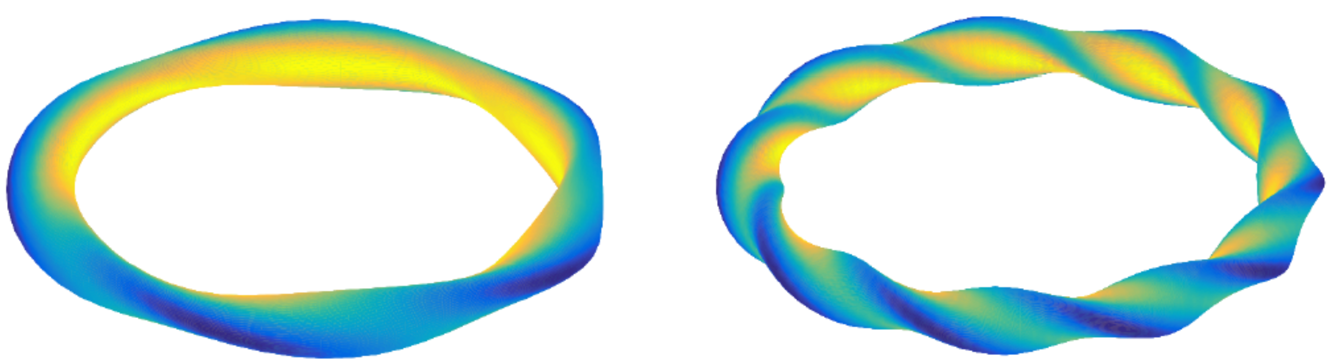
Ideal and relaxed equilibrium β-limits in classical stellarators

Joaquim Loizu

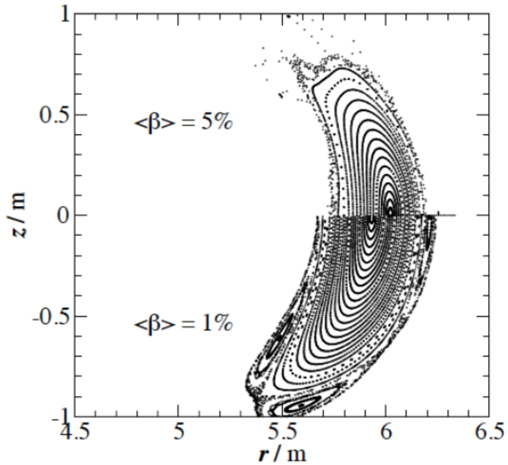
in collaboration with S. Hudson, C. Nührenberg, J. Geiger, P. Helander



ISHW, Kyoto, October 3rd, 2017

Equilibrium β-limit in stellarators is unknown

- β-limit probably determined by the equilibrium, not its stability. [P. Helander et al, PPCF, 2012]
  - Possible degradiation of flux-surfaces at high β.
  - Need for a robust, reliable, and fast code.



[M. Drevlak et al, NF, 2005]

- Ongoing parallel efforts:
  - HINT [Y. Suzuki et al, NF, 2006]
  - SIESTA [S. Hirshman et al, PoP, 2011]
  - SPEC [S. Hudson et al, PoP, 2012]

- SPEC follows the "equilibrium philosophy", namely it addresses the question:

  *What is the **equilibrium** magnetic field that is consistent with the established **equilibrium** pressure and toroidal current profiles?*

- My philosophy to approach an understanding of β-limits:

  *Use numerical experiments to guide our theories towards a **distilling** of the physics*

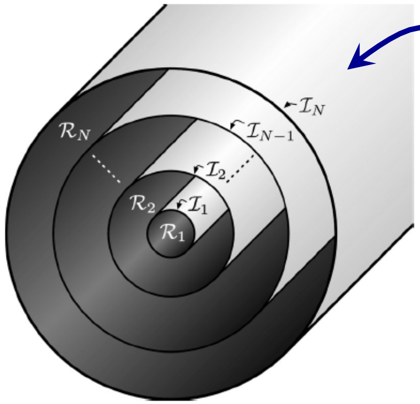
Stepped-Pressure Equilibrium Code (SPEC)

Uses variational principle to find equilibria with islands:

