Plasmoid instability as a tearing instability in time-evolving current sheets

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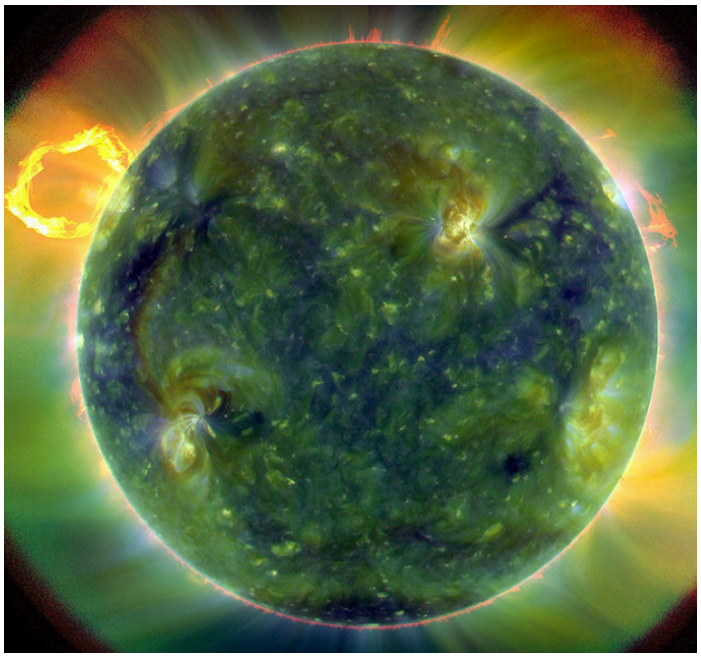
Outline

- Background
  - Motivations to work on this topic
  - Some models of magnetic reconnection
  - Effects of plasmoid formation
- Plasmoid Instability in Sweet-Parker Current Sheets
  - Scalings assuming Sweet-Parker
  - Problems with this approach
- A More Complete Theory of the Plasmoid Instability
  - Principle of least time for plasmoids
  - Scaling laws
- Concluding Remarks
- Supplement: effects of plasma viscosity

Why are we interested in magnetic reconnection?

Magnetic reconnection is thought to play a key role in many phenomena in laboratory, space and astrophysical plasmas.

Solar flares and CMEs



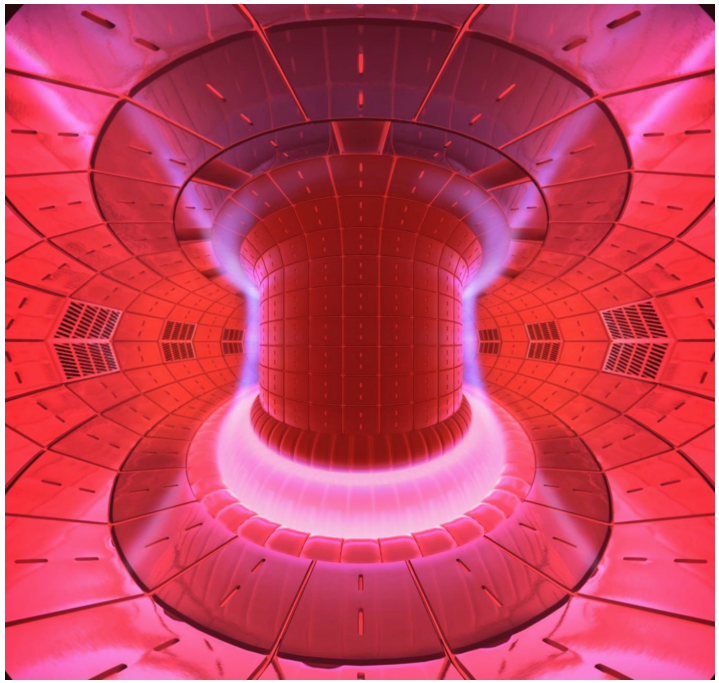
Multiwavelength image of the Sun taken by Solar Dynamics Observatory

Magnetospheric substorms

Aurora Australis recorded from the International Space Station

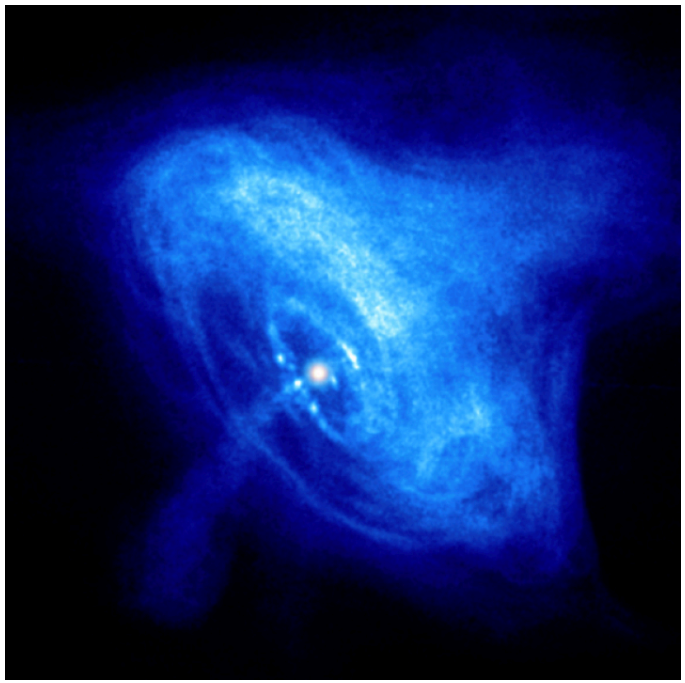
Why are we interested in magnetic reconnection?

Sawtooth crashes



Interior of a tokamak device
with a hot plasma

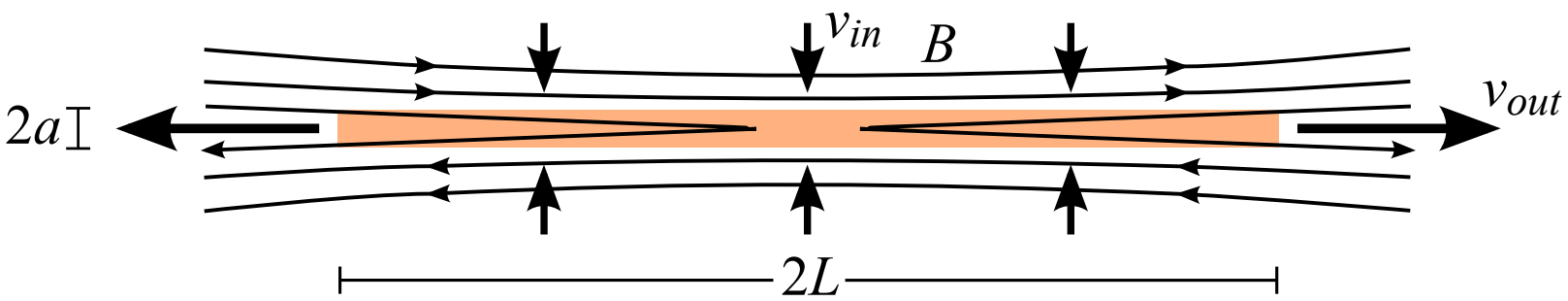
Gamma-ray flares



X-ray image of the Crab Nebula
taken by Chandra X-ray Observatory

Many other systems where magnetic reconnection is important!

Classical Sweet-Parker model



- The Sweet-Parker model assumes a resistive current sheet, 2-D, $\partial/\partial t \approx 0$ and $\nabla \cdot \mathbf{v} \approx 0$.
- It gives some important quantities as a function of the Lundquist number $S := Lv_A/\eta$:

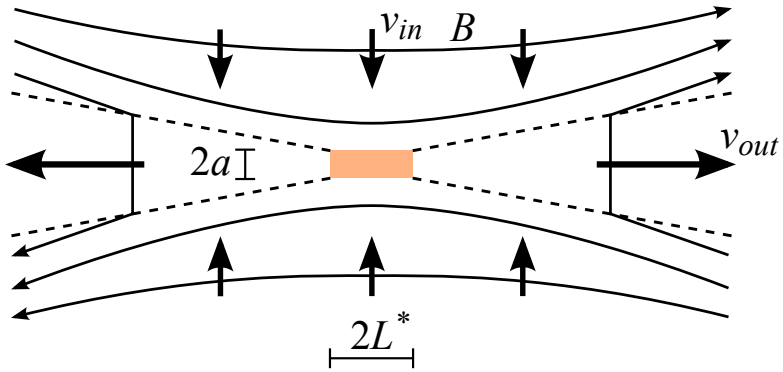
$$a \approx L/\sqrt{S}\,, \quad v_{out} \approx v_A\,, \quad v_{in} \approx v_A/\sqrt{S}\,. \tag{1}$$

  - For solar corona parameters $S \sim 10^{14}$, $\tau_A = L/v_A \sim 1\,\mathrm{s}$ $\Rightarrow \tau_{SP} = L/v_{in} \sim 10^7\,\mathrm{s}$ (solar flares last $10^2 - 10^3\,\mathrm{s}$)

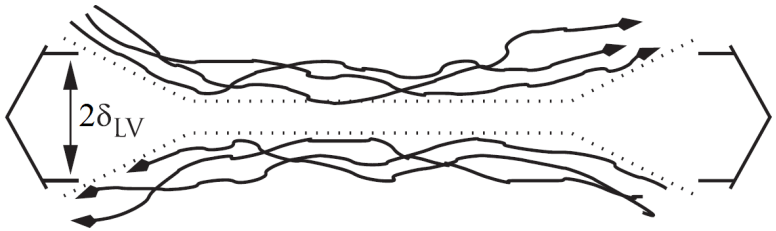
Several other models

Other models have been proposed to explain "fast" reconnection

Petschek model

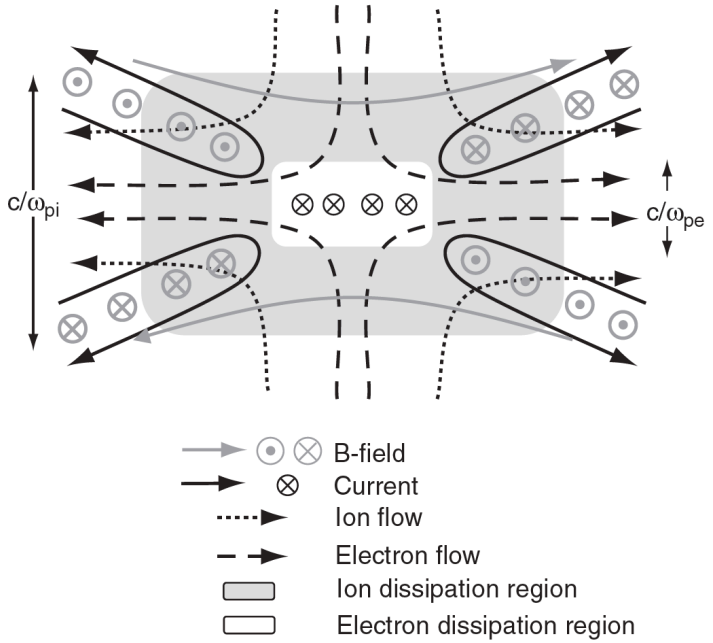


Turbulent models (e.g. LV99)



