A Cross-Benchmarking and Validation Initiative for Tokamak 3D Equilibrium Calculations

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A cross-benchmarking initiative was initiated in the context of the OFES FY 2012 Theory Milestone ("Joule Milestone"):

"Effects of Nonaxisymmetric Fields in Tokamak Equilibria, with Particular Focus on ITER-Relevant Effects".

The idea:

Run a case amenable to analysis by broad spectrum of stellarator and tokamak codes, and compare solutions.

- Work with stellarator symmetric cases to allow full participation of stellarator codes.
  - Symmetry of the magnetic field with respect to combined reflection in the poloidal and toroidal angles;
  - Requires axisymmetric component with up-down symmetry (double-null divertor).
- Tokamak perturbative equilibrium codes require nonaxisymmetric component of **B** small relative to axisymmetric component.

Motivated some later experiments:

Fortuitous Alignment with FY 2014 "Joint Research Target (JRT)" led to a day of dedicated shots on DIII-D

"Joint Research Target (JRT)"– national experimental milestone, on plasma response to 3D perturbation

- Strong analysis component in JRT quarterly milestones.
- Cross-benchmarking initiative merged into JRT.
- Ran a day of dedicated shots on DIII-D in May for the purpose of generating data for validation.
- Diagnostics included more than 100 new magnetic field sensors positioned on the high and low field sides of the tokamak.
- Surprising observation that high field side response for double null is factor 3 to 4 times smaller than for single null plasma.
- Participated in writing JRT quarterly reports and final report.

Return to: FY 2012 Milestone:

"No Battle Plan Survives the First Contact with the Enemy"

––– military strategist Von Moltke

The Plan:

- Work with DIII-D shots from RMP ELM suppression experiments.
  - Externally imposed $\delta B/B \approx 10^{-3}$.
  - Use shots with balanced double-null plasmas.
- Run a broad range of different codes.
  - 1^{st} calculate equilibrium, initially with:
    - VMEC: Used routinely to analyze and plan stellarator experiments
    - widely used tokamak perturbative equilibrium codes: (MARS-F, IPEC, linear M3D-C1)
  - Then use equilibrium to calculate neoclassical and turbulent transport, local and global stability.

In Practice:

- Encountered disagreement between VMEC and perturbative codes.
- Investigated the equilibrium issue for three years

Initiative involves codes spanning broad range of physics and numerical models.

- Calculations involved 7 codes from 4 different institutions:
  - o tokamak perturbative 3D equilibrium codes IPEC (J-K Park) , MARS (Turnbull, Lanctot, Liu);
  - o time-dependent extended MHD code: M3D-C1, linear & nonlinear (Ferraro);
  - o stellarator equilibrium codes: VMEC (Lazarus, Lazerson), NSTAB (Garabedian's code: Cerfon, McFadden), HINT (Suzuki in Japan) , PIES

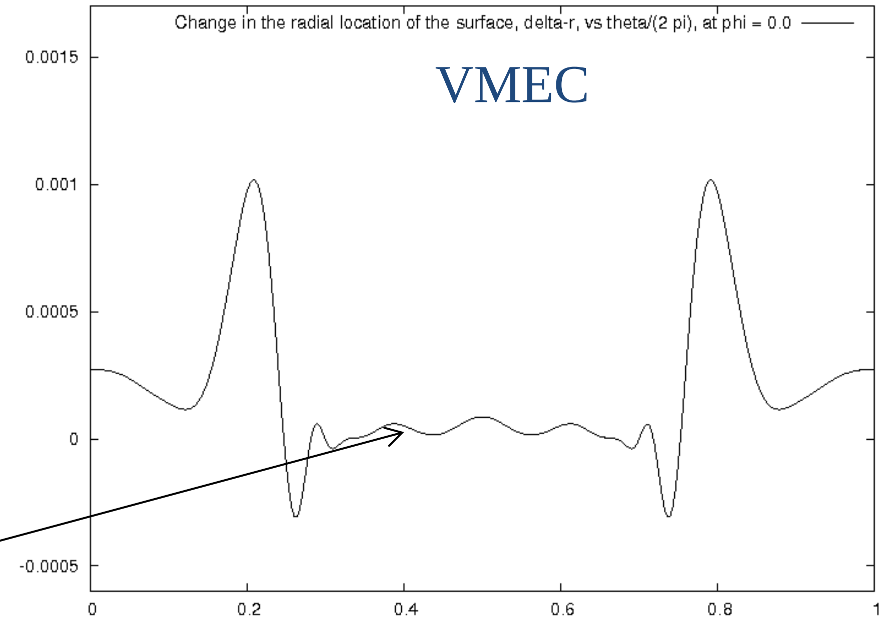
Initial Calculations were done for DIII-D shot 142603

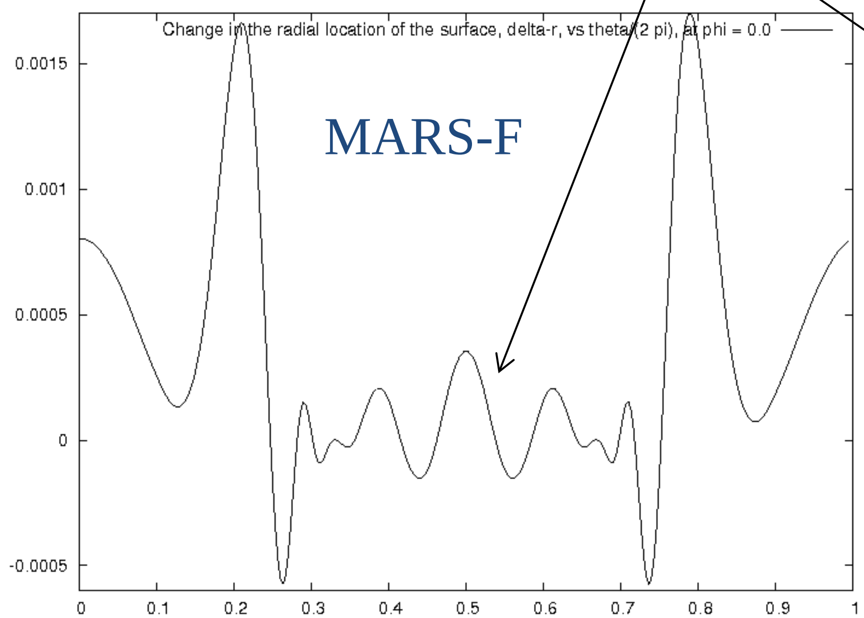
- Externally imposed $n = 3$ perturbation of order $\delta B/B \approx 10^{-3}$.
- Stellarator symmetric: balanced double-null plasma.

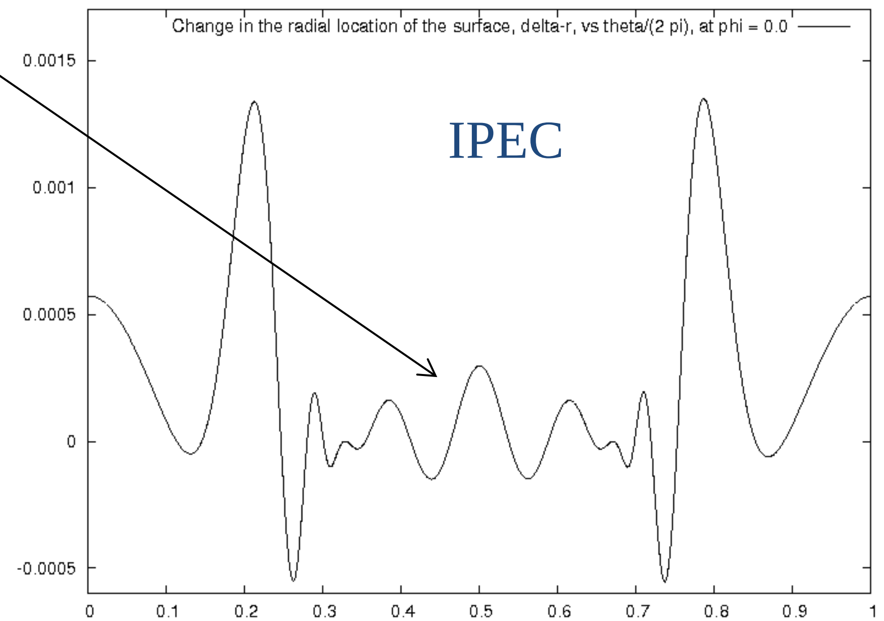
VMEC and perturbative 3D equilibrium codes disagree on calculated flux surface displacement on high field side for shot 142603.

