High Energy Density Matter- Building a relativistic quantum model

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PPPL- January7, 2016

A Talk in Two Acts

- ACT 1: Statistical Construction of a quantum fluid starting from the quantum equation of the elementary constituents

- ACT 2: “Deriving" a nonlinear quantum mechanics starting from a fluid energy momentum tensor

The ‘advanced’ Statistical mechanics of Quantum particles in an arbitrary electromagnetic field

- Follow the standard statistical QM route but with EM interaction fully built in
- The quantum plasma, that may serve as a model for high energy density matter, will be constructed in the following steps:
- First solve the relativistic quantum equations as eigenvalue problems: Find the microscopic energy spectrum and the corresponding eigenfunctions,
- Use the calculated energy spectrum to define the "Boltzmann" factor defining the probability of a given micro state
- We will find that the "Boltzmann" factor will be qualitatively different because the loud presence of the EM field
- Define fluid variables (fluxes, currents, and energy momentum tensor etc) by taking quantum statistical averages
- The quantum fluid equations are, then, simply the standard conservation laws.
- Representative QM system- assembly of EM immersed Klein-Gordon (KG)/ Dirac Fields

The new 'quantum': QPF= Quantum Particle+ EM field

- We will dub the quantum fluid element as QPF- An effective composite of the particle and the EM field.
- For analytical convenience, we will explore the circularly polarized wave EM wave(CPEM) described by the four potential A^μ $(A^0 = 0 = A^z)$,
  $\mathrm{A}^x = A\cos(\omega t - kz), A^y = -A\sin(\omega t - kz) = -A\sin\zeta,$
  $\mathbf{E}= \omega A(\hat{e}_x \sin\zeta + \hat{e}_y \cos\zeta), \mathbf{B} = kA(-\hat{e}_x \cos\zeta + \hat{e}_y \sin\zeta),$

  where A=constant amplitude, the four wave vector $k^\mu = [\omega, 0, 0, k]$ in the lab frame, and the Minkowski signature tensor $\eta^{\mu\nu} = \mathrm{diag}[1, -1, -1, -1]$.
- CPEMs are most amenable to analytical calculations because

$$A^\mu A_\mu = (A^0)^2 - A^i A_i = A^2 = Constant, \tag{1}$$

- A K-G particle of mass m and charge q, interacting with the CPEM, obeys ($\hbar = 1 = c, p^\mu = [p^0 = E, \mathbf{p}]$ is the energy momentum four vector)

$$(p_\mu + qA_\mu)(p^\mu + qA^\mu)\Psi = (i\partial_\mu + qA_\mu)(i\partial^\mu + qA^\mu)\Psi = m^2\Psi \tag{2}$$

Dynamics of the QPF

- The composite QPF entity, for the specified CPEM, obeys ($\zeta = \omega t - kz$)

$$\partial_t^2 \Psi - \nabla^2 \Psi - 2iqA\left[\cos\zeta\partial_x\Psi - \sin\zeta\partial_y\Psi\right] + \mathcal{M}^2\Psi = 0, \tag{3}$$

  with the field-renormalized effective mass $\mathcal{M} = m\sqrt{1 + q^2A^2/m^2} = m\Gamma_f$.
- Caution: Γ_f (and $\mathcal{M}$) are **Lorentz scalars** not to be "conceptually identified" with standard kinematic γ which is the zero component of the four velocity.
- Realizing that (3) has three ignorable coordinates - x, y, and $\rho = kt - \omega z$ (conjugate to ζ), we seek solutions of the form

$$\Psi(x, y, z, t) = e^{iK_\perp(\cos\varphi x + \sin\varphi y)} e^{iK_\rho \rho}\psi(\zeta) \tag{4}$$

  which reduces the eigenvalue problem to a Mathieu equation in ζ

$$\frac{d^2\psi}{d\zeta^2} + (\mu + \lambda\cos\zeta)\,\psi = 0, \tag{5}$$

$$\mu = K_\rho^2 + \frac{\mathcal{M}^2 + K_\perp^2}{\omega^2 - k^2}, \quad \lambda = \frac{2qAK_\perp}{\omega^2 - k^2}, \tag{6}$$

  K_ρ is the conserved "momentum" conjugate to the hybrid direction ρ

The Mathieu equation

- The Mathieu equation allows solutions of the Floquet (Bloch) form

$$\psi = \psi_0 e^{is\zeta} f(\zeta), \tag{7}$$

$f(\zeta)$ is a periodic function of ζ, and s is determined from the exact equation

$$\sin^2(\pi s) = \Delta_0(\mu, \lambda) \sin^2(\pi\sqrt{\mu}) \tag{8}$$

where $\Delta_0(\mu, \lambda)$ is an infinite determinant, a complicated function of μ and λ.

- Regions of Instability: there are regions in $\lambda - \mu$ plane where s becomes complex - the wave function becomes grows (or decays) with $\zeta = \omega t - kz$.
- This class of solutions $\rightarrow$ totally new uncharted phenomena. Particle trapping (a quantum particle always has a finite probability for tunneling out) in the trough of the high amplitude wave (analogous to Brillouin zones)
- Today, however, I will, analytically, illustrate salient features of this approach by concentrating on the untapped particles.
- I will further work with an approximate WKB solution.