Collisionless models

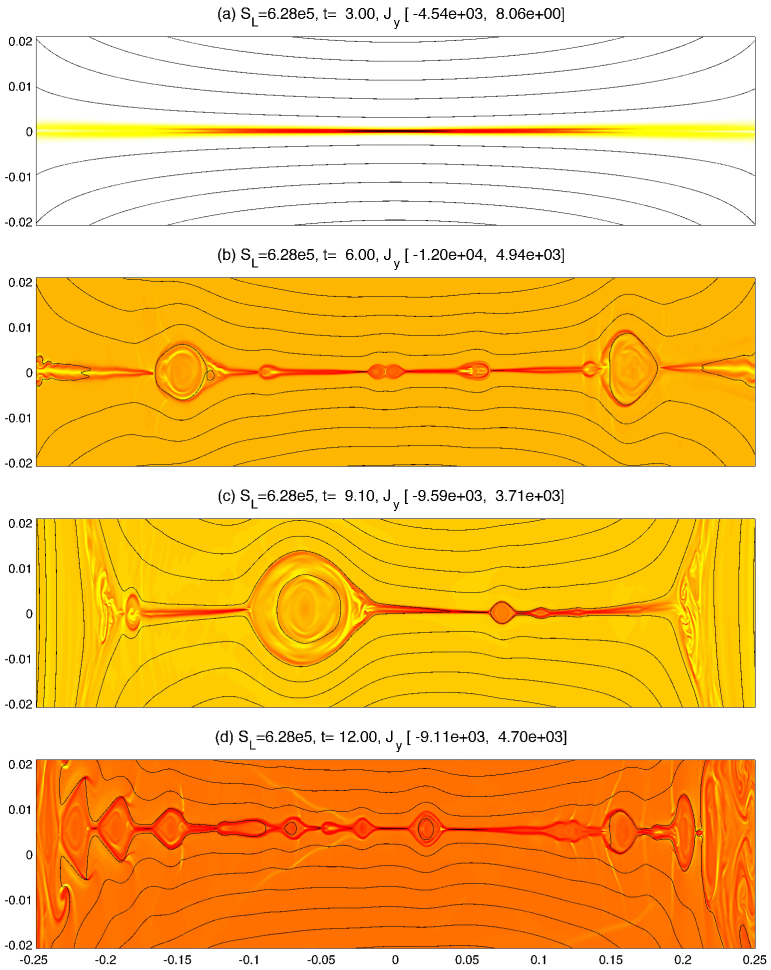
(many contributions...)

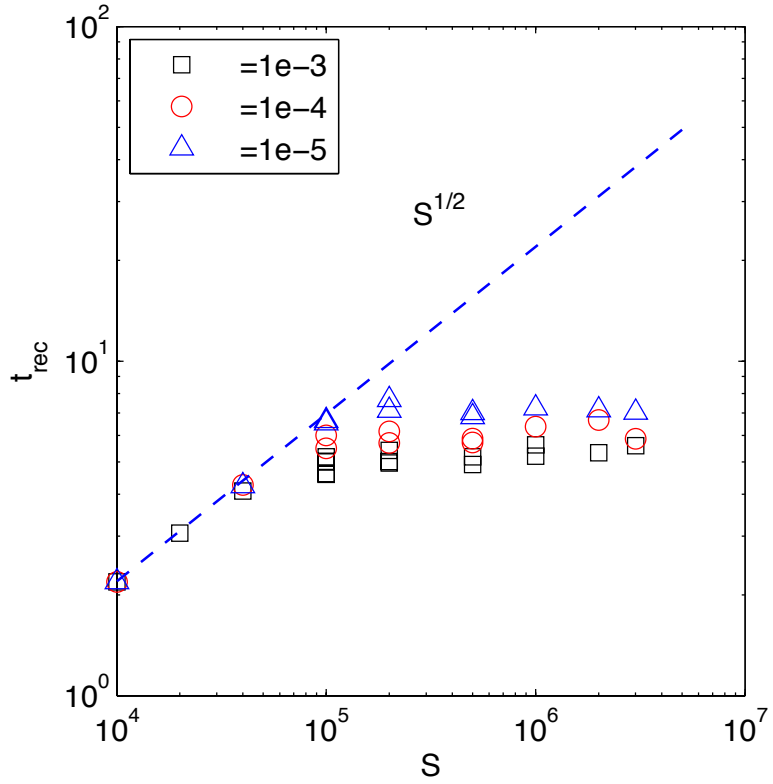


Drake & Shay, 2007

Sweet-Parker at large S is too slow to be true...

- Sweet-Parker scalings do not hold for $S = Lv_A/\eta > S_{critical}$ because of the *plasmoid instability*!



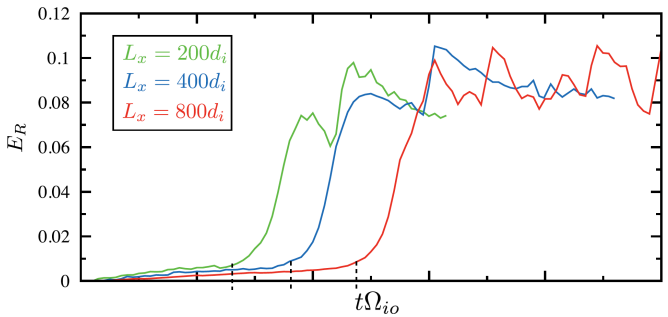


Bhattacharjee *et al.*, PoP 2009

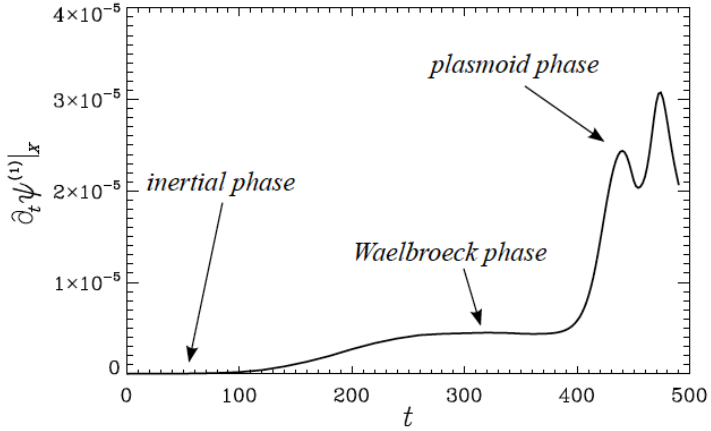
Huang & Bhattacharjee, PoP 2010

Breakdown of the Sweet-Parker model at large S

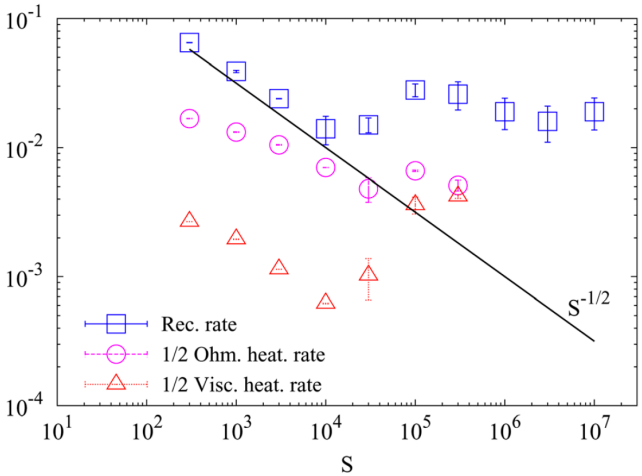
- Speed-up of the reconnection process due to plasmoid formation has been shown by many other research groups



