1}{\sqrt{\beta}}$ , plasma goes firehose unstable.

Note that these can have  $\delta B_{\perp}/B_0 \ll 1$ !



# What happens at these wave-amplitude thresholds?

1. Wave is “interrupted” and can’t oscillate/propagate.

$$\nabla \cdot \left[ \hat{\mathbf{b}} \hat{\mathbf{b}} \left( \frac{B^2}{4\pi} + P_{\perp} - P_{\parallel} \right) \right]$$

↑  
magnetic  
tension
↑  
nullified if this is  $-B^2/4\pi$

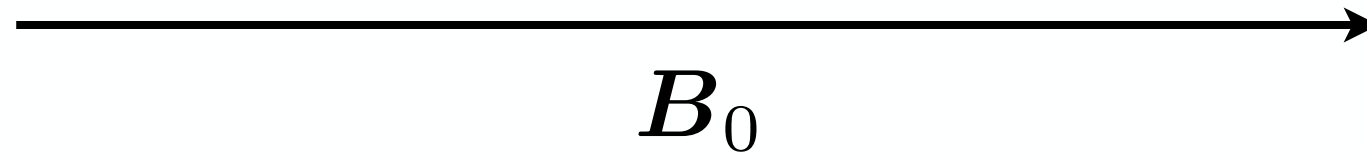
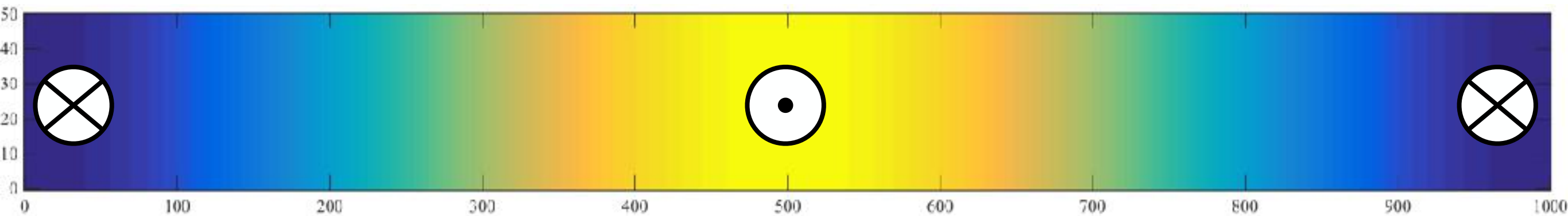
Alfvén wave nonlinearly removes its own restoring force.

2. Plasma is unstable to a sea of ion-Larmor-scale fluctuations, which trap and scatter particles and viscously decay the wave.

Let's see this in action...

linearly polarized, standing Alfvén wave

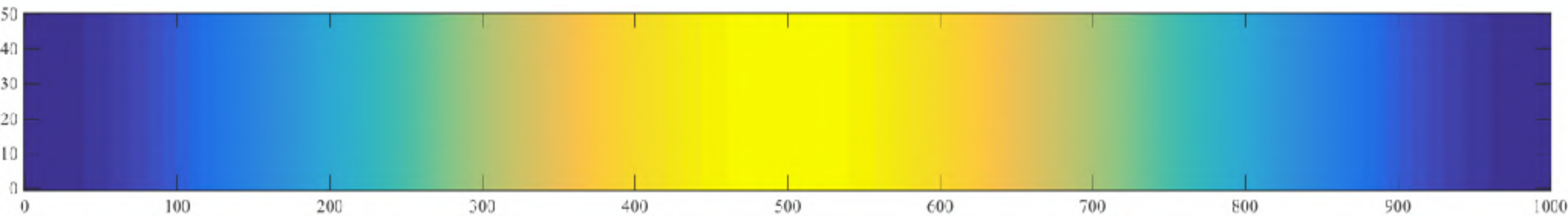
MHD:



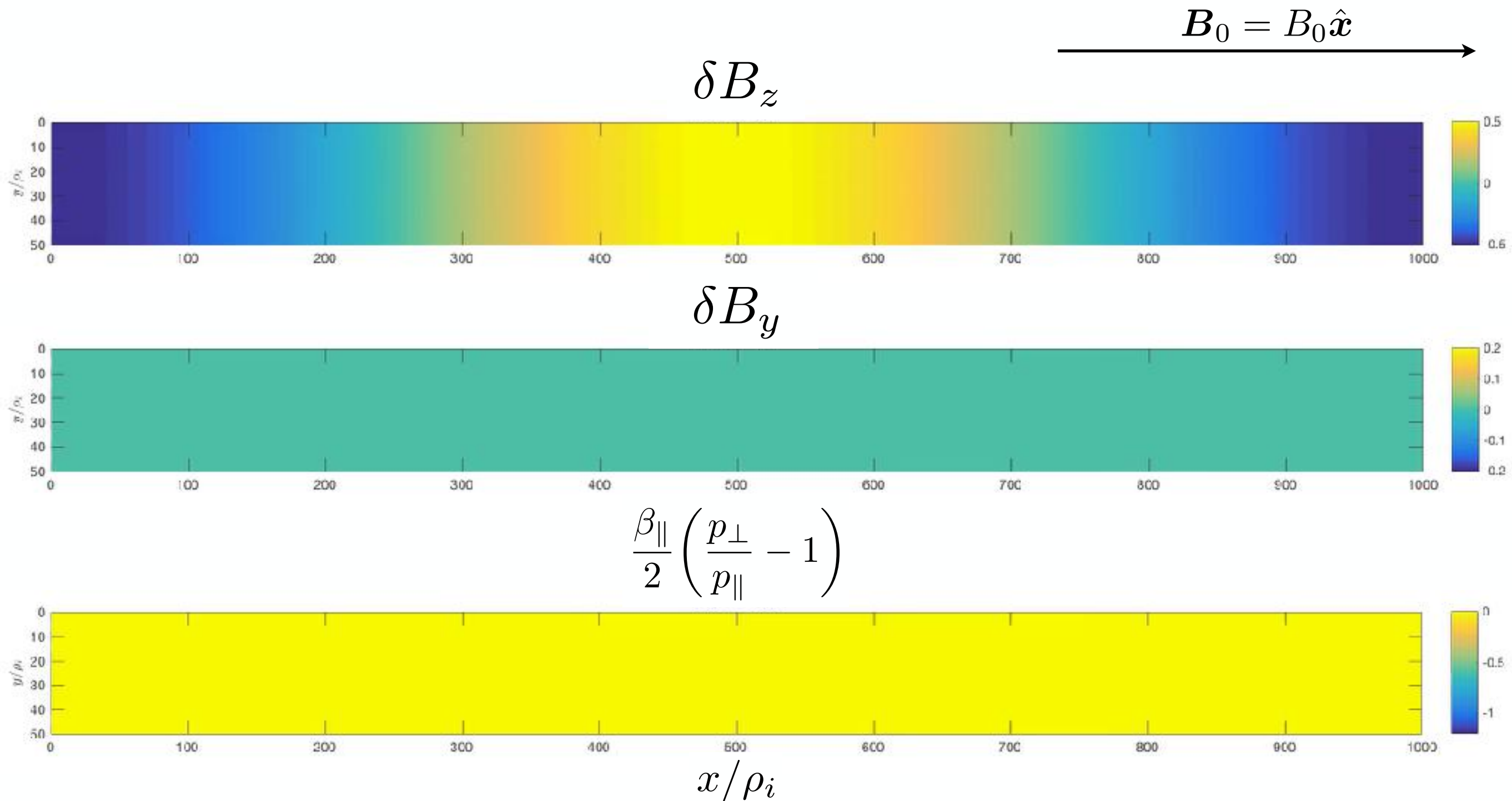


linearly polarized, standing