$$\mathcal{R}_l : \quad \nabla \times \mathbf{B} = \mu_l \mathbf{B}$$

$$\mathcal{I}_l : \quad [[p + B^2/2]] = 0$$

$l = 1, 2, \ldots N$

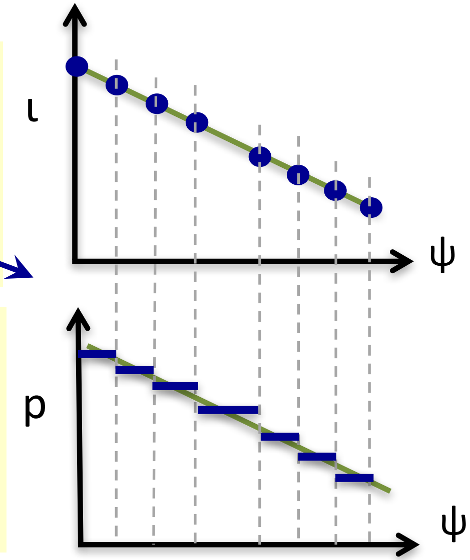


Input

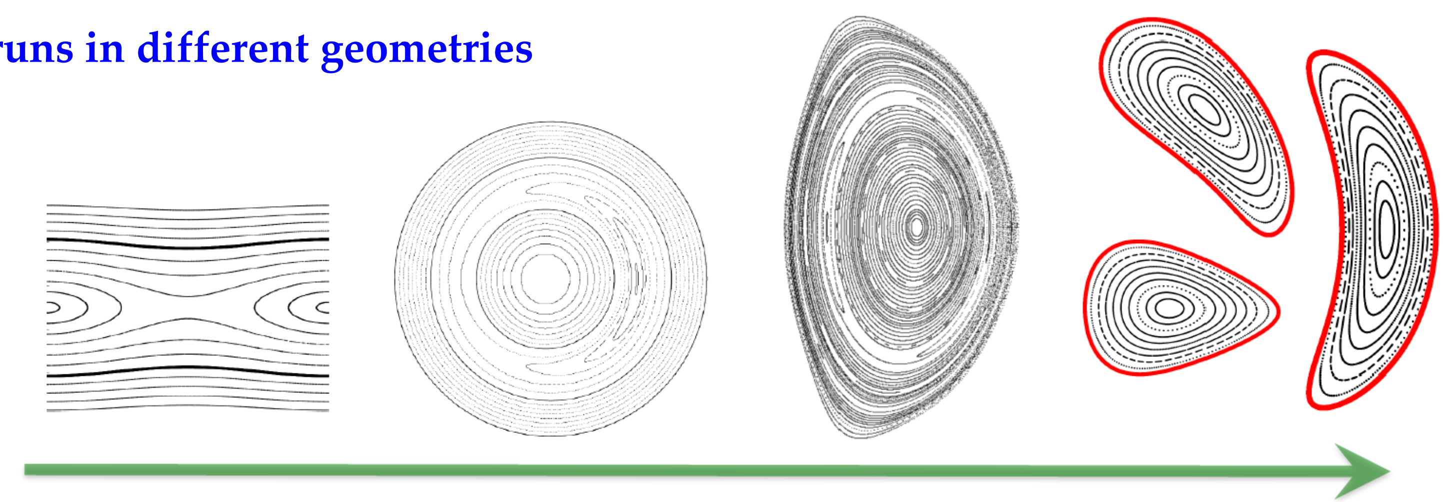
Boundary geometry
\+
Two profiles

Output

B-field in each volume
\+
Shape of KAM surfaces



SPEC runs in different geometries

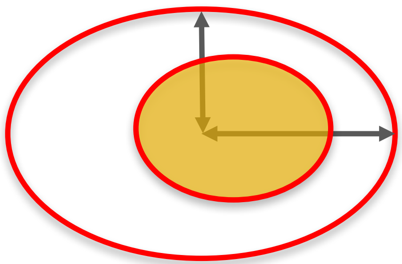


Slab **Cylindrical** **Tokamak** **Stellarator**

Consider $l = 2$ stellarator with simple pressure pedestal

Boundary

Consider a *rotating-ellipse* stellarator:



$$
\begin{aligned}
R(\theta,\varphi) &= R_{00} + \cos\theta + 0.25\cos(\theta - N_p\varphi) \\
Z(\theta,\varphi) &= -\sin\theta + 0.25\sin(\theta - N_p\varphi)
\end{aligned}
$$

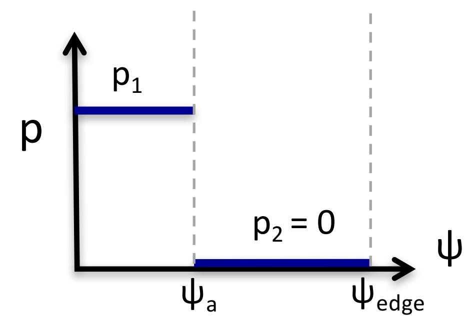
Ideal β-limit scaling:

$$\beta \sim \epsilon\, \iota_v^2 \sim \frac{N_p^2}{R_{00}}$$

[Freidberg, Ideal MHD, 2014]

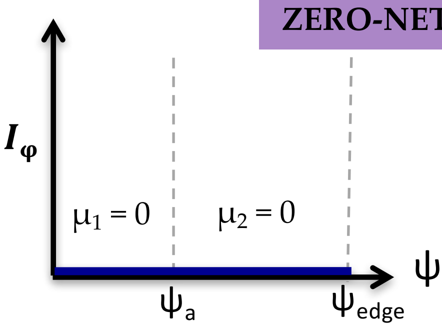
Pressure vs Flux

- Simplest model of a pedestal
- 2 maximally-relaxed force-free volumes
- SPEC naturally describes this system
- VMEC can be used with *steep-but-not-stepped* pressure



Current vs Flux

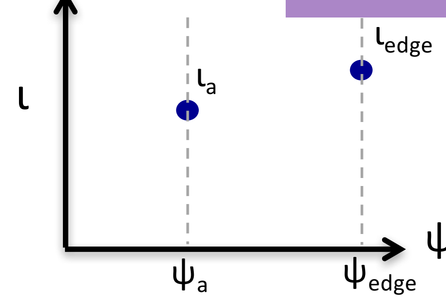
ZERO-NET-CURRENT



- Zero-net-toroidal-current
- No control on transform
- Expect $\iota_a^+ \neq \iota_a^-$