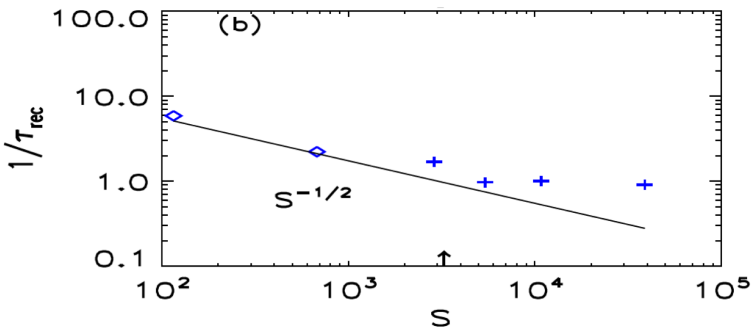
Daughton *et al.*, PRL 2009



Comisso *et al.*, PoP 2015



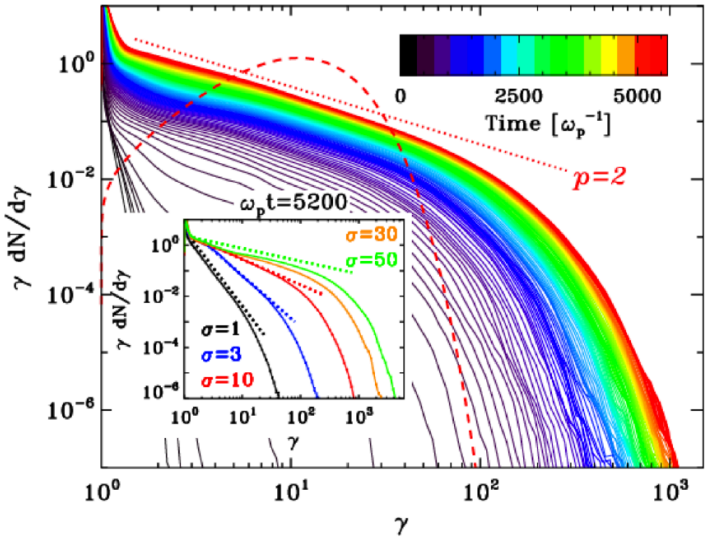
Loureiro *et al.*, PoP 2012



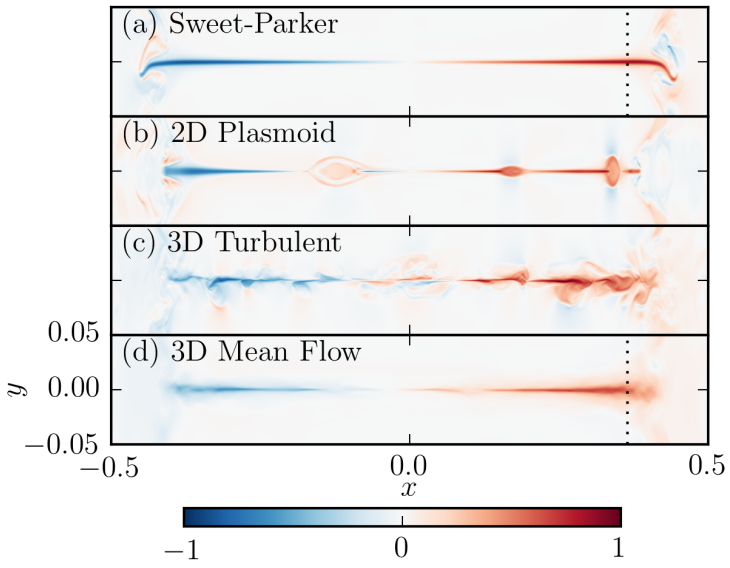
Ebrahimi & Raman, PRL 2015

Other important effects of the plasmoid formation

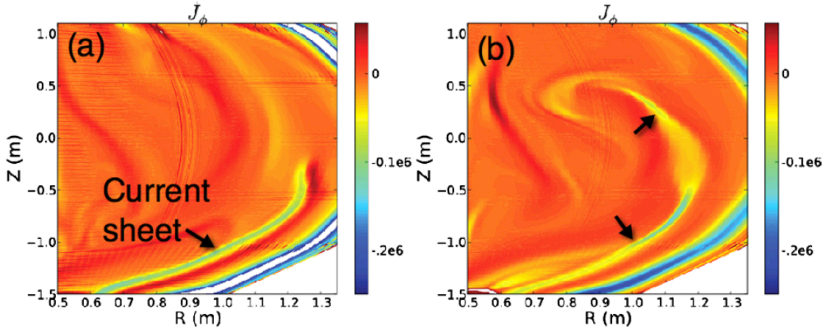
- Other crucial implications:
  - Particle acceleration
  - Turbulent reconnection
  - Fast flux closure
  - .....



Sironi and Spitkovsky, ApJ 2014



Huang & Bhattacharjee, ApJ 2016



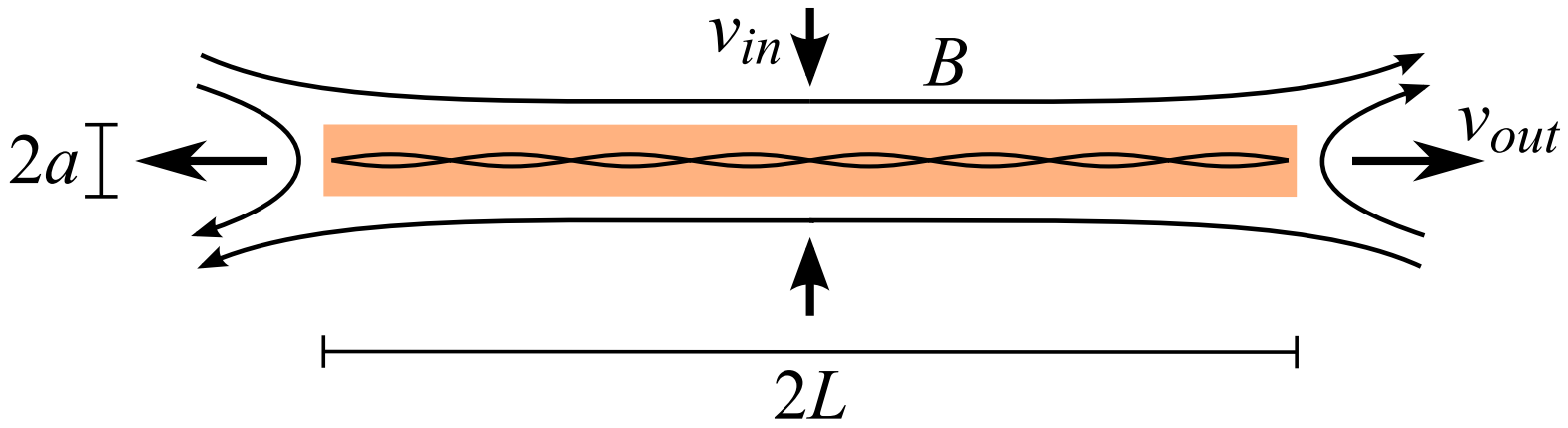
Ebrahimi & Raman, PRL 2015

An important issue to address

- We have seen that the formation of plasmoids plays a crucial role in magnetic reconnection

BUT

- What is the dynamical picture behind the onset and development of the plasmoid instability?



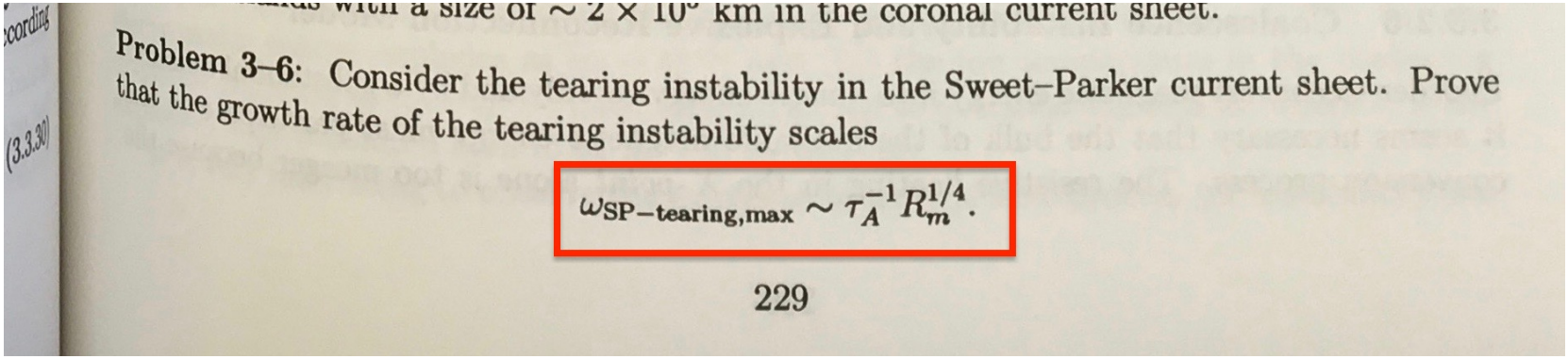
- We will see why the previous knowledge of the plasmoid instability was unsatisfactory

AND

- We will see what are the properties of this instability.

Assuming a Sweet-Parker aspect ratio...

- Tajima & Shibata, *Plasma Astrophysics* (1997)



with a size of $\sim 2 \times 10^5$ km in the coronal current sheet.

Problem 3–6: Consider the tearing instability in the Sweet–Parker current sheet. Prove that the growth rate of the tearing instability scales

$$\omega_{\text{SP-tearing,max}} \sim \tau_A^{-1} R_m^{1/4}.$$

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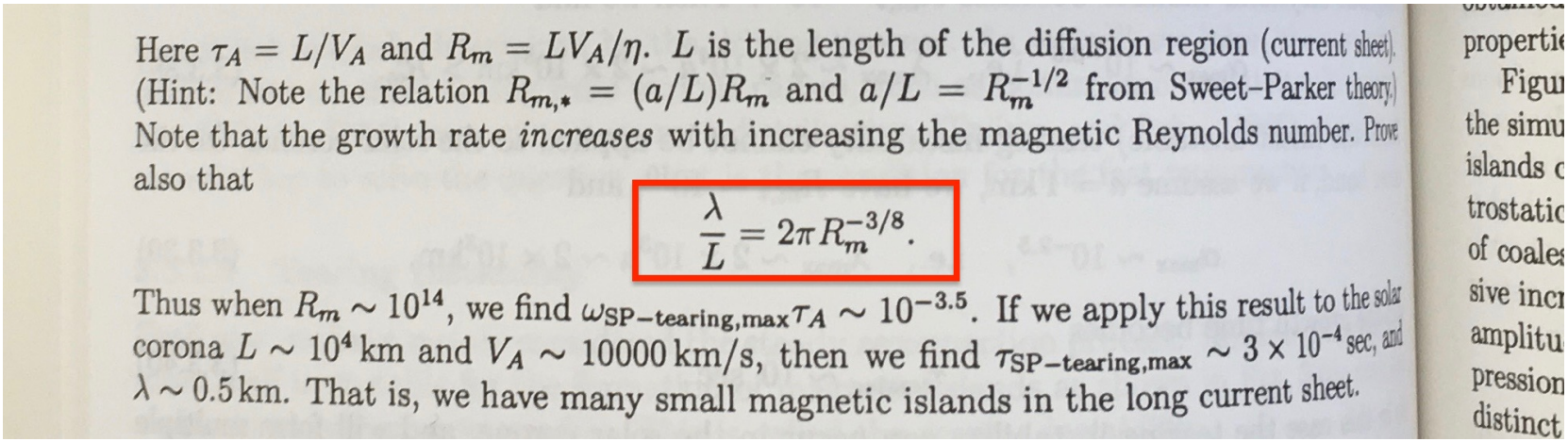