Alfvén wave

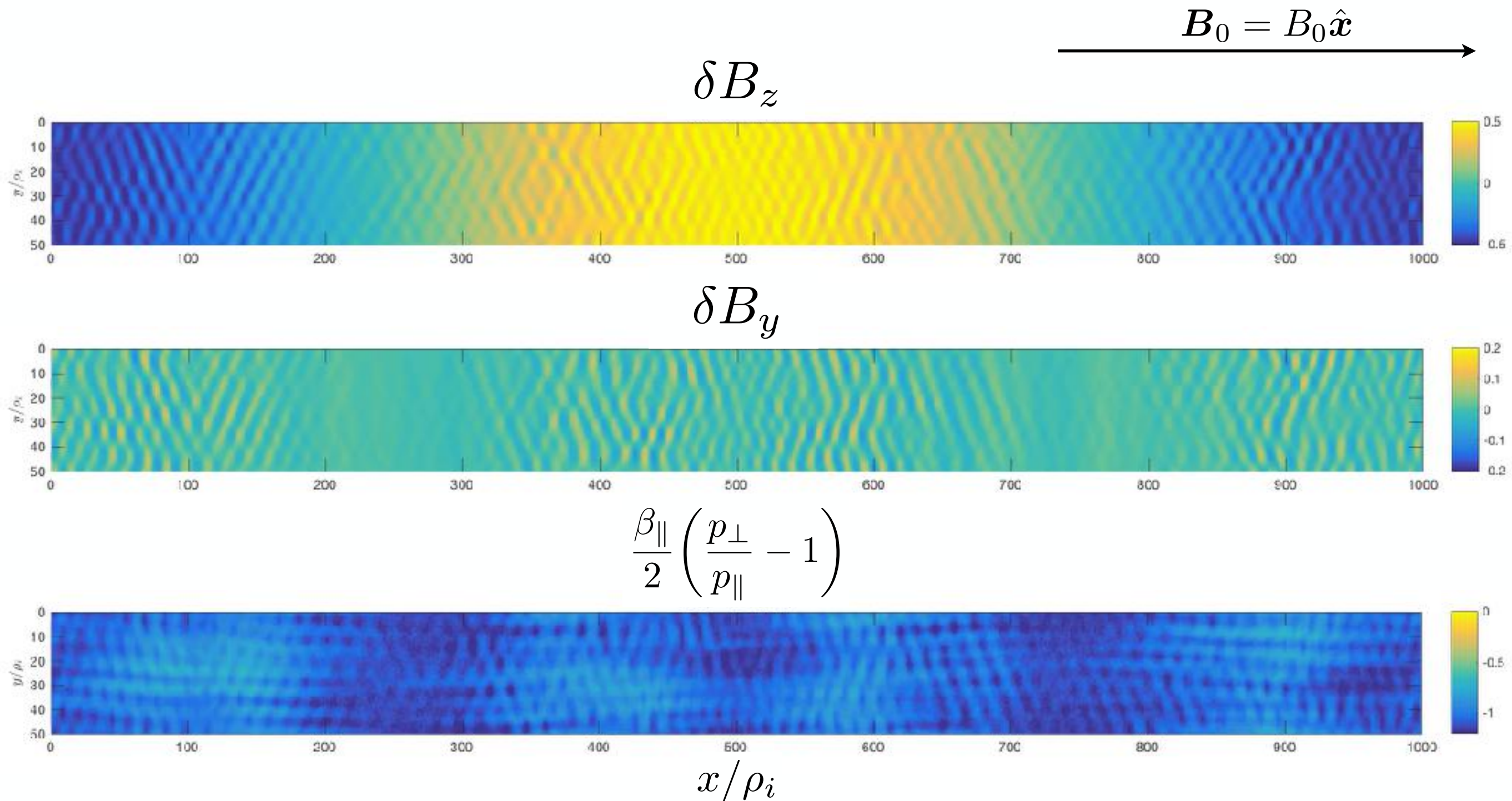
MHD:



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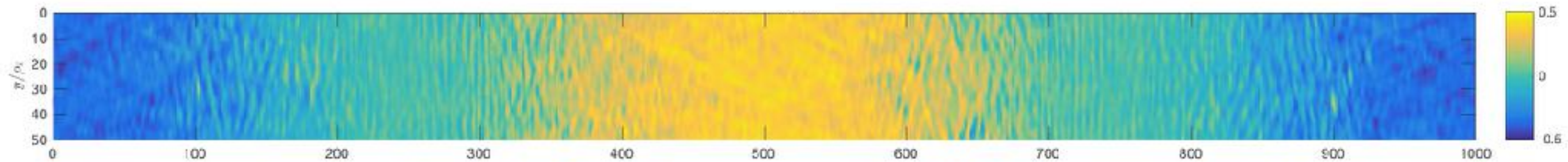




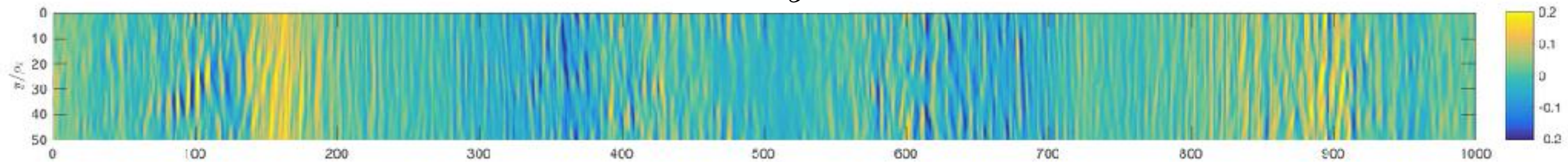
linearly polarized, standing Alfvén wave

