Spline Representations for More Efficient Stellarator Coil Design

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Acknowledgments:
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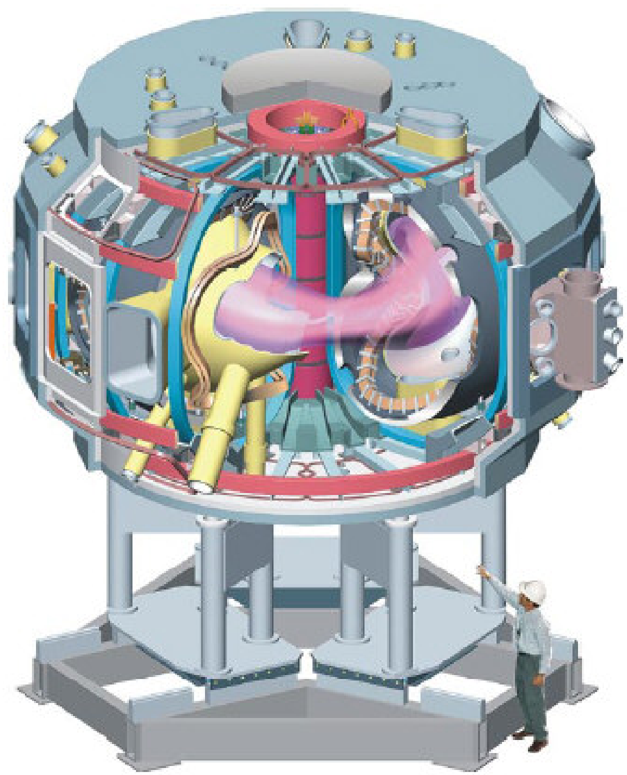
PPPL
PRINCETON PLASMA PHYSICS LABORATORY

Outline

- Motivation
- COILOPT++ features
- Sample applications
  - Straight coils for ARIES-CS: proof-of-concept
  - Retrofitting QUASAR to reduce turbulent transport (work in progress)
- Conclusions & future work

Stellarator Advantages as Fusion Reactors

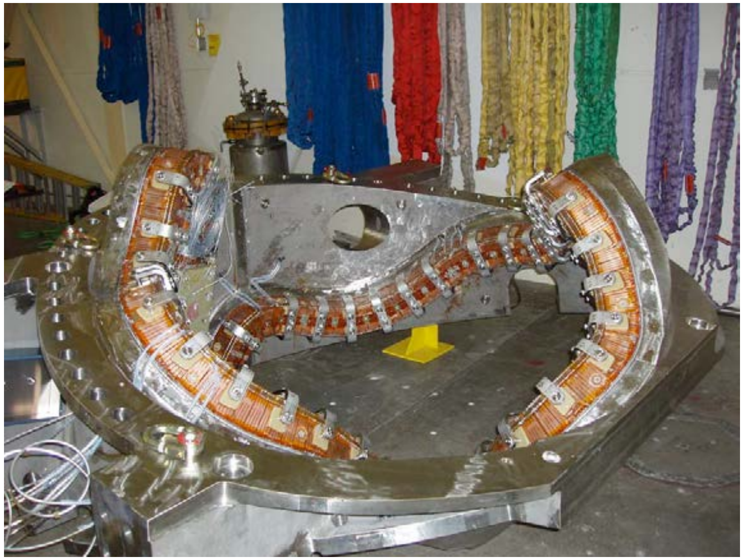
- Unlike tokamaks, stellarator plasmas do not carry significant current.
  - No disruptions
  - Low re-circulating power
- No vertical feedback control required.
- Larger design space allows more optimization.
- Neoclassical transport can be reduced with quasi-axisymmetry (*e.g*. NCSX) or quasi-omnigeneity (*e.g*. W7X).
- Turbulent transport can be reduced with numerical optimization.



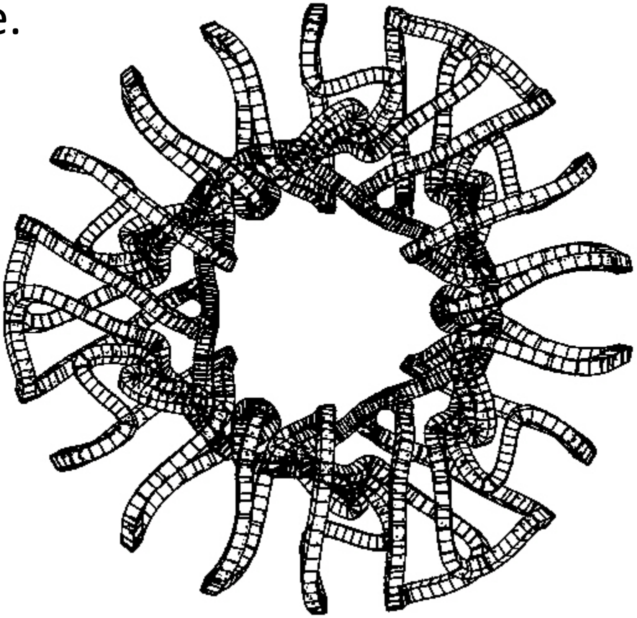
NCSX

Coil Complexity Imposes Challenges

- Numerical optimization yields complex 3D coils.
  - Fabrication and assembly are expensive.



Completed NCSX modular coil

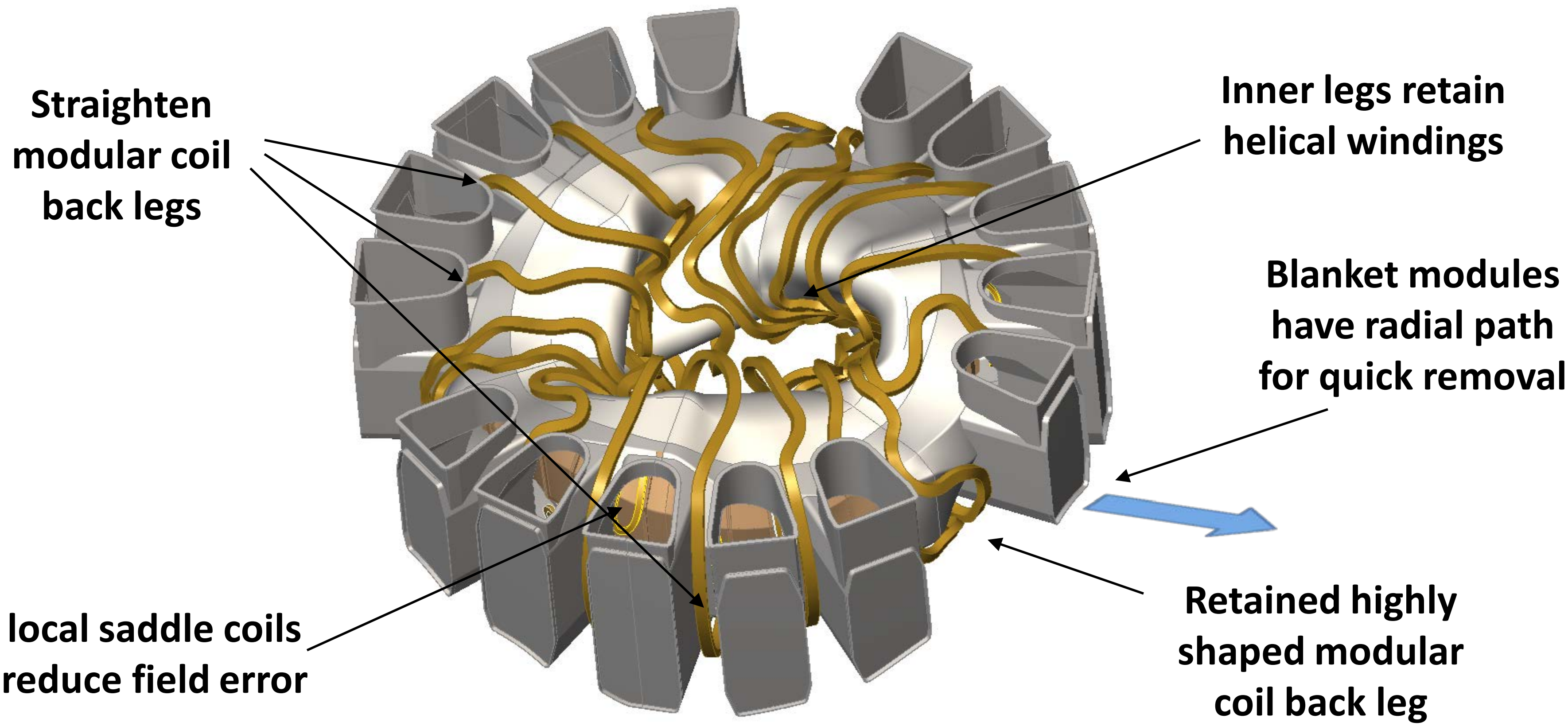


Schematic of final coil assembly

- Interlocked windings impede access for routine maintenance.
  - Maintainability is critical for a reactor

Simplifying coils makes stellarators a more attractive option

- Toroidal blanket and PFC sections should slide out radially.
- One solution: modular coils with straight outer legs.



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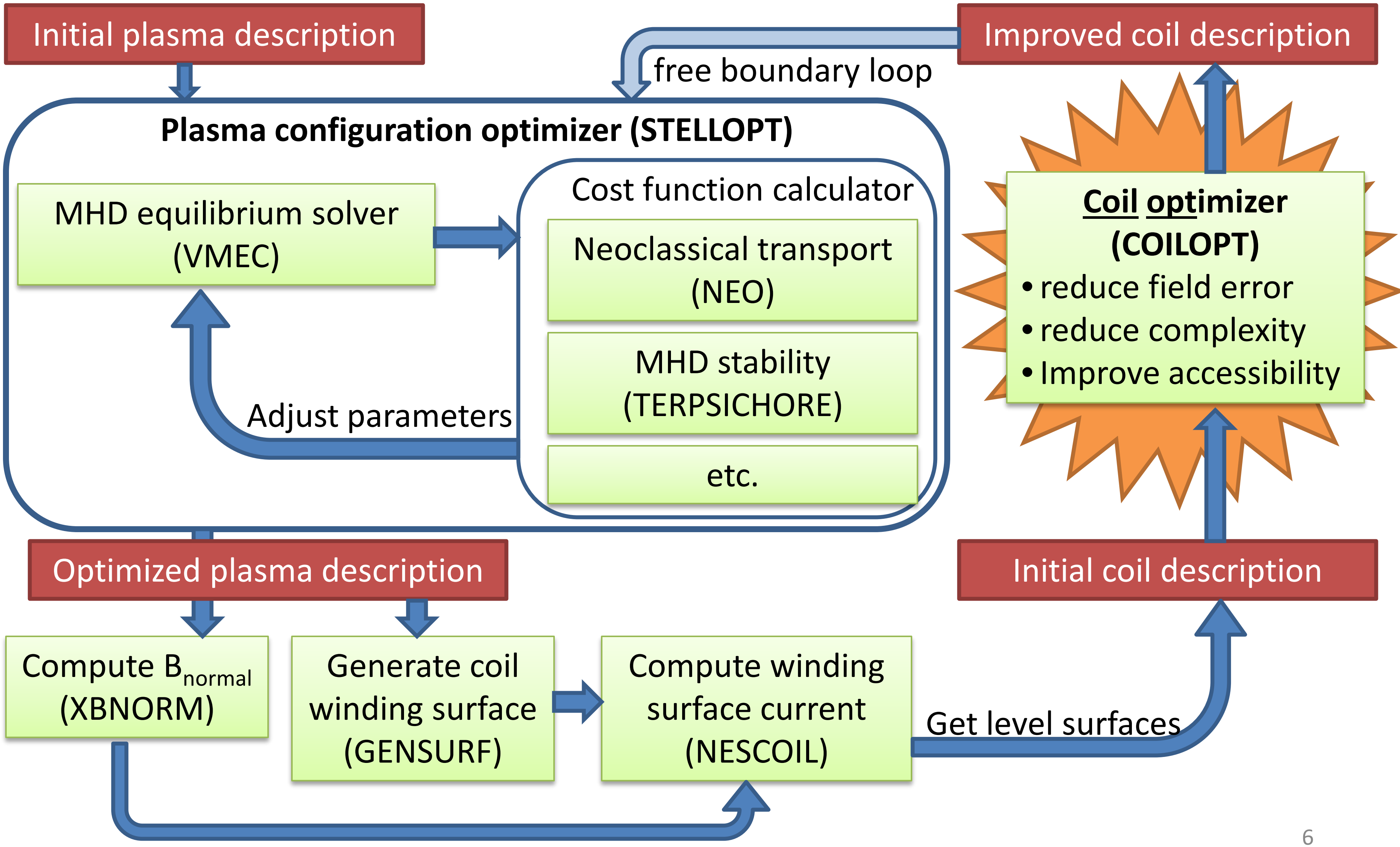
Coil Component of Stellarator Design



Details of the original COILOPT code[1]

- Input: Fourier series describing plasma (control) surface, plasma B_{norm}, winding surface, initial coils

- Primary objective function: cancellation of plasma + coil B_{norm} at control surface

- Output: improved coils, coil vacuum field

- Configuration space: Fourier coefficients for $u=\theta/2\pi$ and $v=N\phi/2\pi$ for each unique coil in the winding surface.
  - Modifying complex local variations is difficult.
  - Introducing a locally straight coil section is extremely inefficient.