Here $\tau_A = L/V_A$ and $R_m = LV_A/\eta$. L is the length of the diffusion region (current sheet). (Hint: Note the relation $R_{m,*} = (a/L)R_m$ and $a/L = R_m^{-1/2}$ from Sweet-Parker theory.) Note that the growth rate *increases* with increasing the magnetic Reynolds number. Prove also that

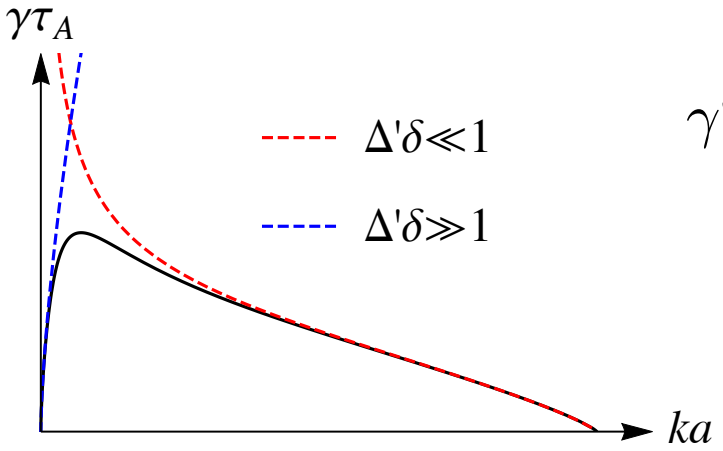
$$\frac{\lambda}{L} = 2\pi R_m^{-3/8}.$$

Thus when $R_m \sim 10^{14}$, we find $\omega_{\text{SP-tearing,max}} \tau_A \sim 10^{-3.5}$. If we apply this result to the solar corona $L \sim 10^4$ km and $V_A \sim 10000$ km/s, then we find $\tau_{\text{SP-tearing,max}} \sim 3 \times 10^{-4}$ sec, and $\lambda \sim 0.5$ km. That is, we have many small magnetic islands in the long current sheet.

(Note that the same result has been re-obtained 10 years later by Loureiro *et al.*, PoP 2007)

Let's do the exercise!

- Tearing mode dispersion relation (Coppi *et al.*, 1976):



$$\gamma^{5/4}\tau_H^{1/2}\tau_\eta^{3/4}\frac{\Gamma\left[\left(\Lambda^{3/2}-1\right)/4\right]}{\Gamma\left[\left(\Lambda^{3/2}+5\right)/4\right]} = -\frac{8}{\pi}\Delta'$$

$$\Lambda := \gamma\tau_H^{2/3}\tau_\eta^{1/3},\ \tau_H = \frac{1}{kv_A},\ \tau_\eta = \frac{a^2}{\eta}$$

- For $\Delta' a \sim 2/ka$ (FKR 1963), the fastest growing mode has

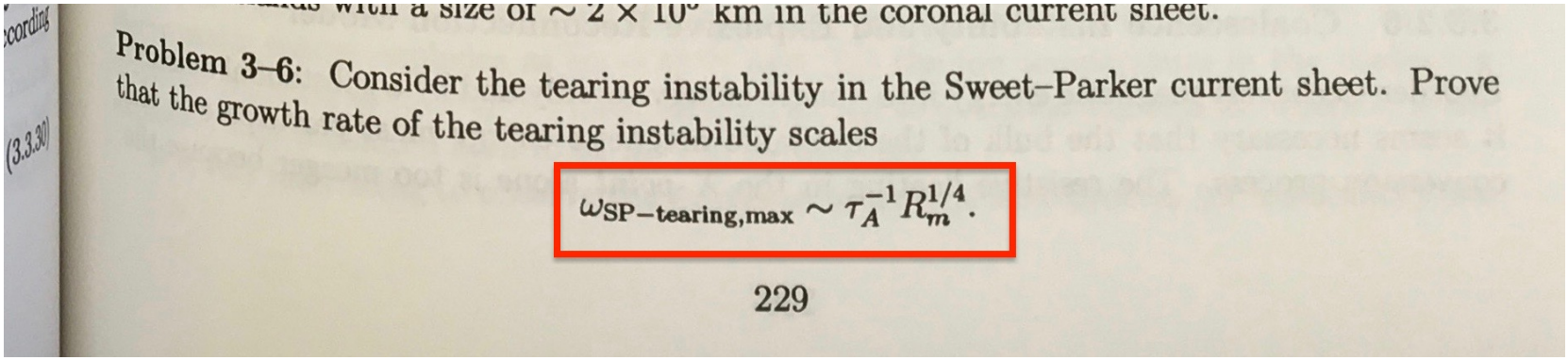
$$k_{max}a \sim S_a^{-1/4}\,, \quad \gamma_{max}\frac{a}{v_A} \sim S_a^{-1/2}\,. \tag{2}$$

- Using $a \sim LS^{-1/2}$ and $S_a \sim (a/L)S$, one easily obtain

$$k_{max}L \sim S^{3/8}\,, \quad \gamma_{max}\frac{L}{v_A} \sim S^{1/4}\,. \tag{3}$$

But there is a problem with this result...

- Tajima & Shibata, *Plasma Astrophysics* (1997)



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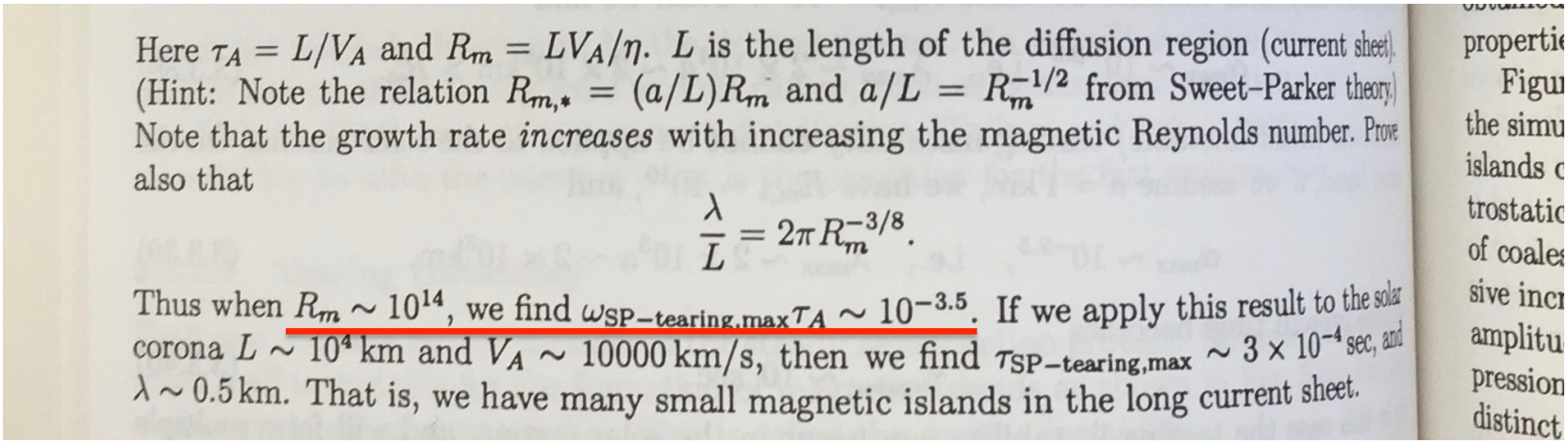

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The instability growth rate is too fast for large S-values!

Sweet-Parker sheets *cannot form* in large S plasmas!

What if we take into account plasma viscosity?

- Introducing the (perpendicular) Prandtl number $P_m = \nu/\eta$:

$$a \approx \frac{L(1+P_m)^{1/4}}{\sqrt{S}}\,, \quad v_{out} \approx \frac{v_A}{\sqrt{1+P_m}}\,, \quad v_{in} \approx \frac{v_A}{\sqrt{S}(1+P_m)^{1/4}}\,.$$

- Plasma viscosity modifies the previous scalings as (Comisso & Grasso, PoP 2016)

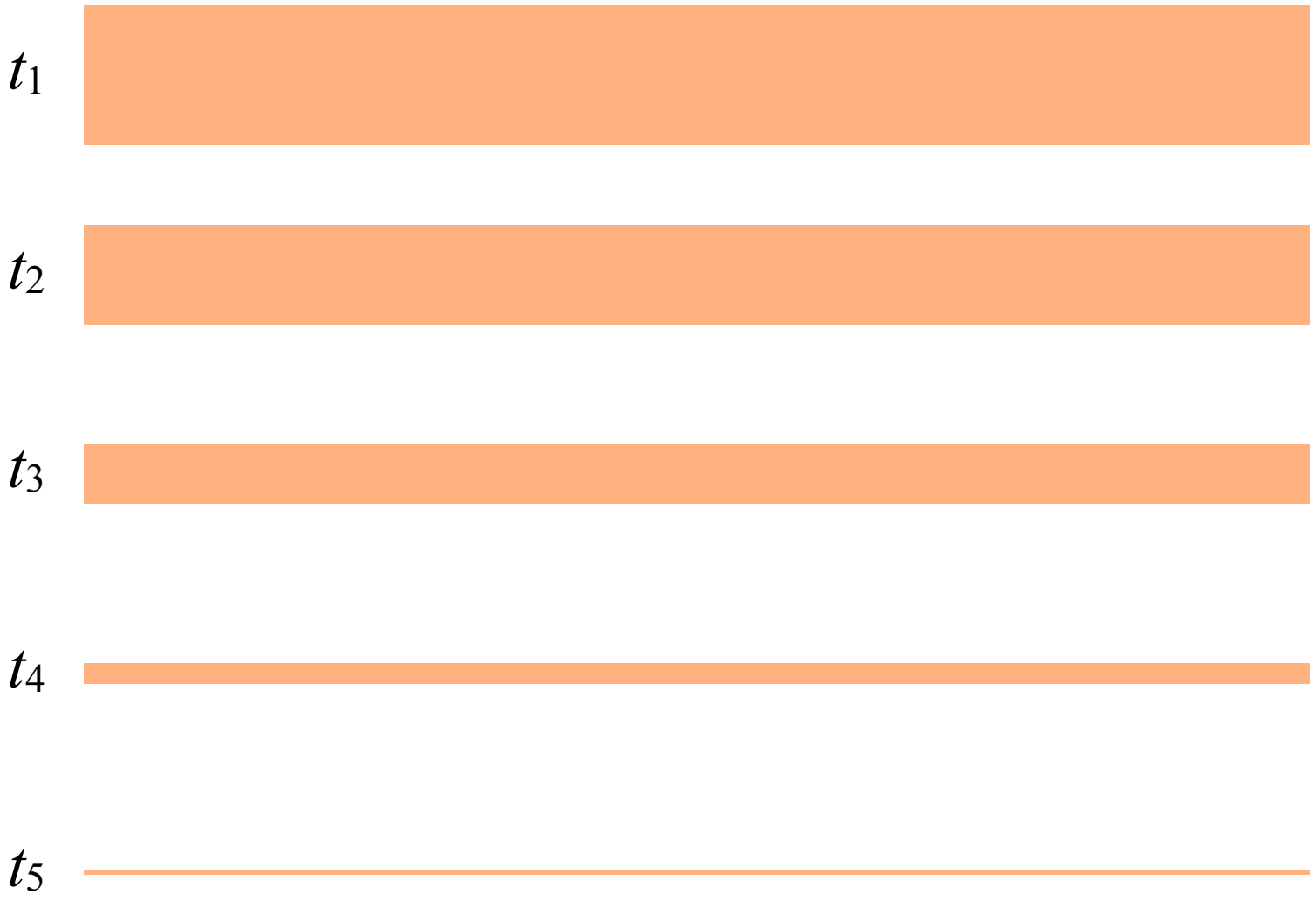
$$\text{growth rate} \Rightarrow \quad \gamma\tau_A \sim S^{1/4}(1+P_m)^{-5/8} \tag{4}$$