$$\mathbf{B}_0 = B_0 \hat{\mathbf{x}}$$

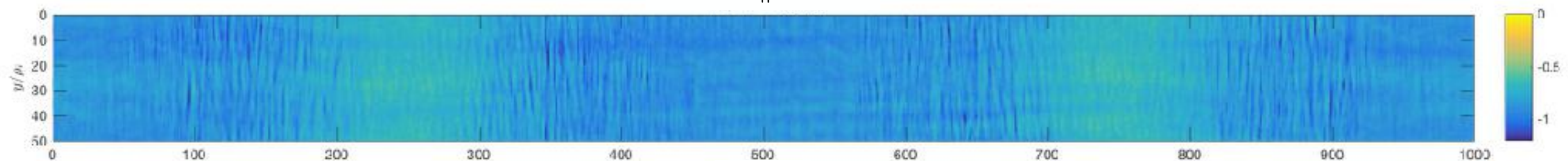
$$\delta B_z$$



$$\delta B_y$$

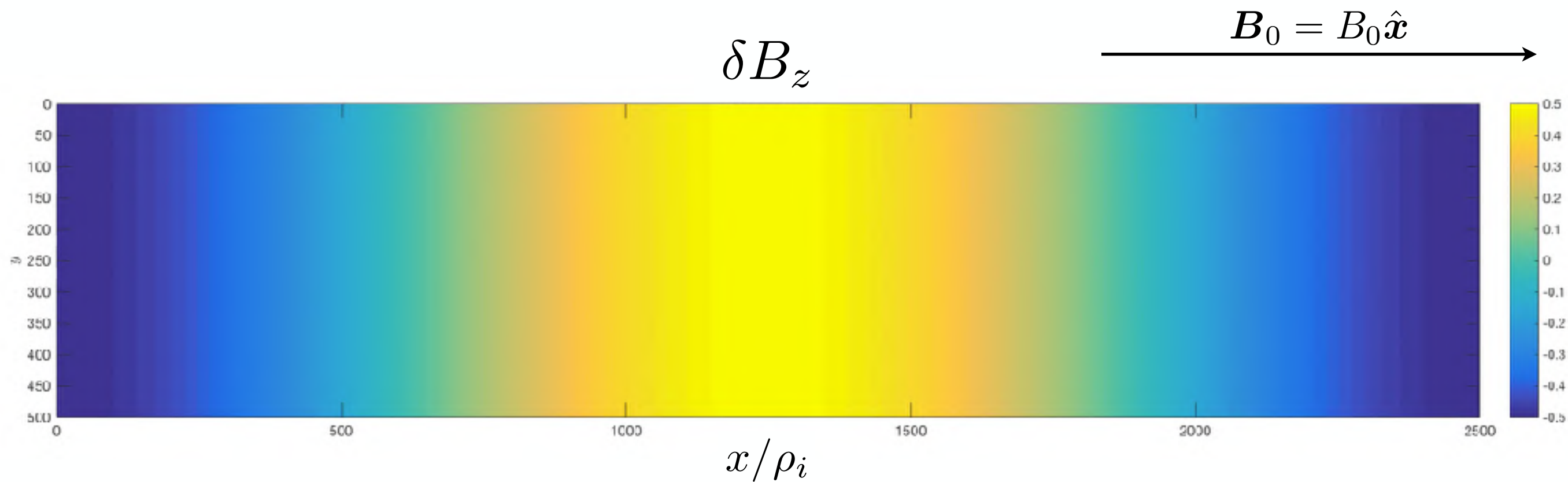


$$\frac{\beta_{\parallel}}{2} \left( \frac{p_{\perp}}{p_{\parallel}} - 1 \right)$$