[1]D.J. Strickler, L.A. Berry, and S.P. Hirshman, *Fusion Sci. and Technol*. **41**, 107 (2002).

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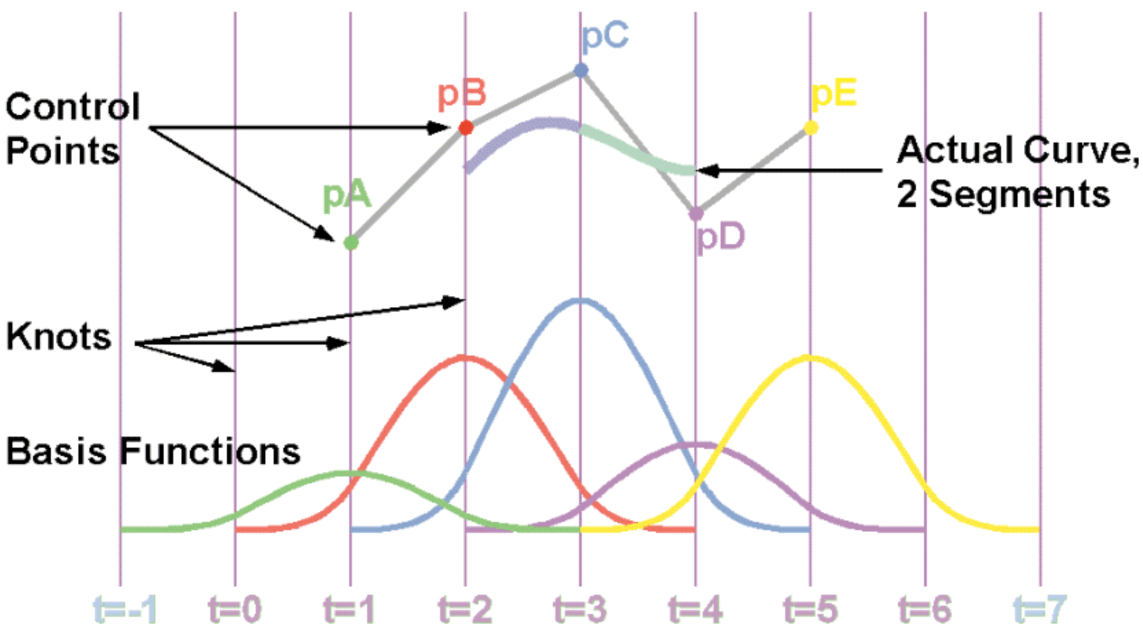
Cubic B-Splines are better for coil optimization

- Splines are piecewise polynomials with arbitrarily high continuity at selected “knots”.

- B(asis)-splines have minimal support for a given polynomial degree. Cubic B-splines maintain 2^{nd}-order continuity with a support of four.



  - Dense packing of knots allows efficient representation of local features.
  - Spline series for coils can be smoothly matched on to straight (constant v) sections.

New Tool: COILOPT++

- Goal: Make COILOPT more object-oriented, flexible.
  - Allow arbitrary mixing of Fourier, spline, and spline+straight section representations of coils on multiple winding surfaces.
  - Allow greater experimentation with different global and local optimization algorithms without modifying other parts of the code.
  - Optimizers are all fully parallelized using MPI.
  - Take advantage of the local nature of the spline representation to accelerate computation of the cost function.

- The results shown in this presentation are preliminary exercises of the new COILOPT++ component of the stellarator design workflow.

Sample Components of the COILOPT++ Cost Vector

Field Error:

$$f_{i,j} = w_{bnorm} \Im_{i,j} \frac{B_{norm,plas}(u_i, v_j) + B_{norm,coils}(u_i, v_j)}{\left|\mathbf{B}_{coils}(u_i, v_j)\right|}$$

$$f_{b\max} = w_{b\max} \max_{i,j} \left| \frac{B_{norm,plas}(u_i, v_j) + B_{norm,coils}(u_i, v_j)}{\left|\mathbf{B}_{norm}(u_i, v_j)\right|} \right|$$

where $u_i = i/N_u$, N_u=2*(max poloidal mode number) and $v_j = j/N_v$, N_v=2*(max toroidal mode number)

Length:

$$f_{len,j} = w_{len} \int_0^1 \left| \frac{\partial \mathbf{x}_j}{\partial s} \right| ds$$

Max. curvature:

$$f_{curv,j} = w_{curv} \max_{0 \le s < 1} \left(\frac{\left| \frac{\partial \mathbf{x}_j}{\partial s} \times \frac{\partial^2 \mathbf{x}_j}{\partial s^2} \right|}{\left| \frac{\partial \mathbf{x}_j}{\partial s} \right|^3} \right)$$

where $\mathbf{x}_j(s)$ is the Cartesian coordinate vector at point s on unique coil j.

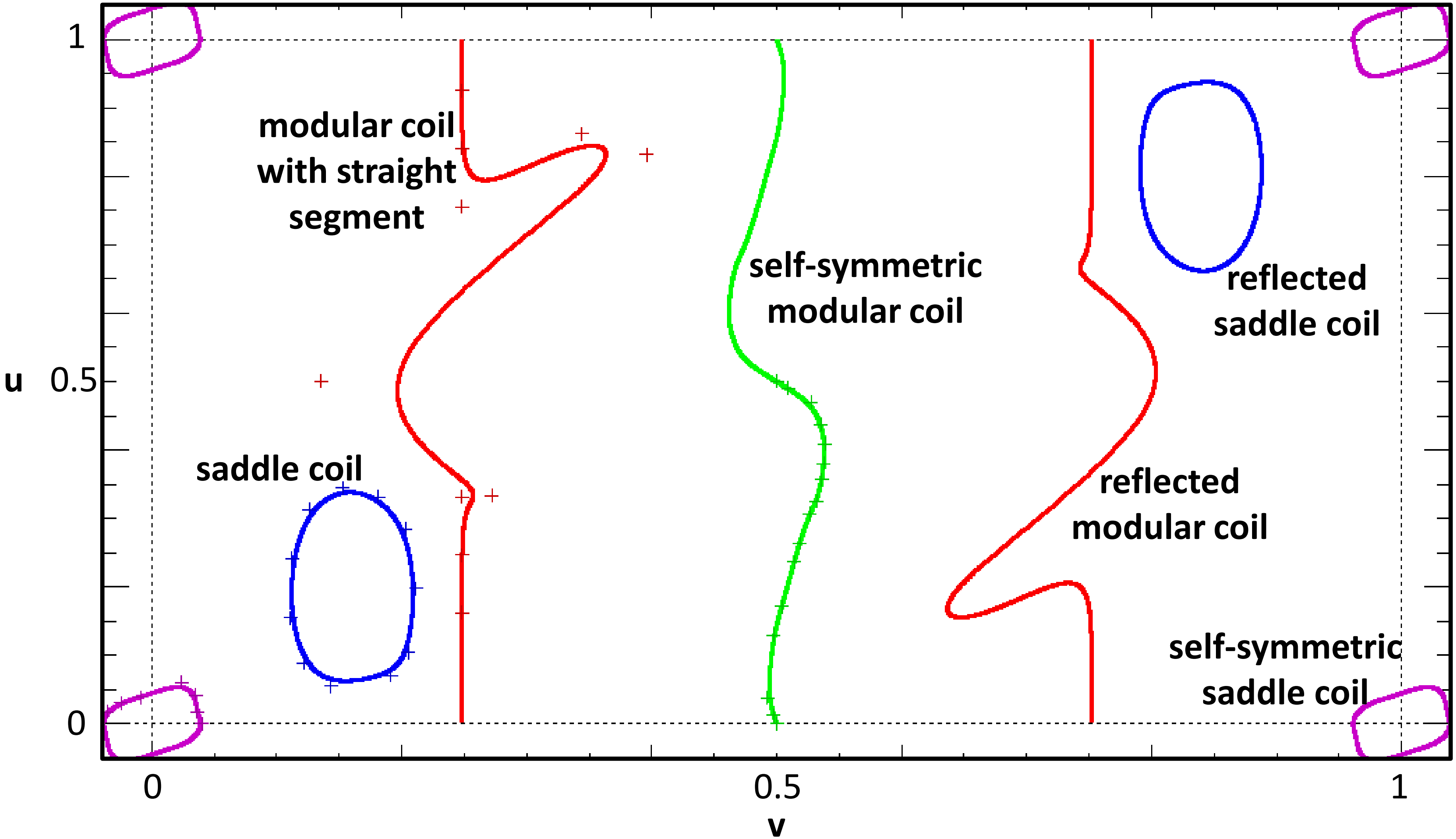
Min. coil-coil separation:

$$f_{cc} = w_{cc} \left(\frac{\lambda}{l_{\min} + \varepsilon} \right)^2$$

where λ is a parameter giving the minimum acceptable separation between coils, l_{min} is the actual distance of closest approach of any two coils, and ε is a small number.

Others: saddle circularity, modular coil torsion, self-intersection, excursion from target region, etc.

Coils types in u-v space



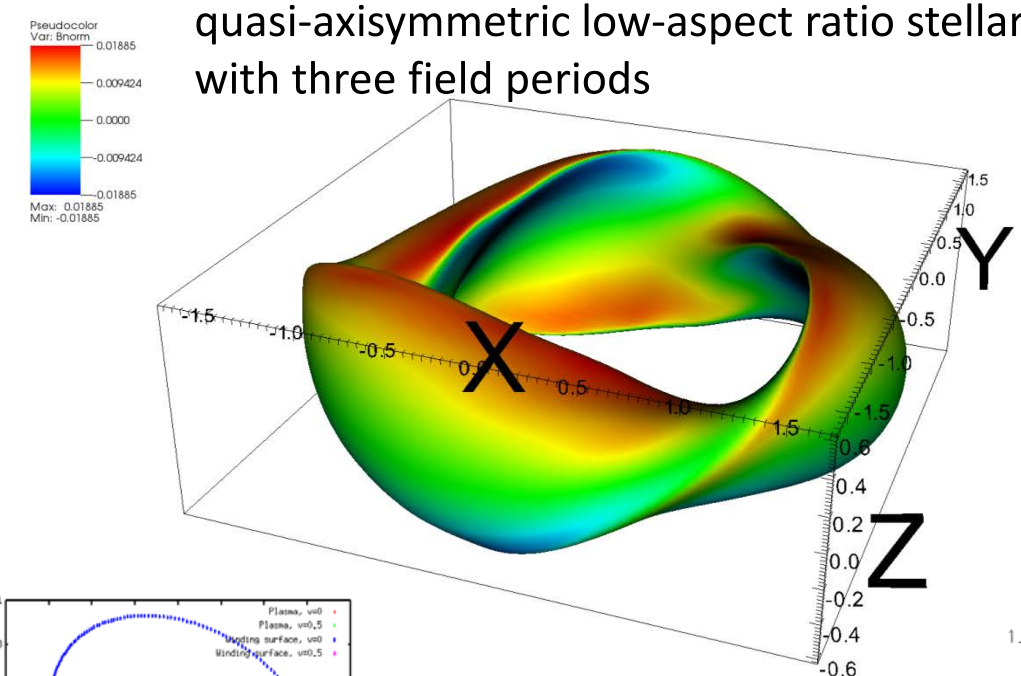
Optimizers

Minimize cost function over configuration space consisting of spline coefficients describing coil shapes within each (fixed) winding surface + coil currents.

