Plasmas simulation using real-time lattice QED

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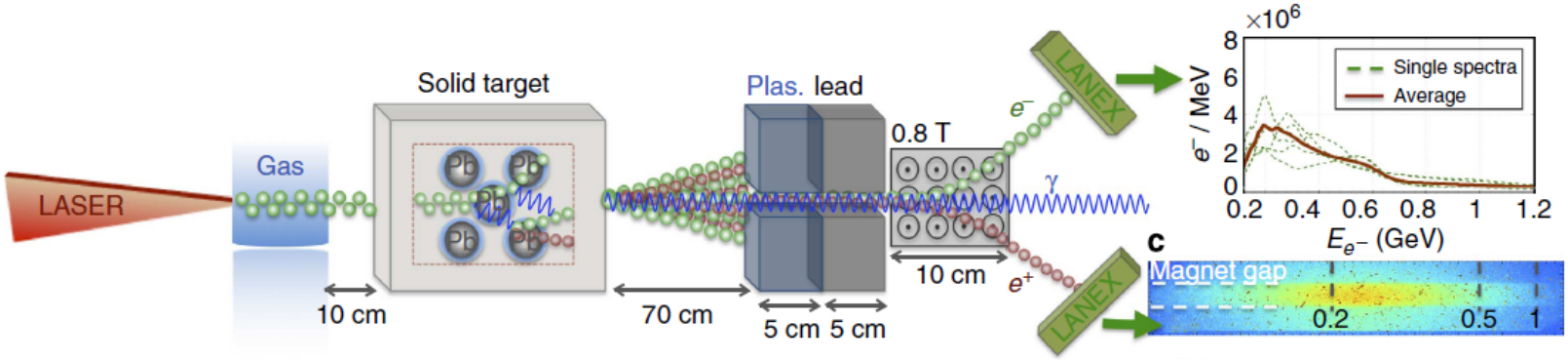
Plasma meets high-energy physics: indirect

Indirect laser-plasma pair production

- Plasma physics: laser wakefield acceleration $\Rightarrow e^-$ beam
- High-energy physics: e^- beam interact with solid $\Rightarrow \gamma, e^-/e^+$
- e^-/e^+ pair plasma? $\Rightarrow$ pulsar, gamma-ray burst, Big-Bang...
- Plasma & high-energy both important?

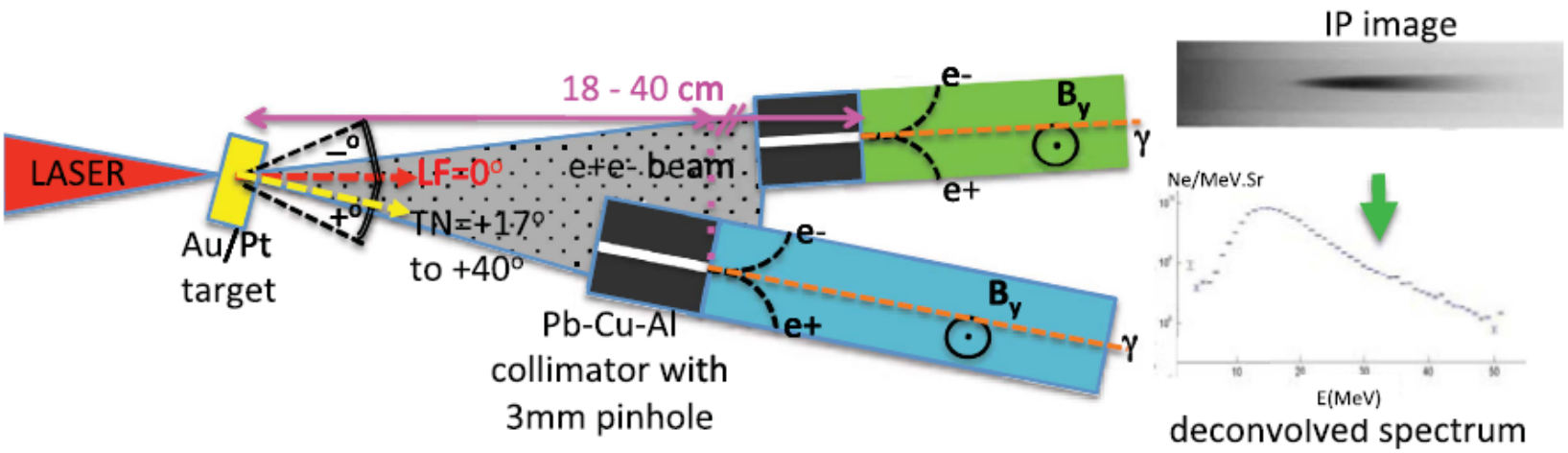


[1] Figure: G. Sarri, K. Poder, J. Cole, *et al.*, Nat. Commun. 6 (2015).

Plasma meets high-energy physics: direct

Direct laser-target pair production

- Laser heat/accelerate electrons in high-Z targets
- Pair production via Trident and Bethe-Heitler processes
- ◦ Plasma & high-energy both important?

