Turbulent optimization of stellarators & tokamaks

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-Introduction:

-Up to now, “transport-optimized” stellarators have been designed to minimize neoclassical (nc) transport, while mitigating turbulent transport (usually the dominant channel in both stellarators and tokamaks) has been little addressed.

-However, with the advent of 2 numerical tools, viz,
-gyrokinetic (gk) codes valid for 3D nonlinear simulations such as GENE[1,2], and
-stellarator optimization codes such as STELLOPT[3],
also optimizing for turbulent transport has become a realistic possibility.

-Using these tools, we have demonstrated[4,5] that stellarators & tokamaks with appreciably reduced transport can be evolved from configurations without this optimization, raising the prospect of new classes of stellarators and tokamaks with greatly improved overall confinement.

[1] F. Jenko, W. Dorland, et al., *Phys. Plasmas* **7**, 1904 (2000).
[2]P.Xanthopoulos, W.Cooper et al., *Phys.Plasmas* **16**, 082303 (2009).
[3] A. Reiman, G. Fu, S. Hirshman, L. Ku, et al, *Plasma Phys. Control. Fusion* **41** B273 (1999).
[4]H. Mynick, N. Pomphrey, P. Xanthopoulos, *Phys. Rev. Letters* **105**, 095004 (2010).
[5]H. Mynick, N. Pomphrey, P. Xanthopoulos, *Phys. Plasmas* ***18***, 056101 (2011).

-Optimization, with STELLOPT:

-Attempts to minimize cost function $\chi^2(\mathbf{z}) = \sum_i \chi_i^2 = \sum_i w_i^2 \hat{\chi}_i^2$ (1)

In "shape space" $\mathbf{z} \equiv \{z_{j=1,..N_z}\}$

where for fixed-boundary equilibria, **z** specializes to the set of Fourier amplitudes specifying the boundary.

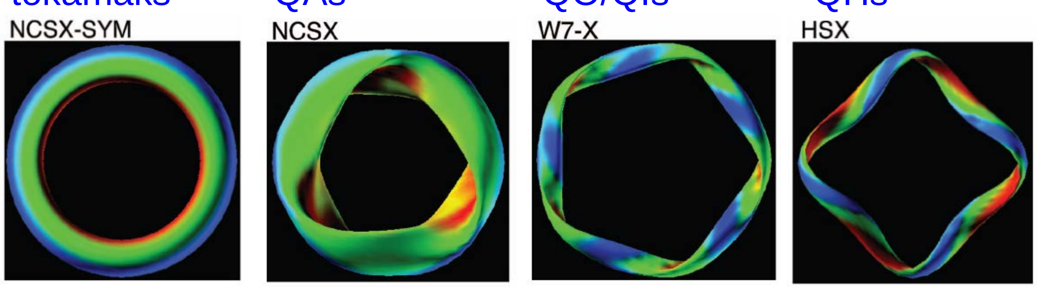
To these we have added χ^2_{txp} = turbulent transport measure.

-For a figure of merit, can take χ_{txp}= Q = radial heat flux.

-Using the averaged heat flux Q_{gk} from GENE nonlinear runs is too computationally expensive: Many 100s of equilibria examined in a typical STELLOPT run, & takes ~ 100 cpu-days for a GENE run for a single flux tube for ITG-ae turbulence, much more for ITG-ke or TEM turbulence. To overcome this, use a "proxy function" Q_{prox} to stand in for Q_{gk}.

-Q_{prox} developed from analytic theory & from GENE studies in a range of toroidal configurations[6]:

tokamaks QAs QO/QIs QHs

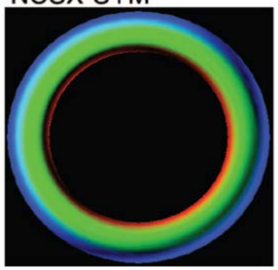


[6] H. Mynick, P.Xanthopoulos, A.Boozer, *Phys.Plasmas* **16**, 110702 (2009).

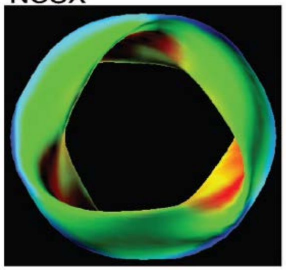
-NC-transport-optimized configurations:

	tokamaks	QAs [a,b]	QO/QIs [c,d]	QHs [a,e]
$\varepsilon_h/\varepsilon_t$:	0	<<1	~1	>>1

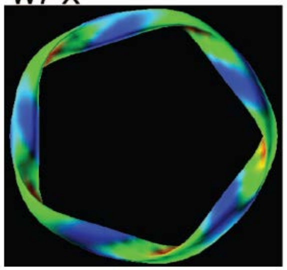
NCSX-SYM



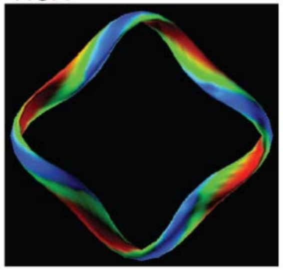
NCSX

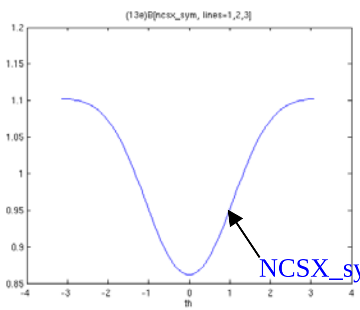


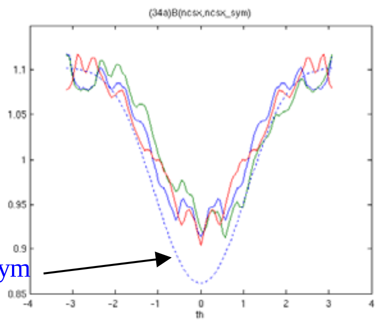
W7-X

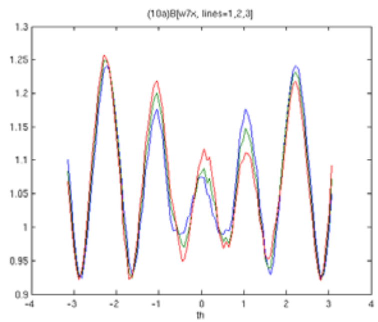


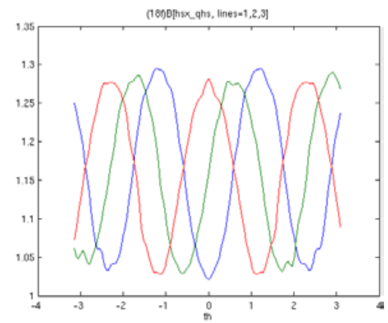
HSX











$$B(\theta \mid \alpha = cst)$$

[a] A. H. Boozer, Phys. Fluids **26,** 496 (1983).
[b]J. Nührenberg, W. Lotz, and S. Gori, in *Theory of Fusion Plasmas*, (Bologna, 1994), P. R. Garabedian, Phys. Plasmas **3,** 2483 (1996).
[c] H. E. Mynick, T. K. Chu, and A. H. Boozer, Phys. Rev. Lett. **48,** 322 (1982).
[d]J. Nührenberg, R. Zille, Phys. Lett. **114A,** 129 (1986).
[e]J. Nührenberg, R. Zille, Phys. Lett. A **129,** 113 (1988).