The Mathieu equation- an approximate WKB solution

- The WKB solution may be succinctly written as

$$\Psi = \left[\frac{(\mu+\lambda)}{16K(r)^2(\mu+\lambda\cos\zeta)}\right]^{1/4} \exp\left[i\mathbf{K}_\perp\cdot\mathbf{x}_\perp + iK_\rho(kt-\omega z) + iQ\right] \tag{9}$$

$$Q = \int(\mu+\lambda\cos\zeta)^{1/2}d\zeta = 2\sqrt{\lambda+\mu}\,\mathbf{E}\left(\frac{\zeta}{2},\gamma\right), \tag{10}$$

$\gamma = \sqrt{2\lambda/(\lambda+\mu)}$ = argument of the elliptic functions: $K(\gamma)$, $\mathbf{E}(\zeta/2,\gamma)$.

- Since t, z) appear explicitly in the dynamics, the bare particle"energy" and z momentum are not conserved. For the QPF, the expectation values: E and K_z:

$$E = i\int\left(\psi^*\frac{\partial\psi}{\partial t}\right)d\zeta, K_z = -i\int\left(\psi^*\frac{\partial\psi}{\partial z}\right)d\zeta$$

will serve as the energy, z-momentum. Thus the detailed QPF energy momentum relation becomes

$$E^2 - K_z^2 = (\omega^2-k^2)\left[-K_\rho^2 + \frac{(\lambda+\mu)\pi^2}{(2K(\gamma))^2}\right] \tag{11}$$

The Eigen value- Anistropic spectrum

- The exact WKB is still hard to pursue analytically. We approximate further the eigenvale

$$E^2 = K_z^2 + \mathcal{M}^2 + (1-\alpha)K_\perp^2 \equiv m^2 + q^2A^2 + K_z^2 + a^2K_\perp^2 \tag{12}$$

with $a^2 = 1 - \alpha$, and

$$\alpha \approx \frac{3q^2A^2}{2(\mathcal{M}^2 + K_\perp^2)} \tag{13}$$

measures the anisotropy in the energy contributed by the momenta along and perpendicular to the direction and propagation. In general, the anisotropy index is a very complex function of parameters (qA, ω, k)

- Two fundamental features, in addition, to the field renormalized mass

  1) The particle spectrum is anisotropic

  2) neither the energy E nor the momentum K_z are constants of motion though $K_\perp$ and K_ρ are: we have written K_ρ in terms of the easier to relate E and K_z

The Dirac Particle In CPEM

- Though much more complicated, the eigenvalue problem for the Dirac particle is conceptually similar. One needs to solve The D.E

$$\gamma^{\mu}(i\partial_{\mu} + qA_{\mu})\Psi = m\Psi \tag{14}$$

where Ψ is the four component spinor

$$\Psi = \begin{pmatrix} \varphi_1 \\ \chi_1 \\ \varphi_2 \\ \chi_2 \end{pmatrix} \tag{15}$$

γ^{μ} are the four by four Dirac matrices.

- for a 1D model with

$$\varphi_1 = P_1 e^{i\zeta/2 - i(Et - K_z z)}, \quad \varphi_2 = P_2 e^{i\zeta/2 - i(Et - K_z z)}$$

$$\chi_1 = Q_1 e^{-i\zeta/2 - i(Et - K_z z)}, \quad \chi_2 = Q_2 e^{-i\zeta/2 - i(Et - K_z z)}$$

one obtains the energy eigenvalue

$$(W_-^2 - K_-^2 - \mathcal{M}^2)(W_+^2 - K_+^2 - \mathcal{M}^2) = q^2 A^2 (k^2 - \omega^2) \tag{16}$$

where $W_{\pm} = E \pm \omega/2$ and $K_{\pm} = K_z \pm k/2$ are, respectively, the shifted energy and z-momentum of the particle;

- the shift is caused by the electromagnetic wave that affects different spin states in different ways

Building the Quantum fluid

- Analytical "solutions" - the energy eigenvalues in CPEM define the micro-state of the constituent QPF particles. Statistical mechanics of weakly interacting QPF will lead to the Macroscopic characteristics - the fluxes, the energy momentum tensor, the averaged induced current etc- of the quantum fluid.
- We will display the KG-QPF calculations in detail- just mention Dirac fluids
- Though KGQPF are bosons, for explicit analytic calculations the high temperature limit -The covariant relativistic M-B distribution for will be used

$$f = Ne^{-\frac{K_\mu U^\mu}{T}} \tag{17}$$

where U^μ= bulk four velocity, and $K^\mu = (E, aK_x, aK_y, K_z)$ is the “anisotropic" microscopic QPF four vector obeying $K^\mu K_\mu = \mathcal{M}^2$; the field renormalized $\mathcal{M}$ is the effective invariant mass. Averages calculated using the rest frame $f = f_R$:

$$f_R = Ne^{-\frac{E}{T}} = N \exp\left(-\zeta\sqrt{1+p^2}\right) \tag{18}$$

$\zeta = \mathcal{M}/T$, adjusted momentum variables $p_z = K_z/\mathcal{M}$, $p_x = aK_x/\mathcal{M}$, $p_y = aK_y/\mathcal{M}$, ($p_0 = E/\mathcal{M}$), and $p^\mu p_\mu = 1$.