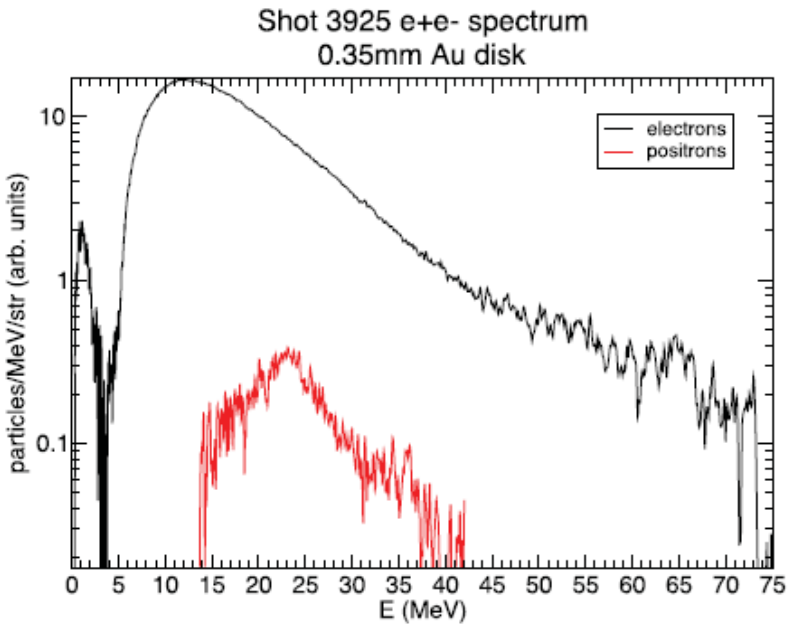


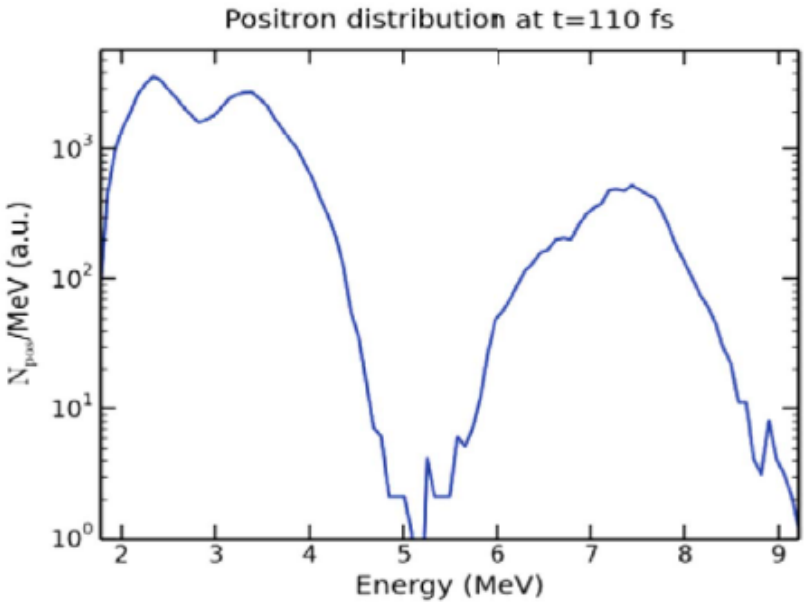
[1] Figure adapted from: E. Liang, T. Clarke, *et al.*, Sci. Rep. 5 (2015).

Evidence plasma & high-energy both important

Semiclassical model fails to capture spectrum

- PIC/Vlasov solver: capture collective plasma effects
- QED cross sections: source terms for particles/photons
- High-energy photons as free streaming particles





[1] Figure adapted from: E. Liang, T. Clarke, *et al.*, Sci. Rep. 5 (2015).

Could semiclassical plasma model work?

Advantages

- $\surd$ Scale separation

  Laser $\sim 10^{-6}$ m
  Plasma $\sim 10^{-9}$ m
  QED $\sim 10^{-12}$ m

- $\surd$ QED cross sections well-known in vacuum
- $\surd$ Plasma kinetic models well developed

Limitations

- $\times$ Energy-momentum conservation
  - What happens to fields after pair production?
  - What happens to electrons after radiating γ-photons?
- $\times$ Interplay between processes, quantum interference
  - Schwinger pair $\leftrightarrow$ laser pulse?

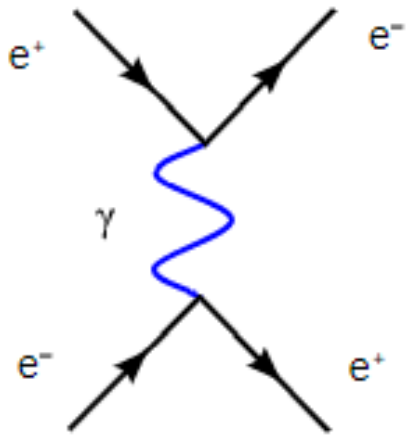
Overcome limitations: beyond semiclassical mode

First principle models

Quantum electrodynamics (QED)

- Dirac model: charged Fermions

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



- Klein-Gordon model: charged Bosons

$$\mathcal{L} = \overline{(D_\mu\phi)}(D^\mu\phi) - m^2\bar{\phi}\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

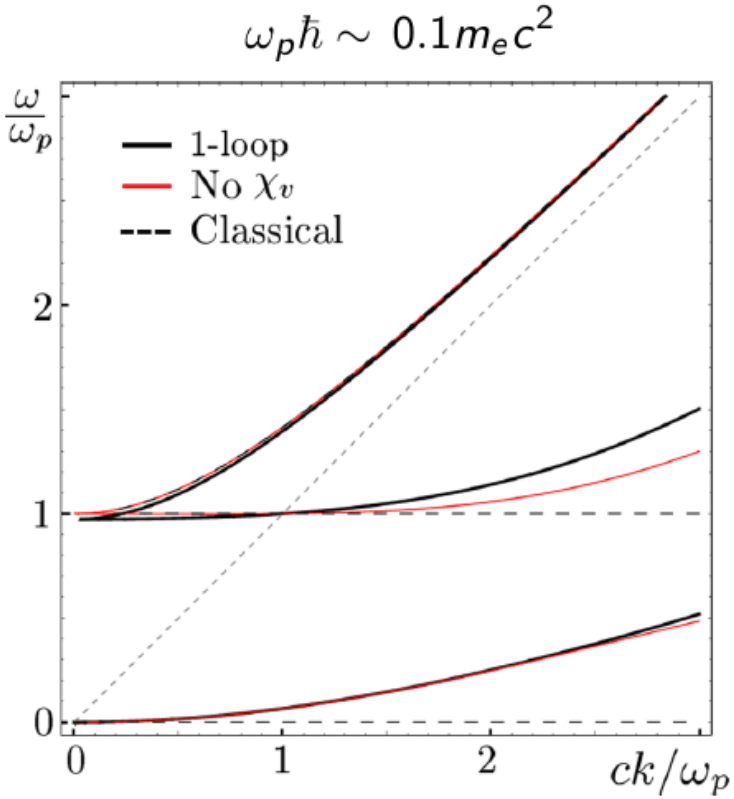
- EM fields enter through field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