-Proxy function $\chi_{txp}{}^2$:

-Take $\chi_{txp} = Q_{prox}$, a proxy model for the radial ion heat flux.
For proxy-1: use simple ITG turbulent model, from the quasilinear (ql) expression

$$Q_i = -\chi n_0 g^{xx} T_i', \; \chi = \sum_{\mathbf{k}} D_k, \; D_k \equiv (\omega_{*s} L_n)^2 \langle | \frac{e\phi_{\mathbf{k}}}{T_s} |^2 \rangle \gamma_k / |\omega|^2 \simeq \gamma_k / k_\perp^2, \qquad (2)$$

with $\omega_{*s} \equiv -(ck_y T_s/eB)\kappa_n, \; \kappa_n \equiv L_n^{-1} \equiv -\partial_r \ln n_0.$

-GENE studies identify radial curvature $\kappa_1 \equiv \mathbf{e}_r \cdot \boldsymbol{\kappa} \approx a\partial_r B/B$, local shear $s_l \equiv \partial_\theta (g^{ry}/g^{rr})$ as key geometric quantities affecting turbulence.

-Use simplified expression for γ_k from ITG dispersion eqn,

$$\gamma_{\mathbf{k}} \simeq (\omega_{*i}/\kappa_n)|\tau\kappa_1(\kappa_p - \kappa_{cr})|^{1/2} H(\kappa_p - \kappa_{cr}) H(-\kappa_1). \qquad (3)$$

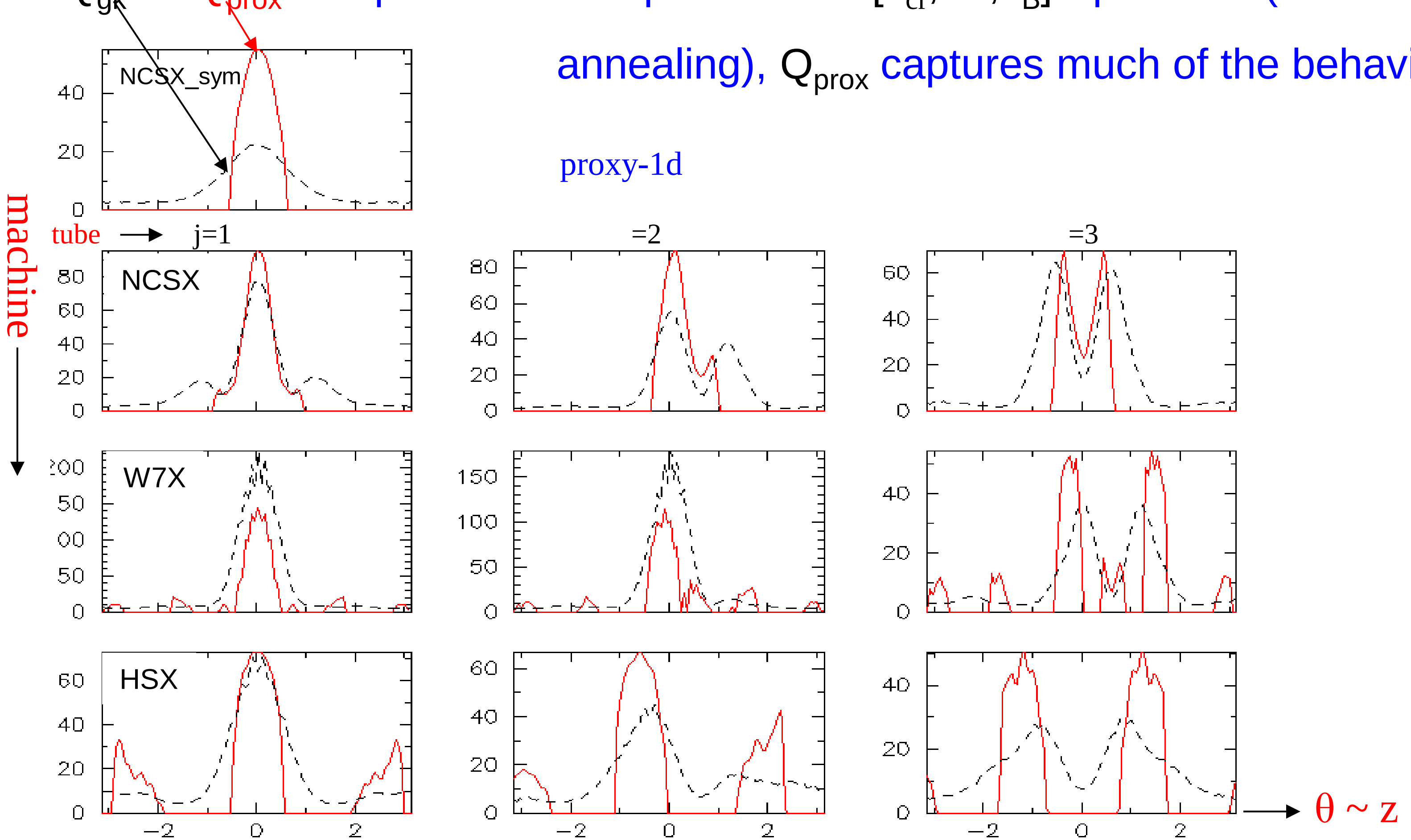
-Model $k_\perp^{-2}$ to reflect the suppressive effect of local shear s_l :

$$k_\perp^{-2}(s_l) \simeq \rho_i^2 + \rho_i L_p / [1 + \langle (\tau_s s_l)^2 \rangle_{\Delta z}], \qquad \text{with} \qquad (4)$$

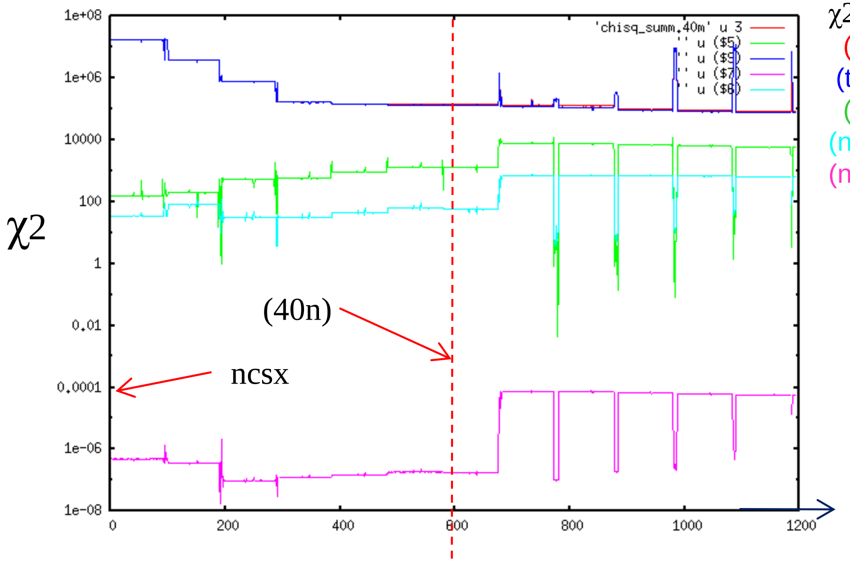
$\tau_s \equiv$ empirical constant, $\langle .. \rangle_{\Delta z} \equiv$ mode-amplitude weighted avg along field line.