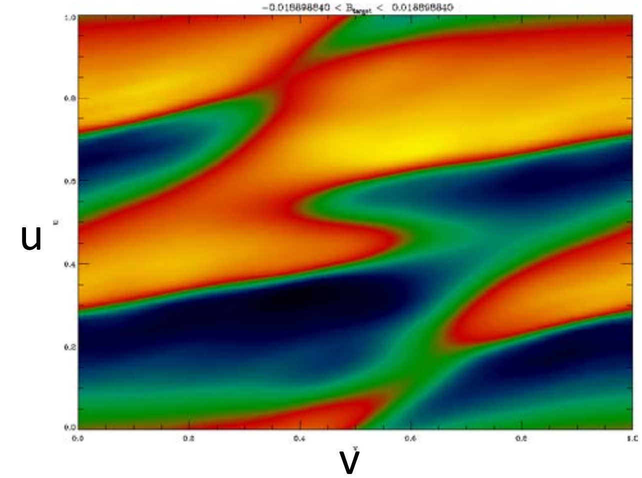
- Global Optimizers
  - Simulated annealing (slow): Takes random walk in configuration space, maintaining thermal equilibrium at steadily decreasing temperature.
  - Differential evolution (medium): Genetic algorithm evolving a population of candidate solutions, eliminating the worst, breeding the best.
  - Particle swarm (fast): Maintains a moving "flock" of trial solutions with inertia, accelerating toward the most favorable regions and each other.
- Local Optimizer
  - Levenberg-Marquardt (very fast): Descend as rapidly as possible down local gradient computed by taking small steps in each direction.

Code verification test case: li383

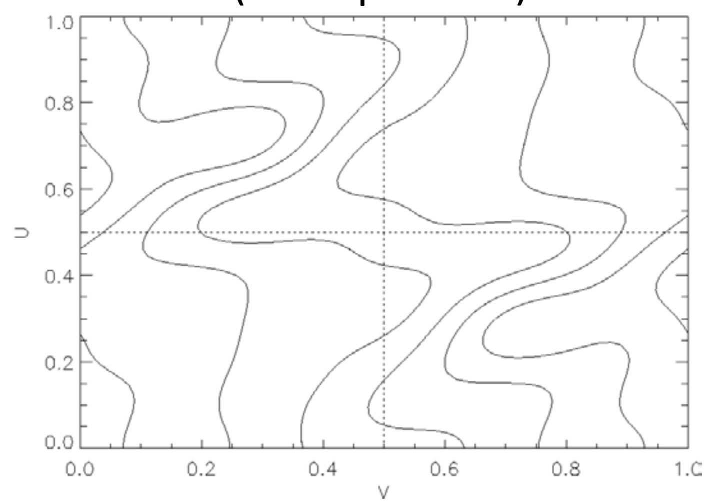
Standard configuration for NCSX
quasi-axisymmetric low-aspect ratio stellarator
with three field periods

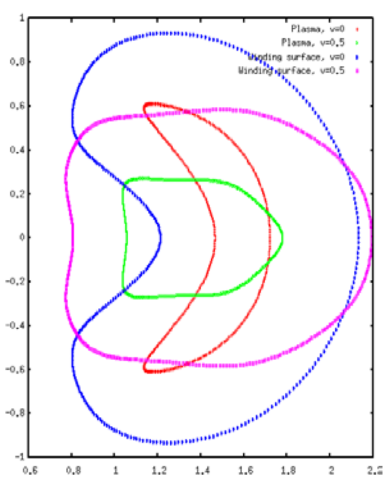


Target B_{normal}



Nominal coil geometry
(3 unique coils)



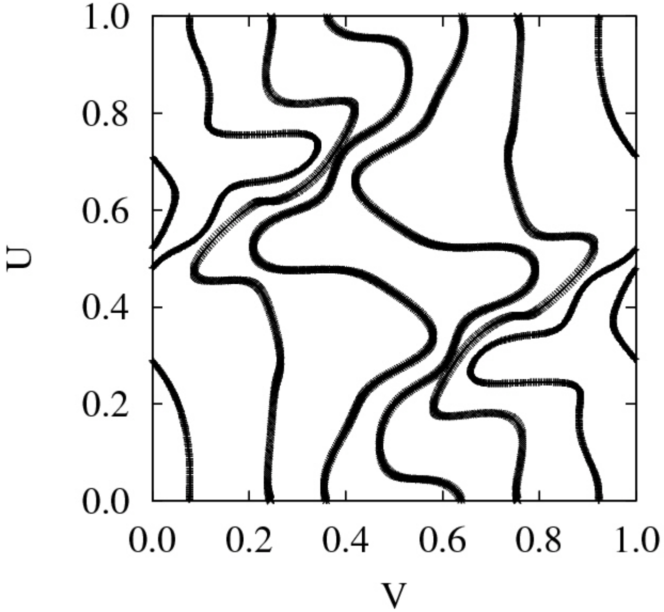


Plasma surface, v=0
Plasma surface, v=0.5
Winding surface, v=0
Winding surface, v=0.5

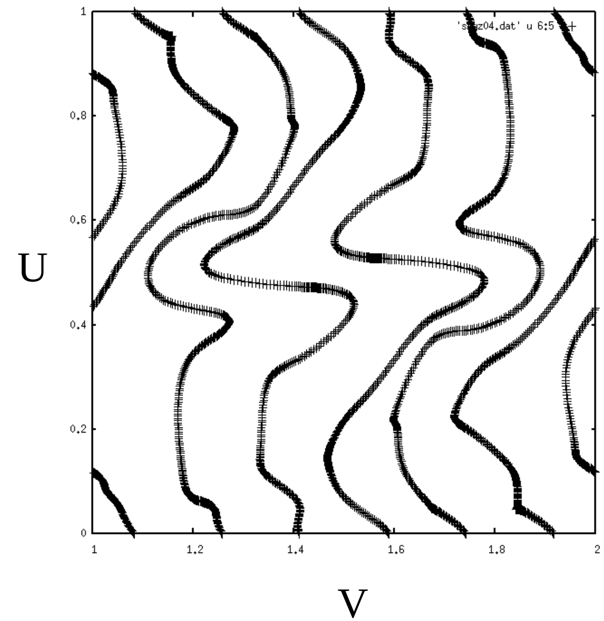
Cross-sections

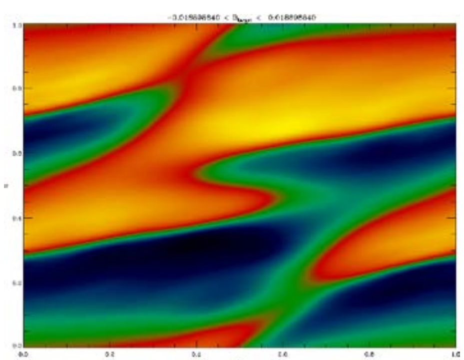
COILOPT++ Approximately Recovers COILOPT Solution

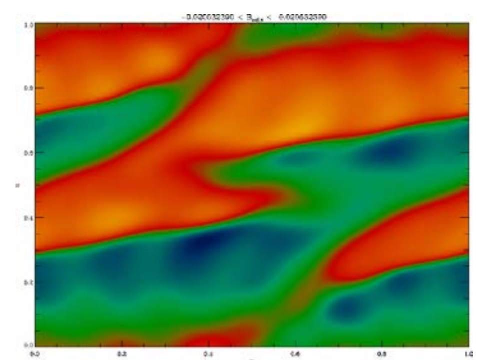
COILOPT (Fourier) optimized coils



COILOPT++ (Spline) optimized coils (DE)

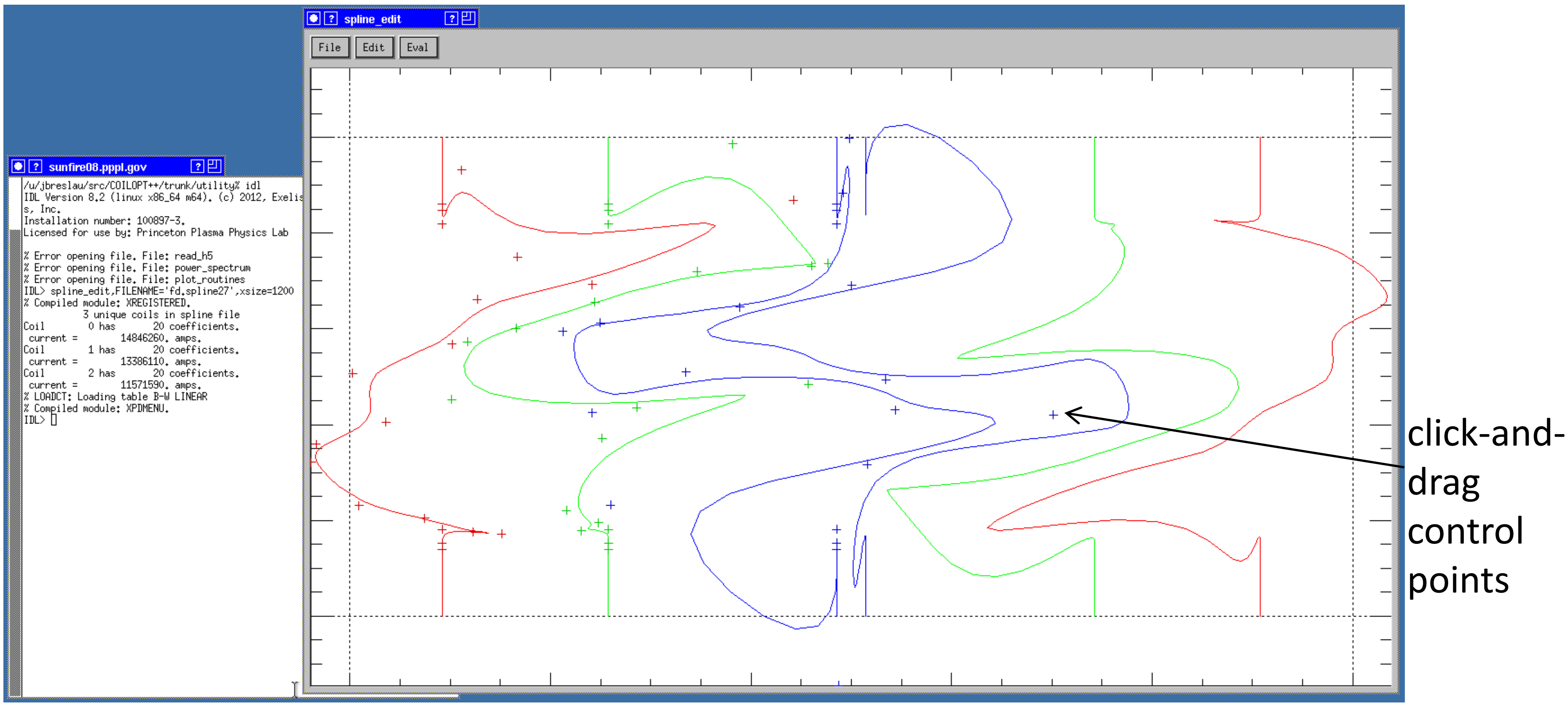






GUI-based spline editor in IDL

- Solves problem of coil kinks and snarls that are resistant to being optimized away.



