Nonlinear quantum electrodynamics in strong laser fields: From basic concepts to electron-positron photoproduction

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MAX-PLANCK-GESELLSCHAFT

MPI for Nuclear Physics, Heidelberg

- Why should we study Strong-Field QED?
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
- Lasers as a tool to study the critical field
  - Nonlinear Compton scattering
  - Nonlinear Breit-Wheeler pair production
- From a single vertex to a QED cascade
  - QED-PIC approach
  - Formation region and hierachy of scales
- Radiative corrections
  - Quantum dressing: exact wave functions
  - Fully nonperturbative regime of QED

More details can be found, e.g., in:

A. Di Piazza, et al., Rev. Mod. Phys. **84**, 1177–1228 (2012)
W. Dittrich, H. Gies, Probing the Quantum Vacuum (Springer, 2000)
E.S. Fradkin, D.M. Gitman, S.M. Shvartsman, QED with Unstable Vacuum (Springer, 1991)
V. I. Ritus, J. Sov. Laser Res. **6**, 497–617 (1985)

Motivation: Why do we want to test nonlinear QED?

QED: eletrons, positrons and photons

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - e\mathcal{A}\!\!\!/ - m\right)\psi - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}, \quad \mathcal{F}^{\mu\nu} = \partial^\mu\mathcal{A}^\nu - \partial^\nu\mathcal{A}^\mu$$

- Here, ϵ_0, $\hbar$ and c are set to unity (sometimes restored for clarity)
- The characteristic scales of atomic physics and QED are determined by the electron mass (m) and charge ($e < 0$)

QED

$$\mathcal{E} = mc^2 \sim 10^6\,\mathrm{eV}$$

$$\lambda\!\!\!^{-}_C = \hbar c/(mc^2) \sim 10^{-13}\,\mathrm{m}$$

$$E_{\mathsf{cr}} = (mc^2)^2/(|e|\,\hbar c) \sim 10^{16}\,\mathrm{V/cm}$$

Atomic physics

$$\mathcal{E}_H = (Z\alpha)^2\mathcal{E}/2 \sim Z^2 \times 10\,\mathrm{eV}$$

$$a_B = \lambda\!\!\!^{-}_C/(Z\alpha) \sim Z^{-1} \times 10^{-10}\,\mathrm{m}$$

$$E_{\mathsf{eff}} = (Z\alpha)^3 E_{\mathsf{cr}} \sim Z^3 \times 10^{10}\,\mathrm{V/cm}$$

$\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$: fine-structure constant, Z: atomic number

Conceptual changes

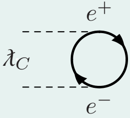
Energy	$\mathcal{E}$	nonrelativistic vs. relativistic description
Length	$\lambda\!\!\!^{-}_C$	classical vs. quantum field theory
Field	E_{cr}	vacuum vs. nonlinear QED

Sauter-Schwinger vacuum instability

- A pure electric field $E \geq E_{\mathrm{cr}}$ is unstable, it decays spontaneously

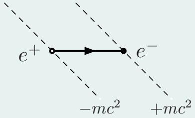
  First observation: Sauter (1931), First modern calculation: Schwinger (1951)

Vacuum fluctuations



Instead of being empty, the vacuum is filled with quantum fluctuations

Heuristic tunneling picture



"Tilted" energy levels $\longrightarrow$ tunneling
Probability: $\sim \exp\left(-\pi E_{\mathrm{cr}}/E\right)$

- Heuristic derivation of the critical field $E_{\mathrm{cr}} = 1.3 \times 10^{16}$ V/cm:
  - Spatial extend of the fluctuations (Heisenberg): $\sim \lambda\!\!\!^{-}_C = \hbar/(mc)$
  - Energy gap between virtual and real (Einstein): $\sim mc^2$
  - Work by the field (Lorentz force): $\sim E\,|e|\,\lambda\!\!\!^{-}_C \longrightarrow E_{\mathrm{cr}} = mc^2/(|e|\,\lambda\!\!\!^{-}_C)$
- In vacuum $I_{\mathrm{cr}} = 4.6 \times 10^{29}\ \mathrm{W/cm^2}$ is not achievable in the near future:

	$\sim \hbar\omega$	Future facilities	I (intensity)	current
optical	$1\,\mathrm{eV}$	CLF, ELI, XCELS,...	$10^{24-25}\ \mathrm{W/cm^2}$	$10^{22}\ \mathrm{W/cm^2}$
x-ray	$10\,\mathrm{keV}$	LCLS-II, XFEL,...	$10^{27}\ \mathrm{W/cm^2}$ (goal)	$10^{18}\ \mathrm{W/cm^2}$

Light-Light interaction in the local limit

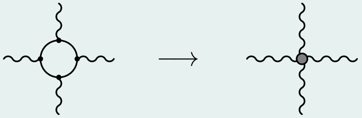
- In vacum (i.e. without real charges and currents) the Lagrangian density for the electomagnetic field is given by $\mathcal{L} = \left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)/2$. Accordingly, the field equations are linear (superposition principle):

$$\nabla \boldsymbol{E} = 0, \quad \nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t, \quad \nabla \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{B} = \partial \boldsymbol{E}/\partial t$$

- In quantum field theory photons couple via virtual electric charges. Effectively, we obtain nonlinear terms in the Lagrangian:

Euler-Heisenberg Lagrangian density (1936)

$$\mathcal{L} = \frac{1}{2}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) + \frac{2\alpha}{45\, E_{\mathrm{cr}}^2}\left[\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)^2 + 7\left(\boldsymbol{E}\boldsymbol{B}\right)^2\right] + \dots$$



Leading-order contribution to the EH-Lagrangian (local limit)

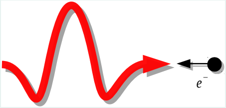
This description is applicable if:

- The wave length is much larger than the Compton wavelength
- The field strength is much smaller than the critical field

How to reach the critical field with existing technology

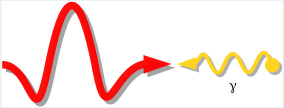
- The laser intensity I is not a Lorentz scalar ($I' \sim \gamma^2 I$, $\gamma = \epsilon/m$)
- Critical intensity $I_{\rm cr} = 4.6 \times 10^{29}\,\mathrm{W/cm^2}$ is obtainable in the boosted frame if $\gamma \sim 10^3 - 10^4$ even if $I \lesssim 10^{22}\,\mathrm{W/cm^2}$ (optical Petawatt system)

Electron-Laser interactions



Electrons with an energy $\epsilon \gtrsim \mathrm{GeV}$ are obtainable via laser-wakefield acceleration

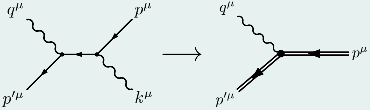
Light-by-light scattering



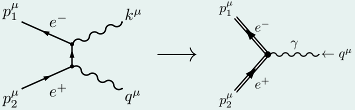
Photons with an energy $\hbar\omega_\gamma \gtrsim \mathrm{GeV}$ are obtainable via Compton backscattering

- For very strong fields the simultaneous interaction with several laser photons becomes important – describable using "dressed" states:

Compton scattering



Breit-Wheeler pair production



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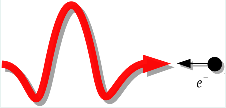

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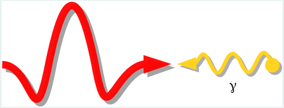

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- For very strong fields the simultaneous interaction with several laser photons becomes important – describable using "dressed" states:

q^μ

p'^μ

<u>Current laser-plasma wakefield acceleration record</u>:

$\epsilon = 4\,\mathrm{GeV}$ ($\gamma \sim 10^4$) W. P. Leemans, et al., PRL **113**, 245002 (2014)

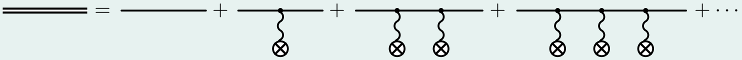
<u>Current Compton-backscattering photon energy record</u>:

$\hbar\omega_\gamma = 2.9\,\mathrm{GeV}$ N. Muramatsu, et al., NIMA **737**, 184–194 (2014)

Dressed states and the classical intensity parameter

- Dressed states are solutions of the interacting Dirac equation:

$$(\mathrm{i}\partial\!\!\!/ - m)\,\psi_p = 0,\ \psi_p = \qquad\qquad (\mathrm{i}\partial\!\!\!/ - eA\!\!\!/ - m)\,\Psi_p = 0,\ \Psi_p =$$



The dressed propagator/external line includes an arbitrary number of interactions with the classical background field

- A single interaction with the background scales as $\sim \xi$ $(\xi = a_0)$

$$\xi \sim \frac{|e|\sqrt{\langle -A^2\rangle}}{mc} \sim \frac{|e|E}{mc\,\omega}, \qquad = \frac{p\!\!\!/ + m}{p^2 - m^2} \sim \frac{1}{m},$$

$$= -\mathrm{i}eA\!\!\!/ \sim |e|\sqrt{-A^2}$$

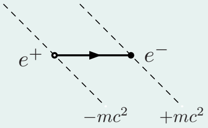
Intensity parameter — Free propagator — Coupling vertex

(E, ω: field strength and angular frequency of the laser field, respectively)

Perturbative regime $\xi \ll 1$	Each coupling suppressed by ξ^2 (probability) n-photon absorption scales as ξ^{2n}
Nonperturbative regime $\xi \gtrsim 1$	Dressing becomes important $[I \gtrsim 10^{18}\ \mathrm{W/cm^2}$ for optical lasers $(\hbar\omega \sim 1\,\mathrm{eV})]$
Semiclassical regime $\xi \gg 1$	Probability amplitude is highly oscillating, classical interpretation of stationary points

Pair production and the quantum nonlinearity parameter

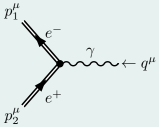
Sauter-Schwinger effect



Spontaneous decay of the vacuum

- Sizable if $E \gtrsim E_{\rm cr} = m^2c^3/(\hbar\,|e|)$ (at the QED critical field)
- Probability: $\sim \exp\left(-\pi E_{\rm cr}/E\right)$ (for a pure electric field)

Breit-Wheeler pair production



Decay of an incoming photon

- Sizable if $\chi \gtrsim 1$ (critical field reached in the boosted frame)
- Probability: $\sim \exp\left[-8/(3\chi)\right]$ (if $\chi \ll 1$ and $\xi \gg 1$)

Electron-positron photoproduction depends crucially on the quantum nonlinearity parameter

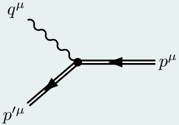
$$\chi \sim \frac{|e|\,\hbar}{m^3c^4}\sqrt{\langle q^\mu F^2_{\mu\nu} q^\nu\rangle} \sim (2\hbar\omega_\gamma/mc^2)(E/E_{cr})$$

[$\hbar\omega_\gamma$: energy of the incoming photon; last relation assumes a head-on collision]

- The photon four-momentum is transfered at the vertex
- Pair is produced ultra relativistic, background field is boosted

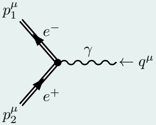
From a single vertex to a QED cascade

Photon emission



In general an electron can radiate more than only once

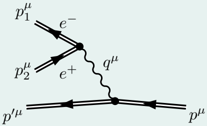
Pair production



The survival probability of a photon can become exponentially small

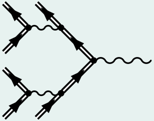
- The total probability $P \sim \alpha\xi N$ for the fundamental processes can become very large [$\alpha \approx 1/137$, N: number of laser cycles]
- At a certain point processes with many vertices become important
- Starting from a single particle a cascade developes