-Instead of the heuristic form for $k_\perp^{-2}$ in (4), one can use the eikonal form from ballooning theory $k_\perp^2 = k_y^2\, g^{yy}$, with metric coef g^{yy}, and $y \sim \theta - \iota\varsigma$ the binormal coordinate.
-Metric coefficient $g^{xx} \sim |\nabla \psi|^2$ in (1) was omitted from proxy-1.
-From these, one may construct various proxy-1 variants, eg, proxy-1c $=\langle \gamma_k\, g^{xx} / g^{yy} \rangle$, proxy-1d $=\langle \gamma_k\, g^{xx} \rangle$, proxy-1e $=\langle \gamma_k / g^{yy} \rangle$, or proxy-1f $=\langle \gamma_k \rangle / \langle g^{yy} \rangle$.
-All are plausible, but heuristic, selected by which agrees better with the gk results.

- Q_{gk} vs Q_{prox} comparison: With parameters [κ_{cr}, Δz, c_B] optimized (*via* simulated annealing), Q_{prox} captures much of the behavior of Q_{gk}:

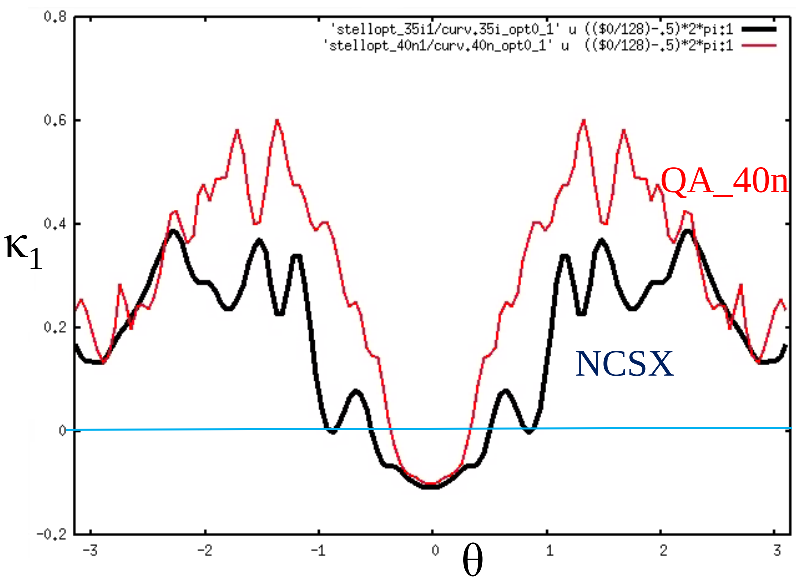


-STELLOPT runs: Start with NCSX (LI383) configuration, β=4.2%.

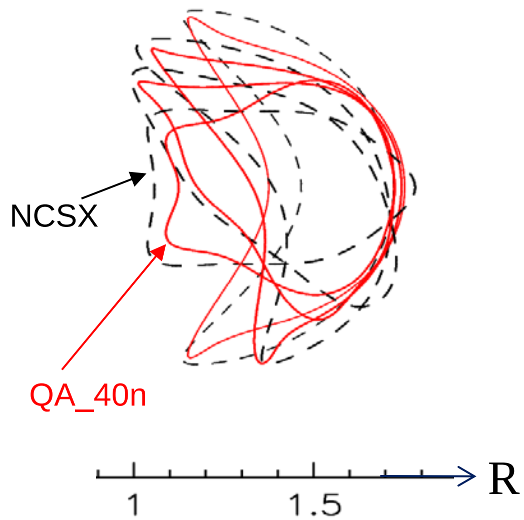


χ2 from ( ) :
(total)
(turbulent transport)
(beta constraint)
(nonaxisymmetry measure)
(neoclassical transport)

-New configurations improve χ by boosting $\kappa_1 \approx a\partial_r B/B$ in region of worst curvature .

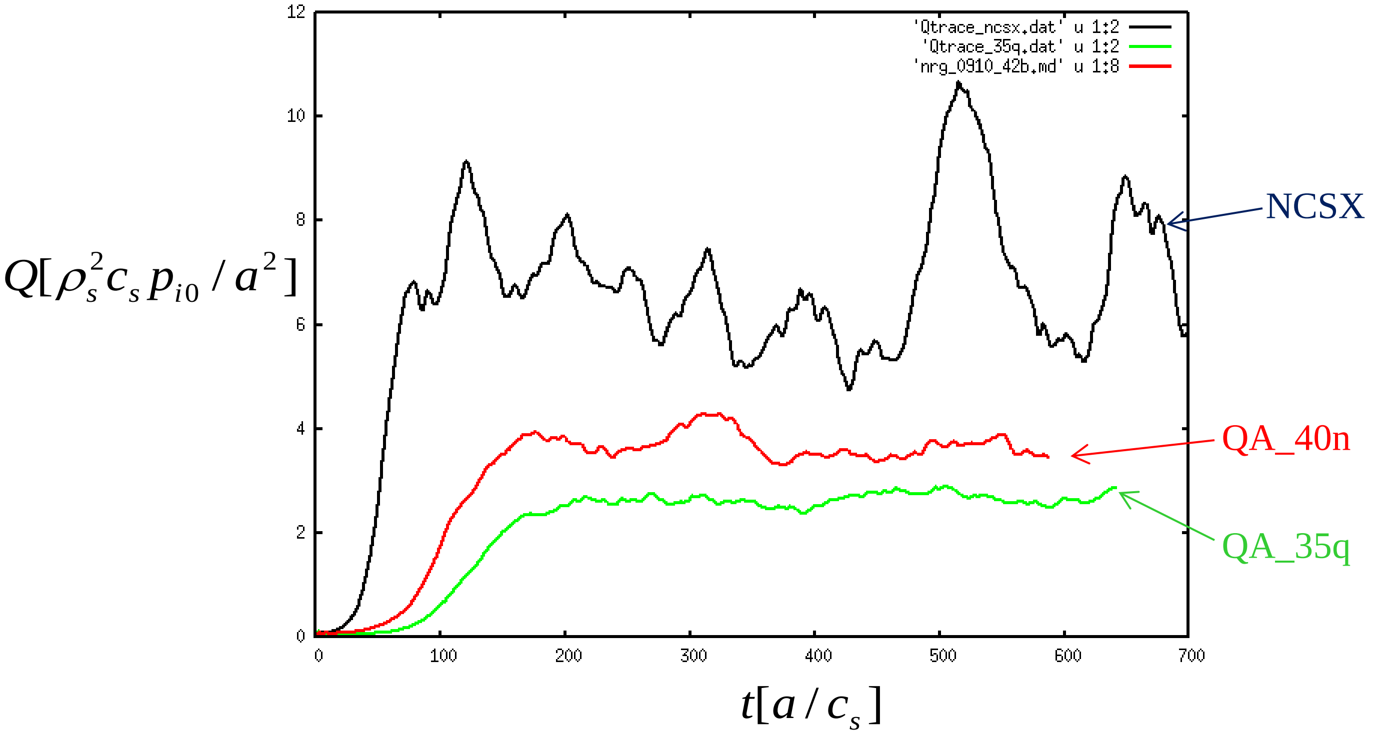


Compare poloidal cross-sections with those of NCSX:



-GENE confirms transport reduced, by factor ~ 2-2.5.