$$x/\rho_i$$

linearly polarized, *traveling* Alfvén wave



interrupts, slows way down, then viscously decays

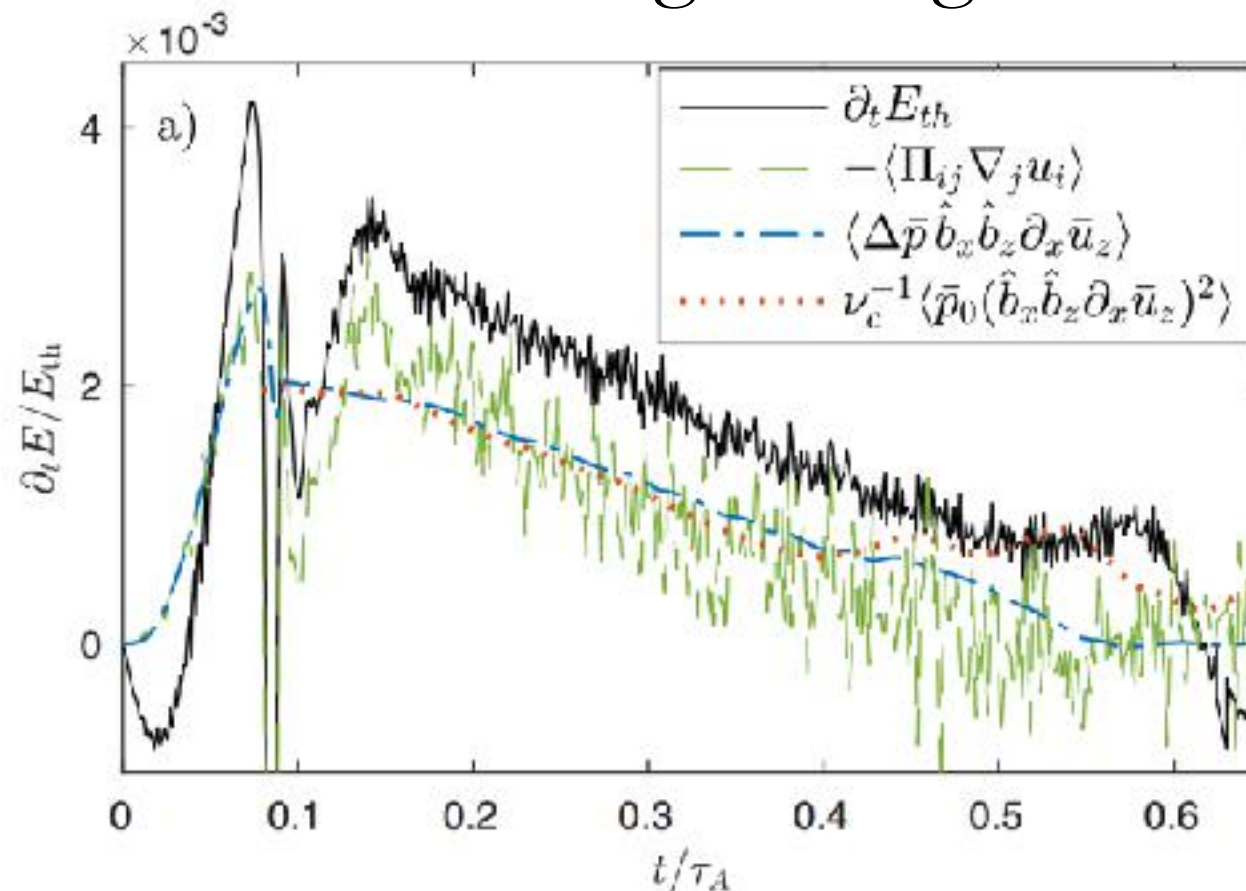
Squire, Kunz, Quataert & Schekochihin (2017), *Phys. Rev. Lett.*