Trident pair production



Simplest cascade process

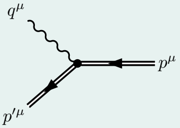
QED cascade



Exponential increase of particles

From a single vertex to a QED cascade

Photon emission



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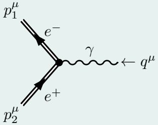

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Trident pair production

QED cascade

Seminal SLAC E-144 experiment:

$\epsilon = 46.6\,\mathrm{GeV}$ ($\gamma \sim 10^5$), $\hbar\omega = 2.4\,\mathrm{eV}$, $I \sim 10^{18}\,\mathrm{W/cm^2}$ ($\xi \approx 1$)

Nonlinear Compton scattering: C. Bula, et al. PRL **76**, 3116 (1996)

Trident pair production: D. L. Burke, et al. PRL **79**, 1626 (1997)

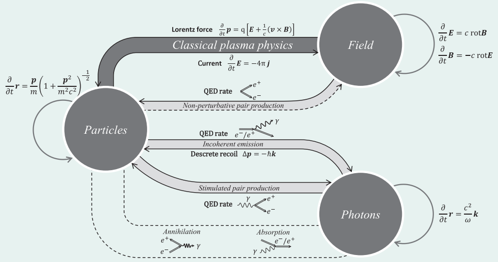
S-matrix approach

- Ab initio calculation, all effects included, arbitrarily precise
- Only asymptotic probabilities, no description of the dynamics
- Complicated if many vertices must be taken into account

PIC-approach

- Separates quantum processes from classical propagation
- Intuitive picture, complicated processes can be considered
- No reliable error estimates, question of applicability

PIC scheme



A. Gonoskov, et al. Phys. Rev. E **92**, 023305 (2015)

PIC approach to QED cascades

S-matrix approach

- Ab initio calculation, all effects included, arbitrarily precise
- Only asymptotic probabilities, no description of the dynamics

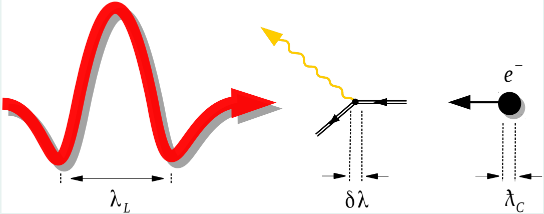
PIC-approach

- Separates quantum processes from classical propagation
- Intuitive picture, complicated processes can be considered

Many recent papers on QED cascades:

Grismayer, Vranic, Martins, Fonseca, and Silva, arXiv:1511.07503 (2015)
Tamburini, Di Piazza, and Keitel, arXiv:1511.03987 (2015)
Gelfer, Mironov, Fedotov, Bashmakov, Nerush, Kostyukov, and Narozhny, PRA (2015)
Gonoskov, Bastrakov, Efimenko, Ilderton, Marklund, Meyerov, et al., PRE (2015)
Green and Harvey, CPC (2015)
Lobet, Ruyer, Debayle, d'Humières, Grech, Lemoine, and Gremillet, PRL (2015)
Vranic, Grismayer, Martins, Fonseca, and Silva, CPC (2015)
Bashmakov, Nerush, Kostyukov, Fedotov, and Narozhny, POP (2014)
Mironov, Narozhny, and Fedotov, PLA (2014)
Narozhny and Fedotov, EPJST (2014)
Ridgers, Kirk, Duclous, Blackburn, Brady, Bennett, Arber, and Bell, JCP (2014)
Tang, Bake, Wang, and Xie, PRA (2014)
...