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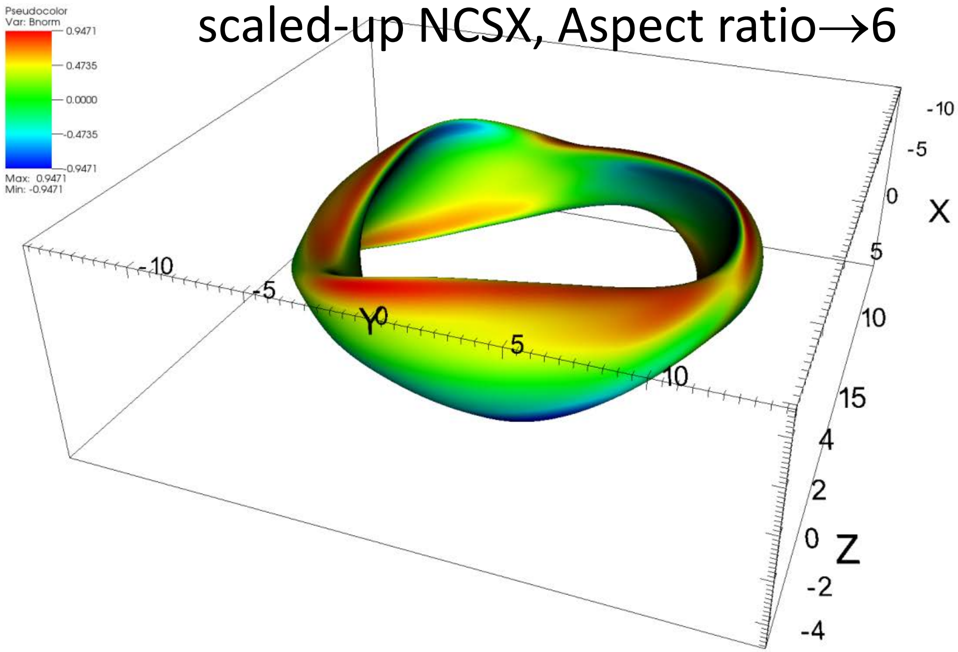
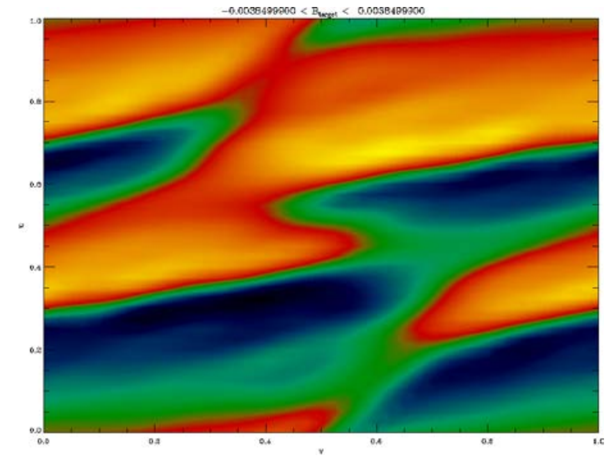
Modified ARIES-CS configuration

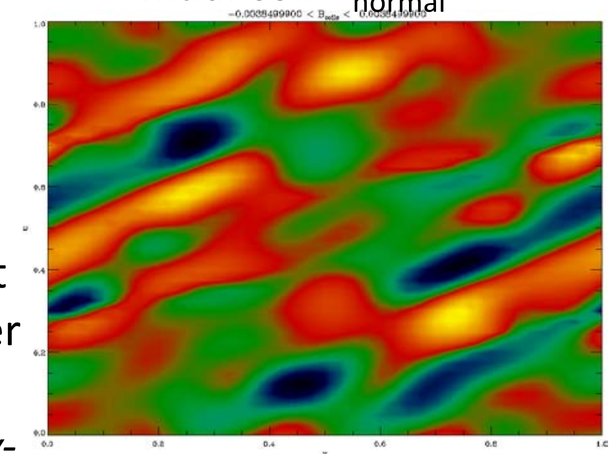
Power plant concept based on scaled-up NCSX, Aspect ratio→6



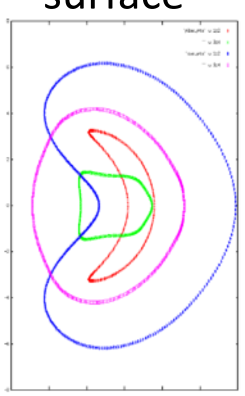
Target B_{normal}



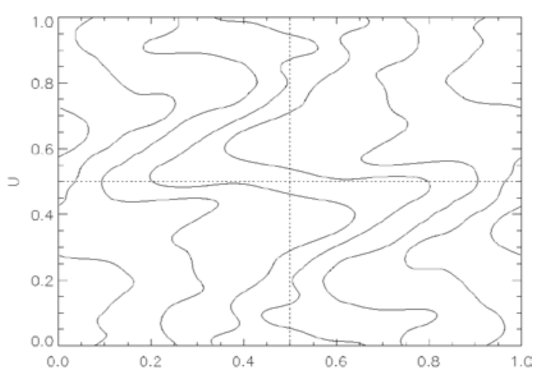
Initial coil B_{normal}



Nominal winding surface



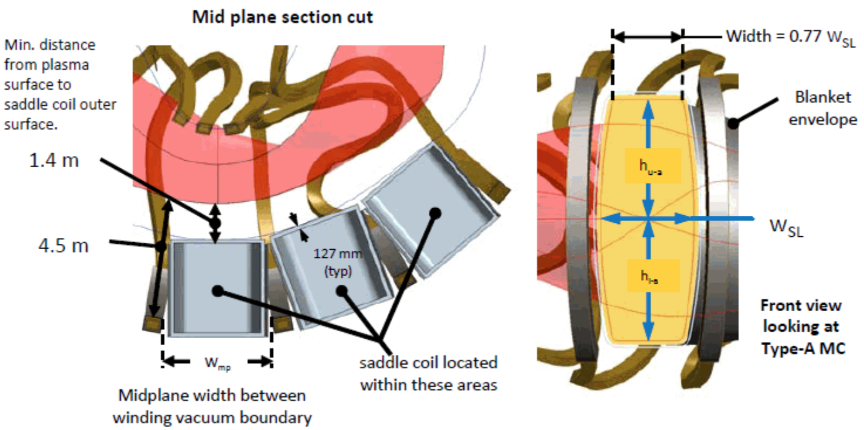
Initial coil geometry* (3 unique coils)



*Note: coils not redesigned after plasma was re-optimized for α-confinement.

Engineering Constraints

Inputs: Total MC current (TC_m, MA)
Number of MC's (N_m)
MC overall current density (CD_m, A/cm^2)
MC winding overall aspect ratio, AR_m
Distance from machine center to plasma outboard midplane surface, (r0, mm)
Plasma surface of MC back leg cryostat distance, (r1, mm)
MC back leg distance to outside of cryostat walls (d1, mm)
MC cryostat horizontal distance to VV boundary (d2, mm)
VV surface to saddle loop containment inner surface (d3, mm)
Saddle loop offset to containment inner surface (d4, mm)
Plasma midplane outer surface to saddle loop containment inner surface (r2, mm)
Assumed thickness of saddle coil containment vessel (t1, mm)
Plasma height at section of interest, (Ph, mm)



Assumed values:

$$TCm := 2.21 \cdot 10^8 A \quad Nm := 18 \quad CDm := 2000 \frac{A}{cm^2} \quad ARm := 1.4 \quad Nm = 18$$

Assumed geometry values:

$$d1 := 100mm \quad d2 := 150mm \quad d3 := 254mm \quad d4 := 100mm \quad Ph := 6.68m$$

$$r0 := 9.40m \quad r1 := 4.50m \quad r2 := 1.4m \quad t1 := 127mm$$

First set of calculations:

$$MC\_overall\_area := \frac{TCm}{Nm \cdot CDm} \qquad MC\_overall\_area = 0.614 m^2$$

$$Winding\_width := \frac{1}{ARm}\sqrt{MC\_overall\_area} \qquad Winding\_width = 0.56 m$$

Calculation of midplane width between vertical windings:

Angle between MC back leg, α $\alpha := \frac{360}{Nm} \qquad \alpha = 20$

Radial distance to MC cryostat corner, rc:

$$rc := \sqrt{(r0 + r1)^2 + (d1 + .5 \cdot Winding\_width)^2} \qquad rc = 13.905 m$$

Winding cryostat half width, w: $w := d1 + .5 \cdot Winding\_width \qquad w = 0.38 m$

Angle to MC cryostat corner, β:

$$\beta := \frac{180}{\pi} asin\left(\frac{w}{rc}\right) \qquad \beta = 1.565$$

Midplane width between winding vacuum boundary, W_{mp}

$$Wmp := 2 \cdot rc \cdot sin\left[\pi \frac{(.5\alpha - \beta)}{180}\right] \qquad Wmp = 4.079 m$$

Maximum saddle loop envelope width, W_{SL}

$$WsL := Wmp - 2 \cdot (d2 + d3 + d4)$$

$$WsL = 3.071 m$$

Saddle loop enverlpe max heigth, H_{SL}

$$HsL := Ph - 2m$$

$$HsL = 4.68 \cdot m$$

Minimum radial position of saddle loop envelope, R_{SL}

$$RsL := r0 + r2 + t1 + d4 \qquad RsL = 11.027 m$$

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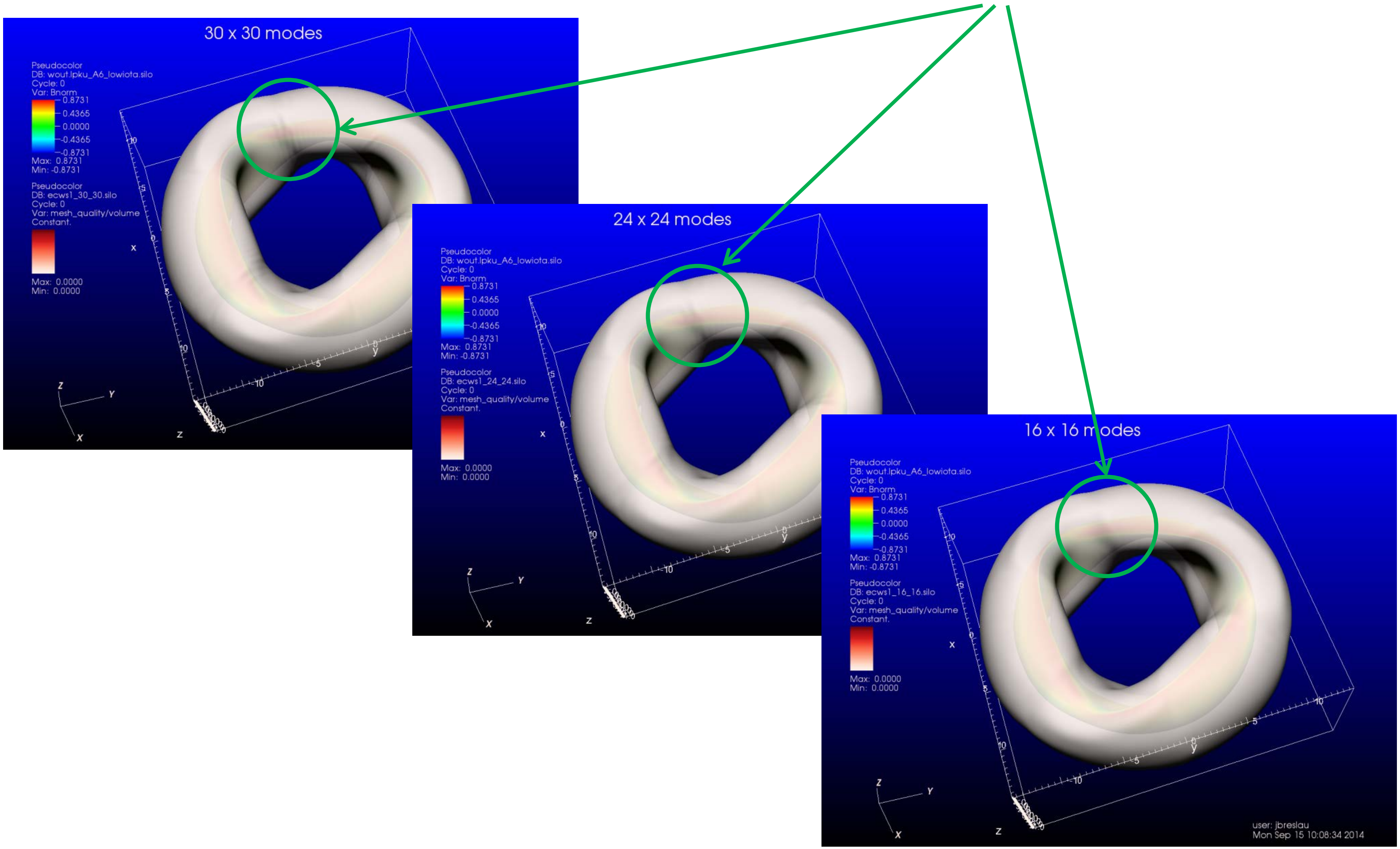
Avoid overfitting winding surface

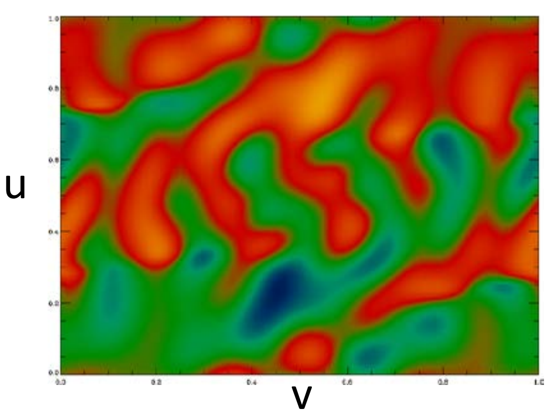
- Surface is fitted to cross-sections determined by engineering constraints.
- Coarser reconstructions generate fewer spurious surface wrinkles/coil wiggles.