# Conclusion:

*linearly polarized Alfvén waves cannot be sustained with amplitudes  $\delta B_{\perp}/B_0 \gtrsim \beta^{-1/2}$ .*

(some evidence for this in the solar wind — not shown here)

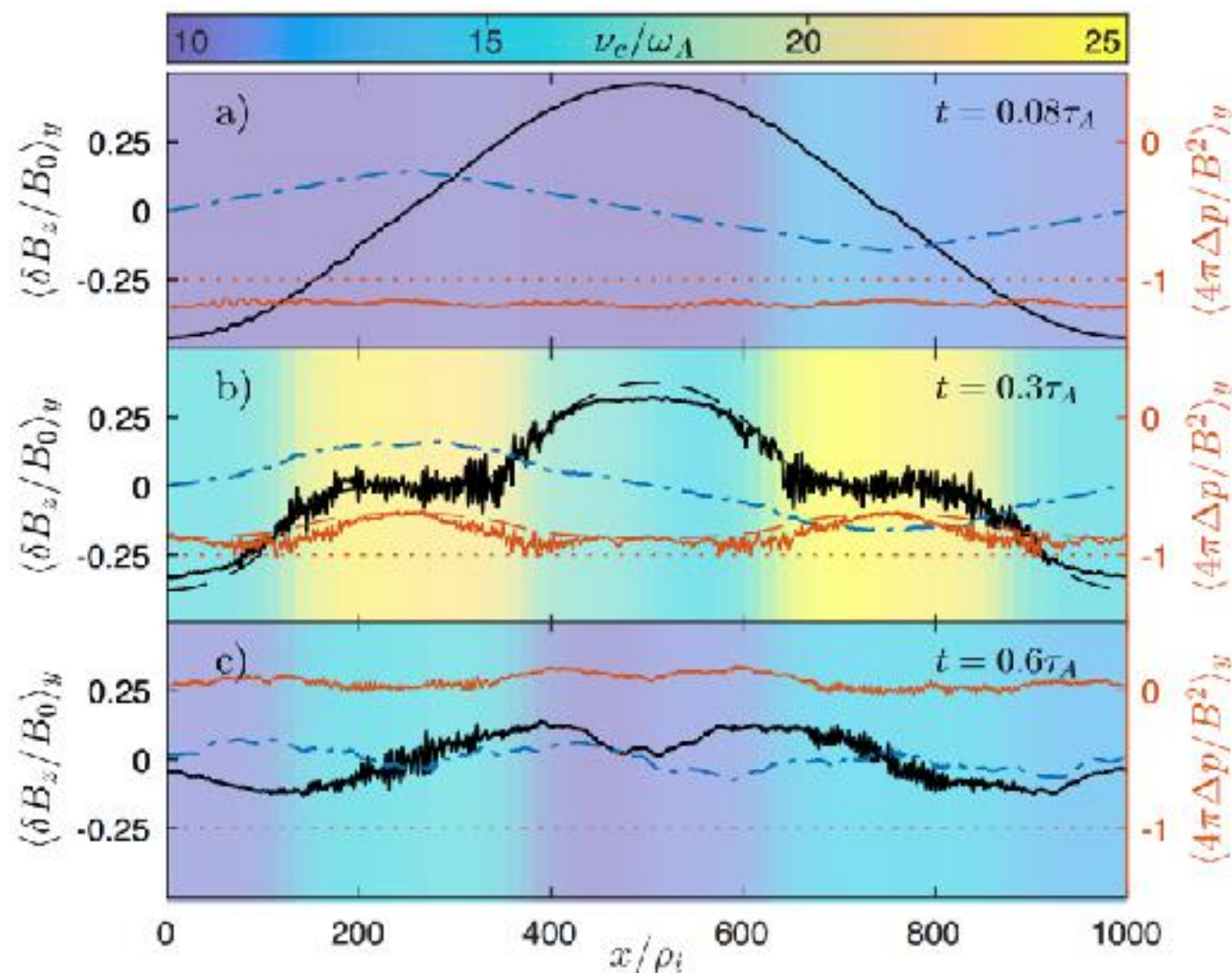
Measured ion viscous heating is Braginskii-like (of practical use)