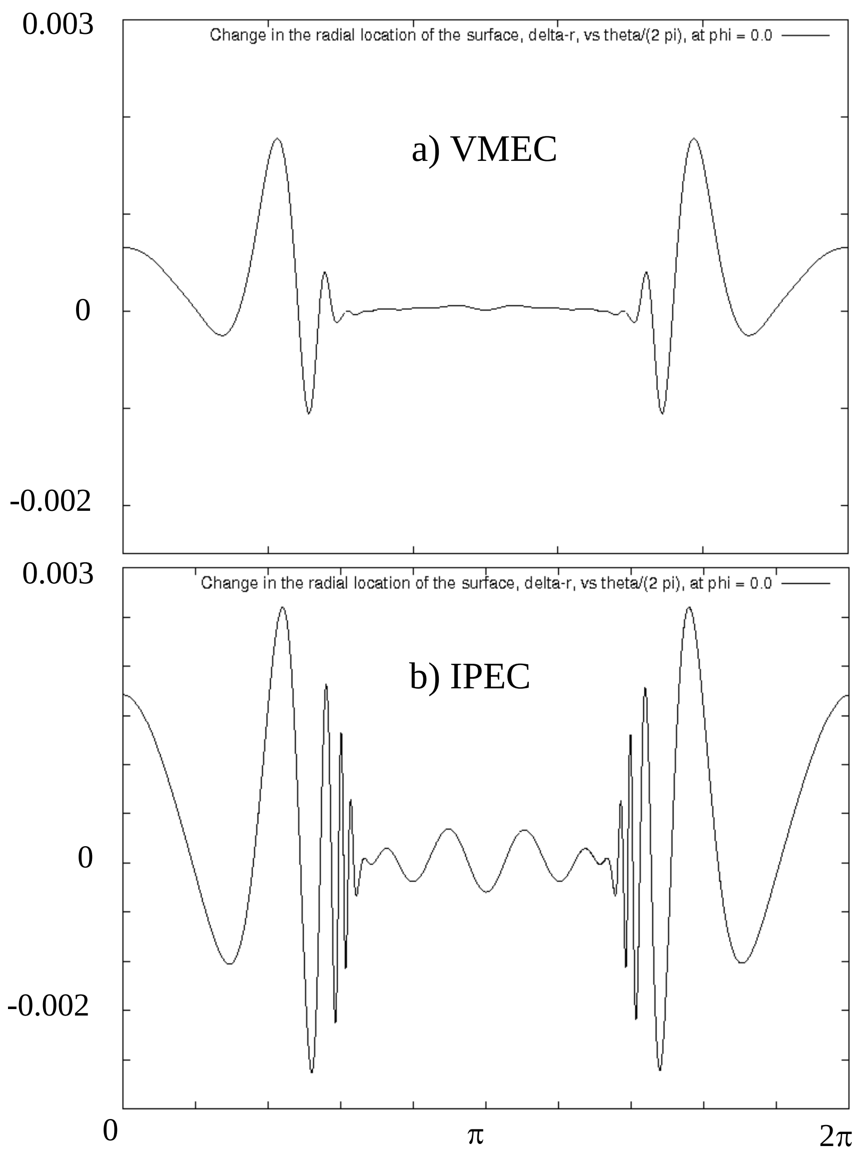
Amplitude of radial flux surface displacement vs. $\theta / (2\pi)$ (poloidal angle) at ϕ=0 for q = 8.5/3 flux surface.

Substantial disagreement on inboard side.









The disagreement becomes more pronounced with increasing minor radius of the flux surfaces.

Comparison of the radial perturbation as a function of poloidal angle for the q=13.5/3 surface as calculated by VMEC and IPEC.

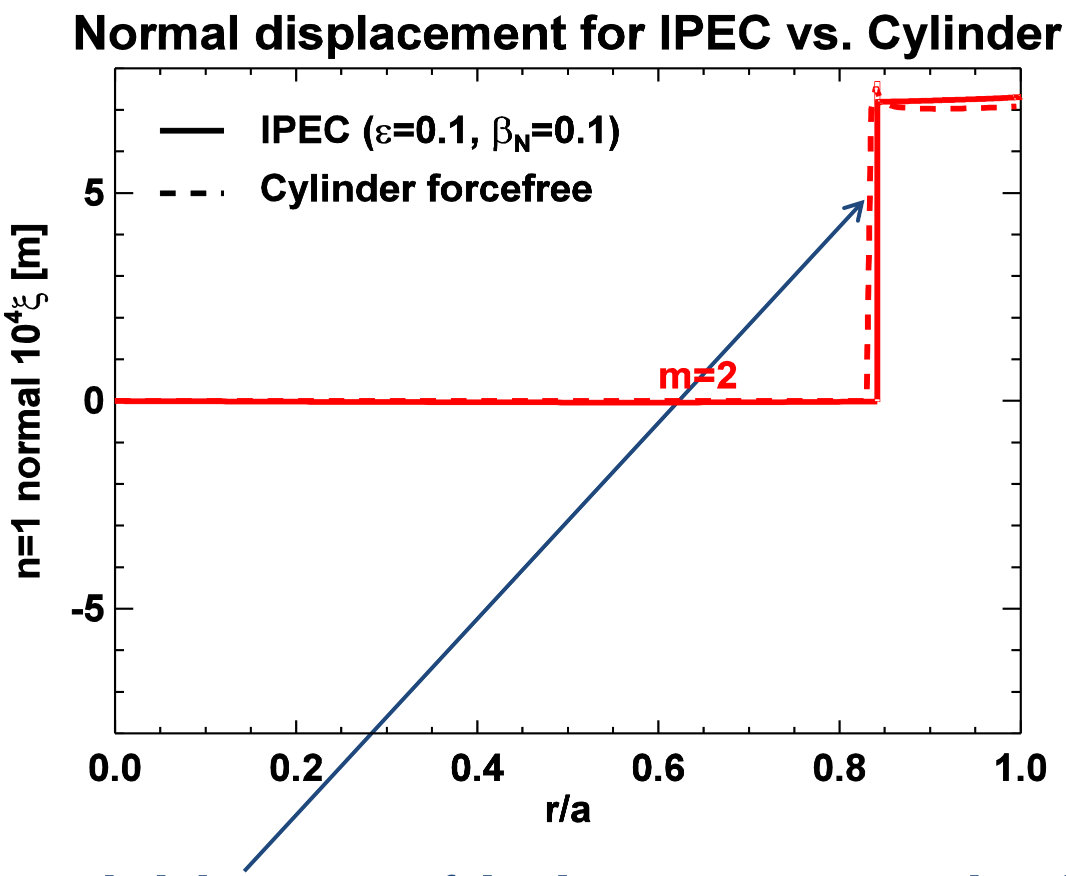
Three approaches pursued to investigate source of disagreement:

1. Calculation of equilibria for simple model cases.
2. Investigation of domain of validity of linearized (perturbed) equilibrium equations.
3. Comparison with calculations from additional codes.

1. Equilibria for Simple Model Cases

IPEC and VMEC solutions quite different for simple model equilibria.