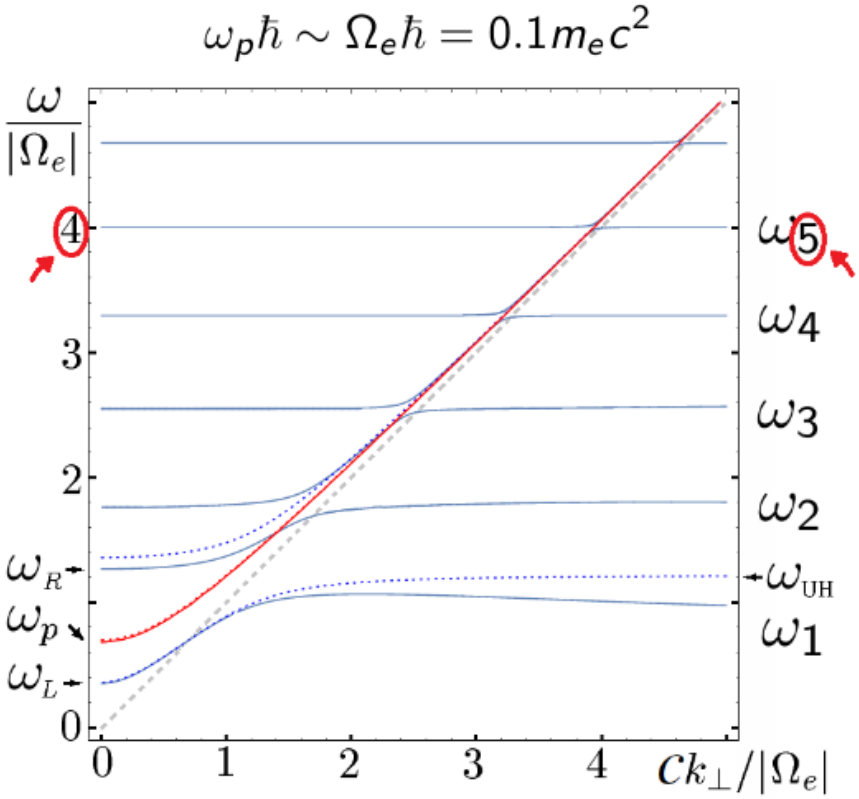
  Couple through covariant derivative $D_\mu = \partial_\mu - ieA_\mu$

What can QED say about plasmas?

Linear wave propagation in exotic environments [1]

- QED for plasma ↔ QM for solids
  Band structure, dielectric response, wave dispersion relation ...





[1] Y. Shi, N. J. Fisch, and H. Qin, Phys. Rev. A 94, 012124 (2016).

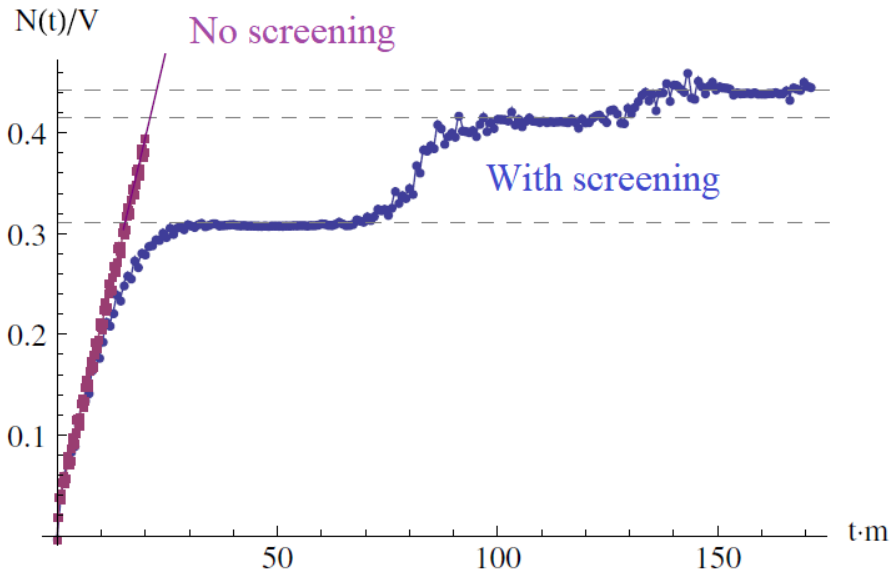
How about nonlinear QED effects?

Background-field QED theory

- Strong field: Schwinger pair, multi-photon processes [1] ...
- Dense plasma: Fermi degeneracy, parametric instability [2] ...

Beyond analytical theory: QED simulations

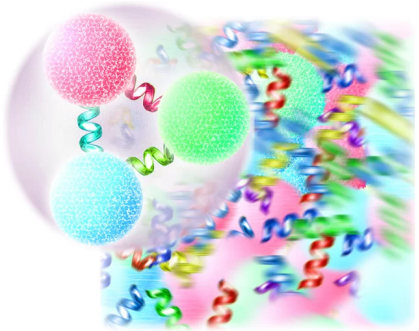
Schwinger pairs screen background E-field [3] ...



[1] S. Meuren, K. Hatsagortsyan, *et al.*, Phys. Rev. D 91, 013009 (2015).
[2] B. Eliasson and P. K. Shukla, Phys. Rev. E 83, 046407 (2011).
[3] V. Kasper, F. Hebenstreit, and J. Berges, Phys. Rev. D 90, 025016 (2014).

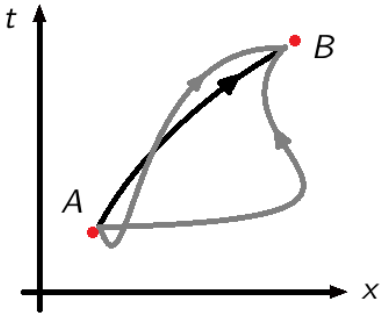
How to do QED simulation?