QED processes in strong laser fields



Important length scales for plane-wave laser fields

1. $L = N\lambda_L$: total length of the laser pulse (N: number of cycles)
   Characterizes the space-time volume which contains a strong field
2. λ_L: laser wavelength (scale on which the field changes its sign)
   Determines the highest possible classical energy transfer
3. $\delta\lambda$: formation region of the basic single vertex QED processes
   Its relation to λ_L determines the qualitative properties of the QED processes
4. $ƛ_C$: electron/positron Compton wavelength
   Fundamental length scale of QED (quantum fluctuations become important)

Equation of motion

- The electron four-momentum P^μ is determined by the Lorentz force:

$$\frac{dP^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} P_\nu \longrightarrow \frac{dP^\mu(\phi)}{d\phi} = \frac{e}{kP_0} F^{\mu\nu}(\phi) P_\nu(\phi),$$

τ: proper time, $\phi = kx$: laser phase, $F^{\mu\nu}$: field tensor, $kP_0 = kP(\phi)$ is conserved

- The position four-vector x^μ is obtained by integrating:

$$x^\mu(\phi) = x_0^\mu + \int_{\phi_0}^{\phi} d\phi' \, \frac{P^\mu(\phi')}{kP_0}, \quad \frac{d\phi}{d\tau} = \frac{kP_0}{m}, \quad F^{\mu\nu}(\phi) = \sum_{i=1,2} f_i^{\mu\nu} \psi_i'(\phi)$$

Solution for a plane-wave field

- Result depends only on the integrated field tensor:

$$P^\mu(\phi) = P_0^\mu + \frac{e\mathfrak{F}^{\mu\nu}(\phi,\phi_0) P_{0\nu}}{kP_0} + \frac{e^2 \mathfrak{F}^{2\mu\nu}(\phi,\phi_0) P_{0\nu}}{2(kP_0)^2},$$

$$\mathfrak{F}^{\mu\nu}(\phi,\phi_0) = \int_{\phi_0}^{\phi} d\phi' \, F^{\mu\nu}(\phi') = \sum_{i=1,2} f_i^{\mu\nu} [\psi_i(\phi) - \psi_i(\phi_0)].$$

Plane-wave dynamics: qualitative properties

Classical electron four-momentum

$$P^\mu(\phi) = P_0^\mu + \sum_{i=1,2}\left\{\underbrace{\frac{e}{kP_0} f_i^{\mu\nu} P_{0\nu}\left[\psi_i(\phi)-\psi_i(\phi_0)\right]}_{\text{transverse acceleration}} + \underbrace{k^\mu \frac{m^2}{2kP_0}\xi_i^2\left[\psi_i(\phi)-\psi_i(\phi_0)\right]^2}_{\text{ponderomotive force}}\right\}$$

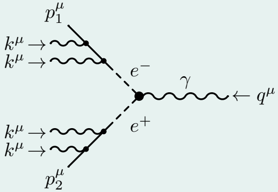
$$F^{\mu\nu}(\phi) = \sum_{i=1,2} f_i^{\mu\nu}\psi_i'(\phi), \qquad f_i^{\mu\nu} = k^\mu a_i^\nu - k^\nu a_i^\mu, \qquad \xi_i = \frac{|e|}{m}\sqrt{-a_i^2}, \qquad \xi = \sqrt{\xi_1^2+\xi_2^2}$$

Normalization: $|\psi_i(kx)|, |\psi_i'(kx)| \lesssim 1$; no dc component: $\psi_i(\pm\infty) = \psi_i'(\pm\infty) = 0$

- **Lawson-Woodward theorem**:
  - no net acceleration $[P^\mu(+\infty) = P^\mu(-\infty)]$
  - Important exception: acceleration of particles created inside the field
- **Momentum and energy scales**:
  - Transverse momentum: linear term, $\sim m\xi_i$
  - Energy absorption: quadratic term (ponderomotive force)
  - Absorption inside $\delta\phi \lesssim 1$ around ϕ_0: $k^\mu[m^2/(2kP_0)][\xi_i\psi_i'(\phi_0)\delta\phi]^2$
- **Conservation of kP**:
  - kP is both classically and quantum mechanically conserved
  - Quantum nonlinearity parameter $[\chi_e = (kP_0/m^2)\xi]$ is conserved
  - Inside a plane-wave field a QED cascade stops at a certain point