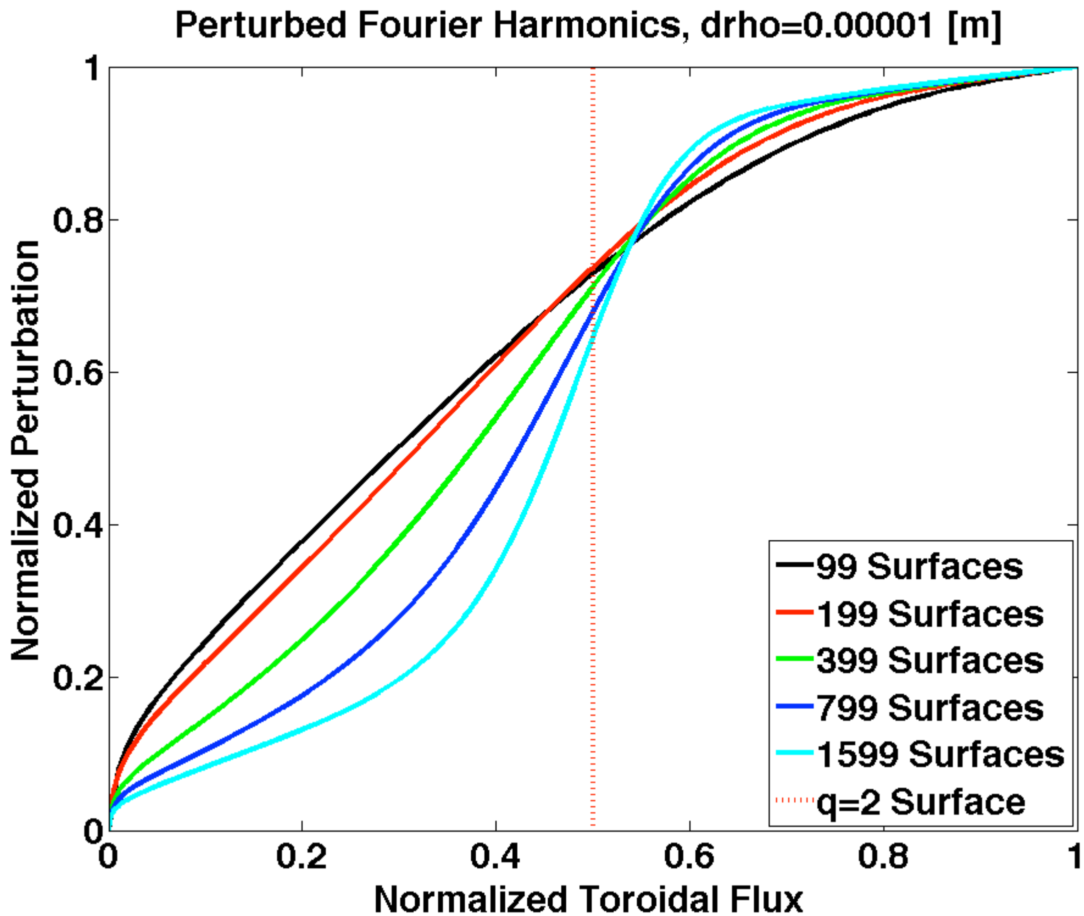
IPEC **(J-K Park)**



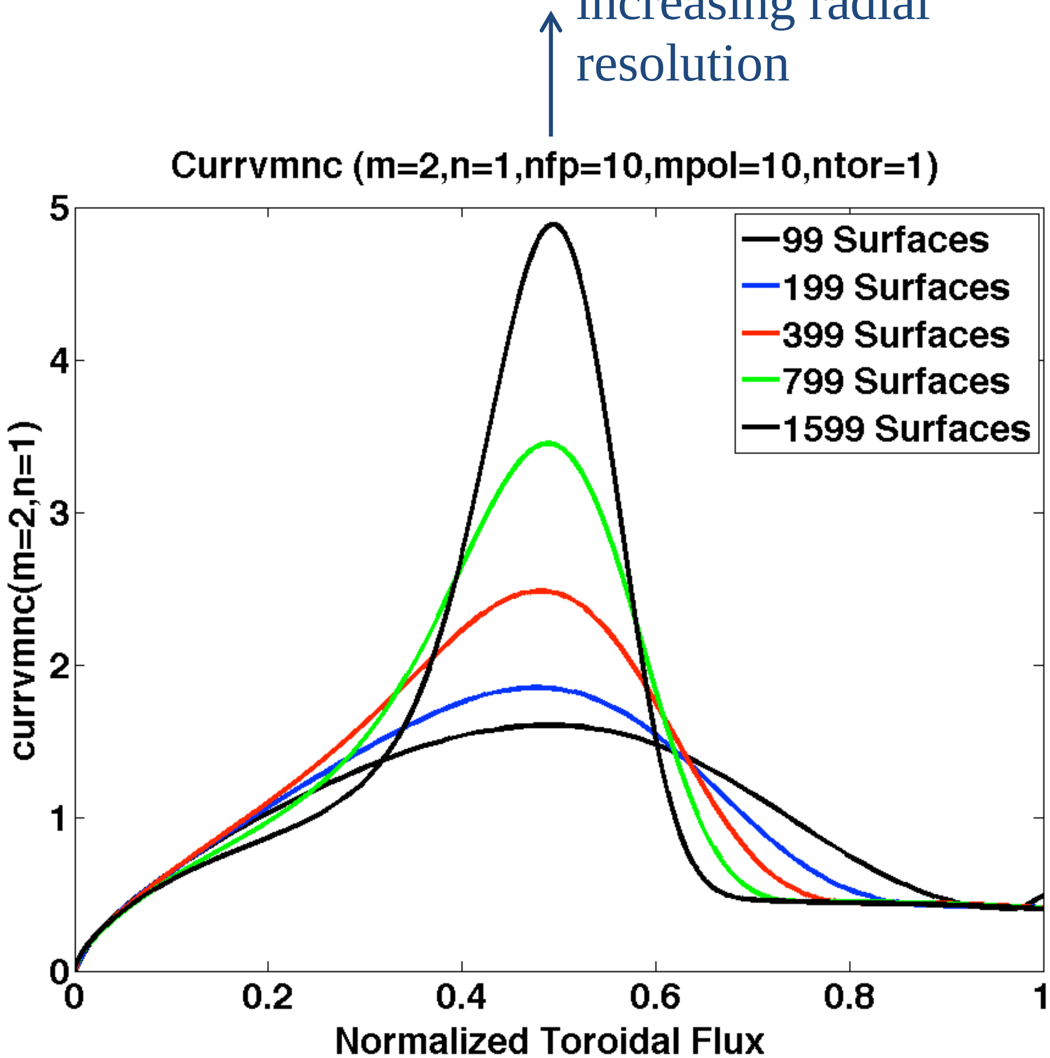
Large radial derivative of displacement associated with large localized current at q = 2 rational surface.

IPEC equilibrium solution for large aspect ratio torus (R/a=10) with circular boundary perturbed by single harmonic.

- The three solutions shown correspond to boundary perturbations having toroidal mode number n=1 and the poloidal mode number, m, indicated in the figure.
- The dashed lines are the corresponding analytic solutions for a perturbed ("infinite aspect ratio") cylinder.


Perturbed Fourier Harmonics, drho=0.00001 [m]
Normalized Perturbation
Normalized Toroidal Flux
0
0.2
0.4
0.6
0.8
1
99 Surfaces
199 Surfaces
399 Surfaces
799 Surfaces
1599 Surfaces
q=2 Surface

increasing radial resolution



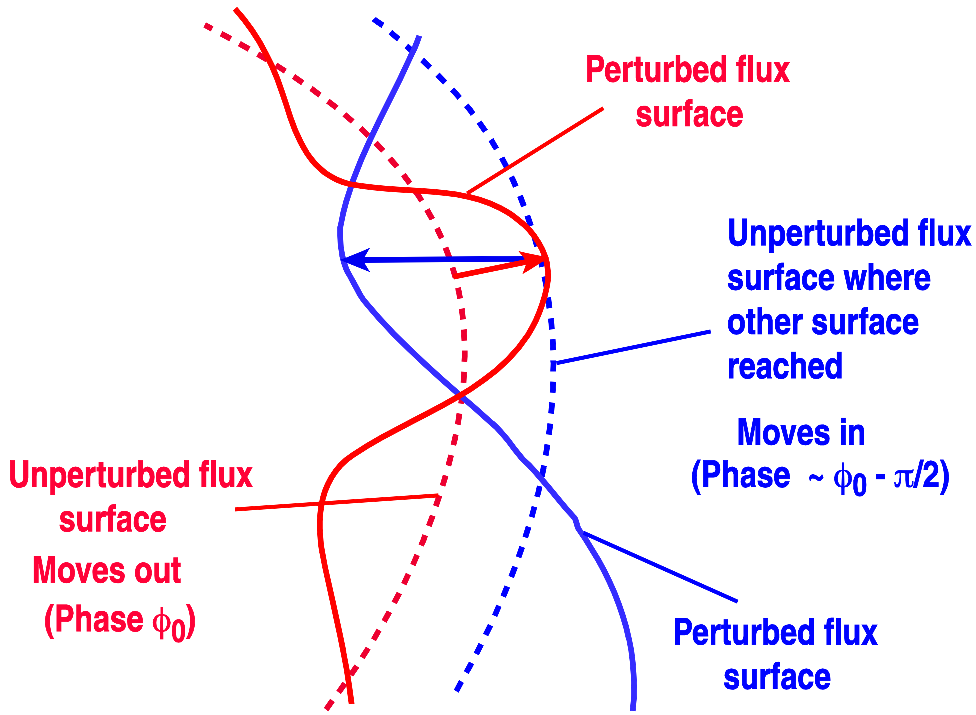
ot of corresponding = 1, *m* = 2 component current density finds at localized current not ell resolved in VMEC lculation. (S. Lazerson)

Dependence on no. of radial grid surfaces consistent with convergence to localized current in limit of infinite radial resolution.

(Similar VMEC study, with similar results, done by Monticello *et al*, US/Japan JIFT workshop, Princeton, NJ, December 2002, https://docs.google.com/file/d/0B3SxUyX3eGoWWHhHbkxTVXNGNGs/edit)

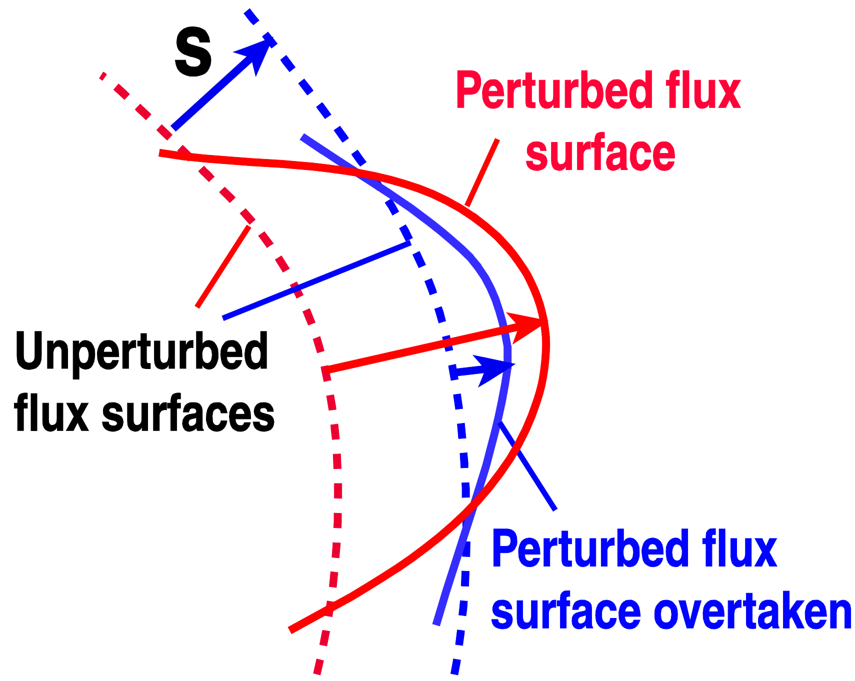
2^{nd} Potential Source of Disagreement:

Validity of perturbative calculation of flux surface displacement breaks down when perturbation gets large enough for flux surfaces to cross.