$$\text{wavenumber} \Rightarrow \quad kL \sim S^{3/8}(1+P_m)^{-3/16} \tag{5}$$

- γ decreases for increasing P_m, but... for realistic values of P_m $(10^{-4} \div 10^4)$ and $S >>> 1$, γ is still super-Alfvénic!
- The chosen equilibrium current sheet (Sweet-Parker sheet) *can not be attained* as current layers disrupt earlier!

Since in reality current sheets form over time...

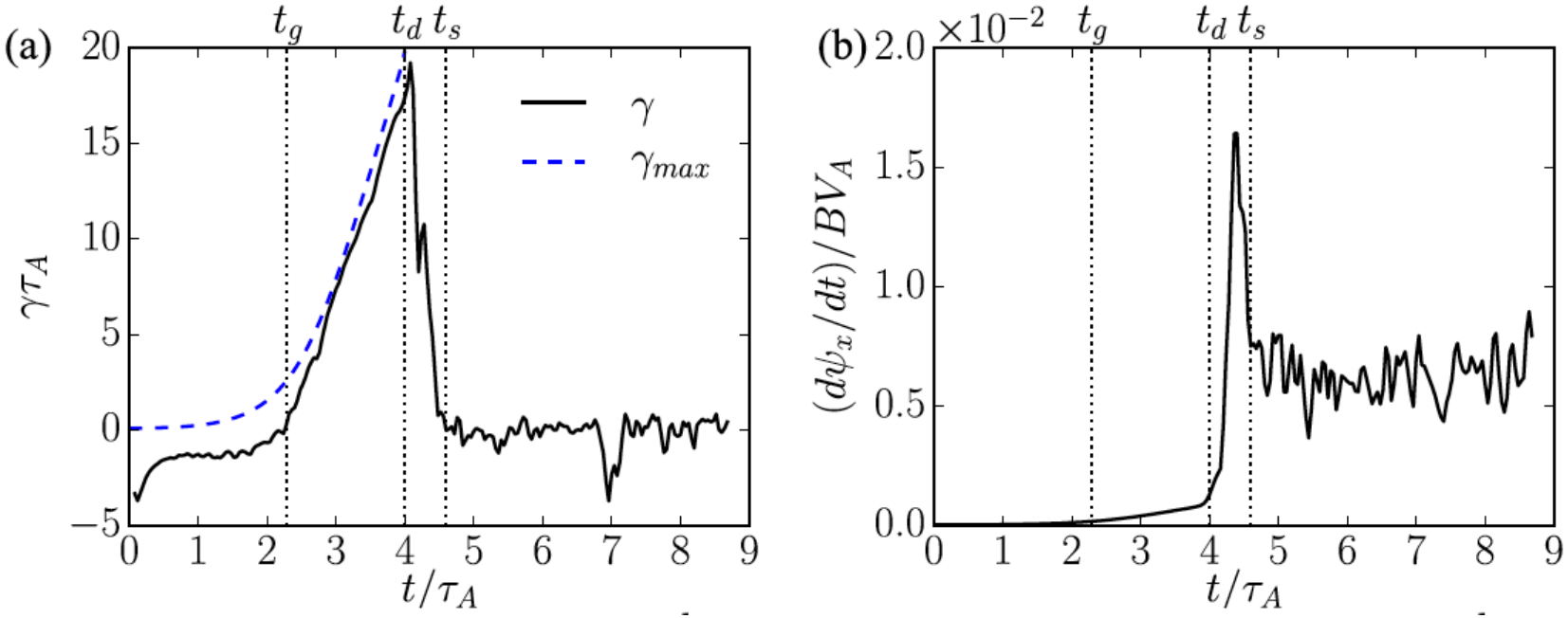
We need to consider a *time-evolving* current sheet



[1] Different recent theories: Pucci, Velli (2014); Uzdensky, Loureiro (2016)

A plot to keep well in mind

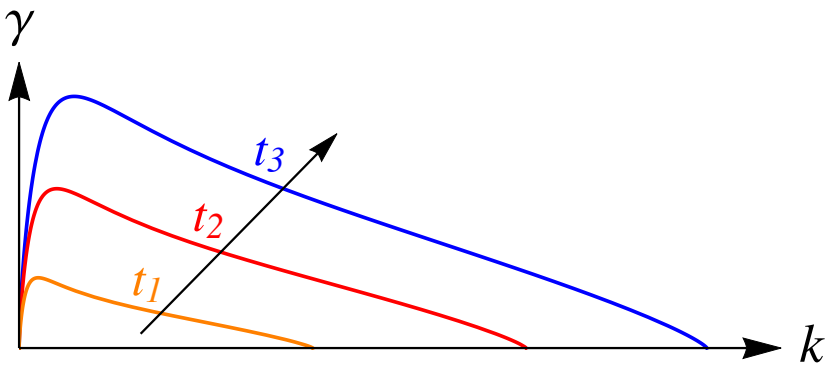
- The plasmoid instability remains quiescent for a certain time, and the fluctuation amplitude starts to grow only when $\gamma_{\max}\tau_A > O(1)$.
- Fast reconnection occurs at $t > t_d$.



Huang *et al.*, to be submitted (2017)

Need for a New Theory... [Comisso *et al.* (2016)]

Tearing modes become unstable at different times and exhibit different instantaneous growth rates $\gamma(k,t)$

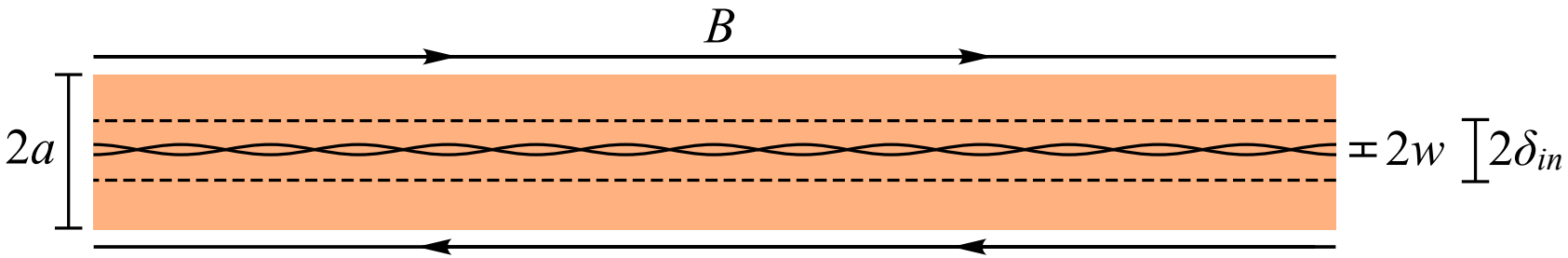


- Their amplitude changes in time according to

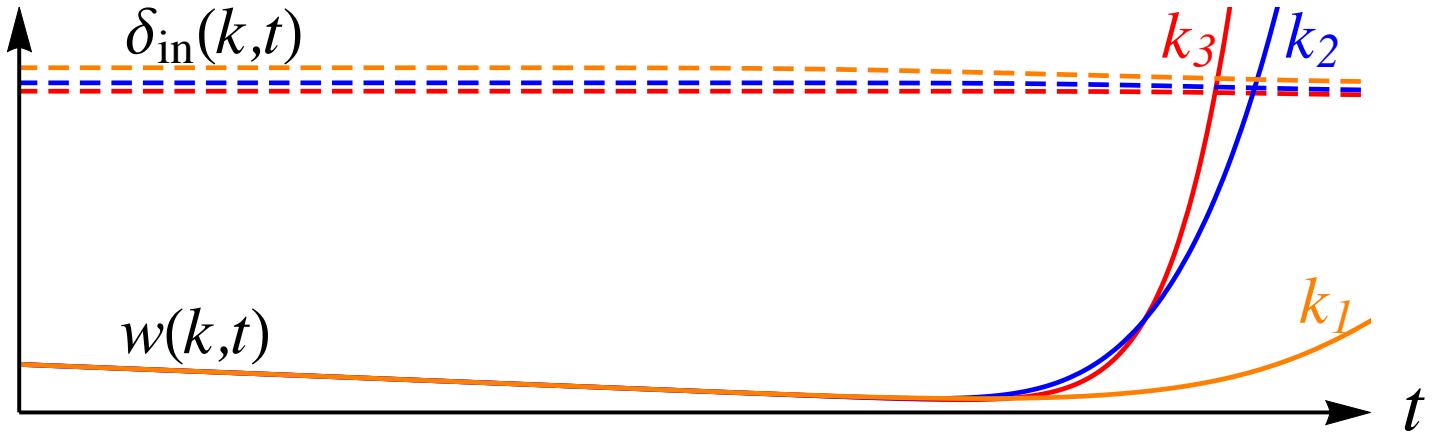
$$\psi(k,t) = \psi_0(k) \exp\left(\int_{t_0}^{t} \gamma(k,t')dt' \right). \tag{6}$$