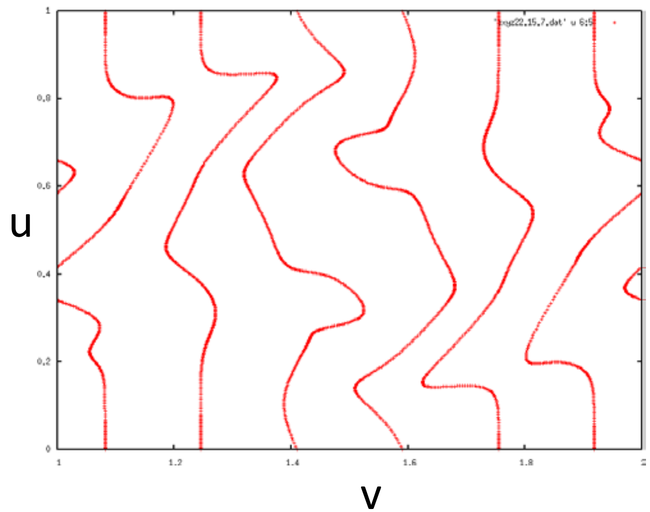
COILOPT++ Results

- Three unique modular coils, two with straight outer legs
- 15 pairs of spline coefficients/coil
- Optimize rms & peak $\delta B/B$, coil length, coil-coil separation, curvature, torsion
- Multiple passes of differential evolution

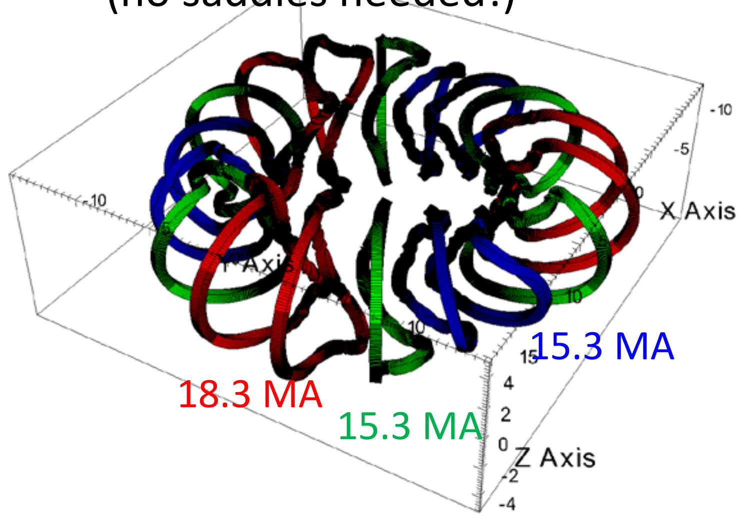
- Final RMS $\delta B/B$ = 0.0227
- Final max $\delta B/B$ = 0.0528



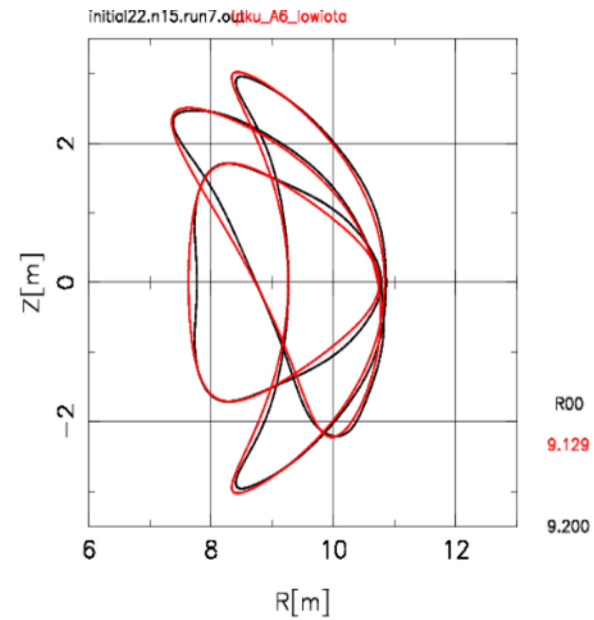
Final modular coils in u-v space



Modular coils in real space (no saddles needed!)

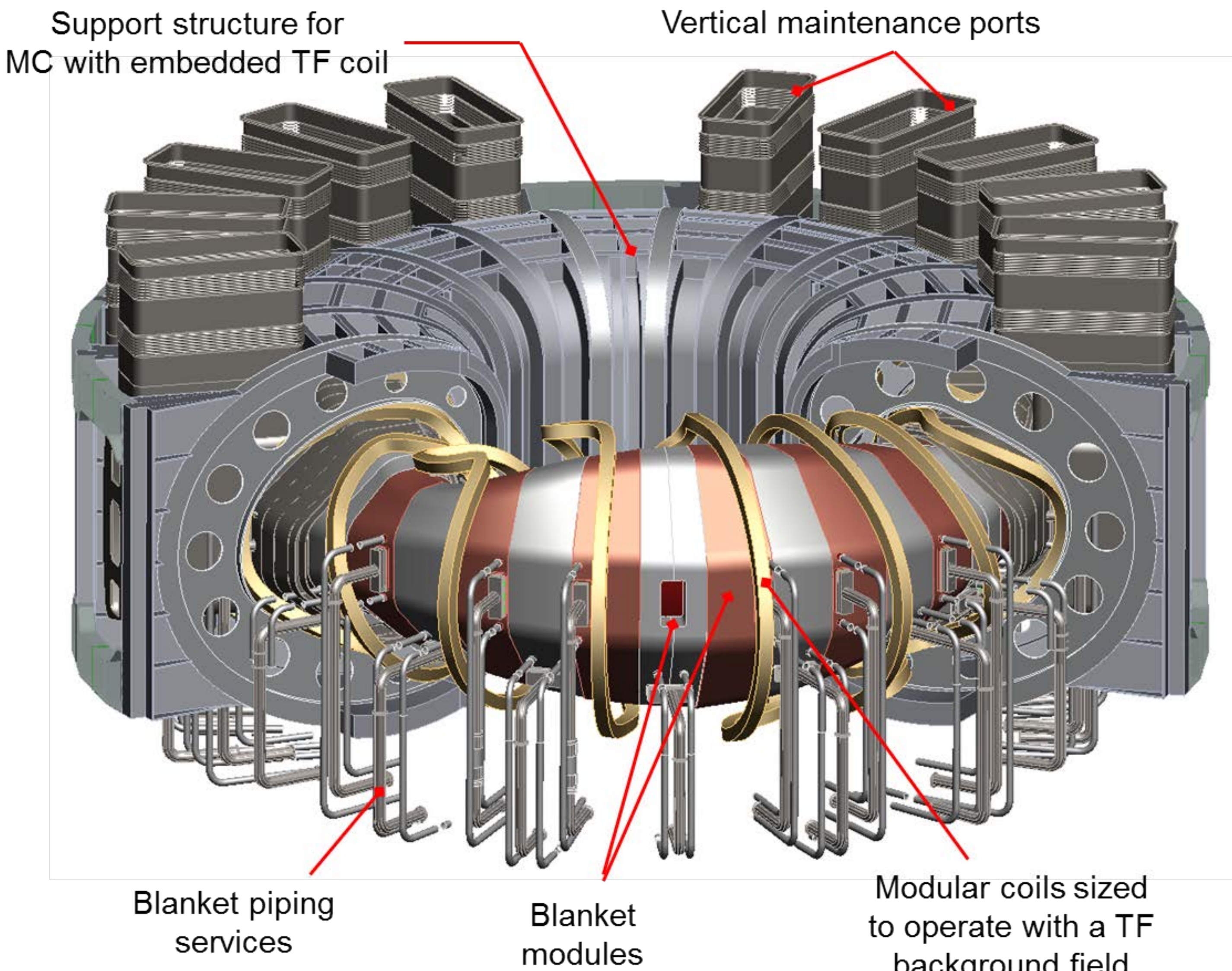


Flux surfaces reconstructed with free-boundary VMEC



N. Pomphrey

Engineering design



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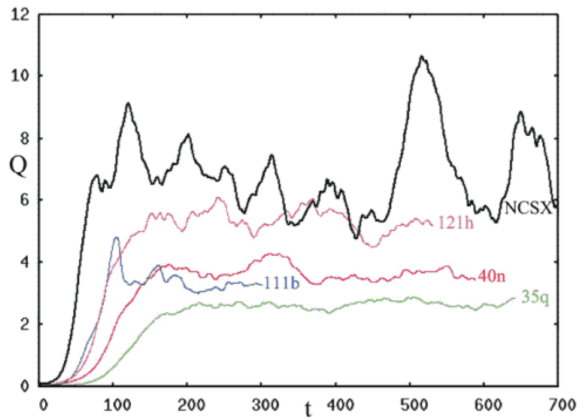
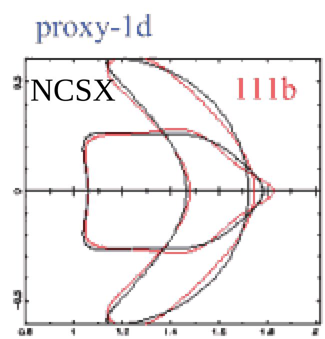
Configuration 111b

- NCSX was designed to minimize neoclassical, but not turbulent transport.
- Harry Mynick has introduced proxy functions[1] that allow relatively rapid heuristic evaluations of the mean turbulent transport of a candidate stellarator equilibrium.
- Turbulent transport can be reduced significantly from an initial NCSX equilibrium by incorporating various such proxies into the STELLOPT cost function and re-optimizing.
- An especially promising configuration is "111b", which shows significantly reduced turbulent heat flux in nonlinear GENE runs with modest changes in plasma shape and other favorable properties.
- Could an appropriate set of trim coils transform NCSX/QUASAR into 111b?