Flux surfaces crossing

Figure 11(a)



Overtaking

Figures from Turnbull *et al.*

Validity of perturbative calculation of flux surface displacement breaks down at surprisingly small perturbation amplitude.

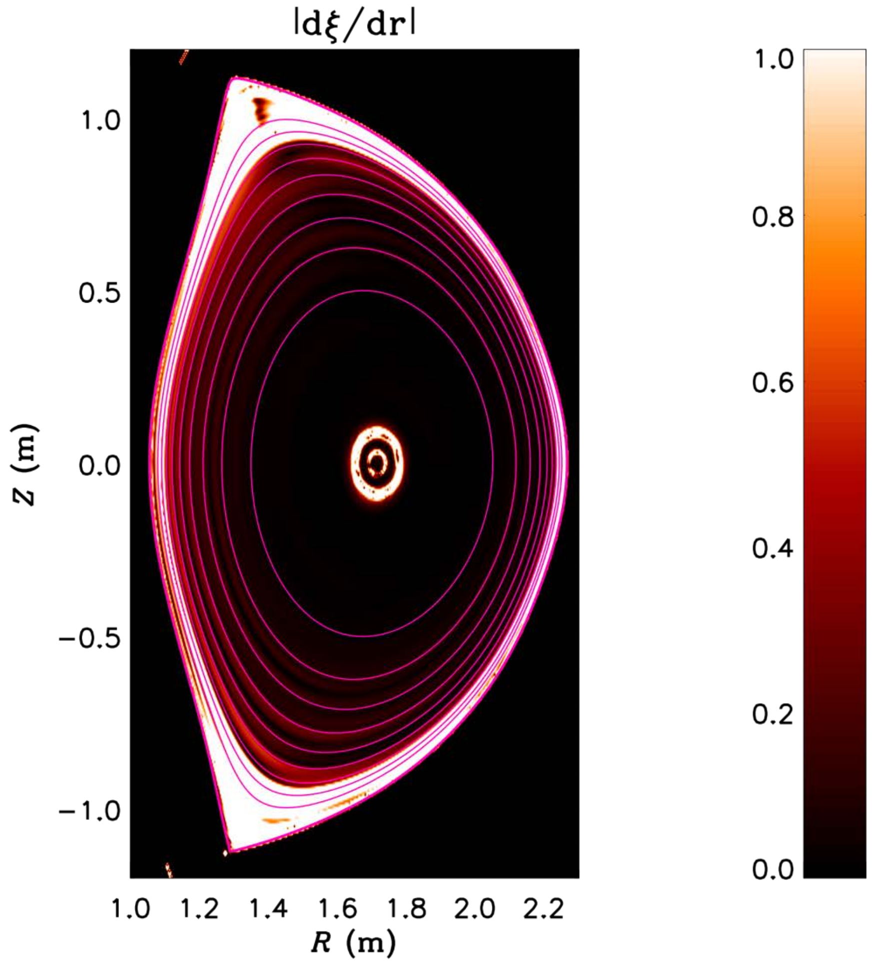
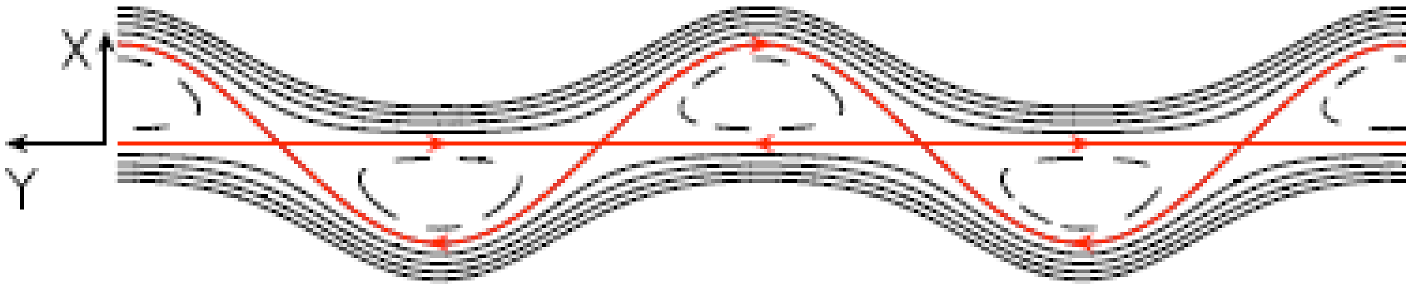


Figure shows evaluation of overlap criterion for linearized M3D-C1 solution for shot 142603. (Ferraro)

- Value of 1.0 indicates overlap. (Overlap condition satisfied.)
- $\delta B / B \approx 10^{-3}$ at plasma boundary.
- Roughly everything outside q = 3.5 surface satisfies overlap condition.
- overlap condition also satisfied in neighbourhood of rational surfaces.

Boozer and Pomphrey have calculated implication of breakdown of linearized (perturbative) solution in neighborhood of rational surface.

A. Boozer and N. Pomphrey, Phys. Plasmas 17, 110707 (2010).



- Spurious residual island with w scaling like $\delta B/B$.
- Linear approximation breaks down in that region.
- Is there similar spurious stochastic region in region near edge where linear approximation breaks down?

Evidence suggests overlap issue does not contribute to disagreement between VMEC and linear codes:

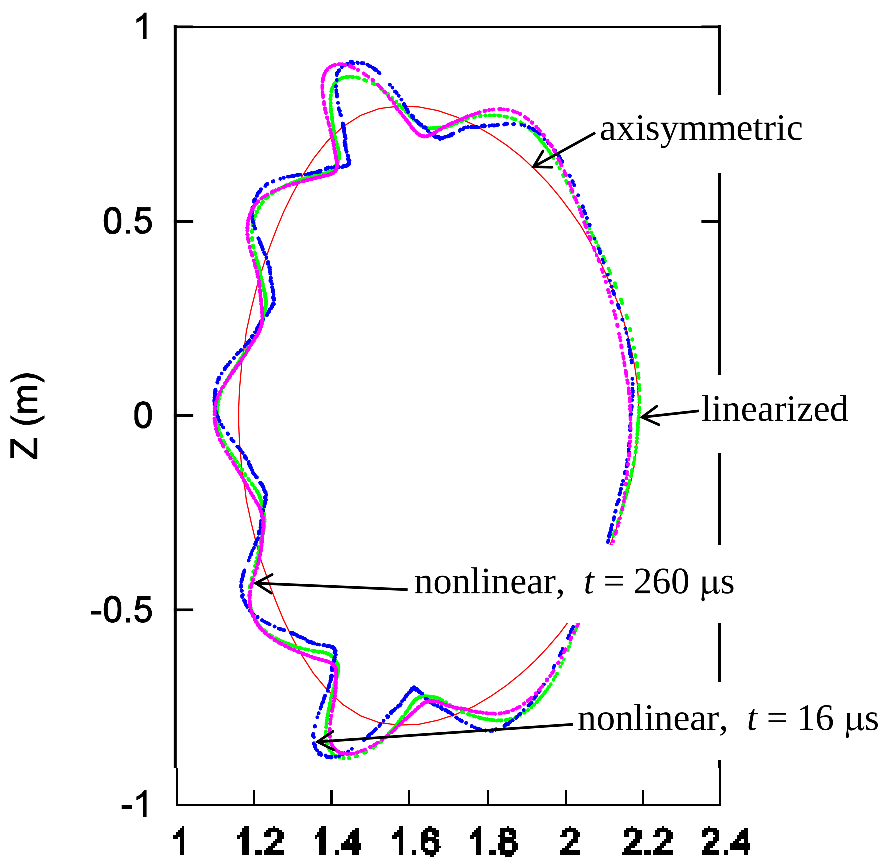
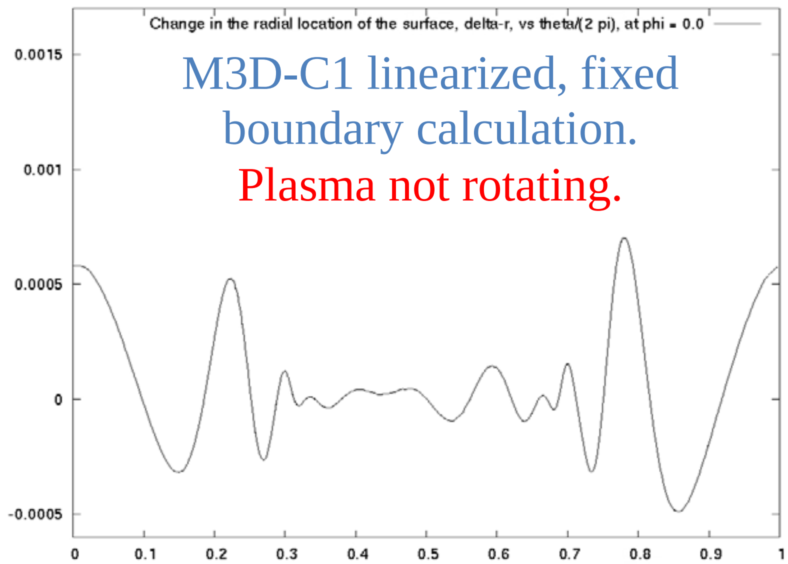
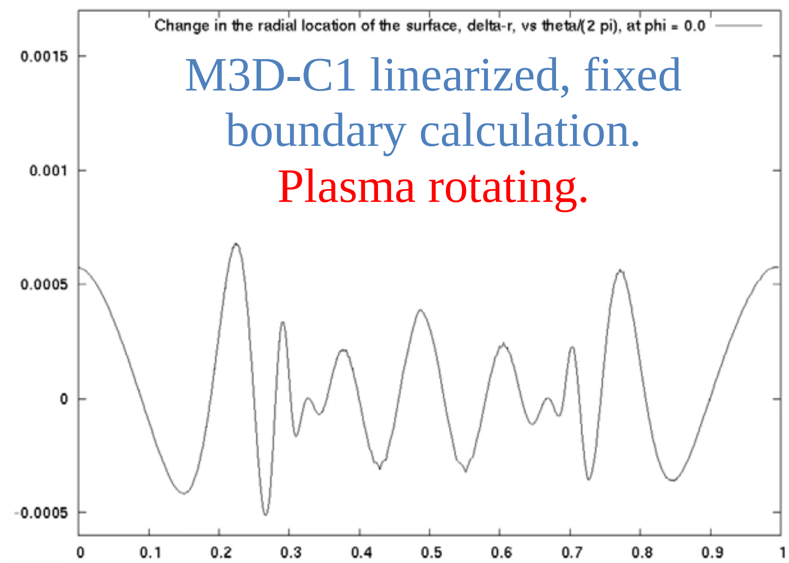