Statistical Averaging-Macroscopic Quantities

- Rest frame density (zero component of the flux Γ)- sets the normalization N

$$n_R \equiv \Gamma^0 = \frac{N\mathcal{M}^3}{a^2}\int d^3p \exp\left(-\zeta\sqrt{1+p^2}\right), \tag{19}$$

$$N = \frac{a^2 n_R \zeta}{4\pi K_2(\zeta)\mathcal{M}^3} \Longrightarrow f_R = \frac{a^2 n_R \zeta}{4\pi K_2(\zeta)\mathcal{M}^3} e^{-E/T} \tag{20}$$

- Evaluate the rest frame En-Mom tensor: $T_R^{\mu\nu} = \int d^3K E^{-1} K^\mu K^\nu f_R$. Only diagonal elements are non zero, $T_R^{\mu\nu} = diag[\mathcal{E}, P_\parallel, P_\perp, P_\perp]$

$$\mathcal{E} = T_R^{00} = \int d^3K E f = \frac{n_R \zeta \mathcal{M}}{K_2(\zeta)} \frac{d^2}{d\zeta^2}\left(\frac{K_1(\zeta)}{\zeta}\right), \tag{21}$$

$$P_\parallel = n_R T, \qquad P_\perp = \frac{n_R T}{a^2} = \frac{P_\parallel}{a^2}$$

- Novel features = the field modified invariant mass $\mathcal{M}$ replacing m. Pressure anisotropy through differentiated parallel /perpendicular response to high intensity CPEM. $T_\perp$=$(T_\parallel/a^2) > T_\parallel$.

General En-Mom Tensor - The QPF fluid

- Invoking Lorentz invariance, one constructs the general En-Mom tensor

$$T^{\mu\nu} = -\eta^{\mu\nu} P_{\parallel} + \beta^{\mu\nu} \left(P_{\parallel} - P_{\perp}\right) + \left(\mathcal{E} + P_{\parallel}\right) U^{\mu} U^{\nu} \,, \tag{22}$$

U^{μ}= fluid four velocity, and $\beta^{\mu\nu}$ is the measure of the anisotropy $(P_{\parallel} - P_{\perp})$.

- For the QPF (circularly polarized CPEM),

$$\beta^{\mu\nu} = \frac{F^{\mu\kappa} F_{\kappa}{}^{\nu}}{F^{\alpha\beta} F_{\alpha\beta}} + \frac{k^{\mu} k^{\nu}}{k^{\alpha} k_{\alpha}} - \frac{A^{\mu} A^{\nu}}{A^{\alpha} A_{\alpha}} \tag{23}$$

is the most general tensor that satisfies all the technical requirements. For the most general case, these equations could be very complicated

- The ideal dynamics of this QPF fluid is, then, contained in

$$\partial_{\mu}(n_R U^{\mu}) = 0, \qquad \partial_{\mu} T^{\mu\nu} = 0 \tag{24}$$

the continuity and the force balance equations

Two Direct application - 1. A Maxwell-QPF system

- First, neglecting anisotropy, The f average of the expectation value of the QPF microscopic vector current $\langle \mathbf{J}_\perp \rangle = (q n_R K_1(\zeta)/\mathcal{M} K_2(\zeta))\mathbf{A}_\perp$ when, inserted in the Maxwell equation,

$$\frac{\partial^2 A_\perp}{\partial t^2} - \nabla^2 A_\perp = -\frac{4\pi q^2 n_R K_1(\zeta)}{\mathcal{M} K_2(\zeta)} A_\perp \,, \tag{25}$$

yield the dispersion relation ($\omega_p = \sqrt{4\pi q^2 n_R/m}$ is the invariant plasma frequency)

$$\omega^2 - k^2 = \frac{{\omega_p}^2}{\Gamma_f \Gamma_{th}} \equiv \omega_{eff}^2 \tag{26}$$

- Of the two Gammas (Lorentz scalars) decreasing ω_{eff}^2

$$\Gamma_f = \mathcal{M}/m \equiv \sqrt{1 + q^2 A^2/m^2}, \qquad \Gamma_{th} = K_2(\zeta)/K_1(\zeta), \zeta = \mathcal{M}/T$$

the first depends only on the EM field, while the second is a temperature-field strength hybrid.

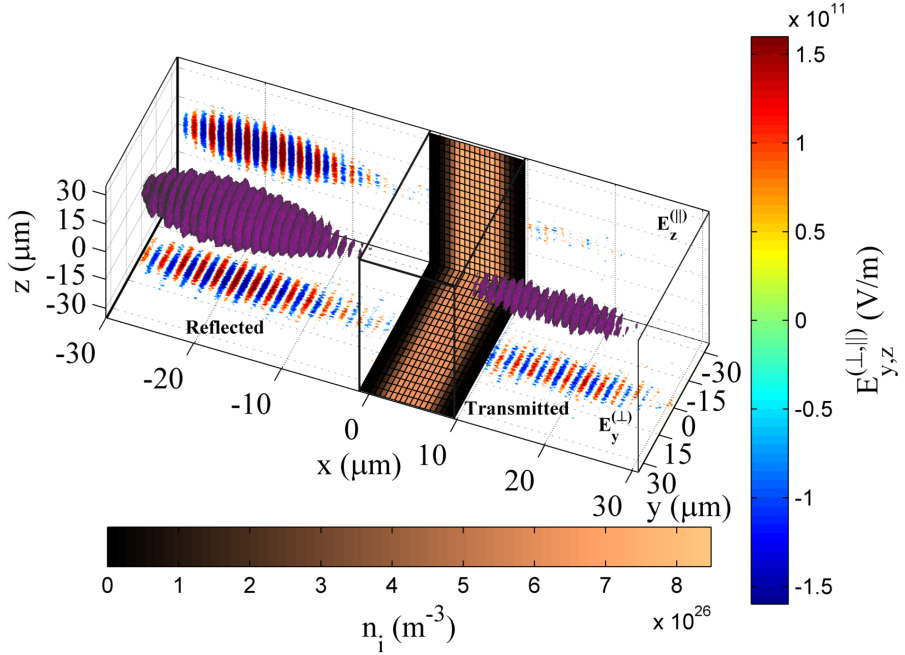
Dispersion Relation- Induced Plasma Transparency