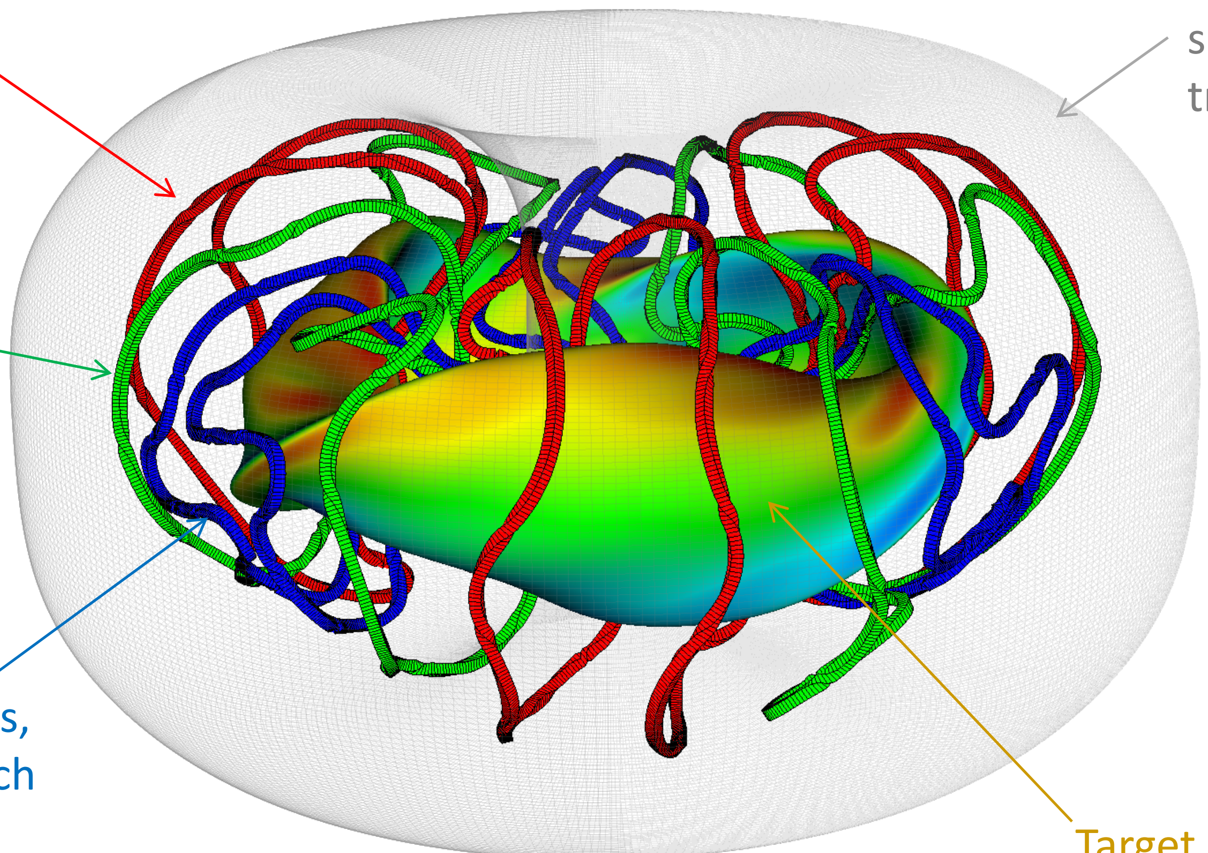




H. Mynick

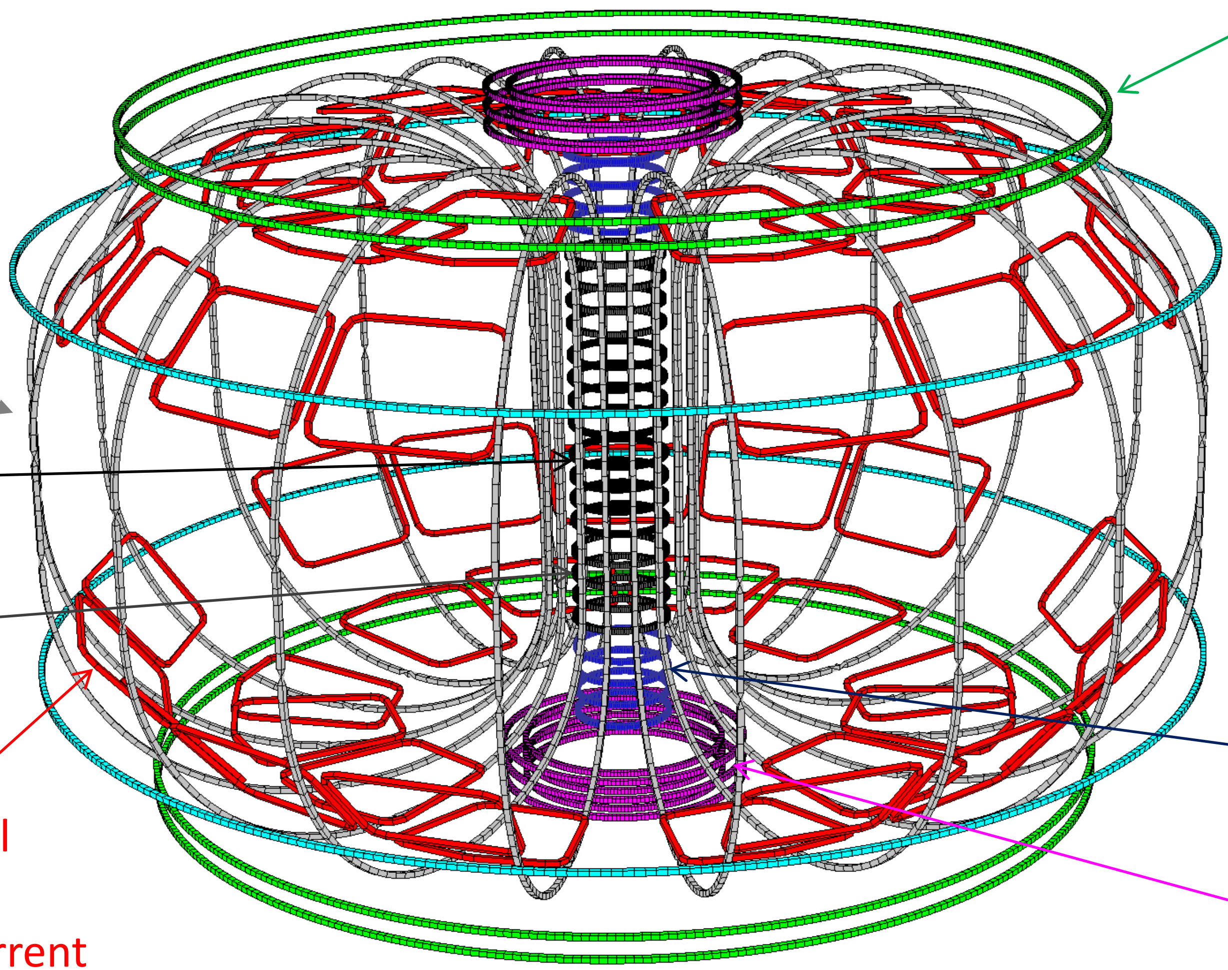
[1]H. Mynick, *et al.*, "Turbulent optimization of toroidal configurations," *Plasma Phys. Control. Fusion* **56** (2014) 094001.

Modular coil set



Net modular coil poloidal current: 11.3994 MA
Target poloidal current for configuration: 11.633 MA

Auxiliary coil sets

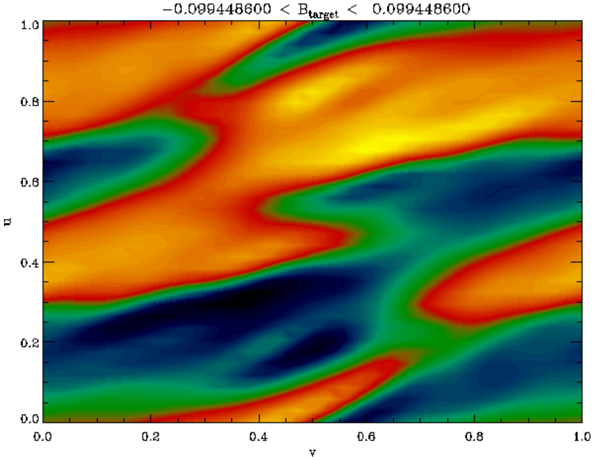


Net TF coil poloidal current: 0.5004 MA
Reduce x 0.4668 to 0.2336 MA to meet target.

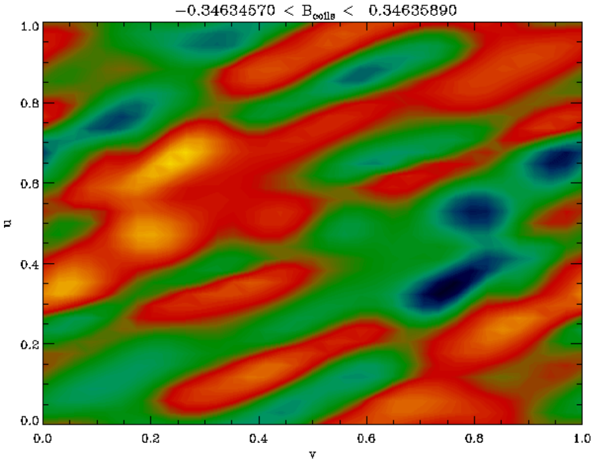
First optimize modular coil currents

- Include net poloidal current target in cost function.
- Differential evolution, pop size = 64 for 3 free variables (360 segments/coil) converged in 116 generations.
- Initial rms $\delta B/B$ = 0.0725; initial max $\delta B/B$ = 0.2460.
- Final rms $\delta B/B$ = 0.0727; final max $\delta B/B$ = 0.1924; Ipol = 11.659 MA.

Target field



Achieved field

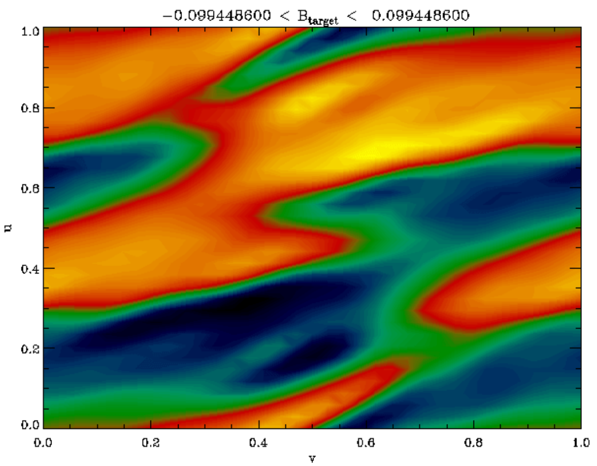


Modular Coil	Initial current (MA)	“Optimized” current (MA)
A	0.6523	0.6524
B	0.6519	0.8263
C	0.5377	0.4645

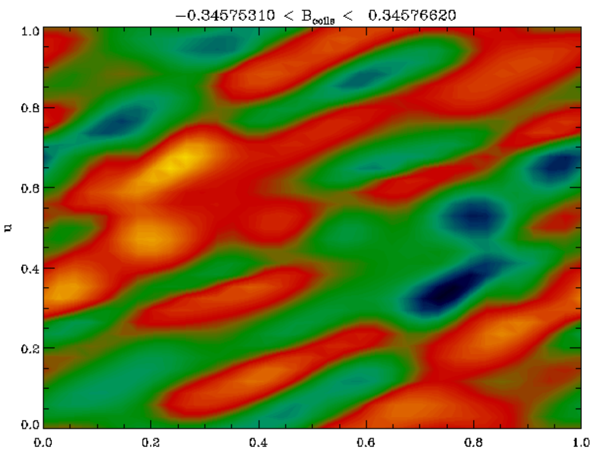
Add TF coils, redo...

- Differential evolution, pop size = 96 for 4 free variables (360 segments/coil) converged in 934 generations.
- Final rms $\delta B/B$ = 0.0728; final max $\delta B/B$ = 0.1918; Ipol = 11.635 MA.

Target field



Achieved field

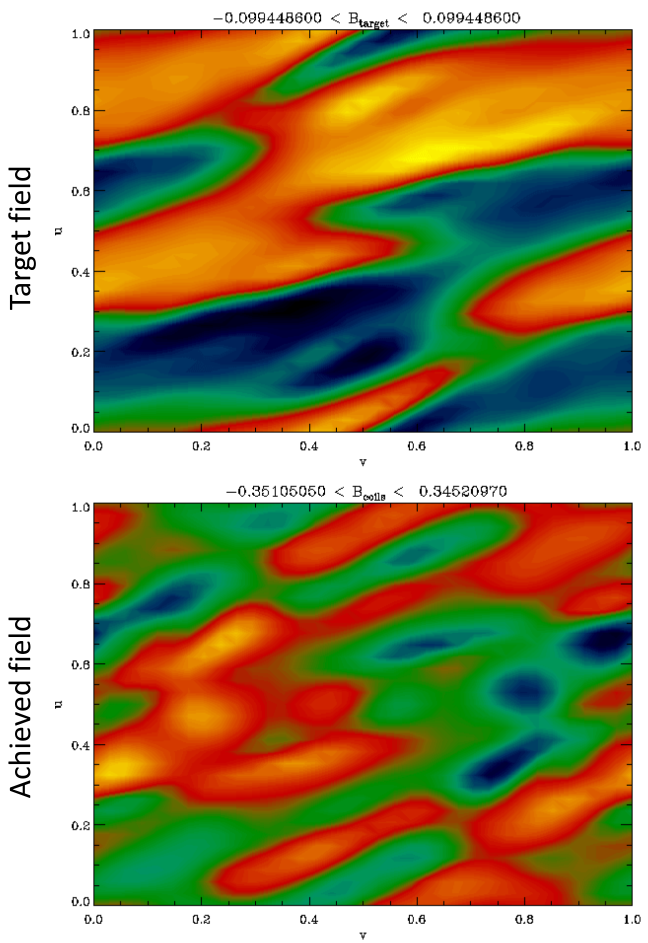


Coil	Initial current (MA)	Optimized current (MA)
Mod A	0.6523	0.6354
Mod B	0.6519	0.8314
Mod C	0.5377	0.4619
TF	0.0455	0.003457

TF coils are inconsequential.