- Their evolution becomes nonlinear when

$$w(k,t) = 2\sqrt{\frac{\psi a}{B}} \quad > \quad \delta_{\rm in}(k,t) = \left[\eta\gamma a^2/(kv_A)^2\right]^{1/4}. \tag{7}$$



Need for a New Theory... [Comisso *et al.* (2016)]



- Principle of Least Time for the Plasmoid Instability, i.e., *the mode of the plasmoid instability that emerges from the linear phase is the one that traverses it in the least time.*
- To implement this formulation, we introduce the function

$$F(k,t) := \delta_{\rm in}(k,t) - w(k,t)\,. \tag{8}$$

- Then, the least time principle is formulated as

$$F(k,t)|_{k_*,t_*} = 0\,, \qquad dt_*/dk_* = 0\,. \tag{9}$$

Need for a New Theory... [Comisso *et al.* (2016)]

- From Eqs. (6)-(9) we obtain the *least time plasmoid Eqs:*

$$\left\{\left(\gamma\bar{t}-\frac{1}{2}\right)\frac{\partial\gamma}{\partial k}+\frac{\gamma}{k}+2\frac{\gamma}{w_0}\frac{\partial w_0}{\partial k}\right\}\Bigg|_{k_*,t_*}=0 \qquad (10)$$

$$\left\{\ln\left(\frac{\delta_{\rm in}}{w_0}\frac{g^{1/2}}{f^{1/2}}\right)-\frac{1}{2}\int_{t_0}^{t}\gamma(t')dt'\right\}\Bigg|_{k_*,t_*}=0 \qquad (11)$$

with:

$$w_0=2\sqrt{\frac{\psi_0 a_0}{B_0}}\,,\quad \bar{t}=\frac{\partial}{\partial\gamma}\int_{t_0}^{t}\gamma(t')dt'\,,\quad f=\frac{a(t)}{a_0}\,,\quad g=\frac{B(t)}{B_0}\,.$$

- From these Eqs. it is possible to arrive at:

γ_* (final growth rate)	L_*/a_* (final aspect ratio)
k_* (final wavenumber)	t_* (elapsed time from t_0)
$\delta_{\rm in*}$ (final inner layer)	τ_p (time from $\hat{\gamma}(\hat{k}_*,\hat{t}_{\rm on})>1/\tau$)

Need for a New Theory... [Comisso *et al.* (2016)]

Until now the equations are very general.

Let's try to be more specific...

- We are interested in the case where:

$$L \approx \text{const.}, \quad B_0 \approx \text{const.}, \quad a(t) = a_0 f(t).$$

- For the moment, we consider an exponentially thinning current sheet (this will be generalized later) of the form

$$\hat{a}(\hat{t})^2 = (\hat{a}_0^2 - \hat{a}_\infty^2) e^{-2\hat{t}/\tau} + \hat{a}_\infty^2 \,, \qquad \text{with } \hat{a}_\infty = S^{-1/2}$$

- Here and in the following:
  - lengths normalized by the current sheet half-length L
  - time normalized by the Alfvén time $\tau_A = L/v_A$
  - magnetic field normalized by the upstream field B_0

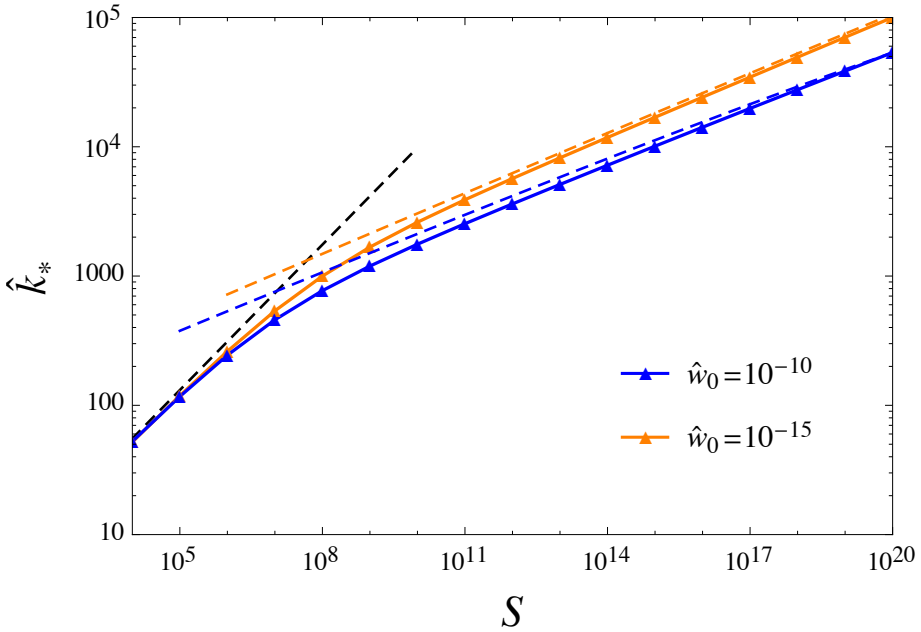
Need for a New Theory... [Comisso *et al.* (2016)]

- With some algebra we can see that there is a *Transitional* S

$$S_T = \frac{1}{\tau^4}\left[\ln\left(\frac{\tau^{9(2-\alpha)/2}}{2^6\varepsilon^3\tilde{\alpha}^{\tilde{\alpha}}}\right)\right]^4 \tag{12}$$

above which the plasmoid instability change behavior!

- The slope of $\hat{k}_*$ substantially *decreases* at large S !!!!!!!!!!



For $S < S_T$:

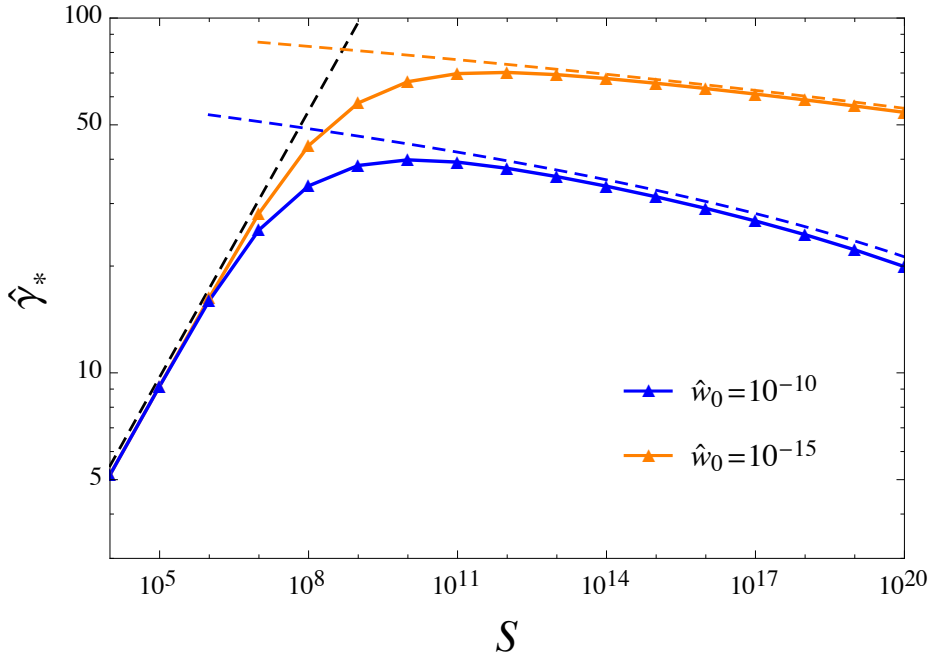
$$\hat{k}_* \sim S^{3/8}$$

For $S > S_T$:

$$\hat{k}_* \simeq c_k \frac{S^{1/6}}{\tau^{5/6}}\left[\ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{5/6}$$

Need for a New Theory... [Comisso *et al.* (2016)]

- The behavior of $\hat{\gamma}_*$ is *non-monotonic* in S !!!!!!!!!!



For $S < S_T$:

$$\hat{\gamma}_* \sim S^{1/4}$$

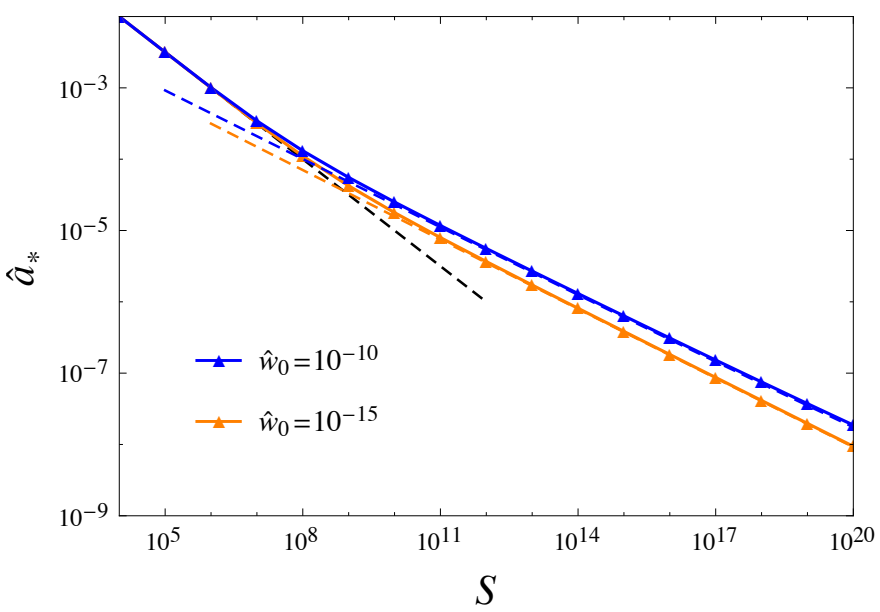
For $S > S_T$:

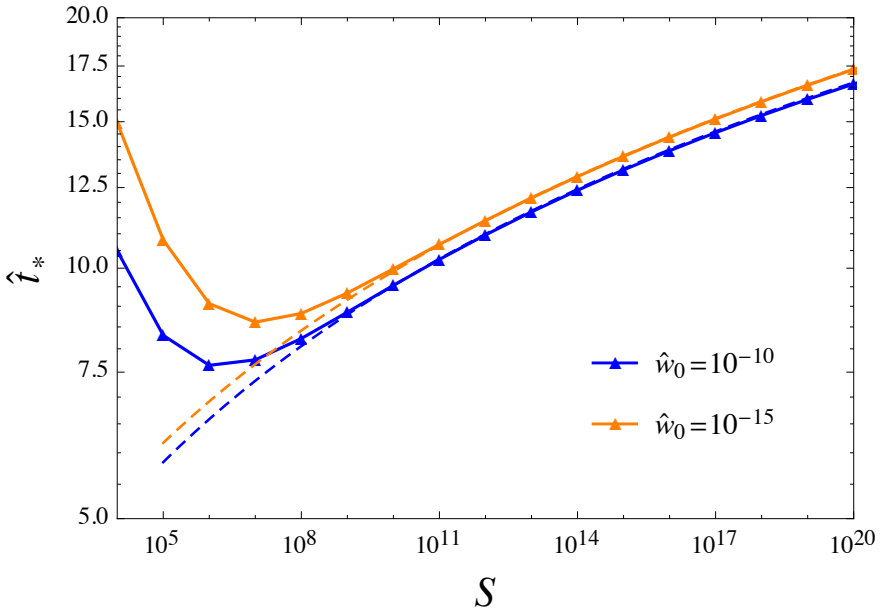
$$\hat{\gamma}_* \simeq \frac{c_\gamma}{\tau} \ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)$$

- The earlier scaling (black dashed line) is *not applicable* for large-S plasmas.
- $\hat{\gamma}_*$ can decrease with S because $\hat{\delta}_{\rm in}$ decreases with S
  $\Rightarrow$ less "space" to accelerate the perturbation growth

Need for a New Theory... [Comisso *et al.* (2016)]

- What about the disruption of the current sheet?

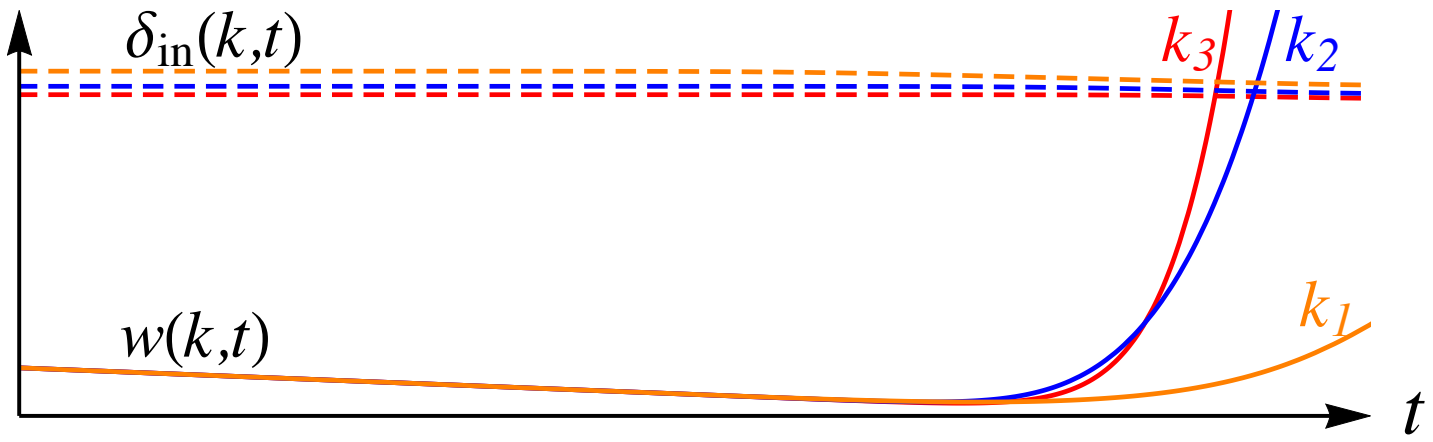




$$\hat{a}_* \simeq \frac{\tau^{2/3}}{S^{1/3}} \left[\ln\left(\frac{\tau}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{-2/3}, \qquad \hat{t}_* \simeq \tau \ln \left\{ \hat{a}_0 \frac{S^{1/3}}{\tau^{2/3}} \left[\ln\left(\frac{\tau}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{2/3} \right\}$$

- Current sheets disrupt before the Sweet-Parker state can be achieved (as expected!)

Need for a New Theory... [Comisso *et al.* (2016)]



- The emergent mode effectively starts growing when

$$\hat{\gamma}(\hat{k}_*, \hat{t}_{\rm on}) > 1/\tau$$

$$(\text{Occurs when } \hat{a} < \hat{k}_* S^{-1/2} \tau^{3/2})$$

- This leads to the *intrinsic* time scale of the instability

$$\tau_p \simeq \tau \ln \left\{ c_k \left[\ln \left(\frac{\tau}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{3/2} \right\} . \qquad (13)$$

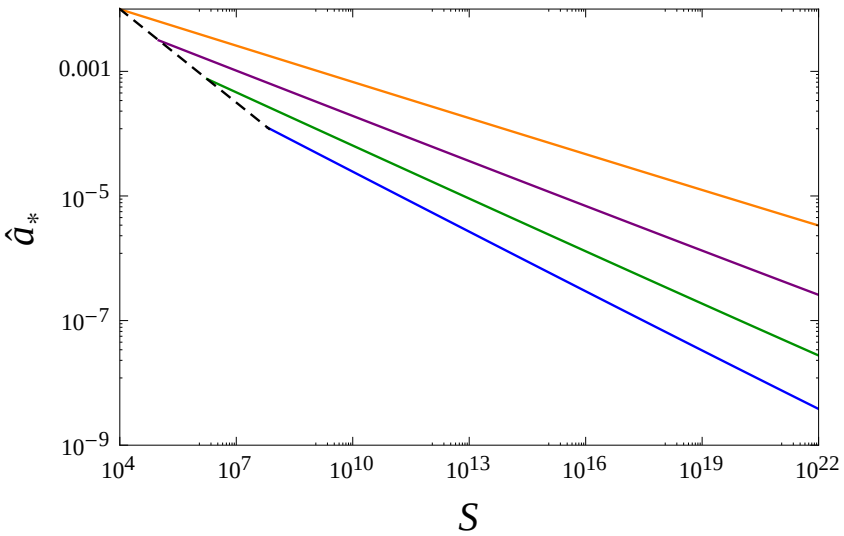
Need for a New Theory... [Comisso *et al.* (2016)]

- To generalize the previous scaling laws, we consider the generalized current thinning function

$$\hat{a}(\hat{t})^2 = (\hat{a}_0^2 - \hat{a}_\infty^2)\left(\frac{\tau}{\tau + 2\hat{t}/\chi}\right)^\chi + \hat{a}_\infty^2 \,, \qquad \text{with } \hat{a}_\infty = S^{-1/2}$$

- The final aspect-ratio ($\frac{1}{\hat{a}_*}$) depends on the thinning process!

$\chi = 2$, $\chi = 4$, $\chi = 10$, $\chi \to \infty$



$[\tau = 1,\ \hat{w}_0 = 10^{-12},\ \hat{a}_0 = 1/2\pi]$

$$\hat{a}_* \sim \hat{a}_0^{\frac{2\zeta}{\chi}} \frac{\tau^\zeta}{S^{\frac{\zeta}{2}}} \left[\ln\left(\frac{\tau^{\frac{3\zeta}{2}}}{S^{\frac{6+3\zeta}{4}}} \frac{\hat{a}_0^{\frac{3\zeta}{\chi}}}{2^6 \varepsilon^3}\right)\right]^{-\zeta}$$

$$\text{where } \zeta := \frac{2\chi}{4+3\chi}$$

Need for a New Theory... [Comisso *et al.* (2016)]

- And finally... also the scaling laws of the plasmoid instability at large S depend on the thinning process!

$$\hat{\gamma}_* \sim S^{(3\zeta-2)/4} \left(\frac{\ln(\theta_R)}{\hat{a}_0^{2/\chi} \tau} \right)^{3\zeta/2} , \tag{14}$$

$$\hat{k}_* \sim S^{(5\zeta-2)/8} \left(\frac{\ln(\theta_R)}{\hat{a}_0^{2/\chi} \tau} \right)^{5\zeta/4} , \tag{15}$$

$$\hat{\delta}_{\mathrm{in}*} \sim S^{-(3\zeta+2)/8} \left(\frac{\hat{a}_0^{2/\chi} \tau}{\ln(\theta_R)} \right)^{3\zeta/4} , \tag{16}$$

where

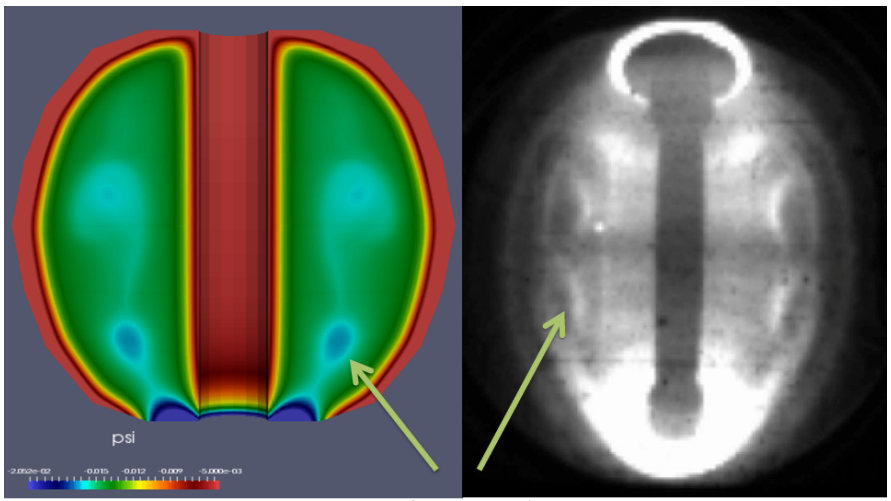
$$\theta_R := \frac{\tau^{3\zeta/2}}{S^{3(2+\zeta)/4}} \frac{\hat{a}_0^{3\zeta/\chi}}{2^6 \varepsilon^3} . \tag{17}$$

Connections with experiments and observations

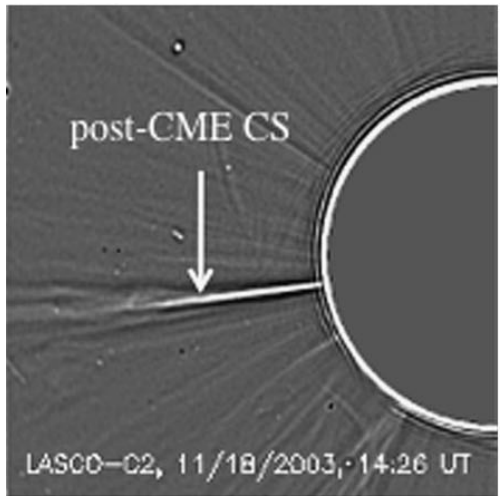
- Now we have a theory that can potentially predict the onset of fast magnetic reconnection.