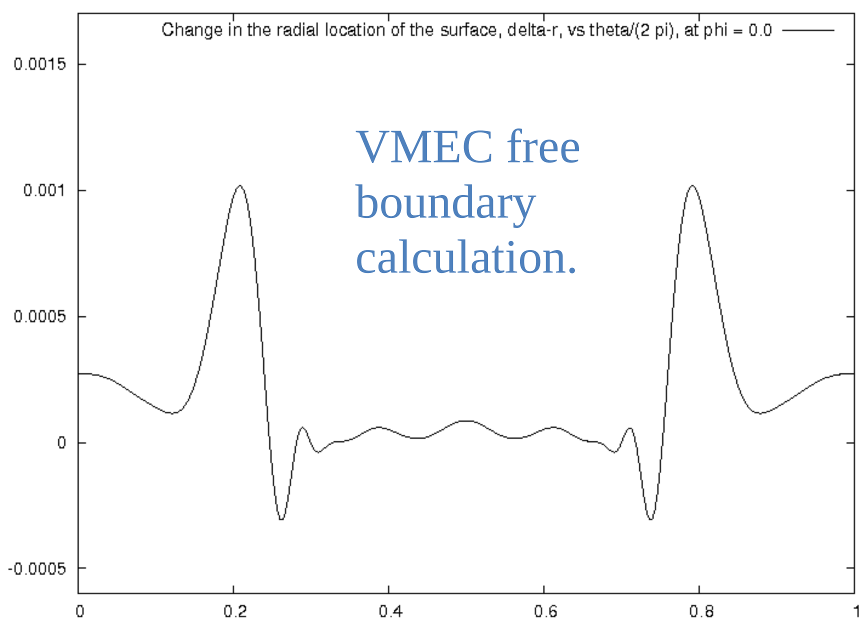
Figure shows perturbed q=2.42 flux surface with perturbation scaled up by factor of 20 to make it visible. (Ferraro)

M3D-C1 calculations find linear solution close to nonlinear solution in interior, despite crossing of flux surfaces for linear solution in region near edge.

M3D-C1 shows effect of localized current on inboard response.







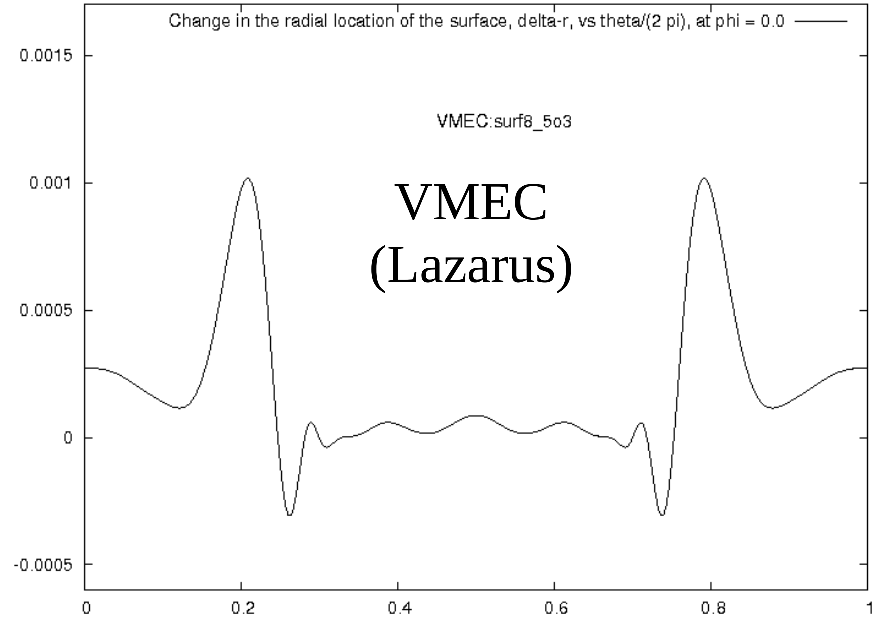
Amplitude of radial flux surface displacement vs. $\theta / (2\pi)$ (poloidal angle) for q = 8.5/3 flux surface.

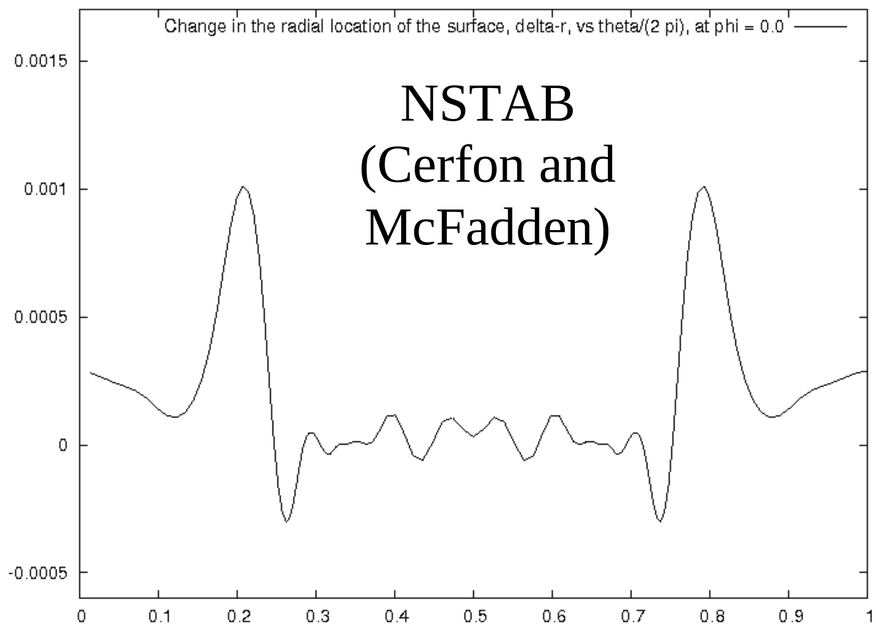
- M3D-C1 perturbative solution for nonrotating plasma gets much better agreement with VMEC than rotating.
- Genarally, screening currents die away rapidly in nonrotating plasma, but can persist in rotating plasma.