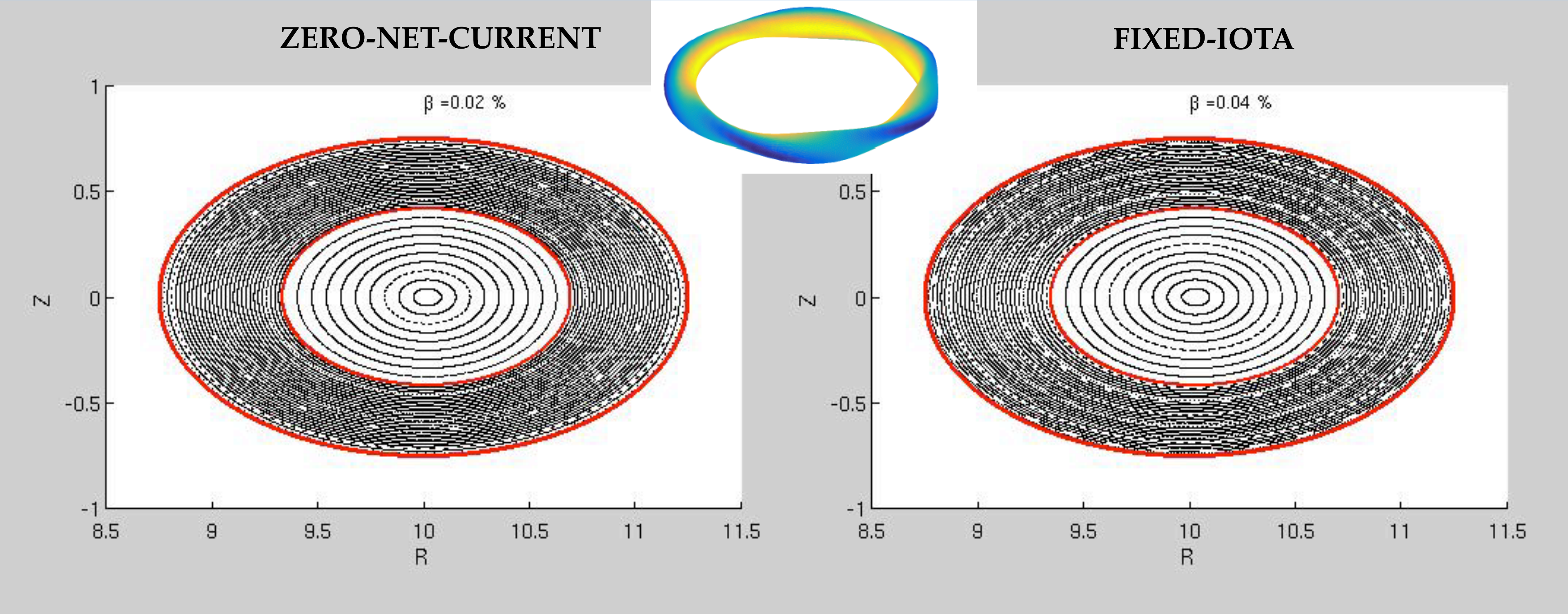
Transform vs Flux

FIXED-IOTA



- Local control on transform
- No control on current
- Expect surface current

Zero-net-current versus Fixed-iota



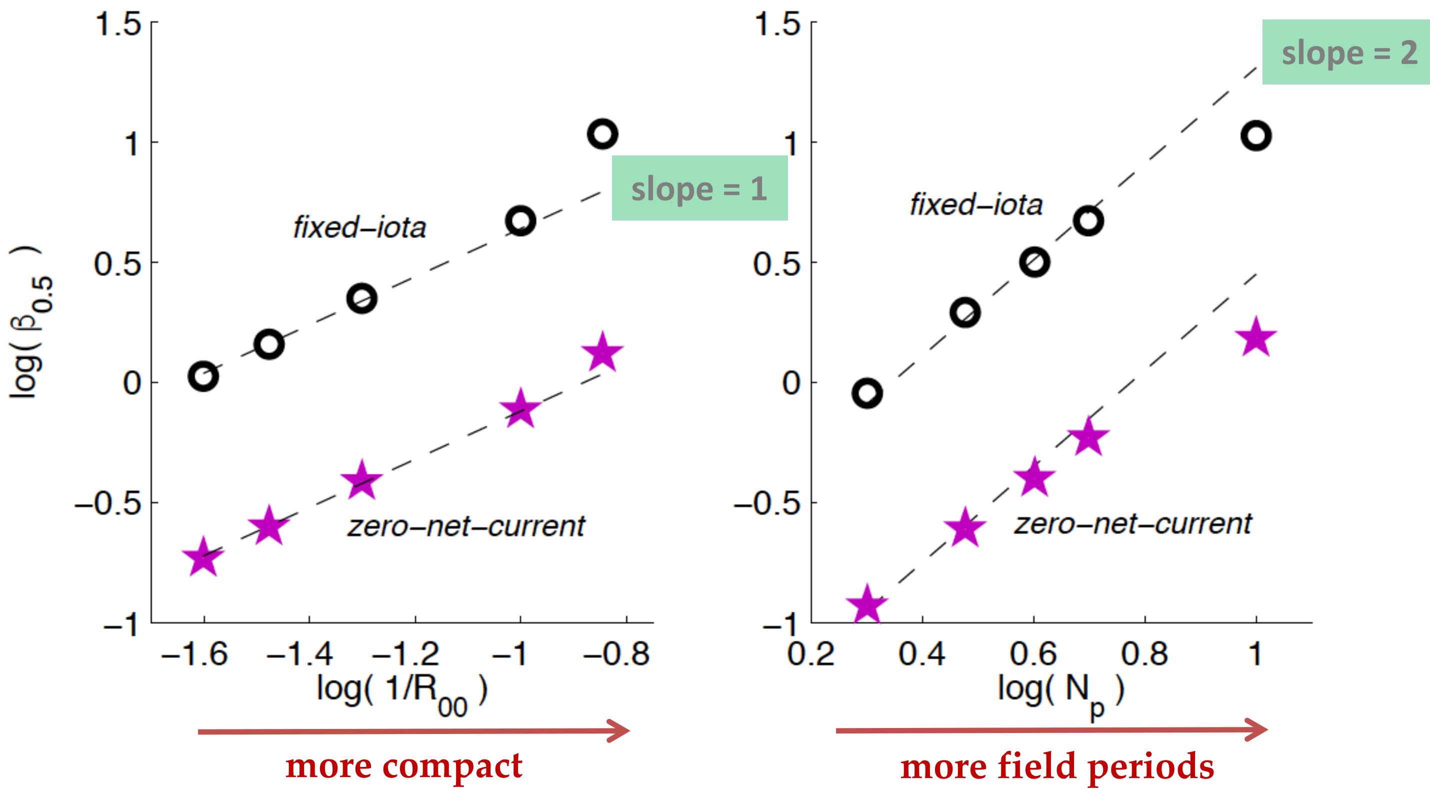
- Shafranov shift increases with β in both cases.
- $\boldsymbol{\Delta}_{ax}$ increases faster for the *zero-net-current* stellarator.
- A separatrix forms at β ≈ 0.4% in the *zero-net-current* stellarator.
- Islands and chaos emerge at β ≈ 1.4% in the *fixed-iota* stellarator.

Expected scaling of $\beta_{0.5}$ is reproduced in all cases

- $\beta_{0.5}$: beta at which **Shafranov shift** of the axis is half the minor radius.