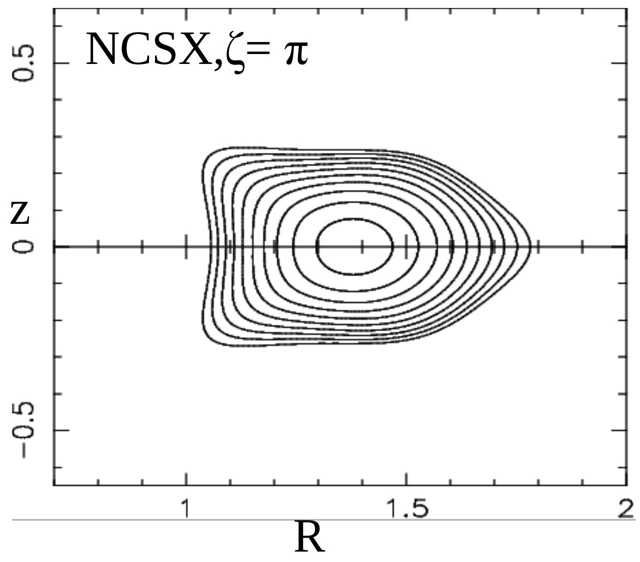
Compare $Q_{gk}(t)$ for QA_35q, QA_40n vs NCSX:

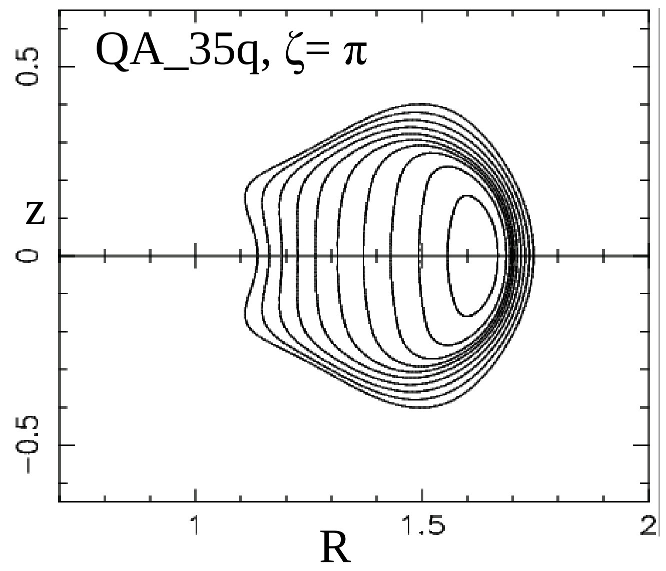


-How does STELLOPT deform NCSX to boost κ_1 ?

-A stellarator with poloidal symmetry number ℓ has rotational transform $\iota[(r/a)] \sim (r/a)^{2|\ell-2|}$ In 1965, Taylor[8] noted that applying a vertical field B_v to a stellarator with ℓ >2 would cause larger shift $\Delta \sim B_v/\iota$ for inner than for outer flux surfaces, resulting in magnetic 'well', $V'' < 0$.]

-Configurations 35q, 40n conform to this picture, NCSX does not:



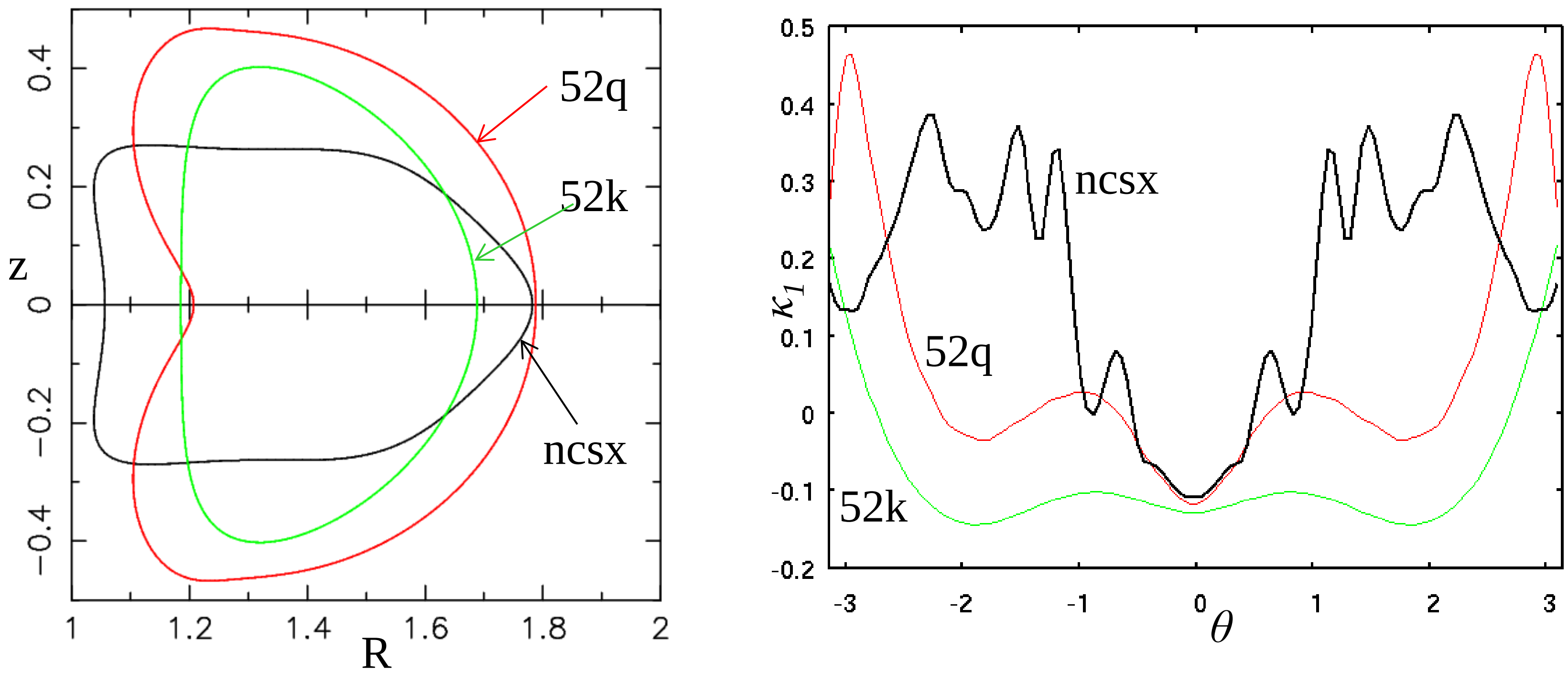


[8] J.B. Taylor, *Phys. Fluids* **8** 1203 (1965).