- Induced transparency, decrease in the effective plasma frequency, is caused by
  1) Γ_f, effective mass enhanced via the EM field,

  2) Γ_{th} further inertia enhancement through thermal effects.
- In Laser-Plasma literature, the transparency induced by Γ_f is, often, called relativistic transparency; it is attributed to the relativistic mass increase - Γ_f , however, is entirely due to the inertia contributed by the electromagnetic field to the particle- electromagnetic system, and has little to do with the kinematic $\gamma = \sqrt{1 + K^2/m^2}$ (which measures the standard kinematic mass increase).
- The thermal enhancement factor $\zeta = \sqrt{(m^2 + q^2A^2)}/T$, registers the field enhanced inertia $\sqrt{(m^2 + q^2A^2)}$ rather than the "bare" mass m.
- That Γ_{th} is a function of $\sqrt{(m^2 + q^2A^2)}/T$ rather than m/T is profound because it resets the scale of temperature. Originally: super relativistic=$T \gg m$, non-relativistic= $T \ll m$ -With $\mathcal{M}$ replacing m, a 10 MeV electron ($m = .5MeV$) temperature becomes "sub-relativistic" for $|qA|/m = 100$!.
- The anisotropy factor a^2 is missing due to simplifying approximations.

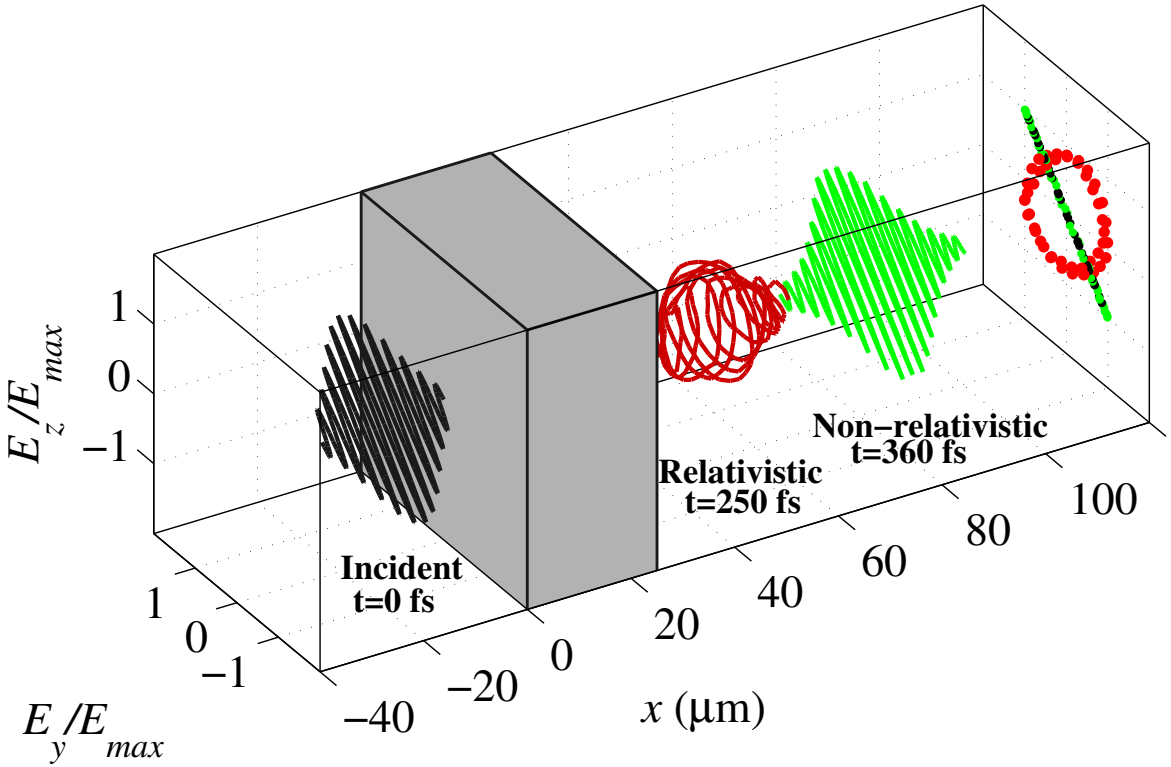
2. Thermal Anisotropy on- differential transparency-A Polarizer at Work-Stark et al, PRL, 115 (2015)-3D PIC

- Simulate using 3D/3V particle in cell code EPOCH a 50 fs, $a = eE_0/m_e c\omega = 0.2$ *cirularly-polarized* laser pulse incident on a 8μm plasma (propagating in the x-direction) of $n = 2.70 n_*$ (classically overdense). Ions are fixed throughout the simulation.
- The reflected and transmitted pulses are shown in the top figure, depicting how only the y-polarization can propagate through the plasma. The polarization aligned along the "hotter" direction is preferentially transmitted.
- We monitored the growth of the Weibel instability through the total magnetic field energy throughout the simulation, along with a measure of the anisotropy in the distribution, $\sqrt{< p_y^2 > / < p_z^2 >}$.
- Here we observe that the anisotropy persists in the plasma over a sufficiently long timescale to be probed. The pulse has already passed through the plasma well before the anisotropy has appreciably diminished.