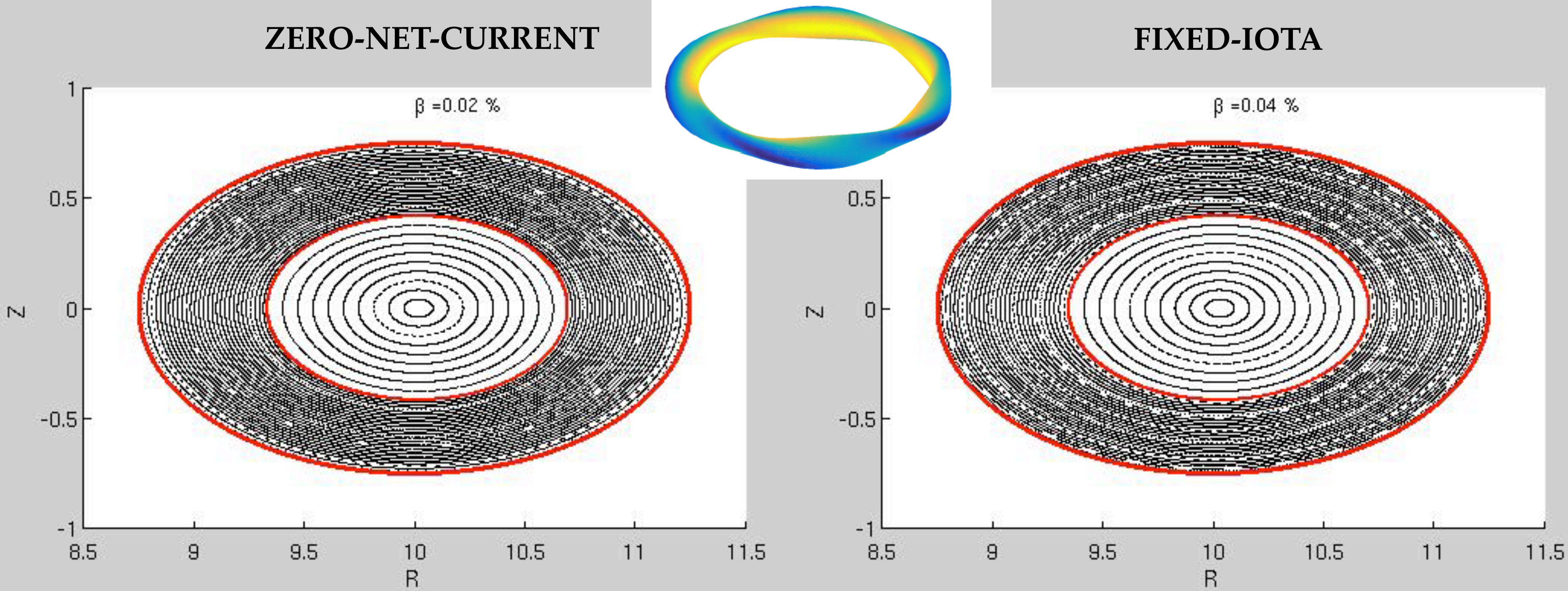
$$\beta_{0.5} \sim \epsilon \, \iota_v^2 \sim \frac{N_p^2}{R_{00}}$$

[Myamoto 2005]



- In all cases, $\beta_{0.5}$ scales as expected in ideal-MHD.
- Small amount of current provides access to higher β.

HBS theory explains macroscopic differences



Ideal MHD: $\beta_{lim} = \epsilon_a \, \iota_v^2 \approx 0.4\%$

$$\iota_a = \iota_v \sqrt{1 - \left(\frac{\beta}{\epsilon_a \iota_v^2} \right)^2}$$

[Freidberg, Ideal MHD, 2014]

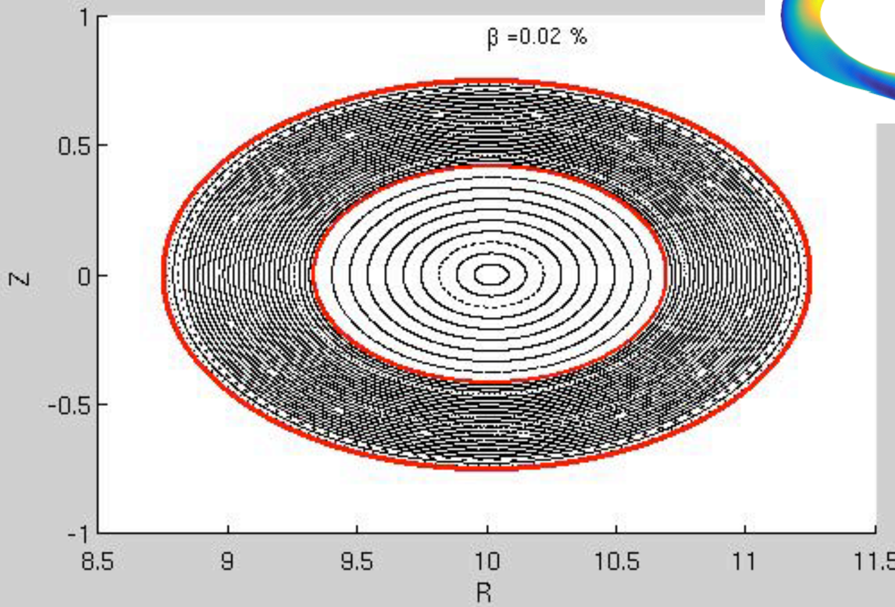
Ideal MHD: no β-limit

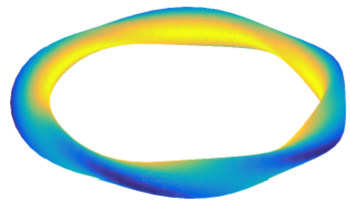
$$\mu_0 I_\varphi = \frac{\iota_v R_0}{2\Psi_a} \left[\sqrt{\frac{1}{2} \left(1 + \sqrt{1 + 4 \left(\frac{\beta}{\epsilon_a \iota_v^2} \right)^2} \right)} - 1 \right]$$

[Freidberg, Ideal MHD, 2014]