-(2): Evolving tokamaks:

-Start from TOK_52k: Shape close to axisymmetrized NCSX, tokamak-like ι(r) profile. Take *n=0* perturbations only.

Evolved config TOK_52q, has inboard indentation.
(This also known[9] to stabilize interchange/ballooning-type modes.)

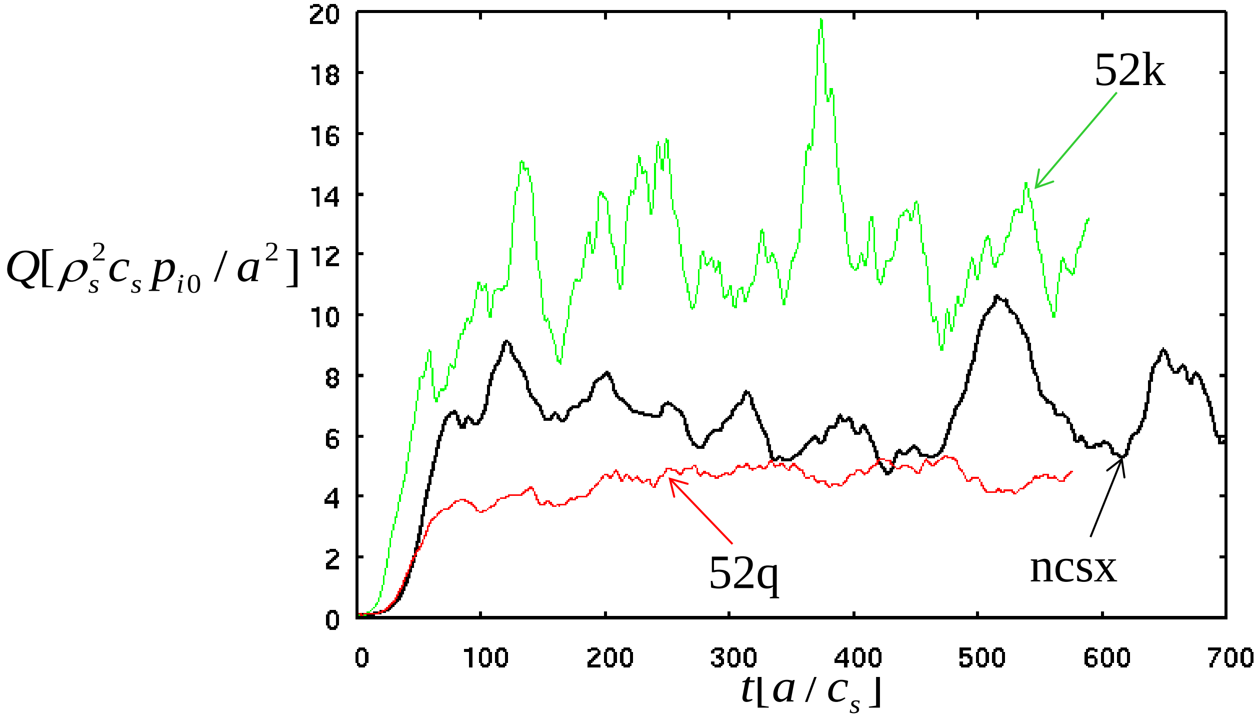


[9] M.S. Chance, S.C. Jardin, T.H. Stix, *Phys. Rev. Letters* **51** 1963 (1983).

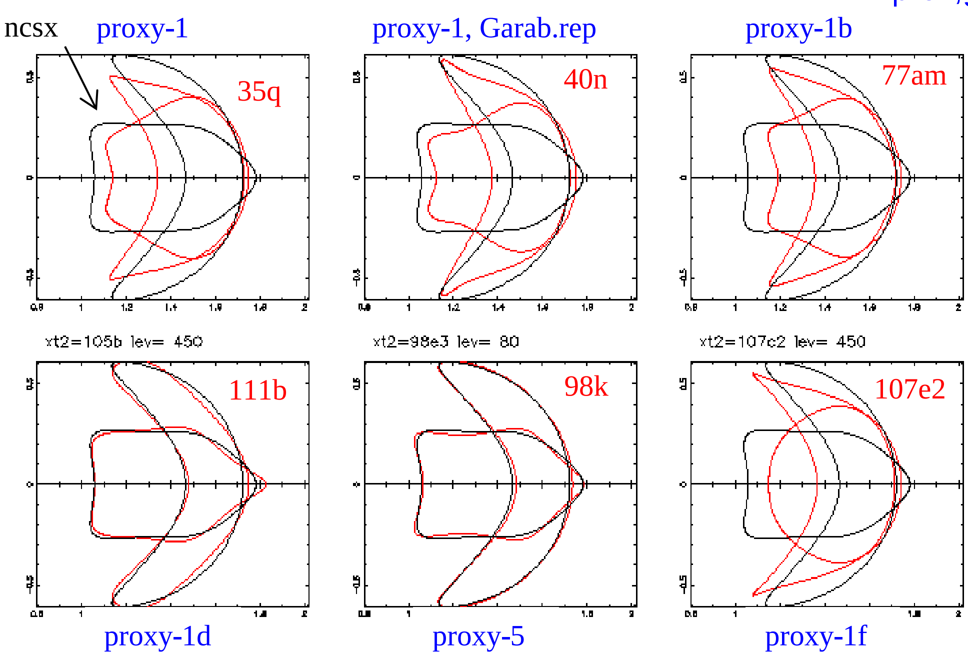
-GENE corroboration:

-Compare $Q_{gk}(t)$ for NCSX, TOK_52k, TOK_52q:

$Q_{gk}(t)$ for TOK_52k reduced by factor 3 for TOK_52q:



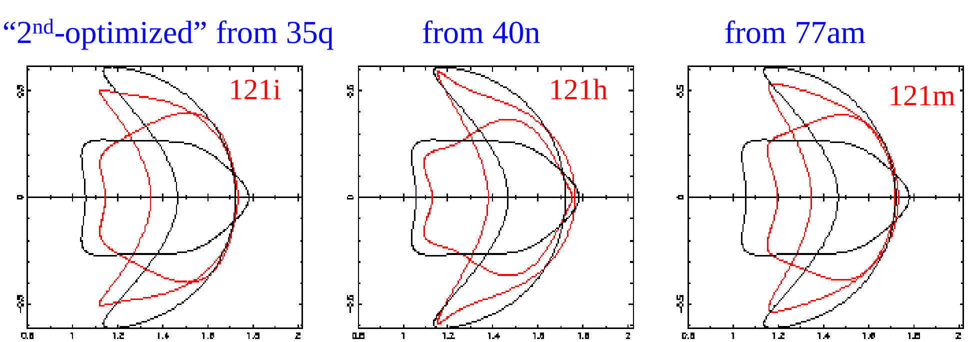
(3)Using these proxies in STELLOPT has produced a growing set of configurations with appreciably reduced $Q_{prox,gk}$. The QAs are the most studied:

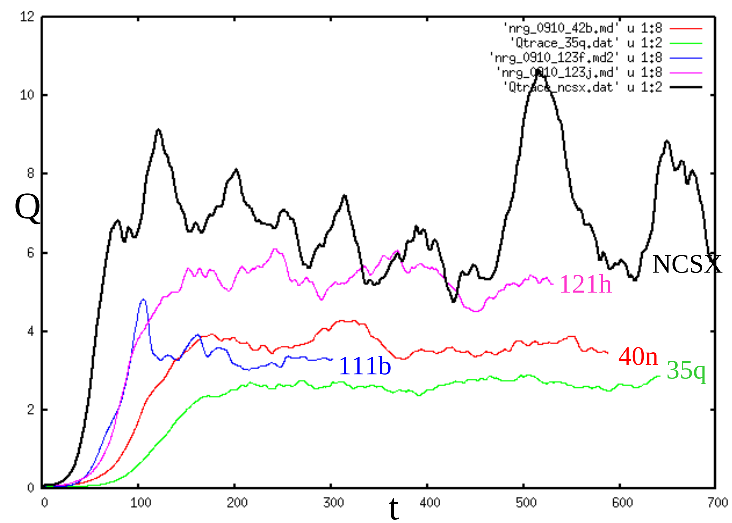


-Note these fall in 3 classes, characterized by poloidal cross-section at $\zeta = \pi$ symmetry plane (compared with "bullet" shape of NCSX):
"breadslice" class: [35q,40n,77am]
"bottle" class: [111b,98k]
"elliptical" class: [107e2].

-Best so far have Q_{gk} reduced from that of NCSX by factor of 2-2.5, some with little or no degradation of nc transport or stability.

-2nd-round of optimization, to improve kink,ballooning characteristics, without loss of good nc & turbulent transport, while maintaining same aspect ratio, beta, R*Btor :

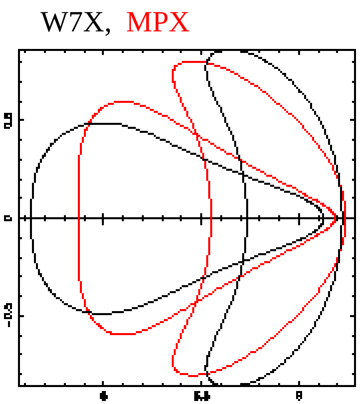


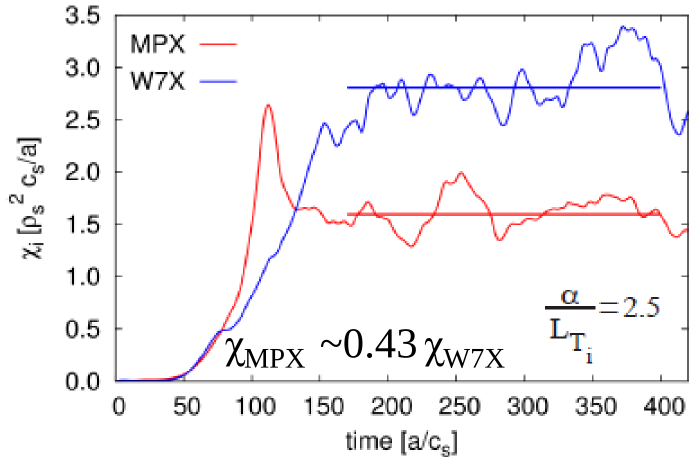


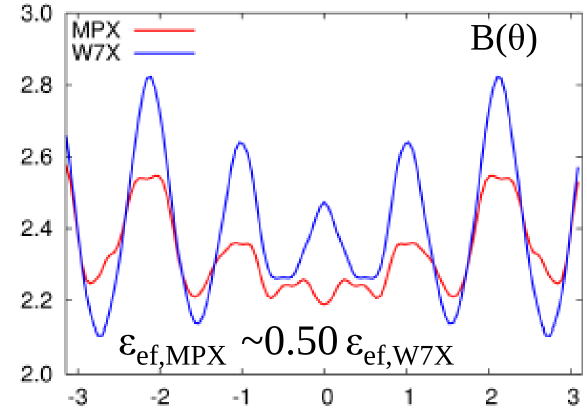
-So far, 111b,121h meet all criteria, including GENE corroboration:

(4)Reduced-ITG configurations have also been obtained for QOs (from W7X):

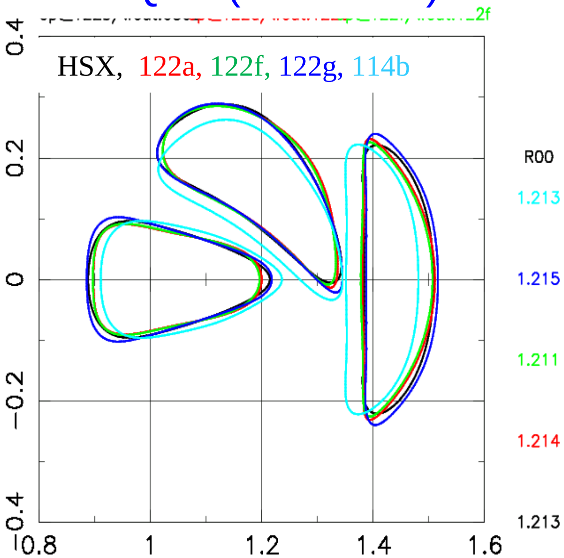
(Very recent MPX results[10], using $Q_{prox} = < \kappa_1 g^{xx}$ or $\kappa_1 (g^{xx})^2 >$ & DE global search algorithm[11]):

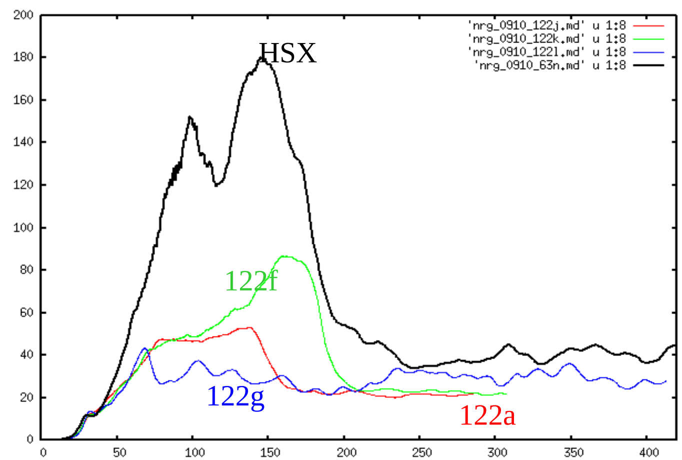






& for QHs (from HSX):





NB. Simulations for ITG w adiabatic electrons, for tube-1 only. HSX operates with quite different parameters, T_e>>Ti for which TEMs should dominate.