3D PIC Simulation- A Waveplate

- *This effect does not just apply to overcritical targets but to underdense plasmas as well.* The phase velocity is strongly determined by the critical density for propagation through a near-critical plasma, so any difference between the polarizations in critical density would show up as a phase shift between two initially identical beams of different polarization.
- Simulate a low-intensity ($a = 0.4$) 50 fs linearly-polarized laser pulse (with $E_y = E_z$) incident on a 25μm plasma of $n = 1.75n_*$. The plasma has the same distribution as before, so this is transparent to both polarizations.
- For comparison, we tested an anisotropic non-relativistic plasma ($\alpha = 500.0$, $\epsilon = 0.45$, $n = 0.7n_*$, $a = 0.1$) to show that it induced a negligible phase shift, confirming that this is a purely relativistic effect.



Examining Filamentation Instability at Relativistic Intensities

- Perform 2D/3V particle-in-cell simulations of a continuous, linearly-polarized laser pulse incident on a constant density plasma. We introduce a periodic perturbation in the intensity

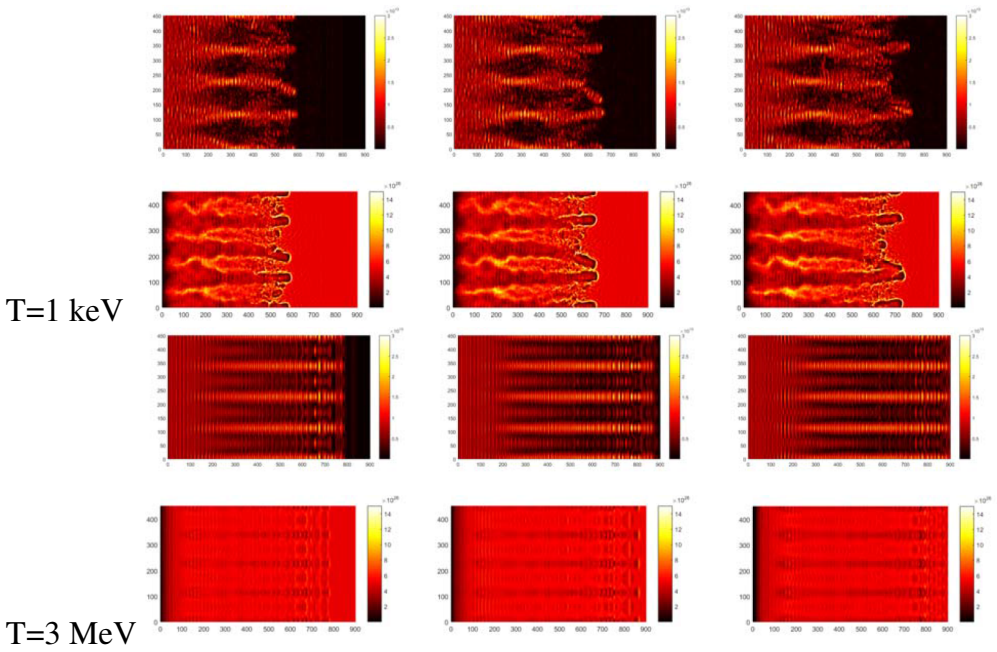
$$I = 1.24 \times 10^{19} \left(1 + \delta\cos(k_y y)\right) \cos^2\left(k_x x - \omega t\right) \text{ W/cm}^2$$

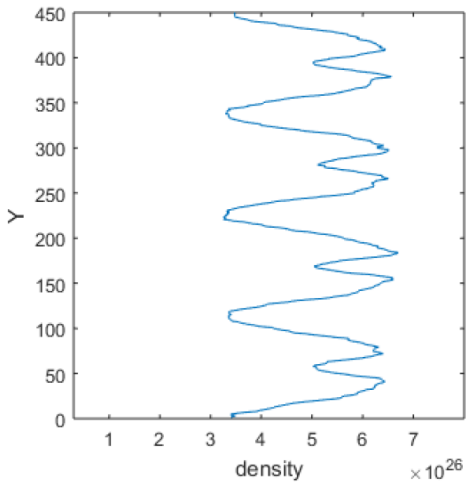
to seed the instability.