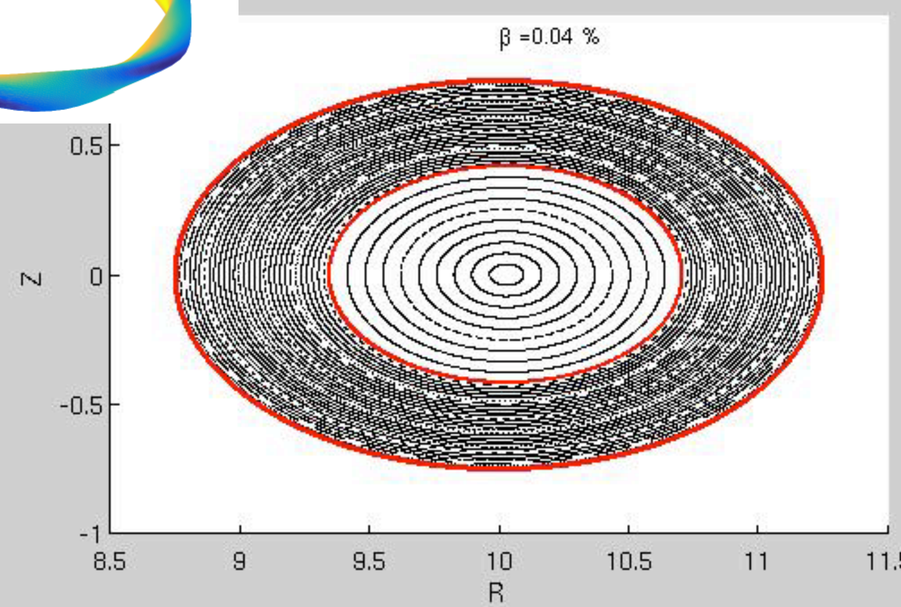
HBS theory explains macroscopic differences

ZERO-NET-CURRENT





FIXED-IOTA



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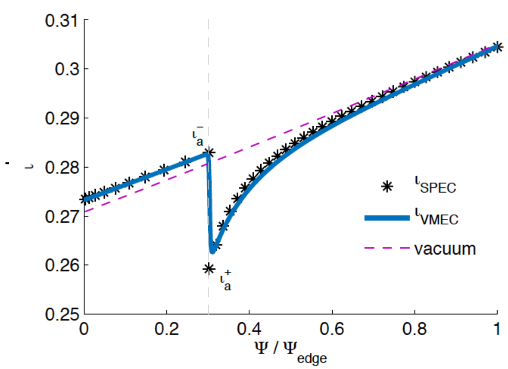

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[Freidberg, Ideal MHD, 2014]

HBS theory explains macroscopic differences

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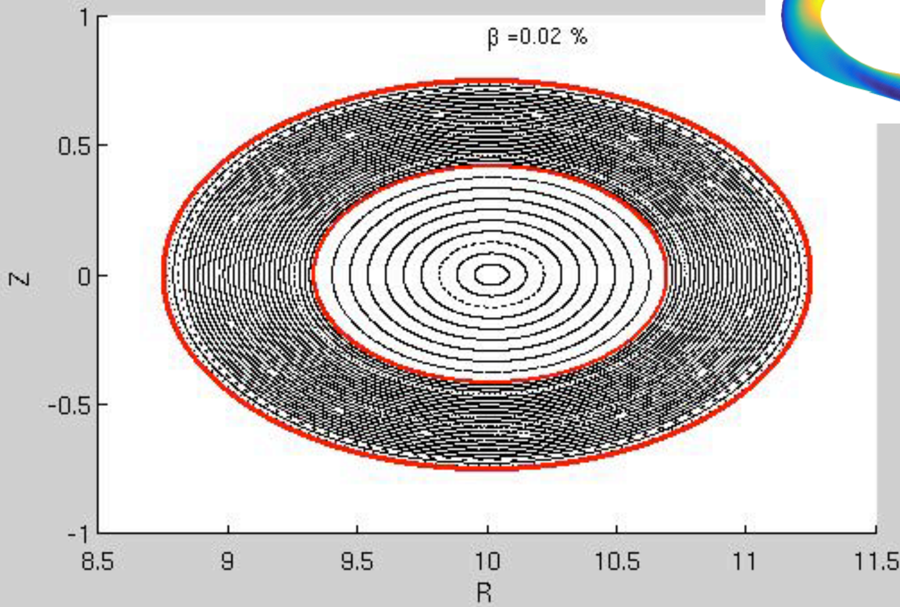
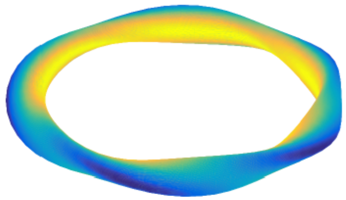
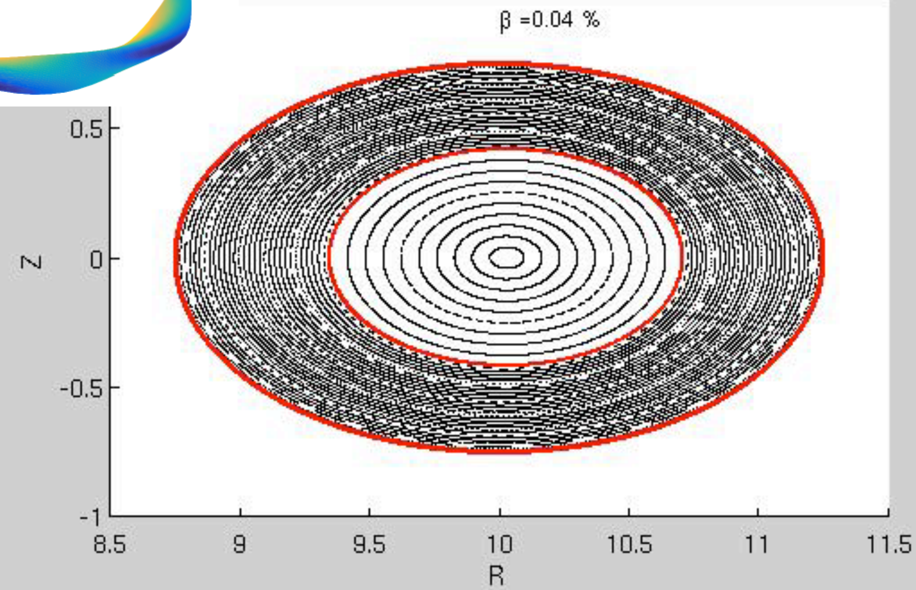