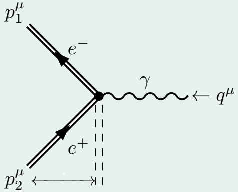
- A plane-wave field depends only on the laser phase $\phi = kx$
- Accordingly, four-momentum is conserved up to a multiple of k^μ:
$$p_1^\mu + p_2^\mu = q^\mu + nk^\mu, \quad n \geq 2m^2/kq \quad (\text{threshold: } p_1^\mu = p_2^\mu)$$
- Within the (small) formation region $\delta\phi$ the four-momentum nk^μ with $n \sim (\xi\delta\phi)^2(m^2/kq)$ can be absorbed (classically) from the laser field:
$$(\xi\delta\phi)^2(m^2/kq) \sim m^2/kq \quad \longrightarrow \quad \delta\phi \sim 1/\xi$$
- Two different regimes can be distinguished:

Multiphoton regime



$\xi \ll 1$: Large formation region, the process “feels” an oscillatory field

Tunneling & classical propagation



$\xi \gg 1$: Small formation region, the process happens instantaneously

Formation region: mulitphoton vs. tunneling process

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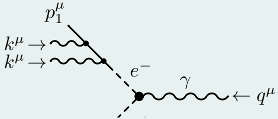

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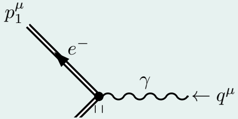
Tunneling & classical propagation

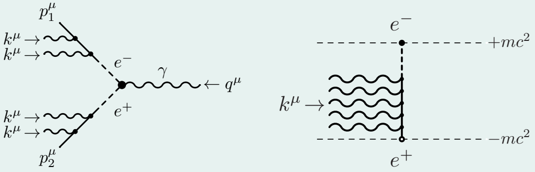


Nonlinear Breit-Wheeler process in the multiphoton regime:
(important application: x-ray lasers)

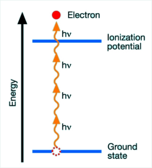
M. J. A. Jansen and C. Müller, arXiv:1511.07660 (2015)
M. J. A. Jansen and C. Müller, PRA **88**, 052125 (2013)

Comparison with ionization in atomic physics

Multiphoton pair production ($\xi \ll 1$)



Multiphoton ionization



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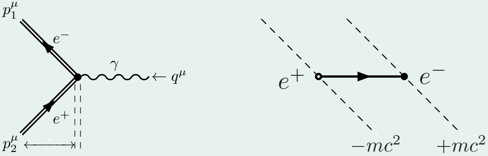
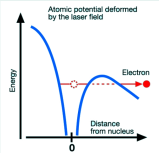
Tunneling pair production ($\xi \gg 1$)



Tunnelionization



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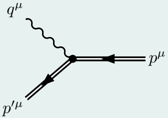
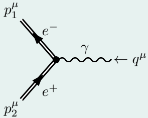
- Pair production is similar to ionization in atomic physics
- The Keldysh parameter distinguishes the two regimes:
  AP: $\gamma = \omega\sqrt{2mI_p}/(|e|E)$, SFQED: $1/\xi = \omega mc/(|e|E)$ $\left(I_p = 2mc^2\right)$
  [ω, E: laser angular frequency/field strength, I_p: atomic ionization potential]

Single vertex processes revisited

Meaning of laser-dressed Feynman diagrams



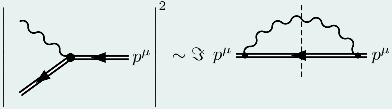
Emission of exactly one photon



Decay into a single e^+e^-–pair

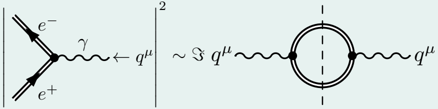
- This interpretation is correct if the total probability remains small
- For large probabilities radiative corrections become important
- The S-matrix is unitary $\longrightarrow$ optical theorem (cutting rules):