Simulation of nuclear matter [1]



- Quarks and gluon: SU(3) particles
  Quantum chromodynamics (QCD)
- Low energy QCD is strongly-coupled, commonly solved by lattice-QCD

Lattice-QED simulation



- Standard lattice simulation: path integral
- Equilibrium simulation: + Wick rotation
- Non-equilibrium: + Schwinger-Keldysh time contours
- ○ Classical-statistic regime: classical path dominate

[1] Image: Lawrence Berkeley National Laboratory

Plasma is in classical-statistic regime

Classical-statistic (CS) regime

- Weak coupling ($e \ll 1$): tree-level effects dominate
- Large occupation ($N \gg 1$): background fields dominate

$\Rightarrow$ Classical fields dominate quantum fluctuations

Classical fields $\Leftarrow$ Minimize action

- Charged fermions: Dirac equation

$$(i\gamma^\mu D_\mu - m)\psi = 0$$
$$j^\mu = e\bar{\psi}\gamma^\mu\psi$$

- Charged bosons: Klein-Gordon equation

$$(D_\mu D^\mu - m^2)\phi = 0$$
$$j^\mu = \frac{e}{i}(\bar{\phi}D^\mu\psi - \text{c.c.})$$

- EM fields: Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$

CS regime: real-time lattice simulation

What can we learn?

- Strong E-field + vacuum: Schwinger pair production [1]
- Strong laser field + dense plasma: laser pair production [2]

$\Rightarrow$ Interplay between collective & QED processes

What is required?

- Resolution: length (m) $\leftrightarrow$ $c\times$ time (s)
  - Compton $10^{-12}(m_e/m)$
  - Laser $10^{-9}(\mathrm{KeV}/\omega)$
  - Density $10^{-9}(10^{21}\mathrm{c.c.}/n_0)^{\frac{1}{3}}$
  - Skin $10^{-7}(10^{21}\mathrm{c.c.}/n_0)^{\frac{1}{2}}$
  - E-field $10^{-12}(E/10^{18}\mathrm{V/m})^{\frac{1}{2}}$
  - B-field $10^{-12}(B/10^{9}\mathrm{T})^{\frac{1}{2}}$
- Difficult in weak-field regime: scale separation
- Indispensable in strong-field regime: scales overlap

[1] F. Hebenstreit, *et al.* Phys. Rev. Lett. 111, 201601 (2013).
[2] **Y. Shi, J. Xiao, H. Qin, and N. J. Fisch, arXiv:1802.00524**

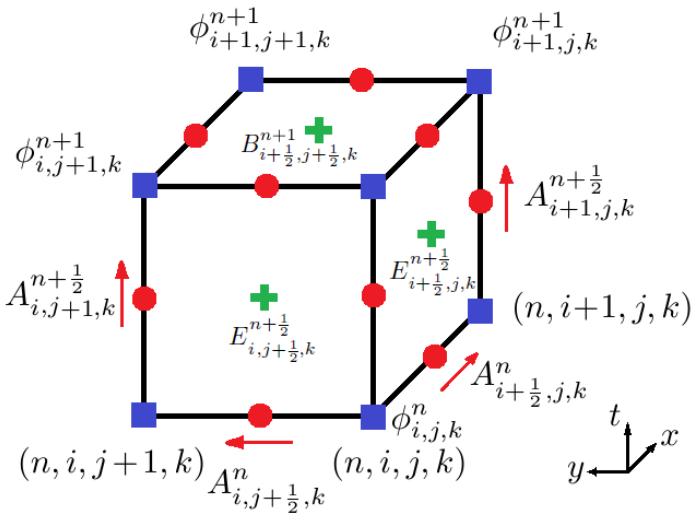
Lattice plasma simulation: solve field equations

Toy model: scalar-QED

- Bosonic plasma: simplest QED plasma
  - Klein-Gordon's Equation: $(D_\mu D^\mu + m^2)\phi = 0$
  - Maxwell's Equation : $\partial_\mu F^{\mu\nu} = j^\nu$

Discretization using exterior calculus (DEC)

- ϕ lives on vertexes $\phi^n_{i,j,k}$
- A_μ, D_μ lives on edges $A^n_{i+1/2,j,k}, A^{n+1/2}_{i,j,k}$
- $F_{\mu\nu}$ lives on faces $E^{n+1/2}_{i+1/2,j,k}, B^n_{i+1/2,j+1/2}$



$\Rightarrow$ Discrete action $S_d = \sum_c L_d[\phi_v, A_e]\Delta V$

$$L_d = (\overline{D_\mu\phi})_e(D^\mu\phi)_e - m^2\bar{\phi}_v\phi_v + \frac{1}{2}(E_f^2 - B_f^2)$$

DEC ensures geometric identities

Discrete Exterior Calculus

- Electric field $E^1 = F_{01}$

$$E^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} = \frac{1}{\Delta t}\left(A^{n+1}_{i+\frac{1}{2},j,k} - A^{n}_{i+\frac{1}{2},j,k}\right) - \frac{1}{\Delta x}\left(A^{n+\frac{1}{2}}_{i+1,j,k} - A^{n+\frac{1}{2}}_{i,j,k}\right)$$

- Magnetic field $B^3 = -F_{12}$

$$B^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} = \frac{1}{\Delta y}\left(A^{n}_{i+\frac{1}{2},j+1,k} - A^{n}_{i+\frac{1}{2},j,k}\right) - \frac{1}{\Delta x}\left(A^{n}_{i+1,j+\frac{1}{2},k} - A^{n}_{i,j+\frac{1}{2},k}\right)$$

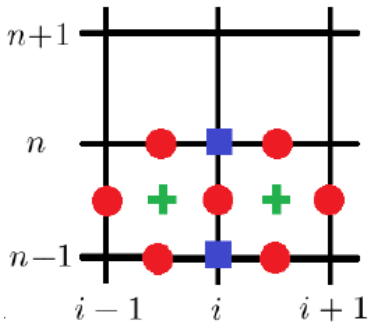
Bianchi identities automatically satisfied

- Exterior derivative $d^2 = 0$. Special case $dF = d^2 A = 0$

- $\nabla \cdot \mathbf{B} = 0 \Rightarrow \frac{1}{\Delta_l}\left(B^n_{r+\frac{l}{2}} - B^n_{r-\frac{l}{2}}\right) = 0$
- $\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \Rightarrow \frac{1}{\Delta t}\left(B^{n+1}_{r-\frac{i}{2}} - B^n_{r-\frac{i}{2}}\right) = \frac{\epsilon_{ijk}}{\Delta_k}\left(E^{n+\frac{1}{2}}_{s+\frac{j}{2}+k} - E^{n+\frac{1}{2}}_{s+\frac{j}{2}}\right)$