Freeze TF & Modular currents, add PF, re-optimize…

- Differential evolution, pop size = 96 for 4 free variables (360 segments/coil) converged in 1004 generations.
- Final rms $\delta B/B$ = 0.0718; final max $\delta B/B$ = 0.1896; Ipol = 11.635 MA.



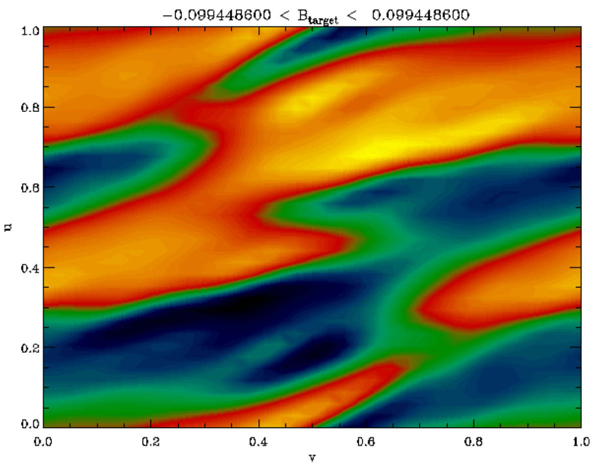
Coil	Initial current (MA)	Optimized current (MA)
PF 3	0.0281	7.4921
PF 4	-0.0548	-0.8408
PF 5	0.0301	0.2911
PF 6	0.0942	-0.2216

Very large jump in PF coil currents!

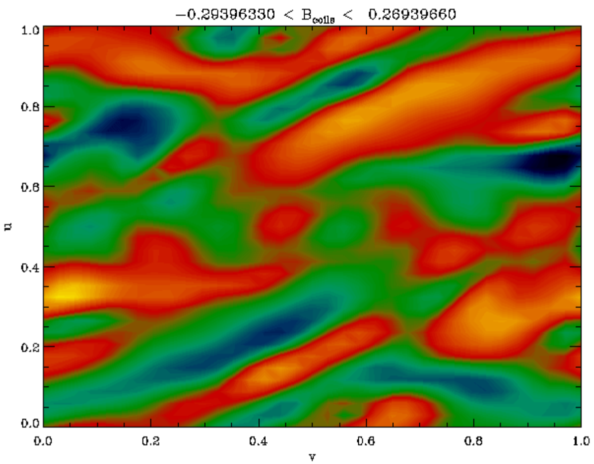
Add minimal trim set, optimize currents…

- DE gave no improvement in 896 generations (pop. size 96).
- Levenberg-Marquardt, 10 free variables (270 segments/coil) converged in 11 iterations.
- Final rms $\delta B/B$ = 0.0568; final max $\delta B/B$ = 0.1639; final I_{pol} = 11.633 MA.

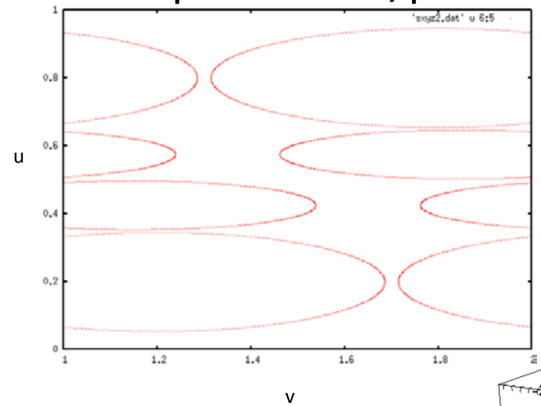
Target field



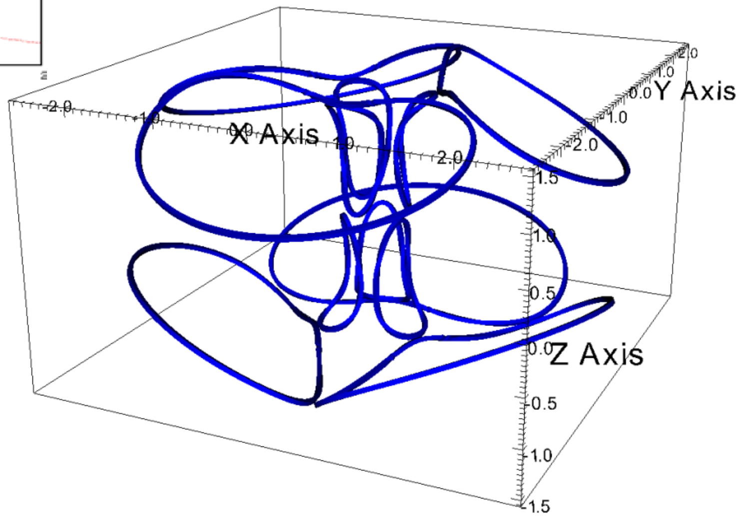
Achieved field



2 unique saddles/period



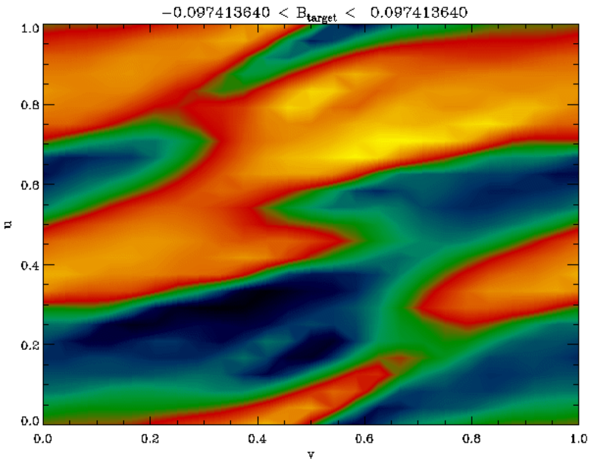
3D view



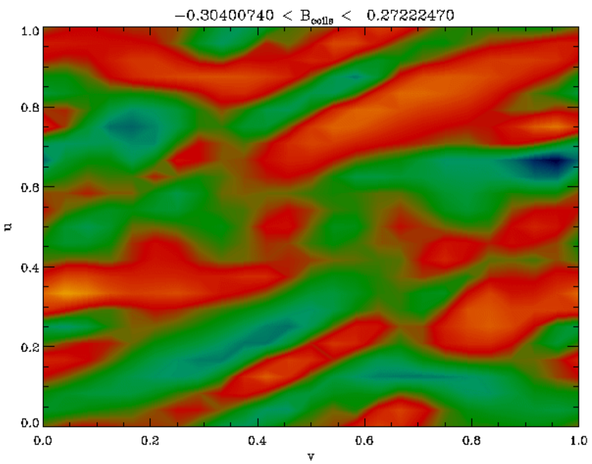
Optimize trim geometry, all currents

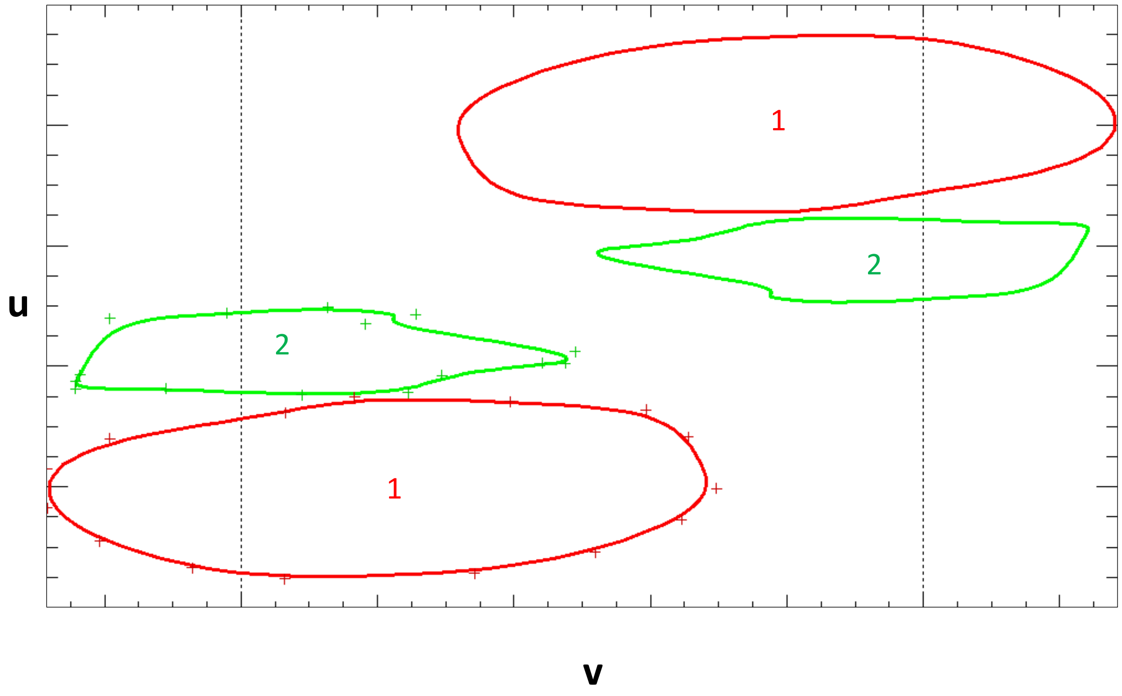
- Levenberg-Marquardt, 70 free variables (192 segments/coil)
- Targeted selfint, bnorm, bmax, Ipol, coilcoil, max curvature, circularity, length.
- Ran on 24 cores for 1:12:47, 16384 iterations.
- Final rms $\delta B/B$ = 0.0551; final max $\delta B/B$ = 0.1577; final I_{pol} = 11.633 MA.