FIXED-IOTA



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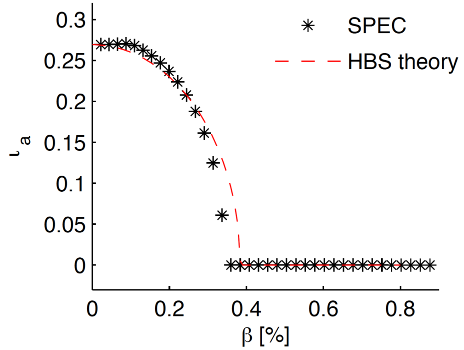
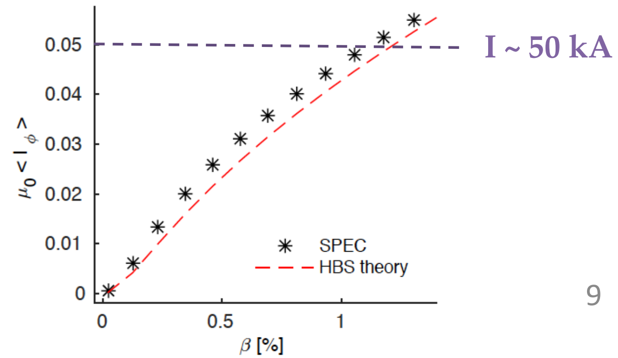
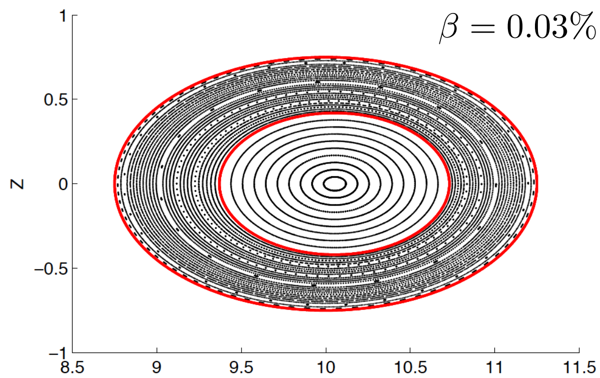
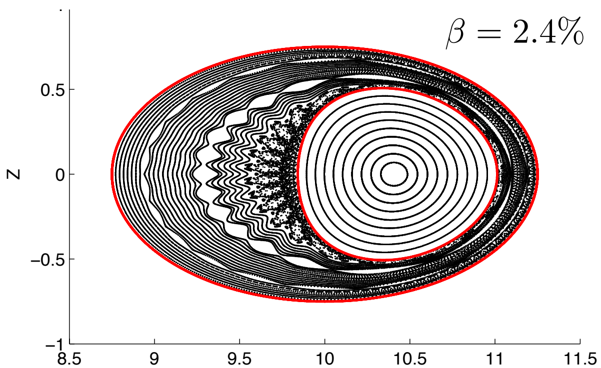


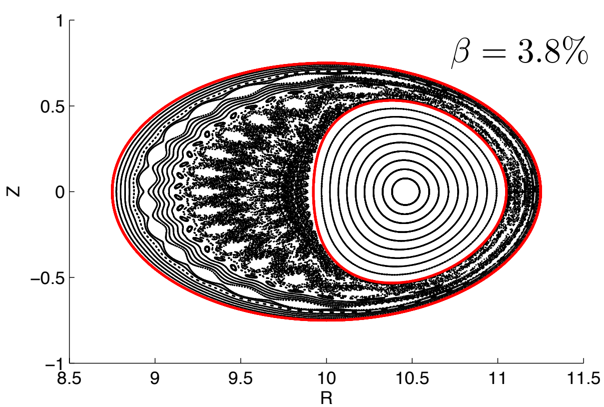
Ideal MHD: no β-limit



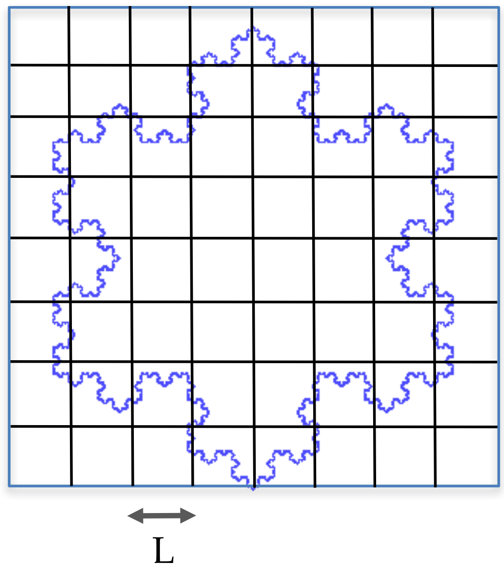
Fractal dimension of field-lines increases with β