Mass operator



Total emission probability

Polarization operator



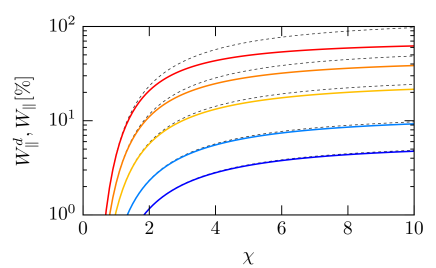
Total decay probability

- The imaginary part of loop diagrams ensures a unitary time evolution

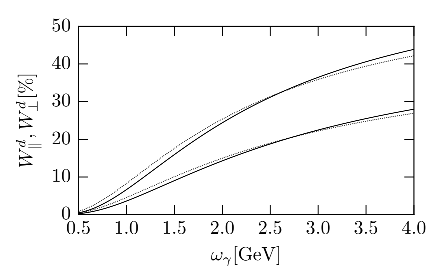
SM and A. Di Piazza, PRL **107**, 260401 (2011)

SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRD **91**, 013009 (2015)

By combining available optical petawatt lasers with existing GeV gamma sources, the pair-production probability can become very large



dashed lines: without wave-function decay
$\xi = 10, 20, 50, 100, 200$ $(N = 5)$
$F^{\mu\nu}(x) = f^{\mu\nu}\sin^2[\phi/(2N)]\sin(\phi)$, $\phi = kx$



solid: $F^{\mu\nu}(x) = f^{\mu\nu}\sin^2[\phi/(2N)]\sin(\phi)$
dashed: $F^{\mu\nu}(x) = f^{\mu\nu}\sin^4[\phi/(2N)]\sin(\phi)$
$\xi = 100$, $N = 5$, $\omega = 1.55\,\mathrm{eV}$

Problem: For certain values of χ, ξ the evaluation of the leading-order Feynman diagram violates unitarity

Solution: The back-reaction of the decay on the photon wave function must be taken into account

SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRD **91**, 013009 (2015)

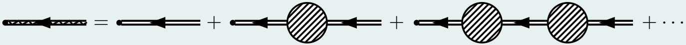
Radiative corrections: exact wave functions

Photon

a) Exact photon wave function (includes radiative corrections)

b) Polarization operator (all one-particle irreducible diagrams)

Electron



c) Exact electron wave function (includes radiative corrections)

d) Mass operator (all one-particle irreducible diagrams)

Exact wave functions obey the Schwinger-Dyson equations, e.g.,

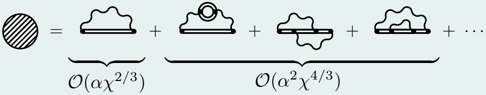
$$-\partial^2 \Phi_q^{\mathrm{in}\,\mu}(x) = \int d^4y\, P^{\mu\nu}(x,y)\Phi^{\mathrm{in}}_{q\nu}(y), \qquad \Phi_q^{\mathrm{in}\,\mu}(x) : \text{incoming photon}$$

Furry-picture approach to strong-field QED:

- Strong background fields ($\xi \gtrsim 1$) are included exactly (dressed states)
- The radiation field (non-occupied modes) is treated perturbatively

$\longrightarrow$ QED becomes a nonperturbative theory (like QCD?) for $\alpha\chi^{2/3} \gtrsim 1$

Full breakdown of perturbation theory



Mass operator: perturbation theory with respect to the radiation field

Different regimes for strong background fields ($\xi \gg 1$):

1. $\chi \ll 1$: **classical regime**
   Quantum effects are very small, pair production is exponentially suppressed
2. $\chi \gtrsim 1, \alpha\chi^{2/3} \ll 1$: **quantum regime**
   Recoil and pair production are important, but the radiation field is a perturbation
3. $\alpha\chi^{2/3} \gtrsim 1$: **fully nonperturbative regime**
   Perturbative treatment of the radiation field breaks down

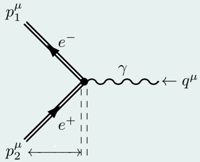
V. I. Ritus, J. Sov. Laser Res. **6**, 497–617 (1985)

- Semiclassical description
  - Classical interpretation of the stationary points
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
- Numerical results
  - Momentum distribution of the created pairs
  - Importance of interference effects

More details can be found in:

SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)
SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRL **114**, 143201 (2015)
SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRD **91**, 013009 (2015)

Leading-order Feynman diagram



- Photon: four-momentum q^μ $(q^2 = 0)$
- Electron: four-momentum p_1^μ $(p^2 = m^2)$
- Positron: four-momentum p_2^μ $(p'^2 = m^2)$

(we do *not* introduce dressed momenta!)