Target field



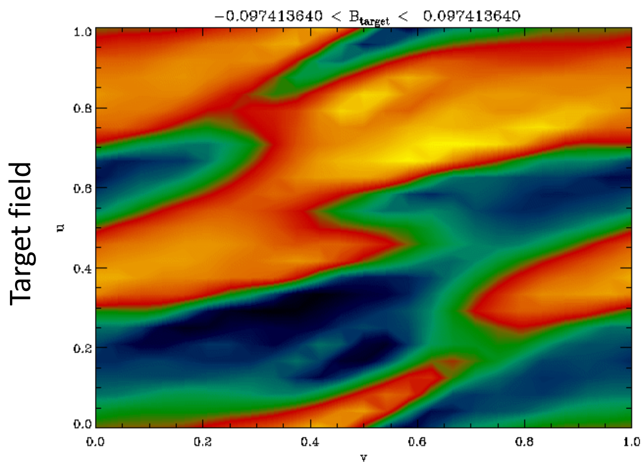
Achieved field

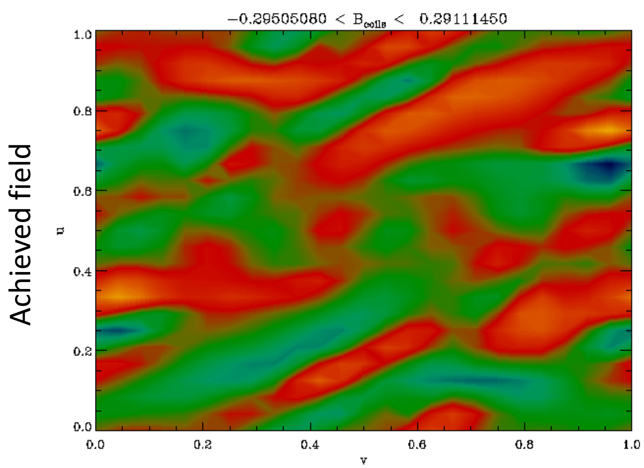


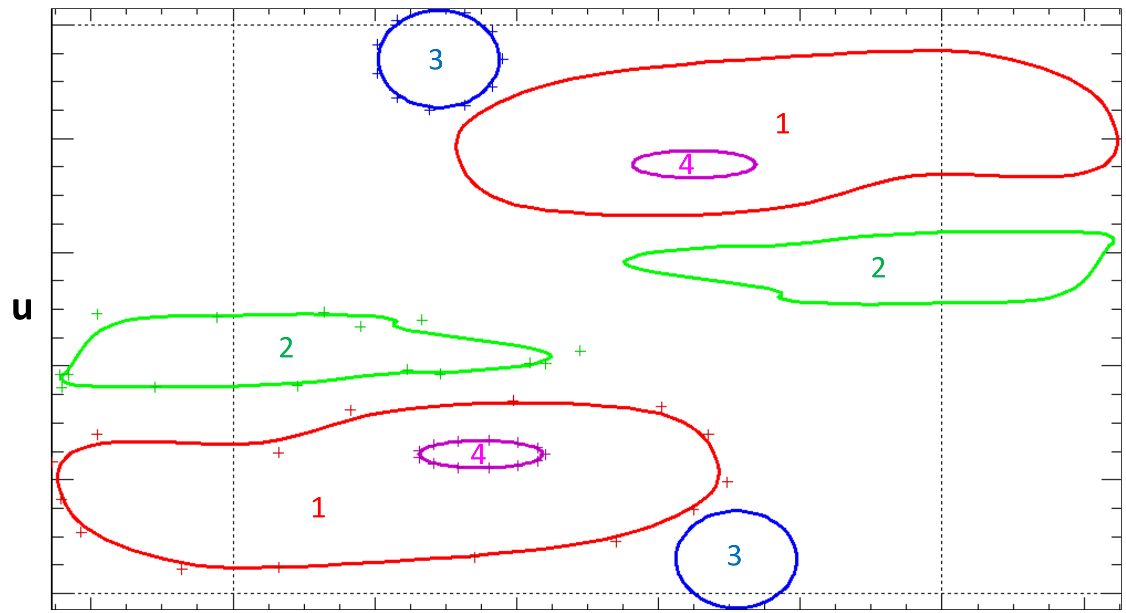


Add more trim coils, optimize trim currents

- **Differential evolution**, pop. size 48 for 4 free variables (192 segments/coil) converged in 552 generations.
- Final rms $\delta B/B$ = 0.05348; final max $\delta B/B$ = 0.1572; final I_{pol} = 11.633 MA.





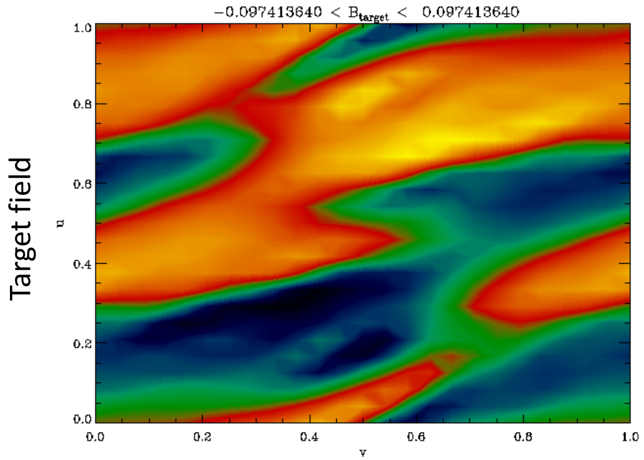


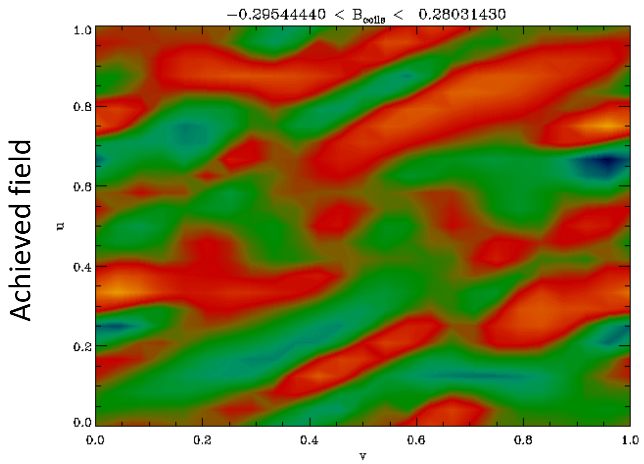
Trim coil	Initial current (MA)	Optimized current (MA)
1	2.1865	2.1727
2	-0.6308	-0.8245

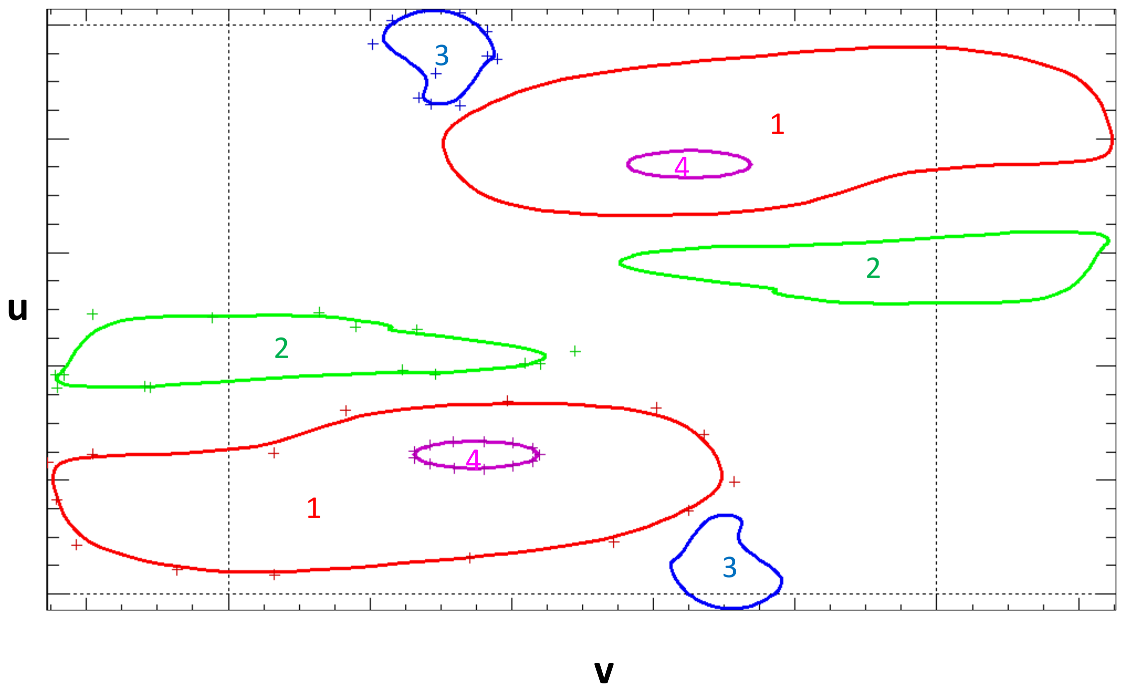
Trim coil	Initial current (MA)	Optimized current (MA)
3	5.0000	0.0430
4	5.0000	0.4121

Optimize trim geometries, all currents

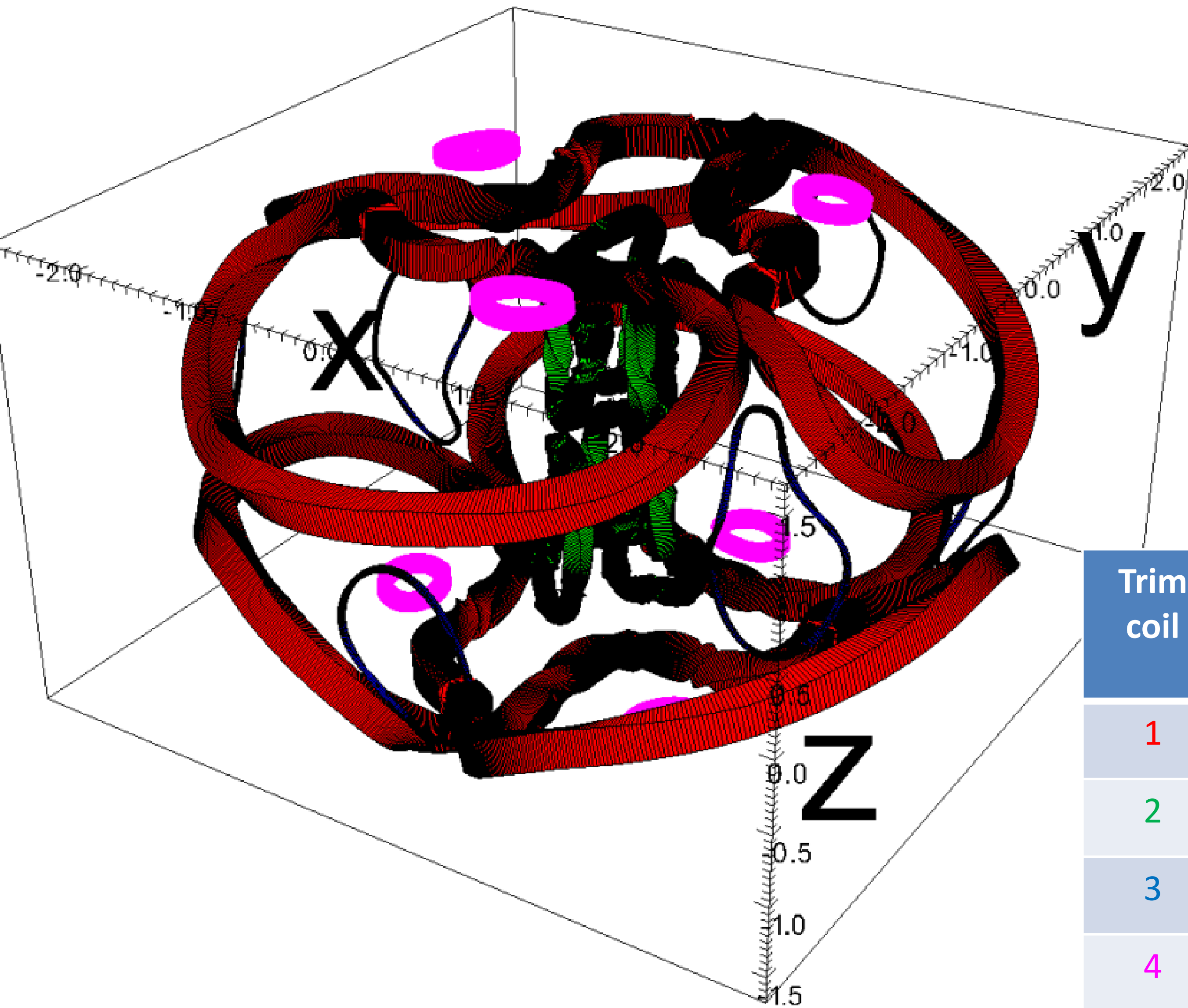
- **Particle swarm** for 120 free variables (192 segments/coil).
- Targeted selfint, bnorm, bmax, Ipol, coilcoil, max curvature, circularity, length.
- Ran on 32 cores for 31:40:32; 16,384 iterations.
- Final rms $\delta B/B$ =0.05264; final max $\delta B/B$ =0.1566; final I_{pol} = 11.633 MA.







Realspace trim coil geometry



Trim coil	Initial current (MA)	Optimized current (MA)
1	2.1727	2.1219
2	-0.8245	-0.8252
3	0.0430	0.0430
4	0.4121	0.4194

Outline

- Motivation
- COILOPT++ features
- Sample applications
  - Straight coils for ARIES-CS: proof-of-concept
  - Retrofitting QUASAR to reduce turbulent transport (work in progress)
- Conclusions & future work

Summary & Conclusions

- A new tool exists for improved stellarator coil design.
  - Spline-based representation allows easy imposition of geometric constraints.
  - Flexible global and local optimizers allow rapid and thorough exploration of design space.
  - COILOPT++ is now fully interfaced with STELLOPT.

- Modular coil sets with straight outer legs may be a way to produce simpler, more maintainable stellarators.

- Reshaping QUASAR into the 111b configuration with trim coils at the TF winding surface is probably impractical. But iteration with STELLOPT might still yield a trim coil set capable of reducing turbulent transport.

Future Work

- Ku and Boozer[1] have suggested isolating those components of the external field distribution that are most sensitive to external perturbations and targeting coils to those components. This technique could be implemented in COILOPT++.

- Options could be added to optimize the shapes of the winding surfaces as well as the coils.

- Much work remains to be done in choosing optimal parameters for the global and local optimizers to improve the robustness of the solutions and accelerate convergence.

[1]Long-Poe Ku and Allen H. Boozer, *Phys. Plasmas* **17**, 122503 (2010).