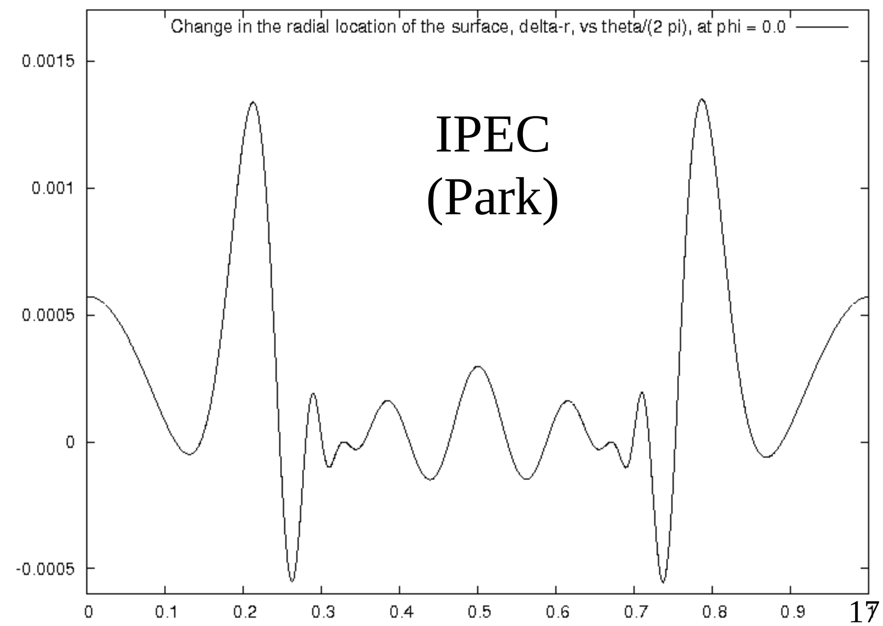
NSTAB code finds further evidence of role of localized currents near rational surfaces in contributing to perturbation on inboard side.

Using VMEC boundary, NSTAB nevertheless gets solution intermediate between VMEC and IPEC.

Perturbation amplitude (radial displacement) vs. $\theta / (2\pi)$



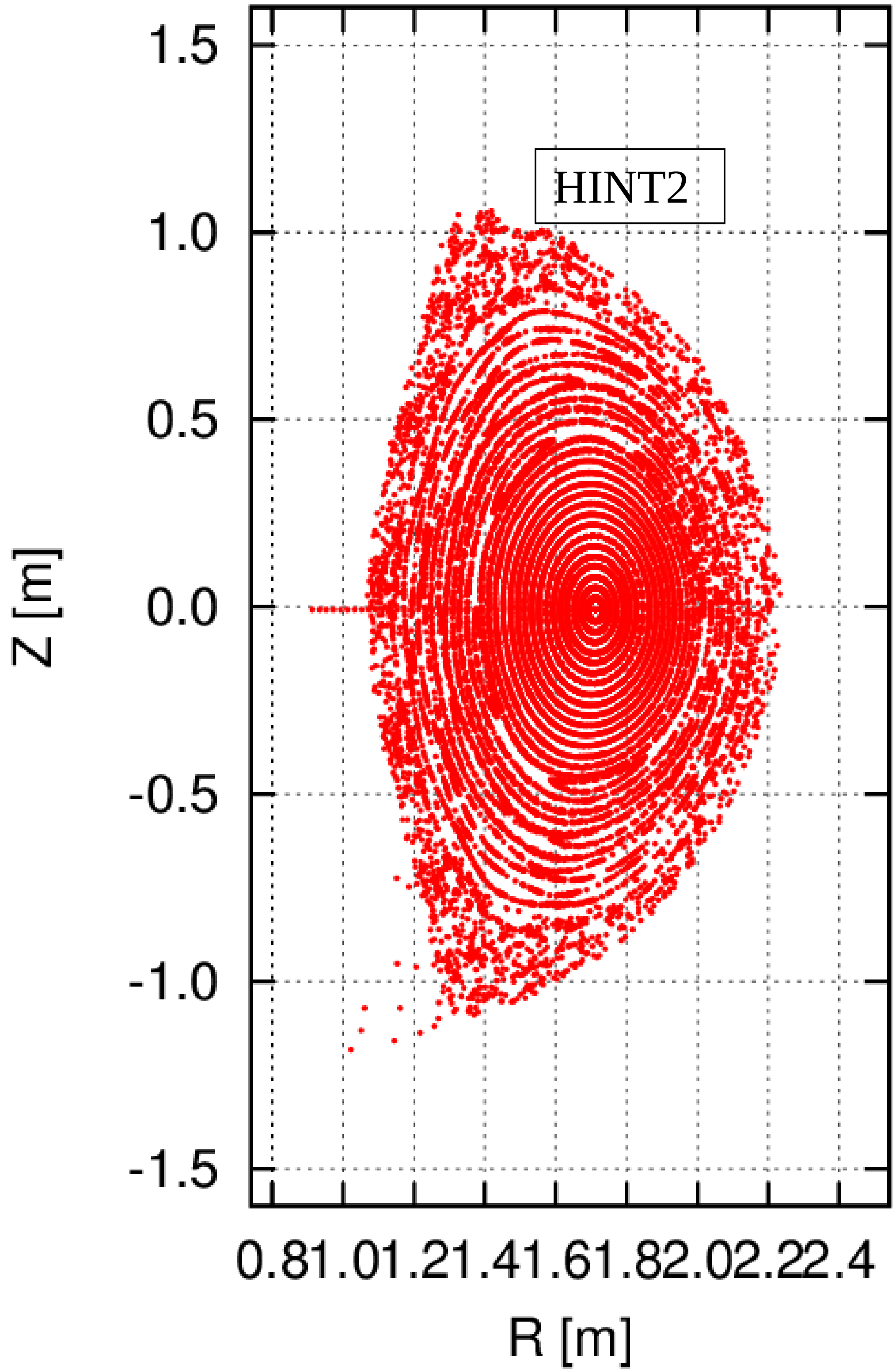




Summary of calculations with nested flux surfaces:

Evidence suggests that source of disagreement between VMEC and perturbative codes is absence of localized currents in VMEC.

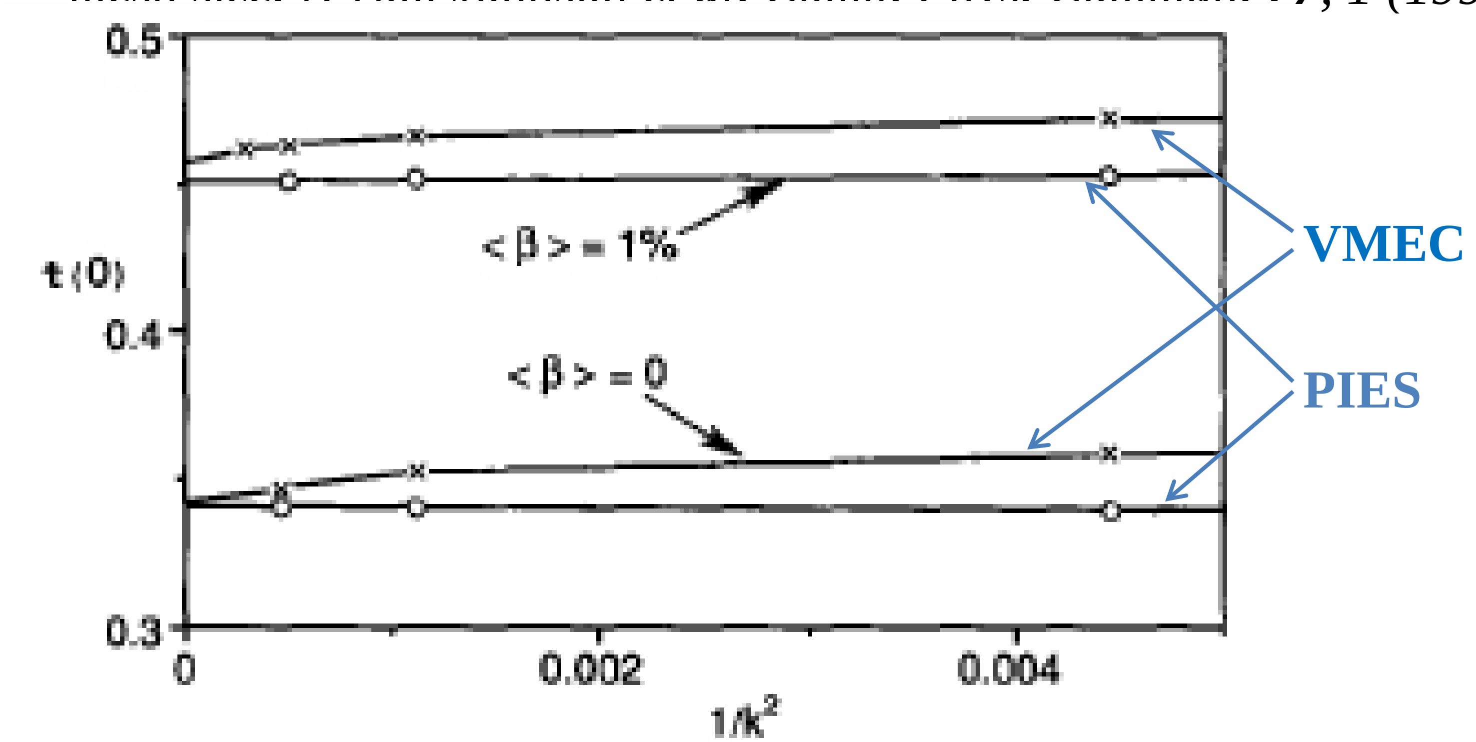
1. VMEC has difficulty with localized current in simple model equilibria.
2. Nonlinear M3D-C1 solution indicates that linear solution is valid in regions not calculated to have overlapping flux surfaces, even though this constraint is violated in substantial region near the plasma boundary.
3. M3D-C1 solution for nonrotating plasma, where screening currents should not persist, gives much better agreement with VMEC than rotating plasma.
4. NSTAB code finds further evidence of role of localized currents near rational surfaces in contributing to perturbation on inboard side:
   - assumes good surfaces, as does VMEC;
   - does nonlinear calculation, similar to VMEC;
   - believed to handle localized currents at rational surfaces more accurately than VMEC;
   - no free boundary capability. Used calculated VMEC plasma boundary.
   - **Inboard perturbation amplitude nevertheless larger than in VMEC.**
     - NSTAB solution intermediate between VMEC and IPEC.