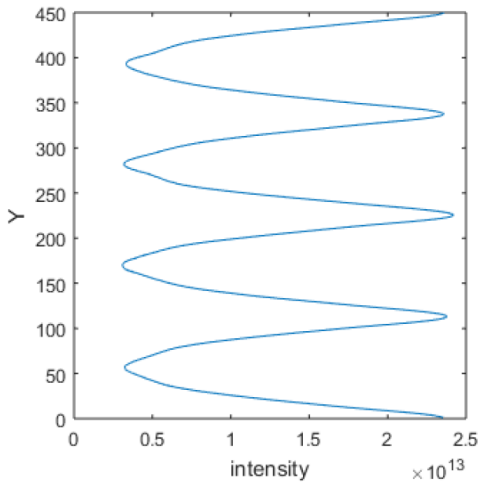
Wavelength- $\lambda = 1.06\mu\text{m}$

Density- $n_e = 0.5 n_c$

- First we tested how initial relativistic temperatures in the plasma affect the development of the instability. Using $\delta = 0.02$ and $k_y = 8\pi/30\mu\text{m}$, we performed $T = 1$ keV, 100 keV, 1 MeV, 3 MeV and observed the filamentation (intensity and density snapshots from 200-250 fs).



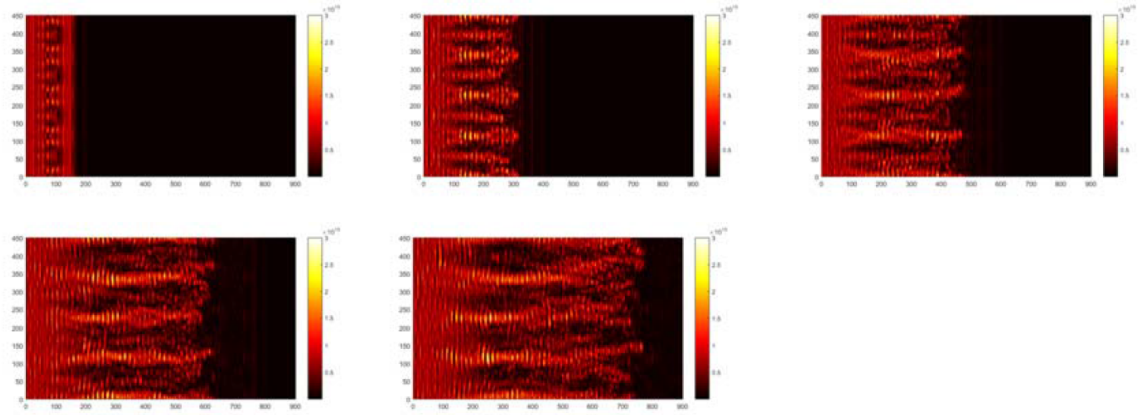




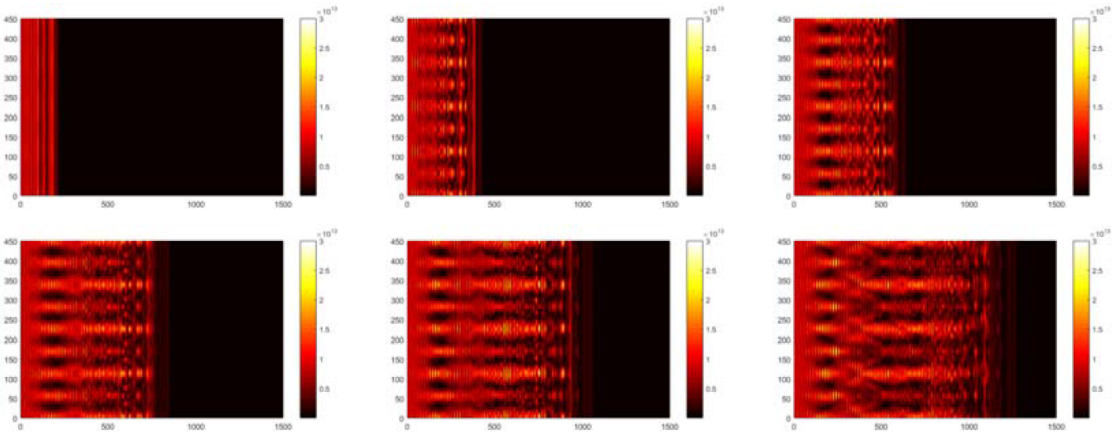
Density and intensity profiles for the 3 MeV simulation reveal the structure within the plasma

Effect of Electron Mass on Filamentation

- We extended the study to look at the dependence of the filamentation on the electron mass: $m = m_e, 2m_e, 3m_e$
- These results demonstrate that the heavier particles can delay the onset of filamentation and additionally prolong the duration of the filamentation period.



$m = m_e$
50-250 fs



$m = 3m_e$
50-300 fs

Summing UP- first part

- Quantum dynamics of a KG (Dirac) particle in an arbitrary amplitude EM wave was interpreted as the dynamics of a a QPF (combined quantum particle+field) system with an effective field enhanced mass, $\mathcal{M}^2 = m^2 + q^2A^2$ and an anisotropic “energy spectrum":

$$E^2 = m^2 + q^2A^2 + K_z^2 + a^2K_\perp^2 \equiv \mathcal{M}^2 + K_z^2 + a^2K_\perp^2, \quad a^2 \neq 1$$

- anisotropy stems from violation of rotational symmetry. The asymmetry parameter, in general, is a complicated function of q^2A^2, ω and k. For the Dirac QPF, the energy spectrum is split; the degeneracy between the positive and negative energy states, and between the spin up and spin down states in either class is broken. Thus the statistical mechanics of the QPF will, again, be profoundly different from that of the standard Dirac electron (without the field).
- A statistical mechanics of weakly interacting QPF was constructed. The "Boltzmann" factor, deciding the probability of a given micro state, was dictated by the energy spectrum of the QPF.
- Relevant macroscopic physical quantity -fluxes (Γ^μ), currents (J^μ), and energy momentum tensor($T^{\mu\nu}$) etc- were calculated. The conservation laws constitute the equations of motion of this effective quantum fluid.
- Two spectacular examples of straightforward applications were given- 1) How the field-enhanced mass sets a very different scale for temperature measurement, 2) The anisotropy in the energy spectrum of the QPF translates into a temperature anisotropy in the QPF fluid, that leads to differential transparency; such an anisotropic fluid is a highly effective polarizer.
- The formalism and the model is ready for criticism, applications and further investigation.