[10] Xanthopoulos, Mynick, Helander, et al, *Phys. Rev. Lett.* **113**, 155001 (2014).
[11] Mynick, Pomphrey, Ethier, *Phys. Plasmas* **9**, 869 (2002).

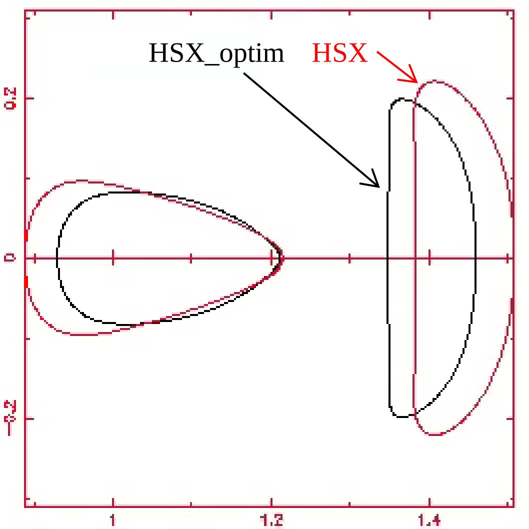
(5) TEM optimization:

-An analogous approach may be applied to TEMs. From recent work[12,13] by Proll, Helander, et al on stability of drift-type modes, the power P_e from the mode to electrons may be written

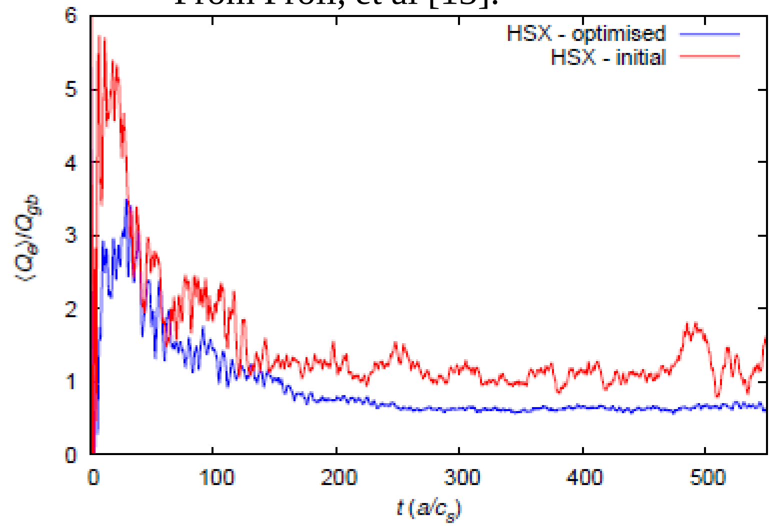
$$P_e = \frac{\pi e^2}{T_e} \int \frac{dl}{B} \int d\mathbf{v} \delta(\omega - \bar{\omega_{de}}) \bar{\omega_{de}} (\bar{\omega_{de}} - \omega_{*e}^T) |J_o \bar{\phi}|^2 f_{e0}$$

-For instability, need this $<$ 0, for which need $\langle \overline{\omega_{de}} \omega_{*e}^T \rangle > 0$

Using this as the TEM proxy fn, & optimizing with STELLOPT from HSX:



From Proll, et al [13]:

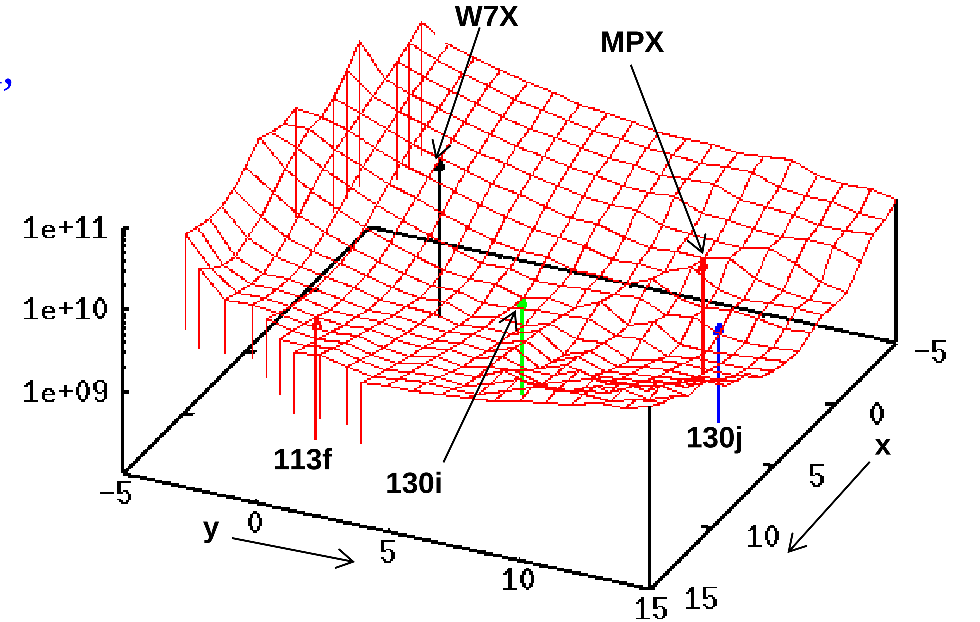


>TEM optimization tends to push in same dir'n in **z**-space as ITG optimization.

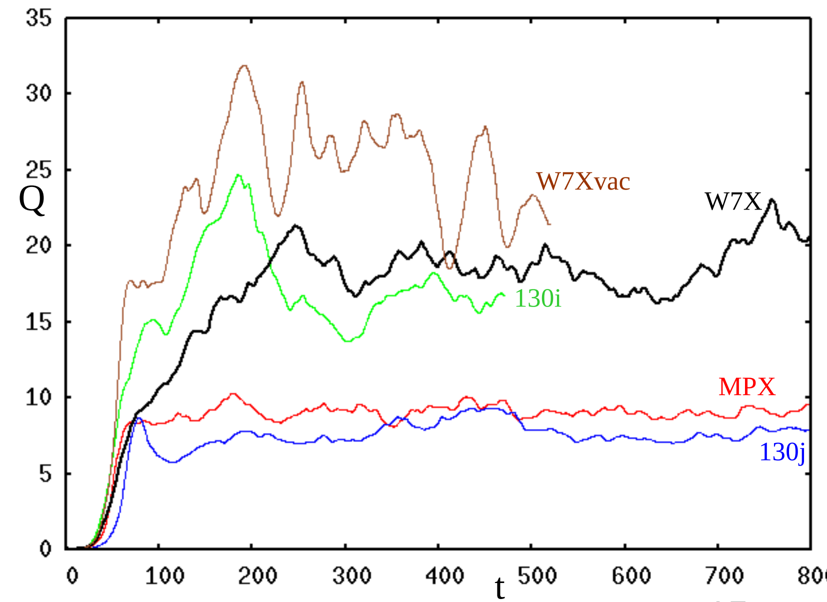
[12] Proll, Helander, Connor, Plunk, *Phys. Rev. Letters* **108**, 245002 (2102).
[13] Proll, Faber, Helander, Lazerson, Mynick, Xanthopoulos, Sherwood Theory Meeting (New York, March 16-18, 2015).