Koch curve

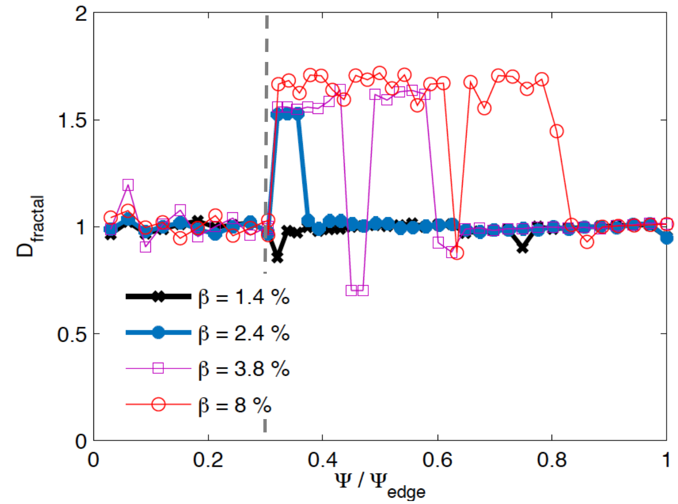


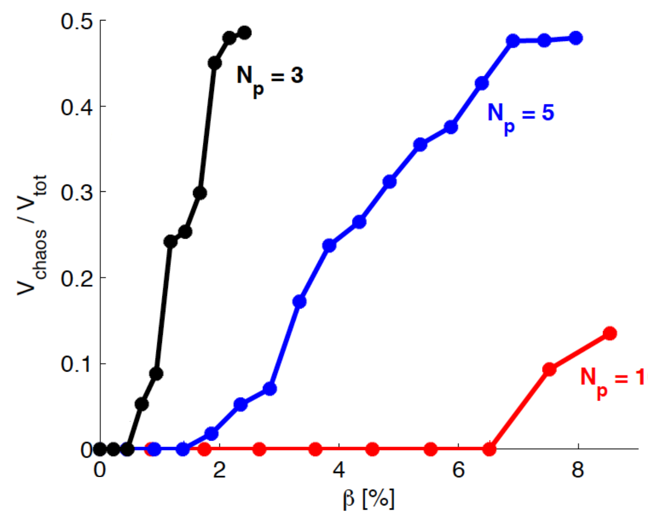
Haussdorf dimension

$$D = \lim_{L \to 0} \frac{\log N(L)}{\log(L)}$$

Volume of chaos

$$V_{chaos} = V_{tot} \sum_{i=1}^{n_{lines}} \frac{(\Psi_i - \Psi_{i-1})}{\Psi_{edge}} \mathcal{H}(D(\Psi_i) - D_{crit})$$





A theory for the non-ideal β-limit

- Expect that chaos emerges due to the overlaping of islands. [Chirikov, Phys Reports, 1979]

- Expected island width due to a resonance is: [Boozer, Rev Mod Phys, 2004]

$$w \sim \sqrt{B_{mn}/(m\iota')}$$

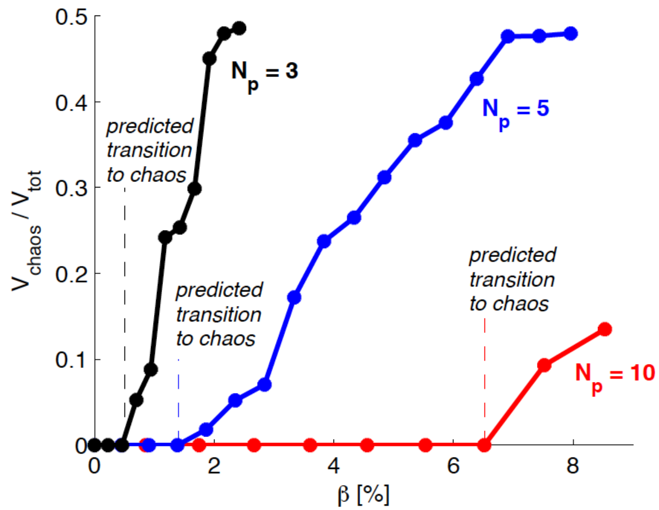
- As β increases, I_φ increases and modifies the rotational transform.

- Hypothesis: islands and chaos will emerge when:

$$\iota_I(\beta) \sim \iota_v$$

namely when

perturbations in the poloidal field due to toroidal current $\sim$ *vacuum poloidal field*



- Inserting ansatz in HBS theory for the current,

$$1 = \sqrt{\frac{1}{2}\left(1 + \sqrt{1 + 4\frac{\beta_{chaos}^2}{\epsilon_a^2 \iota_v^4}}\right) - 1} \Longrightarrow \boxed{\beta_{chaos} = \sqrt{12}\epsilon_a \iota_v^2}$$

Conclusions and perspectives

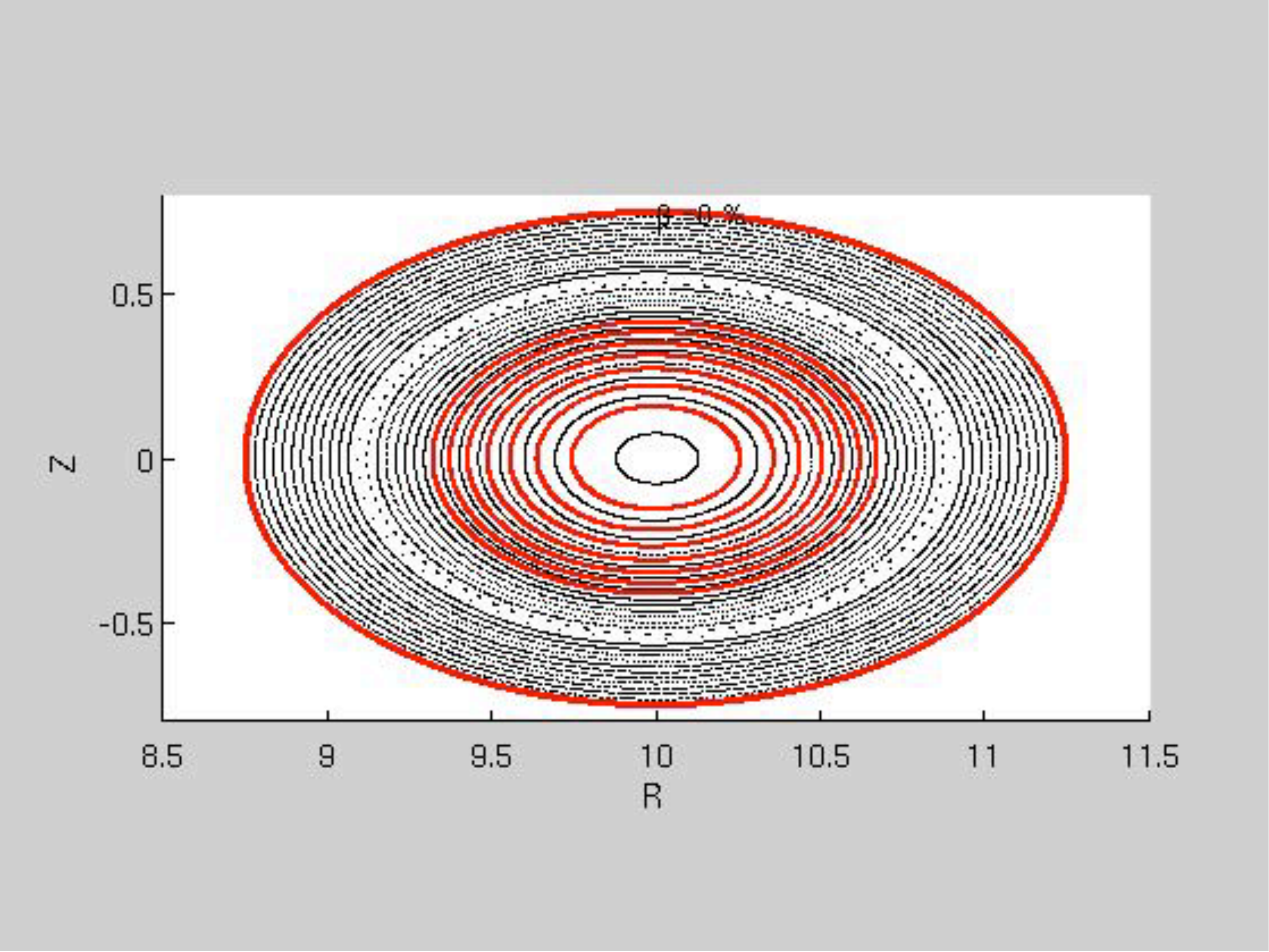
- Basic study of equilbrium β-limit indicates that
  1. Macroscopic features behave as predicted by ideal-MHD
  2. Zero-net-current stellarator behaves "ideally"
  3. Fixed-iota stellarator ($I_\varphi > 0$) shows "non-ideal β-limit"