Nonlinear Quantum Mechanics- Perspective

- The quantum/fluid enterprise has spanned two complementary paths:
- Quantum to Classical: traditional quantum plasmas, and the statistical building up the fluid made from weakly interacting QPF.
- Classical to Quantum- The complementary path where we figure out a way to “quantize" a fluid element when the classical En-Mom tensor $T^{\mu\nu}$/Hamiltonian is known. In contradistinction to the elementary particle quantization, the fluid elements obey a "nonlinear quantum mechanics=NQM".
- The subject of Proposing and examining NQM has a rich history: Mielnik (74), Bialynicki-Birula and Mycielski (1976), Haag and Bannier (1978), Kibble(78), Weinberg (1989), Gisin, Gisin et al (1990 -2000), Jordan (- 2006)- Polemical discussion still on - Can nonlinear QM can appear at a fundamental level?
- Much More modest aim: Can we have a semi-phenomenological QM that may capture the more important collective modes of a very high energy density fluid.
- The theory is both heuristic, and hypothetical, but is consistent and rigorous. We will work with the (intrinsic) spin-less fluid (KG) for simplicity.

Nonlinear Quantum Mechanics of Hot fluids

- Particle QM begins with the prescription ($p^\mu = [E, \mathbf{p}]$ = four momentum)

$$p^\mu = -i\hbar\partial^\mu \tag{27}$$

into the classical dispersion

$$H^2 \equiv p^{02} = \mathbf{p^2} + m^2 \to [\hbar^2\partial_\mu\partial^\mu + m^2]\Psi = 0 \qquad KGE \tag{28}$$

- Seeking a similar prescription for the 'quantum' of a hot fluid, we start with an isotropic relativistic (thermally and kinematically) perfect fluid of particles with no internal degree of freedom: its En-Mom tensor is

$$T^{\mu\nu} = (nf/w)\, p^\mu p^\nu + \Pi\, \eta^{\mu\nu}\ , \tag{29}$$

where p^μ =fluid momentum, Π=pressure, f=enthalpy, n=rest frame number density, and w= a mass constant ($nf/w = U + \Pi$, where $U = T^{00}$)

- Classical dynamics of this fluid is in the continuity and force balance equations

$$\partial_\mu(np^\mu) = 0, \qquad \partial_\mu T^{\mu\nu} = 0 \tag{30}$$

Canonical Momentum-Fluid Quantization

- the kinematic momentum p^μ ceases to be the right dynamic variable in terms of which the equations of motion have a “simple structure". The “canonical momentum" is the thermally modified “effective momentum" ($g = n/fw$)

$$P^\mu = f\, p^\mu \quad \Rightarrow \partial_\mu(gP^\mu) = 0, \qquad \partial_\mu T^{\mu\nu} = \partial_\mu(g\, P^\mu P^\nu + \Pi\, \eta^{\mu\nu}) = 0 \quad (31)$$

- This is the classical dynamics of a fluid made of quasi (non elementary) particles - Fluidons- a thermally dressed electron, for example.
- The transition to the equivalent ‘quantum mechanics of a Fluidon’ is affected by the prescription (c.f Bohm Potential)

$$gP^\mu P^\nu \Longrightarrow g\left(P^\mu P^\nu + \frac{\hbar}{2i}\partial^\mu \frac{\hbar}{2i}\partial^\nu \ln g\right), \quad compare \quad p^\mu = -i\hbar\partial^\mu \quad (32)$$

- The transformed $T_q^{\mu\nu}$ (q for quantum) and the quantum equation of motion:

$$T_q^{\mu\nu} = g\, P^\mu P^\nu + \Pi\, \eta^{\mu\nu} - \frac{\hbar^2}{4} g\partial^\mu\partial^\nu \ln g \quad \Rightarrow QEM, \quad \partial_\mu T_q^{\mu\nu} = 0 \quad (33)$$

Quantum Eq. of Motion of Fluidons- The Wave Function

- The rest is technical manipulation invoking an Eq. of state $\Pi = \Pi(g)$, throwing in the EM field, choosing the irrotational canonical momentum field $P^\mu + qA^\mu = \partial^\mu S$, executing Cole-Hopf transformation via the inverse Madelung decomposition

$$\Psi = \sqrt{n/f}\; e^{iS/\hbar}, \quad \Psi^*\Psi = n/f,$$

one derives the thermally-dictated nonlinear EM-KG equation

$$[-(i\hbar\partial_\mu - qA_\mu)(i\hbar\partial^\mu - qA^\mu) - \lambda\bar{\Pi}(\Psi^*\Psi) - m^2]\Psi = 0, \tag{34}$$