(6) STELLOPT in mapping mode can provide insight, aid in optimization:

-Map over QO z-space plane defined by W7X, MPX, and 113f indicates new “basin”, with configuration 130j with even lower transport than MPX.[14]



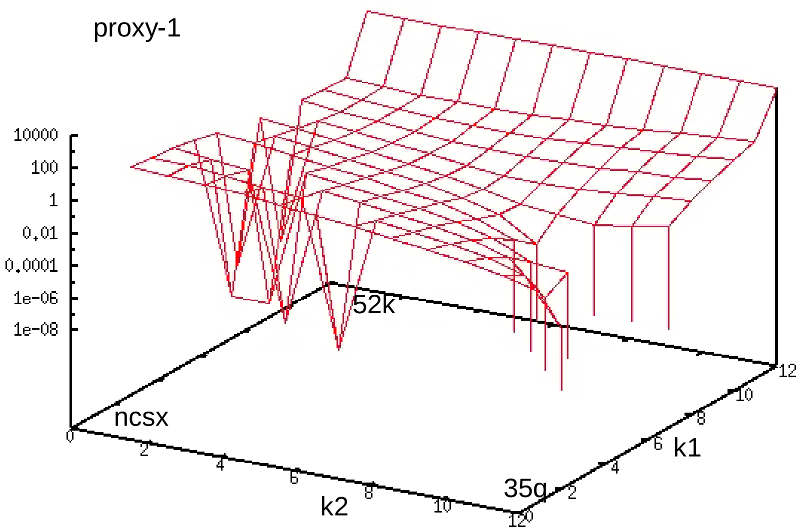
-Nonlinear GENE runs corroborate this:

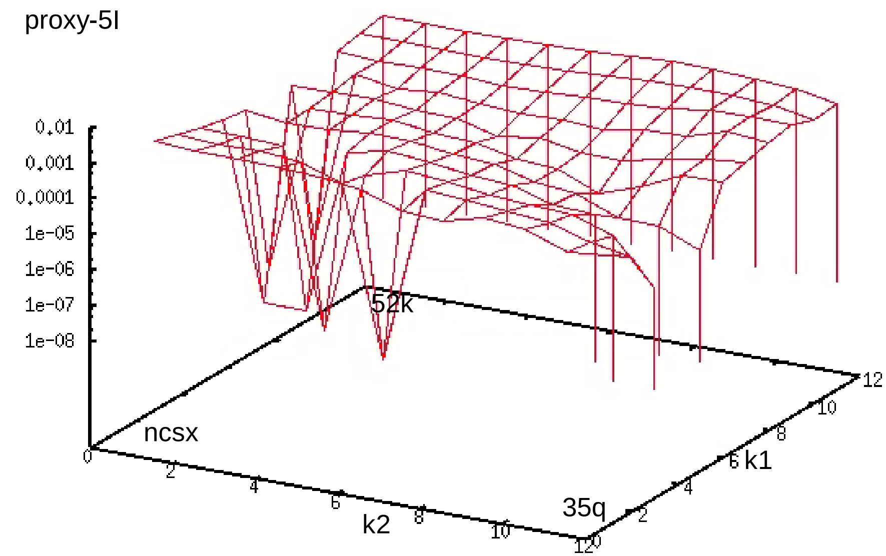


[14] Mynick, Xanthopoulos, Faber, Lucia, Rorvig, Talmadge, *Plasma Phys. Control. Fusion* **56** (2014) 094001.

(7) Proxy extension: GENE-in-STELLOPT:

-If Q_{prox} were computed using actual gk results, it would enhance the accuracy of the proxy and allow assessment of any modes and further physics without further code modification. As noted, nonlinear gk runs are computationally too costly, but since the ql $Q_{prox} \sim \gamma$, and since linear gk calculations are much faster, using GENE to compute γ is becoming feasible. We have extended STELLOPT to use GENE's eigenvalue solver to permit this > proxy-5:





-Summary:

-Using the gk code GENE and optimization code STELLOPT, we have demonstrated that stellarator and tokamak designs can be evolved which have turbulent transport levels well below (factors of 2-3) the starting configurations without this optimization, often without significant degradation of the nc transport.

-Brute force operation by STELLOPT is able to find design approaches sometimes earlier discovered only by difficult analytic work, & sometimes new ones.

-Find TEM optimization tends to push in same dir'n in **z**-space as ITG optimization, because of related physics.

-STELLOPT in mapping mode can provide insight into these approaches, as well as aiding in optimization.

-Developing a reliable proxy fn Q_{prox} serves as a testbed for better turbulence theories, and to clarify which geometric "knobs" are effective in controlling turbulent transport:
$[\kappa_{1}, g^{xx}, \Lambda = g^{xy}/B \sim \int dz\, s_l]$

-Future work:

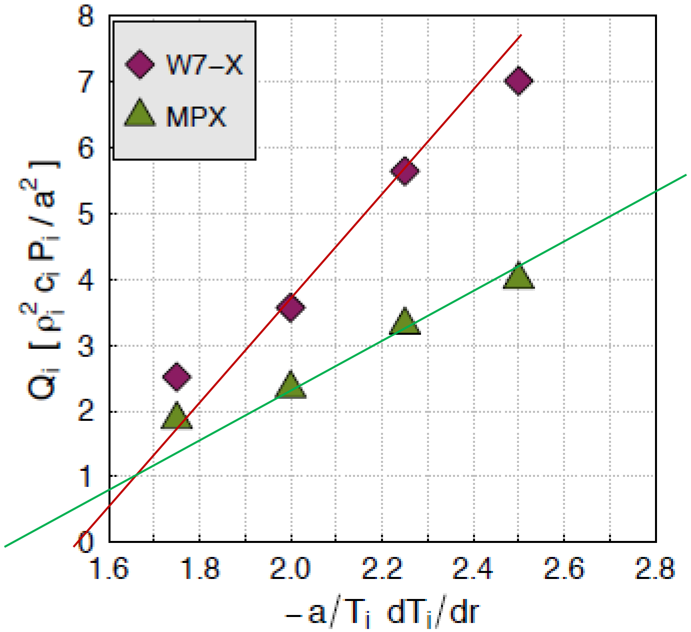
-further improve Q_{prox} .

-adapt STELLOPT to optimize for larger κ_{cr} rather than lower Q at fixed κ.

-better understand the number and effect of independent geometric “knobs” which affect turbulence.

-refine turbulent-optimized configurations to meet criteria for practical designs, &

-find experimental tests of these ideas on current or nearer-term devices.



From [10]: Xanthopoulos, Mynick, Helander, et al, *Phys. Rev. Lett.* **113**, 155001 (2014).