Initial step: Gauss' law solve vector potential

Equation from discretized action S_d



- Variational principle: $\delta S_d / \delta A_s^{n+1/2} = 0$

$$\frac{1}{\Delta_l}\left(E_{s+\frac{l}{2}}^{n+\frac{1}{2}} - E_{s-\frac{l}{2}}^{n+\frac{1}{2}}\right) = J_s^{n+1/2}$$

- Discrete version of $\partial_t \nabla \cdot \mathbf{A} = -\nabla^2 A^0 - \rho$

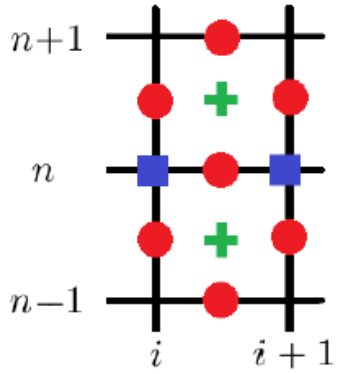
$$\frac{A_{s+l/2}^1 - A_{s-l/2}^1}{\Delta t \Delta_l} = \frac{A_{s+l/2}^0 - A_{s-l/2}^0}{\Delta t \Delta_l} + \frac{A_{s+l}^{1/2} - 2A_s^{1/2} + A_{s-l}^{1/2}}{\Delta l^2} + J_s^{1/2}$$

- Easier to solve than Poisson's equation: RHS known !
  Bidiagonal matrix in 1D $\Rightarrow$ Easier than Poisson's equation

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 \end{pmatrix} \begin{pmatrix} A_{1/2} \\ A_{3/2} \\ A_{5/2} \\ \vdots \\ A_{n+1/2} \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{pmatrix}$$

Field evolution: only need Ampère's law

Equation from discretized action S_d



- Variational principle: $\delta S_d / \delta A^n_{i+1/2,j,k} = 0$

$$\frac{1}{\Delta t}\left(E^{n+\frac{1}{2}}_{s+\frac{i}{2}} - E^{n-\frac{1}{2}}_{s+\frac{i}{2}}\right) = \frac{\epsilon_{ijk}}{\Delta_j}\left(B^n_{r-\frac{k}{2}} - B^n_{r-\frac{k}{2}-j}\right) + J^n_{s+\frac{i}{2}}$$

- Explicit time advance $(A^{n-1}_{s+l/2}, A^{n\pm 1/2}_s, A^n_{s+l/2}; \phi^n_s) \to A^{n+1}_{s+l/2}$

$$\begin{aligned} A^{n+1}_{s+\frac{i}{2}} &= A^n_{s+\frac{i}{2}} + \frac{\Delta t}{\Delta_i}\left(A^{n+\frac{1}{2}}_{s+i} - A^{n+\frac{1}{2}}_s\right) + \Delta t^2 J^n_{s+\frac{i}{2}} \\ &+ \Delta t\left[E^{n-\frac{1}{2}}_{s+\frac{i}{2}} + \epsilon_{ijk}\frac{\Delta t}{\Delta_j}\left(B^n_{r-\frac{k}{2}} - B^n_{r-\frac{k}{2}-j}\right)\right] \end{aligned}$$

- ◦ Discrete Faraday's law automatically satisfied by DEC

$\Rightarrow$ Less dynamical equation than Yee's algorithm

Advance charged particles: KG equation

Equation from discretized action S_d

- Variational principle: $\delta S_d / \delta \bar{\phi}_v = 0$

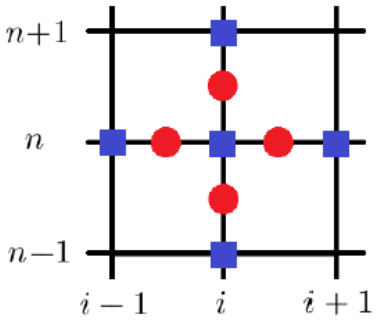
$$\frac{1}{\Delta t^2}\left(\phi_s^{n+1} e^{-iq\Delta t A_s^{n+\frac{1}{2}}} - 2\phi_s^n + \phi_s^{n-1} e^{iq\Delta t A_s^{n-\frac{1}{2}}}\right)$$
$$= \frac{1}{\Delta_l^2}\left(\phi_{s+l}^n e^{-iq\Delta_l A_{s+\frac{l}{2}}^n} - 2\phi_s^n + \phi_{s-l}^n e^{iq\Delta_l A_{s-\frac{l}{2}}^n}\right) - m^2\phi_s^n$$

- Charge density: time-component of 1-form

$$J_s^{n+1/2} = \frac{iq}{\Delta t}\left(\bar{\phi}_s^{n+1} e^{iq\Delta t A_s^{n+\frac{1}{2}}} \phi_s^n - \text{c.c.}\right)$$

- Current density: space-component of 1-form

$$J_{s+\frac{l}{2}}^n = \frac{iq}{\Delta_l}\left(\bar{\phi}_{s+l}^n e^{iq\Delta_l A_{s+\frac{l}{2}}^n} \phi_s^n - \text{c.c.}\right)$$