- In the cold limit (no internal energy), the enthalpy factor $f \to 1$, $\Pi(\bar{\Pi}) \to 0$- the world goes back to sanity- the linear KG. The internal energy, however, changes the world and the effective QM of a fluidon is fully nonlinear. The $A_\mu = 0$ “dispersion relation" is amplitude dependent

$$\mathcal{E}^2 = \mathbf{P}^2 + m^2 + \lambda {\Psi_0}^{2(\Gamma-1)} \tag{35}$$

this result requires summing up of an infinite number of diagrams in High temp field theories The non relativistic limit becomes the Landau-Ginzburg for $\Gamma = 2$

$$i\hbar\frac{\partial\Psi}{\partial t} = [-\frac{\hbar^2}{2\mu}\left(\partial^k - \frac{iq}{\hbar}A^k\right)^2 + q\phi + \lambda|\Psi^*\Psi|^{2(\Gamma-1)}]\Psi, \tag{36}$$

Internal degrees of freedom-Fermions- Dirac system

- How do we generalize the NLQ to include the spin degrees of freedom.
- We will present now a rigorous NLQ generalization of Dirac fluid through its Feynman-GellMan version .
- it is algebraically very complex- However, I will give a short summary
- Start from the relativistic version of the Spin Stress: EnMom tensor

$$\mathcal{T}^{\mu\nu} = gP^{\mu}P^{\nu} + \Pi\eta^{\mu\nu} + g\frac{\hbar^2}{8}\partial^{\mu}M_{\alpha\beta}\partial^{\nu}M^{*\alpha\beta}, \tag{37}$$

with $M_{\alpha\beta} = \Psi^{\dagger}\sigma_{\alpha\beta}\Psi/\Psi^{\dagger}\Psi$; $\sigma_{\alpha\beta} = i\gamma_{\alpha}\gamma_{\beta}$, and γ^{α} are the Dirac matrices.

- Our standard prescription yields the quantum EnMom tensor

$$\mathcal{T}_q^{\mu\nu} = g\left(P^{\mu}P^{\nu} + \frac{\hbar}{2i}\partial^{\mu}\frac{\hbar}{2i}\partial^{\nu}\ln g\right) + \Pi\eta^{\mu\nu} + g\frac{\hbar^2}{8}\partial^{\mu}M_{\alpha\beta}\partial^{\nu}M^{*\alpha\beta} \tag{38}$$

NLQ-Feynman Gell-Mann Eq- Essential Steps

- The FG-NLQ equation of motion is manipulated, first, by defining an extended "vorticity" free four vector

$$P^{\mu} + qA^{\mu} - \varsigma\partial^{\mu}\omega = \partial^{\mu}S \tag{39}$$

with the property

$$\partial^{\mu}P^{\nu} - \partial^{\nu}P^{\mu} = -qF^{\mu\nu} - \hbar\Omega^{\mu\nu}, \Omega^{\mu\nu} = -\partial^{\mu}\varsigma\partial^{\nu}\omega + \partial^{\nu}\varsigma\partial^{\mu}\omega \tag{40}$$

$\Omega^{\mu\nu}$ is a spin specific vorticity curvature.

- With tons of algebra, and manipulations, we derive

$$\left[D_{\mu}D^{\mu} + \frac{q\hbar}{2c}\sigma_{\alpha\beta}F^{\alpha\beta} + m^2 + \lambda\bar{\Pi}(\Psi^{\dagger}\Psi)\right]\Psi = 0, \tag{41}$$

where $D^{\mu} = i\hbar\partial^{\mu} - qA^{\mu}$. Nonlinerarity $\lambda\bar{\Pi}(\Psi^{\dagger}\Psi)$. Notice that F-G operator differs from the KG operator only through the spin coupling- $(q\hbar/2c)\sigma_{\alpha\beta}F^{\alpha\beta}$- to the EM field (cf Gordon decomposition)

SOLITONS- A Nonlinear Necessity

- Let us do a simple 1+1 problem in z and t for the thermal NKG: one seeks

$$\Psi = \psi(z,t)e^{iKz-iWt}, \quad \psi(z,t) = \psi(z-(K/W)t) \equiv \psi(\zeta) \quad (42)$$

  $\psi(\zeta)$ is the envelope function, propagating at the group velocity of the plane wave.

- The NKG , then, becomes

$$\alpha\frac{d^2\psi}{d\zeta^2} + \triangle\psi - \bar{\Pi}(\psi^2)\psi = 0$$

  where $\Delta = W^2 - m^2 - K^2$ and $\alpha = (W^2 - K^2)/W^2$.

- α is positive definite, while Δ can be either sign.

Dark and Localized SOLITONS- Model nonlinearity

- Normal matter for which the simplest model

$$\bar{\Pi}(\psi^2) = 2a\psi^2, a > 0$$

allows us the First integral of the equation for the envelope function

$$\alpha \left(\frac{d\psi}{d\zeta}\right)^2 + \triangle\psi^2 - a\psi^4 = Const.$$

This choice leads to a Kink (dark) soliton:

$$\psi = \sqrt{\frac{\triangle}{2a}} \tanh \sqrt{\frac{\triangle}{2\alpha}}\, \zeta$$

- For exotic matter ($a < 0$) and for $\Delta < 0$, we have a localized soliton

$$\psi = \sqrt{\frac{\triangle}{|a|}} \operatorname{sech} \sqrt{\frac{|\triangle|}{\alpha}} \zeta$$