Nonlinear calculations that do not assume good flux surfaces (HINT2, nonlinear M3D-C1 and PIES) find stochasticity in an outer region of the plasma .

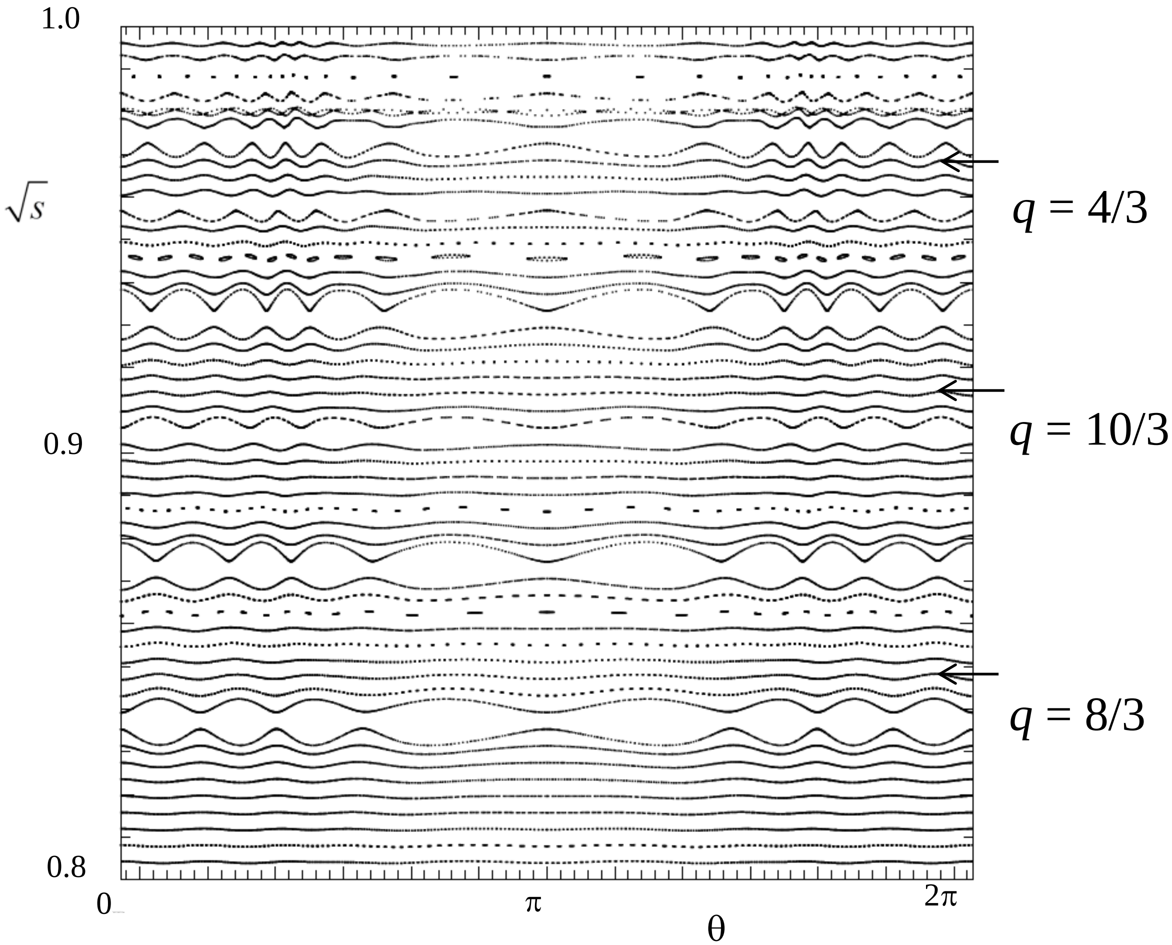
Why does PIES agree with VMEC when island widths are small?

Rotational transform at magnetic axis for ATF stellarator as function of mesh size. [From Johnson et al. Comp. Phys. Commun. ***77****, 1 (1993).]*



- Now routinely use VMEC for initial guess for PIES.
  - o In cases with small islands where solutions disagree, we verify that solutions converge to same value as resolution increased.
- Large neoclassical viscosity in stellarators → rotate more slowly than tokamaks → expect less screening.
- PIES removes screening currents to allow islands to form.

PIES calculation indicates that VMEC solution for this shot not self-consistent.

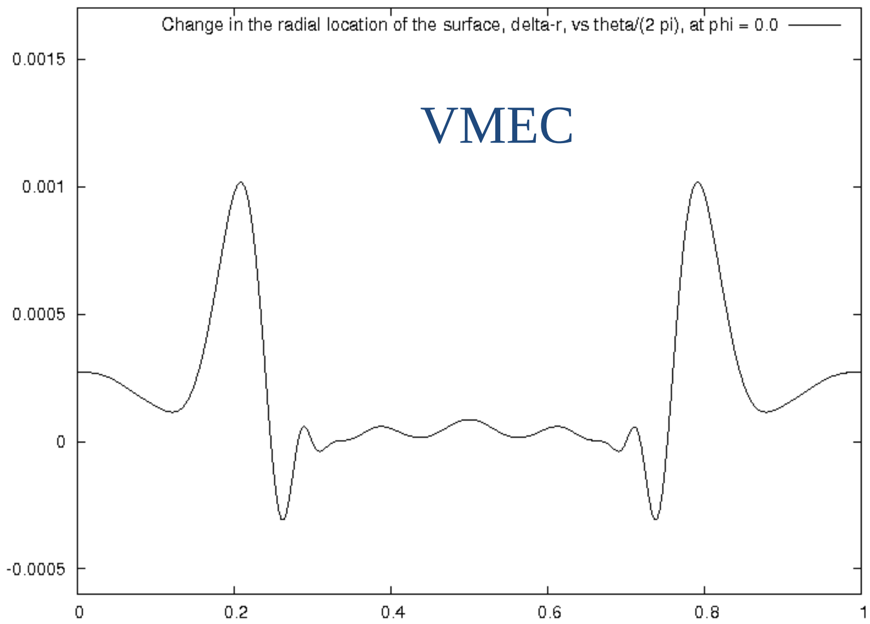


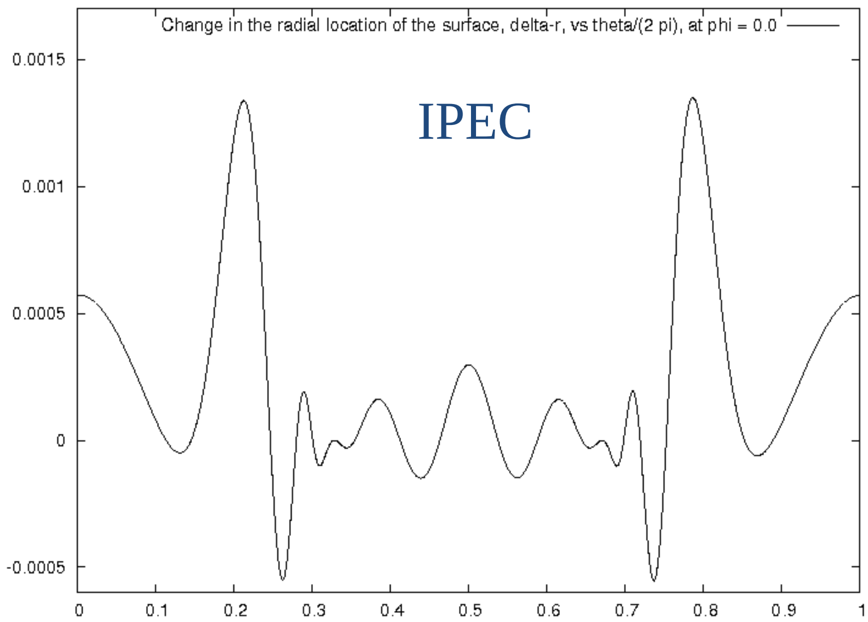
- For comparison with VMEC, fixed boundary PIES run done using VMEC boundary.
- Plot in VMEC coordinates facilitates comparison.
- Islands nevertheless have significant effect on flux surface shape.