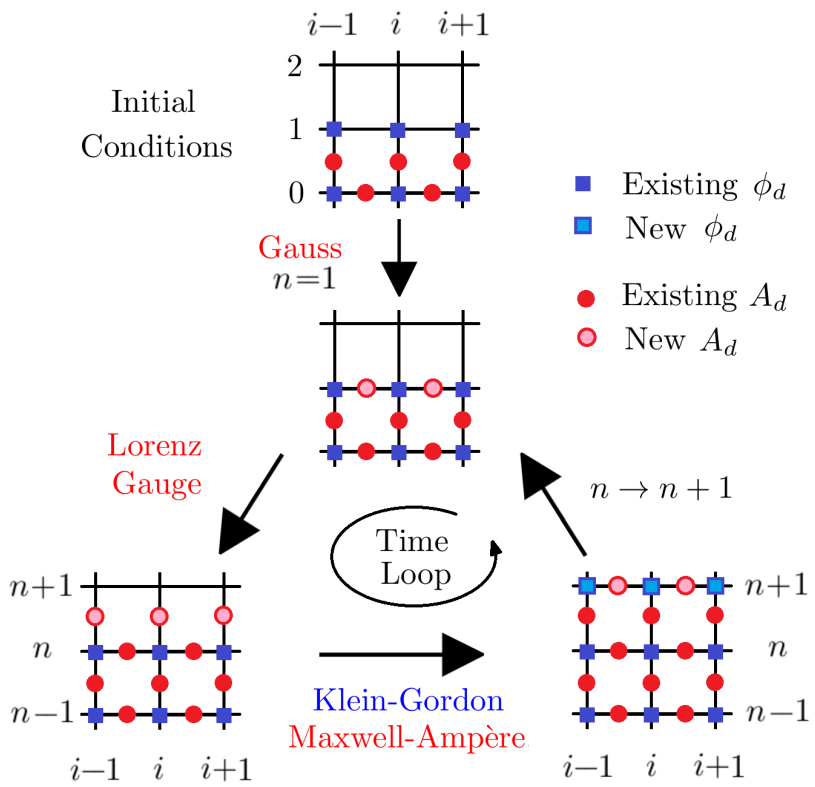


- U(1)-gauge symmetry $\Rightarrow$ Charge conservation

$$\frac{1}{\Delta t}\left(J_s^{n+\frac{1}{2}} - J_s^{n-\frac{1}{2}}\right) = \frac{1}{\Delta_l}\left(J_{s+\frac{l}{2}}^n - J_{s-\frac{l}{2}}^n\right)$$

Numeric scheme: explicit, parallelizable

- Initial values $\phi_s^0, \phi_n^1; A_{s+l/2}^0, A_s^{1/2}$
- Initial step $A_{s+l/2}^1$: Gauss's law
- Time advance:
  - Gauge condition $(A_s^{n-1/2}, A_{s+l/2}^n) \to A_s^{n+1/2}$
  - Klein-Gordon equation $(\phi_s^{n-1}, \phi_s^n; A_{s+l/2}^n, A_s^{n\pm 1/2}) \to \phi_s^{n+1}$
  - Maxwell-Ampère's law $(A_{s+\frac{l}{2}}^{n-1}, A_{s+\frac{l}{2}}^n, A_s^{n\pm\frac{1}{2}}; \phi_s^n) \to A_{s+\frac{l}{2}}^{n+1}$



Resolution required:

$$\Delta t < \Delta x < \frac{1}{H}, \frac{1}{qA}$$

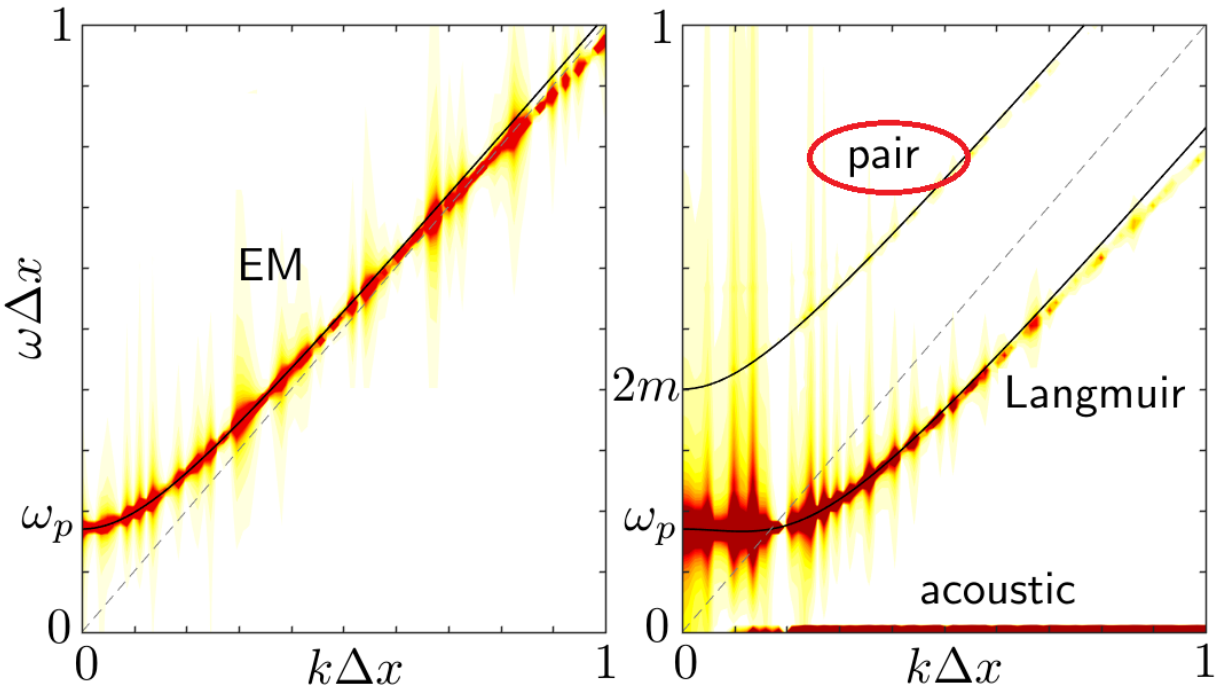
Code verification: capture linear wave spectra

Small amplitude waves in unmagnetized e-i plasma

- Initial conditions: random $\langle A_e \rangle = 0$, $\langle \phi_v \bar{\phi}_v \rangle = n_0/2m$
- Power spectra of E-field components **recover analytic** dispersion relations up to grid resolution $k\Delta x \sim 1$