# Sub-grid model?

Many large-scale aspects of the interrupted AW behavior can be captured using a modified Braginskii model; others with a modified Landau-fluid model.  
(see Squire, Quataert & Schekochihin 2017, NJP)



Jono Squire and I are working further on this.

Snowcluster 2021?



What about compressive fluctuations?

In a magnetized, weakly collisional plasma:  $\omega^2 = k^2 a^2 - i\omega k^2 \mu$

But for (small) viscous losses (and steepening), sound waves propagate just fine

In a magnetized, collisionless plasma:  $\frac{\omega}{kv_{\text{thi}}} Z\left(\frac{\omega}{kv_{\text{thi}}}\right) = -\left(1 + \frac{T_i}{T_e}\right)$

solving this...  $\frac{\gamma}{|\omega|} \sim -1$  if  $T_i \sim T_e$

A. Schekochihin: “[*in a collisionless hot plasma*] no one will hear you scream”

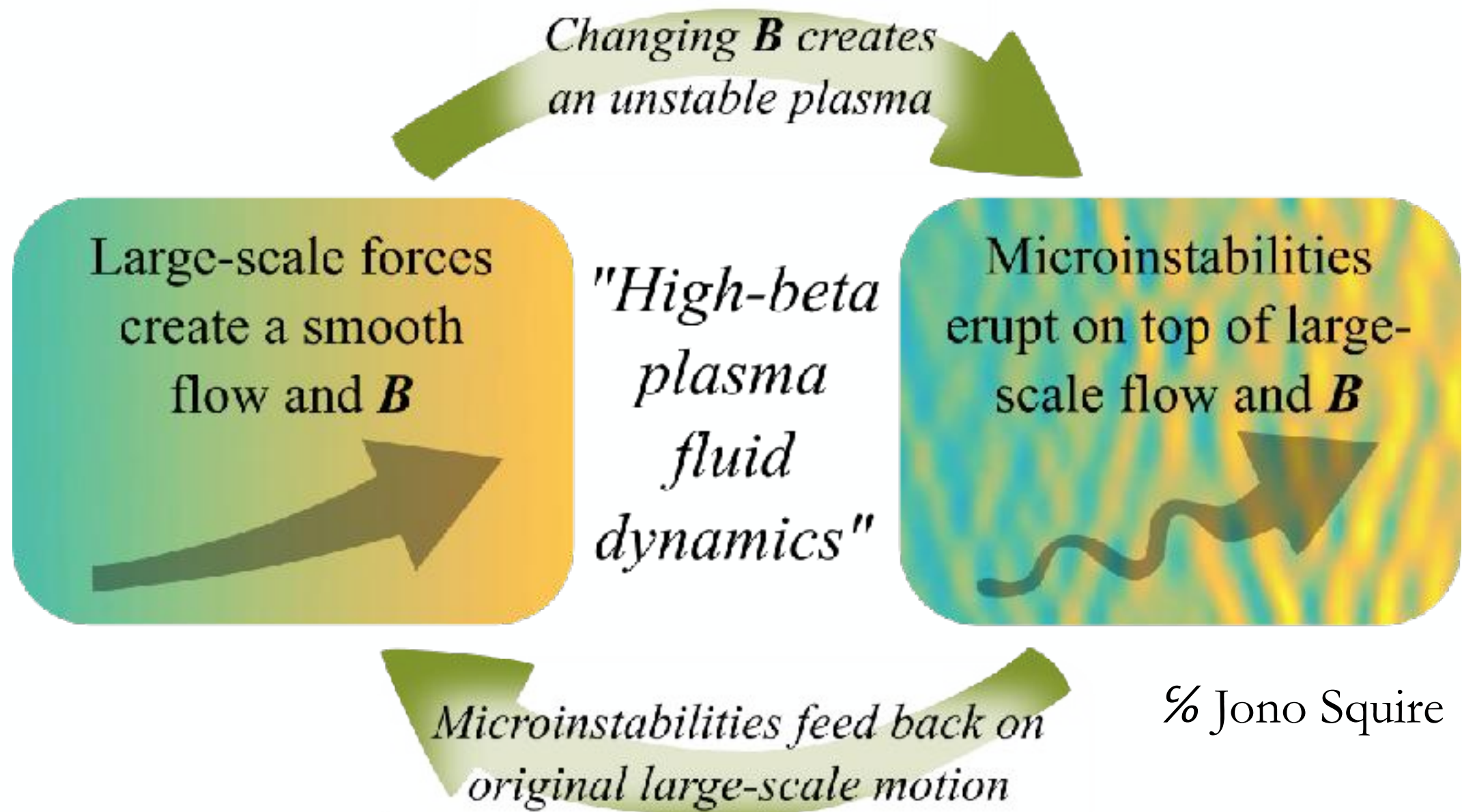
well, not necessarily...

what if compressive fluctuations drives pressure anisotropy,  
which excites mirror/firehose, which makes the plasma act “MHD-like”

redacted

key idea:

these kinetic instabilities restore  
fluid-like behavior to collisionless systems  
by limiting departures from  
local thermodynamic equilibrium



# Implications, Predictions, and Wild Speculation

In a high- $\beta$  low-collisionality plasma...

- There should be a  $\beta$ -dependent maximum amplitude for different polarizations of Alfvén waves (testable prediction in SW, not in ICM...)
- Compressive fluctuations with amplitudes above a  $\beta$ -dependent threshold should propagate easier than they would otherwise
- Direct energy transfer from macroscales to microscale fluctuations and thermal energy, w/o customary scale-by-scale cascade ( $\text{Re} \sim 1$ )  
implies that stable viscous-heating model in Kunz *et al.* (2011) might be right!
- Impact on cosmic-ray-driven instabilities and CR diffusion in ICM; what if CR-driven AWs suffer from interruption physics?
- Modern theories of Alfvén-wave turbulence (e.g., GS95) most likely don't apply at sufficiently high  $\beta$