Discussion

- Use of double-null, RMP ELM suppression experiments from DIII-D for 3D equilibrium calculations allows cross-benchmarking of full suite of stellarator and tokamak 3D equilibrium codes.
- Calculations by VMEC stellarator code and by tokamak perturbed equilibrium codes find significantly different solutions on high field side of flux surfaces.
- Evidence suggests that difference arises from absence of localized currents at rational surfaces in VMEC code.
- Calculation by a suite of codes provides:
  - verification;
  - “error bars”;
  - insight into strengths and weaknesses of each of the codes;
  - insight into the physics.
- An emerging paradigm in computational physics?
  - GEM (geomagnetic environment modeling) reconnection study.
  - Studies with multiple codes in climate modeling.

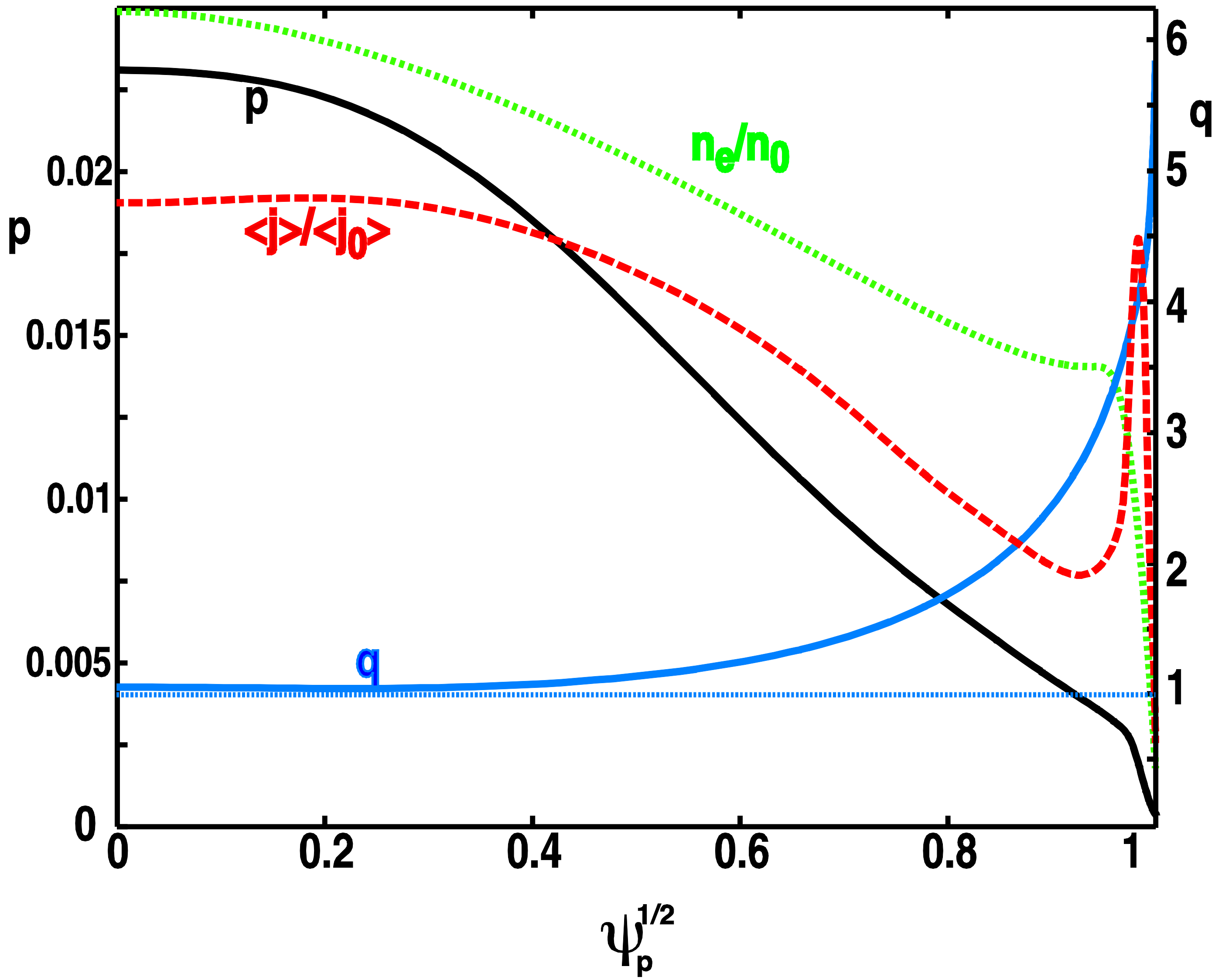
- Calculations indicate that sheet currents at low oder rational surfaces produce global effects on 3D equilibrium solution.
  - Can global differences in observed 3D equilibrium flux surface shape provide new information about localized screening currents in experiments?





Backup Slides

The reconstructed pressure, safety factor, density and current profiles for DIII-D discharge 142603 at 3519 ms. from an axisymmetric kinetic EFIT reconstruction.



The reconstructed rotation and Spitzer resistivity profiles for DIII-D discharge 142603 at 3519ms

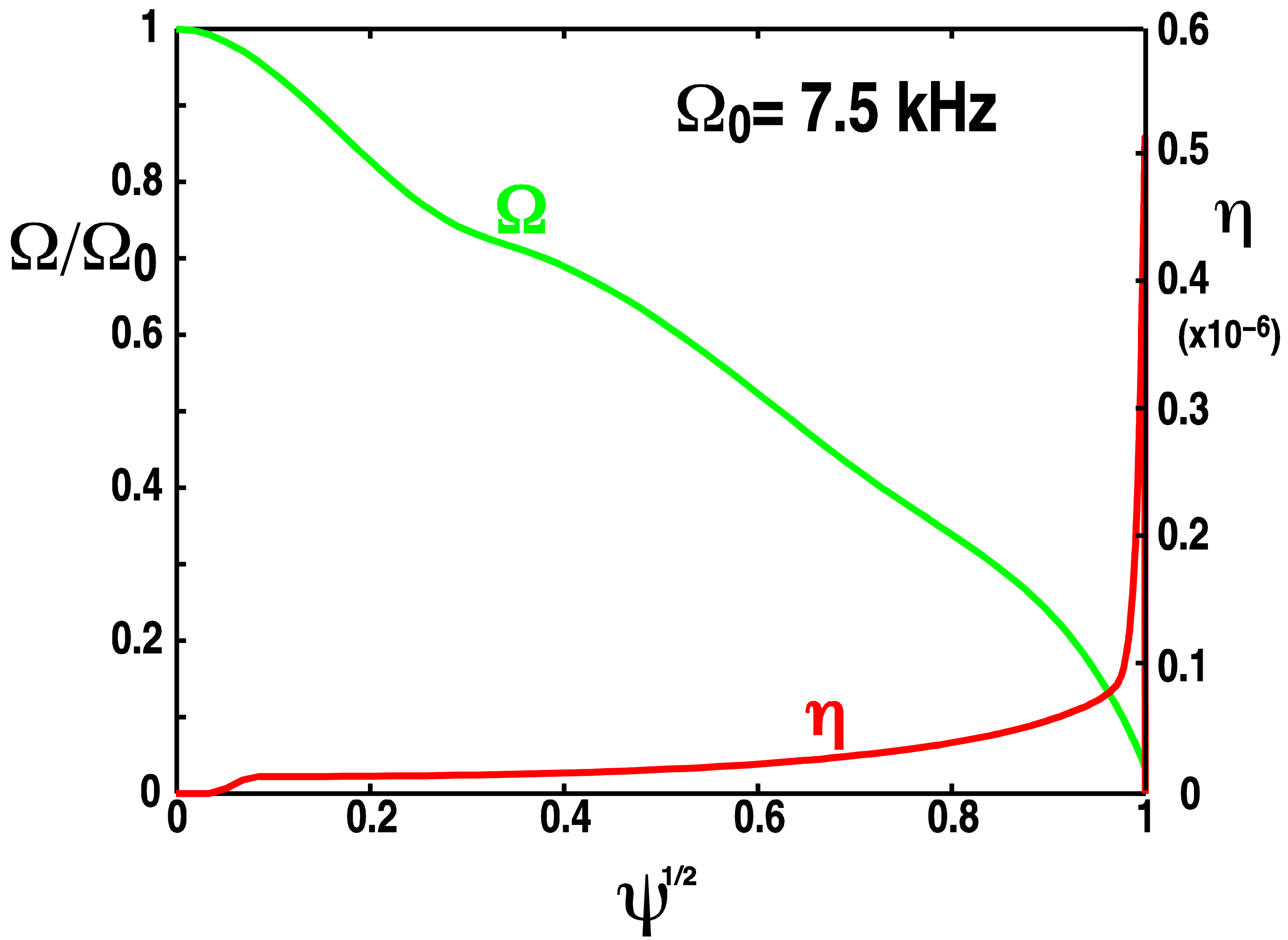


Figure 2(