◦ Transverse waves: gapped EM modes, degenerate

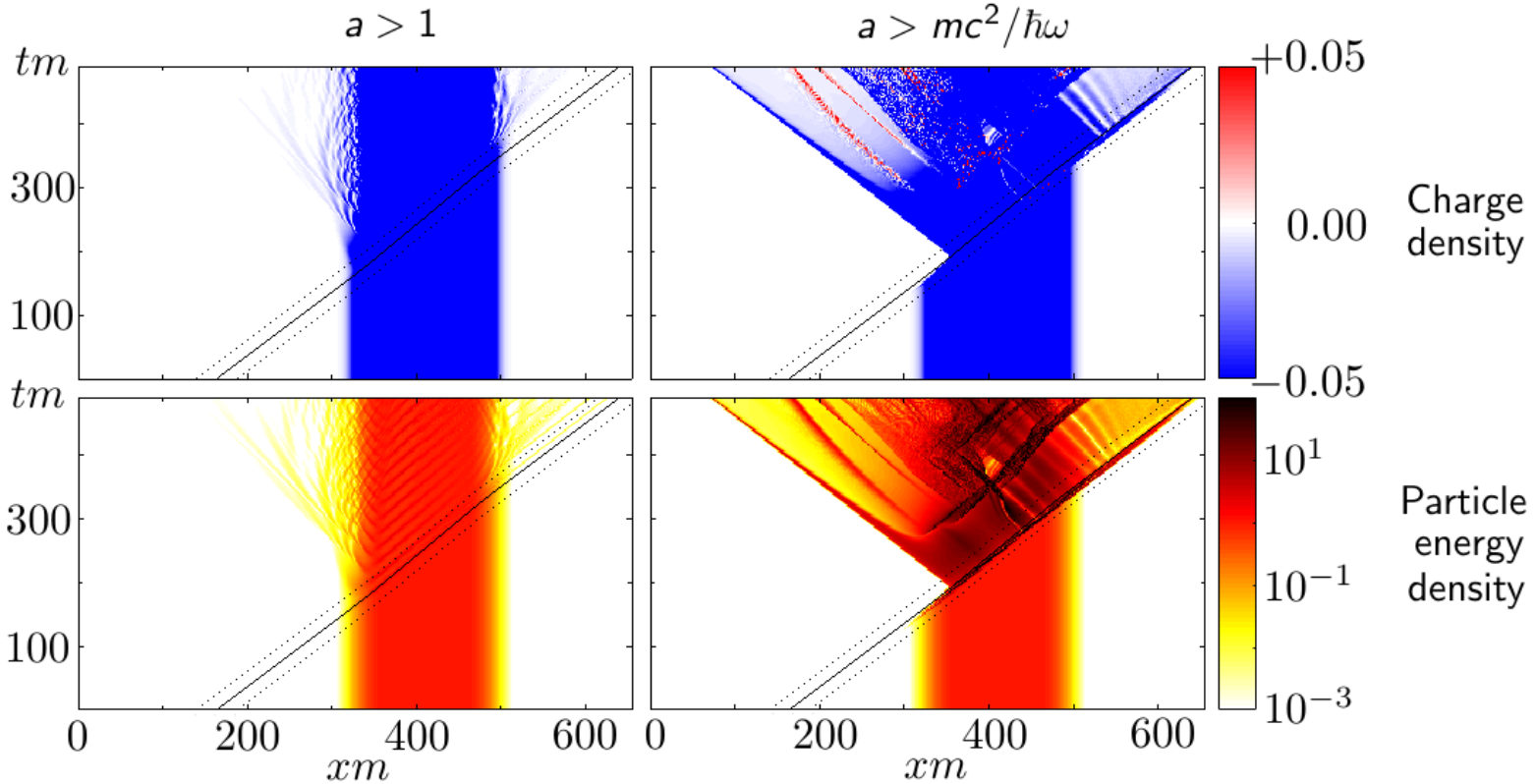
◦ Longitudinal waves: acoustic, Langmuir, pair modes



Numeric example: x-ray laser + 1D slab

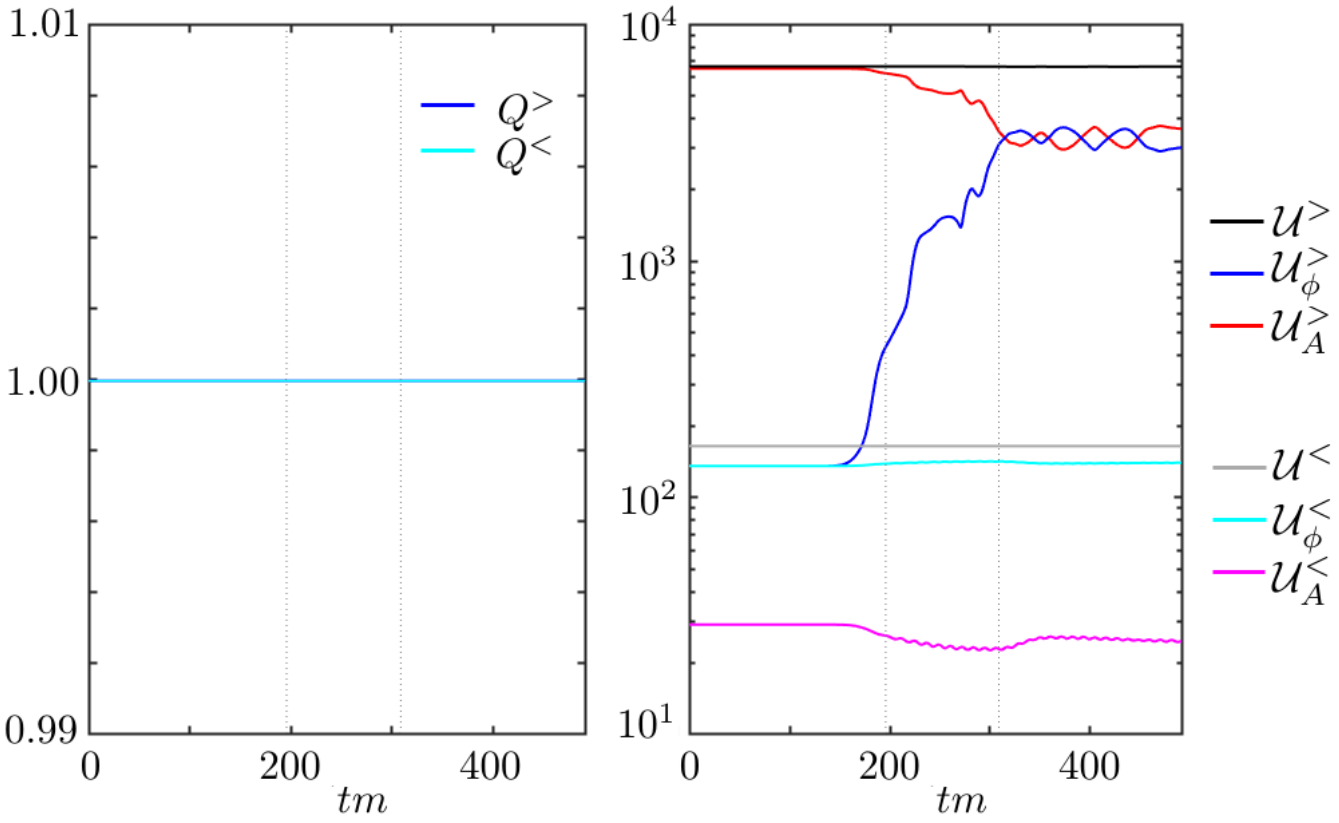
Wakefield acceleration → Schwinger pair production