  Opportunities to check the theory in the real world...

- The "easiest" quantities to check should be $\hat{a}_*$ and $\hat{t}_*$.



Ebrahimi & Raman, PRL (2015)



Lin *et al.*, SSRv (2015)

Concluding Remarks

- The scaling laws of the plasmoid instability *are not simple power laws*, and depend on:
  - The Lundquist number (S)
  - The noise of the system (ψ_0)
  - The characteristic rate of current sheet evolution ($1/\tau$)
  - The thinning process (χ)
  - Viscosity (P_m) and kinetic (to be included) effects.

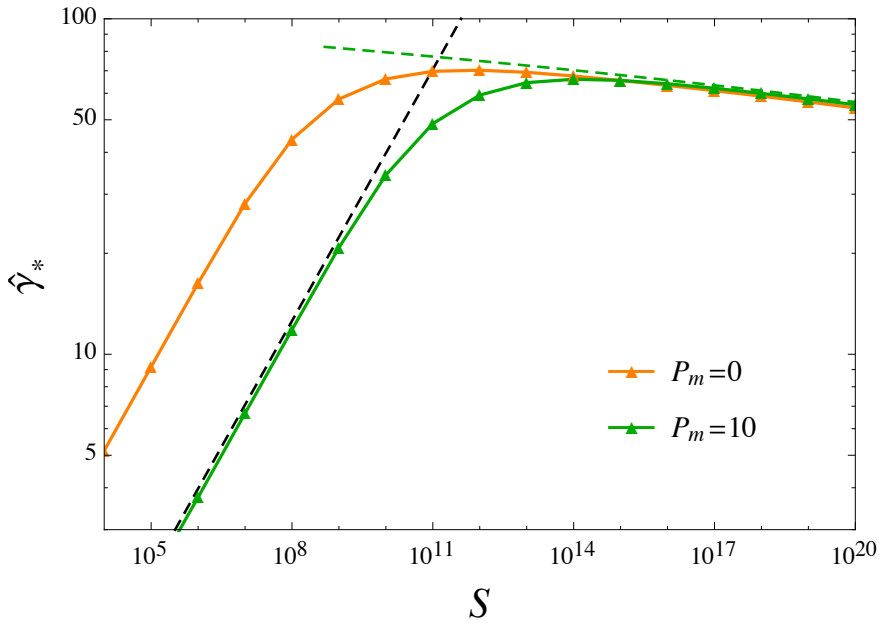
- In astrophysical systems, reconnecting current sheets break up before they can reach the Sweet-Parker aspect ratio.
  - The scaling laws of the plasmoid instability obtained assuming a Sweet-Parker current sheet are *inapplicable* to the vast majority of the astrophysical systems.

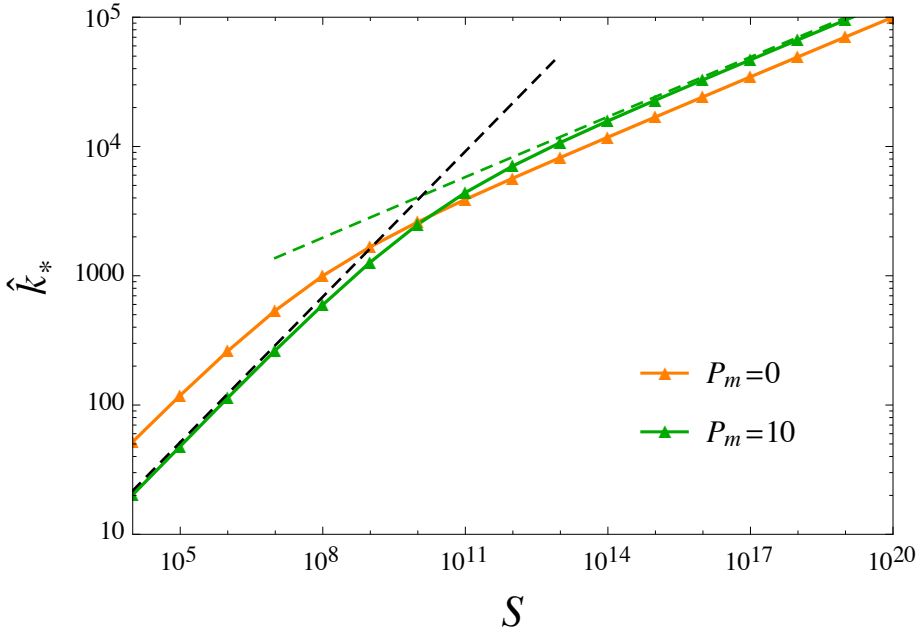
- The plasmoid instability comprises of a relatively long period of quiescence ($\hat{t}_* - \tau_p$) followed by rapid growth over a shorter timescale (τ_p).

Supplement: plasma viscosity

Also plasma viscosity could be important in several systems.

- Plasma viscosity allows to extend the validity of the Sweet-Parker based scalings to larger S-values ($S_T \uparrow$).

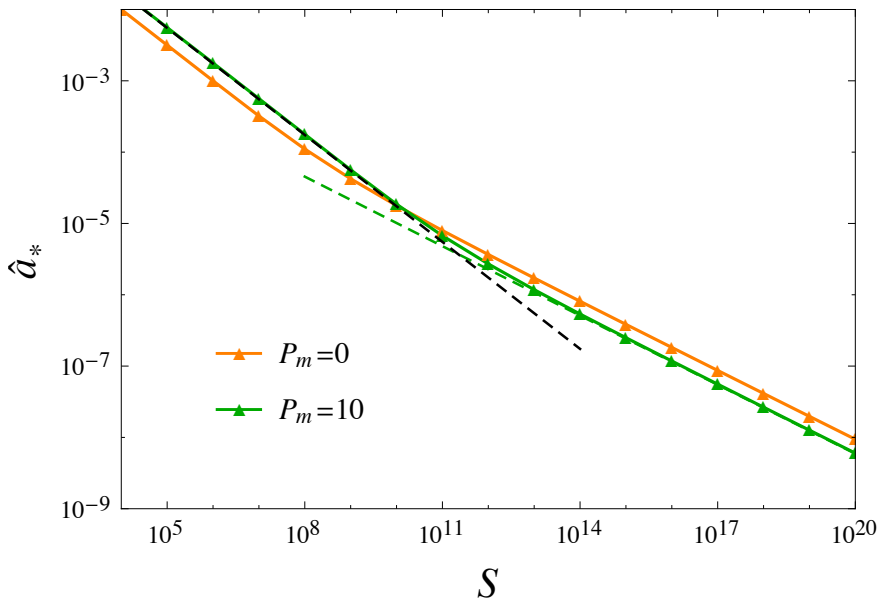


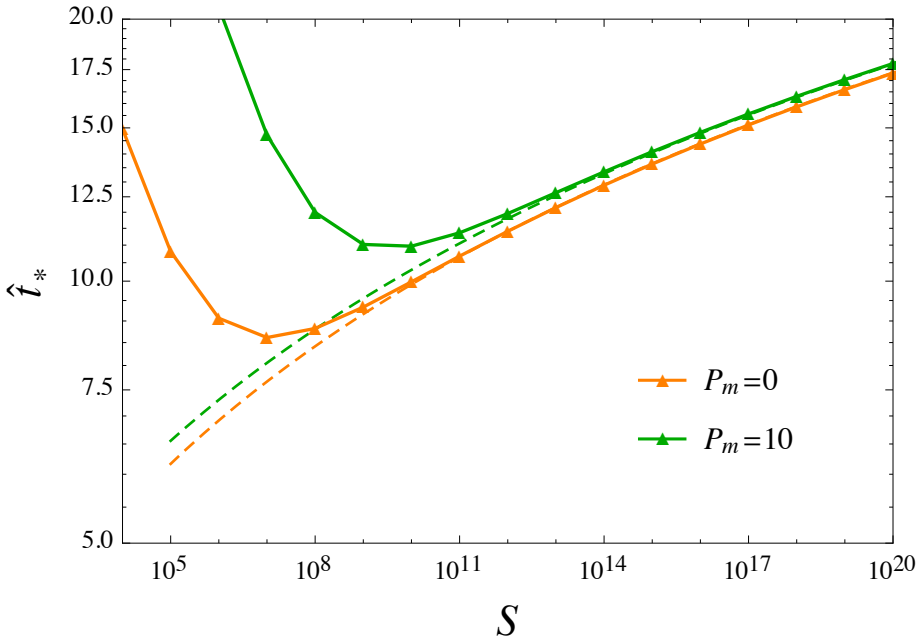


$$\hat{\gamma}_* \simeq \frac{\lambda_\gamma}{\tau} \ln\left(\frac{\tau P_m^{1/2}}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right), \qquad \hat{k}_* \simeq \lambda_k \frac{S^{1/6} P_m^{1/3}}{\tau^{5/6}} \left[\ln\left(\frac{\tau P_m^{1/2}}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{5/6}$$

Supplement: plasma viscosity

- At large S, plasma viscosity allows to reach *larger aspect ratios* of the reconnecting current sheets.





$$\hat{a}_* \simeq \frac{\tau^{2/3}}{S^{1/3} P_m^{1/6}} \left[\ln \left(\frac{\tau P_m^{1/2}}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{-2/3}, \quad \hat{t}_* \simeq \tau \ln \left\{ \frac{S^{1/3} P_m^{1/6}}{\tau^{2/3} / \hat{a}_0} \left[\ln \left(\frac{\tau P_m^{1/2}}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{2/3} \right\}$$