- SPEC has been used to assess whether or not magnetic islands and chaos ***can*** emerge at high β in ***simple*** stellarators configurations.

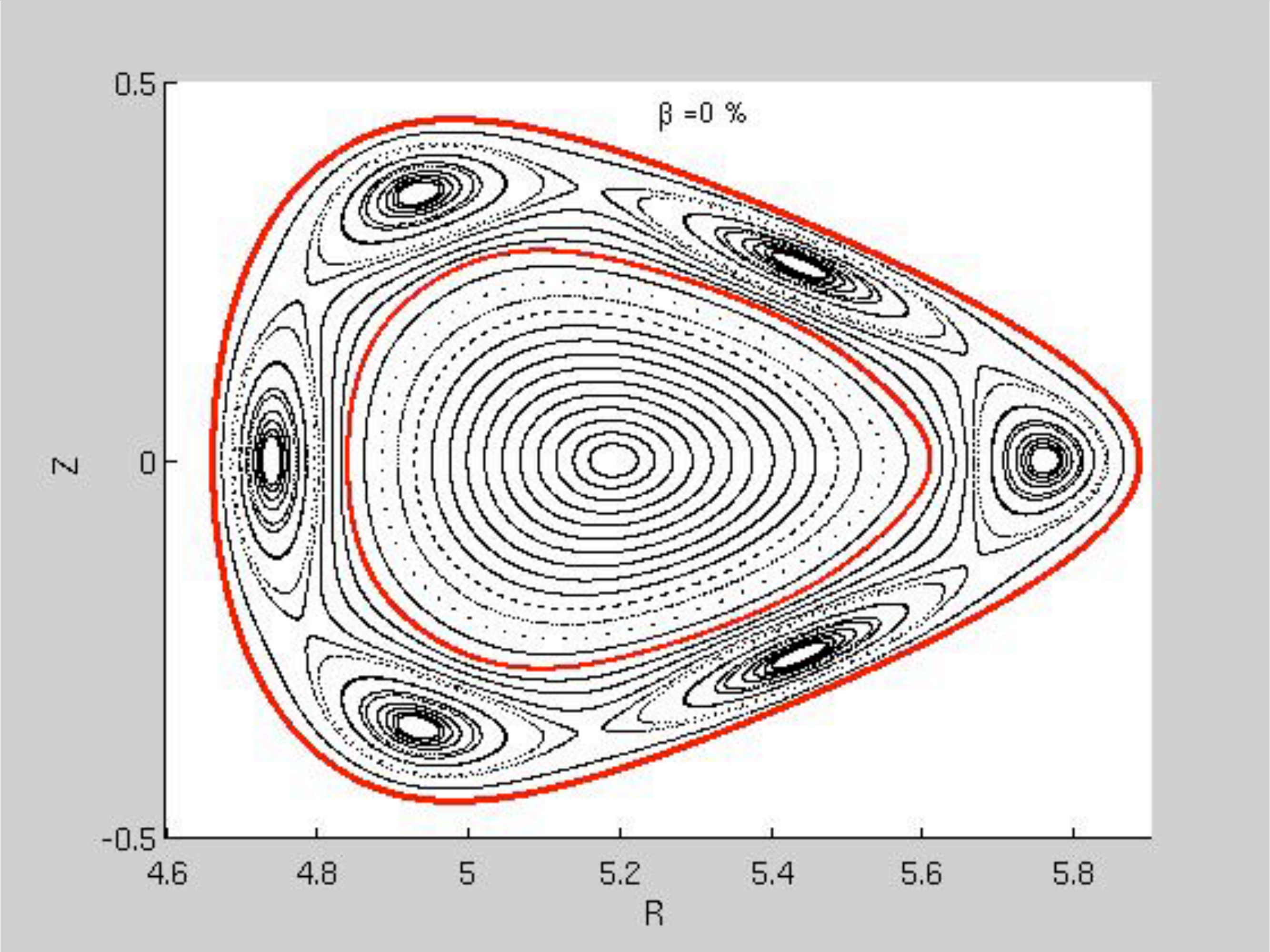
We studied "worst-case-scenario" of maximum relaxation: how to incorporate the possibility of pressure-induced island-healing?

Can we extend the theory to more complex geometries and non-trivial pressure profiles?

Non-trivial pressure profile (low resolution)



W7-X OP1.1 limiter configuration (low resolution)



This is not an experimental prediction. This simlpy emphasizes that the equilibrium β -limit may be determined as in simpler geometries.