- Relativistic intensity $a > 1$: ponderomotive snow-plow, wakefield acceleration, splashing from oscillating boundaries
- Quantum intensity $a > mc^2/\hbar\omega$: + pair production by wakefield and backscattered laser, pair annihilation



Charge conserved/Energy error small

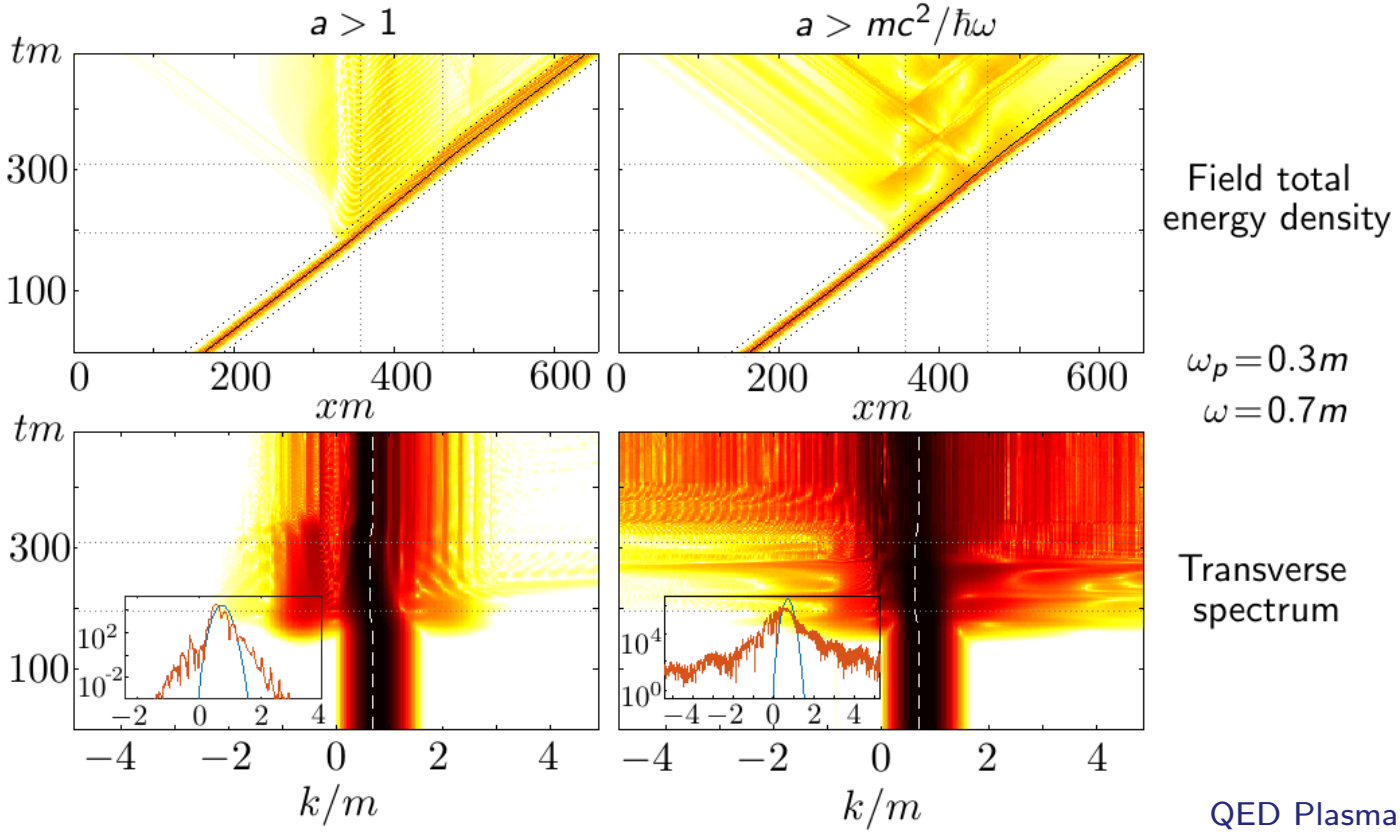
- Total charge is conserved to machine precision
- Total energy error $\mathcal{O}(en\Delta t^2)$. Conservation if $e = 0$



What happens to EM fields?

Parametric instability → Radiation generation

- Relativistic intensity $a > 1$: excite plasma wave, frequency down-shift, Raman scattering, harmonic generation
- Quantum intensity $a > mc^2/\hbar\omega$: + spectral broadening, γ-ray production through pair re-collision



Summary: lattice-QED plasma simulation

Why/When do we need it?

- Collective & high-energy effects both important
- Plasma scales overlap with strong-field scales

How do we simulate?

- Real-time lattice QED in classical statistic regime
- Solve field equations with proper initial/boundary conditions
- ◦ Methods also instructive for classical plasma simulations

What can we learn?

- Plasma in strong laser field, neutron star magnetosphere, ...
- Particle acceleration, e^-/e^+ pair production, ...
- High harmonic, γ radiation generation, ...
- ◦ Open ground for future research

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