Semiclassical approximation

- We assume a strong plane-wave laser pulse $(\xi \gg 1)$
- The S-matrix is solvable analytically (to leading order)
- Stationary-phase analysis: main contribution to the process at $\phi = \phi_k$
- We propagate the final momenta back in time $p_{1,2}^\mu \longrightarrow p_{1,2}^\mu(\phi)$

$$p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu$$

- At the stationary phases ϕ_k $n(\phi) > 0$ is minimal
- Process happens where the pair becomes real as easy as possible!

SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)

Global conservation law

$$p_1^\mu + p_2^\mu = q^\mu + nk^\mu$$

Local conservation law

$$p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu$$

Classical absorption

$$n_{\mathrm{cl}}k^\mu = p_1^\mu + p_2^\mu - [p_1^\mu(\phi_k) + p_2^\mu(\phi_k)]$$

Propagation from the stationary point

Quantum absorption

$$n_{\mathrm{q}}k^\mu = p_1^\mu(\phi_k) + p_2^\mu(\phi_k) - q^\mu$$

Absorption during the creation

- Pair production at ϕ: $n(\phi)k^\mu$ must be absorbed "non-classically"
  $\longrightarrow$ $n(\phi)k^\mu$ is a measure for the effective tunneling distance
- Stationary-phase condition obeyed at $\phi = \phi_k$:
  $\longrightarrow$ $n(\phi_k)$: minimum laser four-momentum needed to be on shell

<u>Implications for the QED-PIC community</u>

- We obtain the scaling laws: $n_{\mathrm{q}} \sim \xi/\chi$ and $n_{\mathrm{cl}} \sim \xi^3/\chi$, respectively
- The energy transver from the laser to the particles is dominated by classical physics (taken into account self-consistently in a PIC code)
- The quantum absorption is not taken into account in a PIC code
  $\longrightarrow$ We have a definite error estimate now!

Four-momenta in the canonical light-cone basis

Characteristic four-vectors of the problem

- The Breit-Wheeler process is characterized by the quantitites:

$$\underbrace{q^\mu,}_{\text{gamma photon}} \qquad \underbrace{k^\mu,}_{\text{laser photons}} \qquad \underbrace{f_1^{\mu\nu}, f_2^{\mu\nu}}_{\text{laser polarizations}} \quad \longrightarrow \quad \Lambda_i^\mu = \frac{f_i^{\mu\nu} q_\nu}{(q f_i^2 q)^{1/2}}$$

- They allow us to construct a canonical light-cone basis:

$$k^\mu, \quad \bar{k}^\mu = q^\mu / kq, \quad e_1^\mu = \Lambda_1^\mu, \quad e_2^\mu = \Lambda_2^\mu, \qquad (q^2 = 0,\ kq \neq 0,\ \Lambda_i^2 = -1)$$

Invariant momentum parameters

- We define the Lorentz-invariant momentum parameters R, t_1 and t_2:

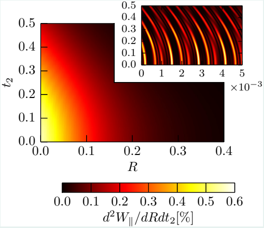
$$p_1^\mu = (1/2 + R) q^\mu + s' k^\mu + t_1 m \Lambda_1^\mu + t_2 m \Lambda_2^\mu, \qquad p_1^2 = m^2,$$
$$p_2^\mu = (1/2 - R) q^\mu - s k^\mu - t_1 m \Lambda_1^\mu - t_2 m \Lambda_2^\mu, \qquad p_2^2 = m^2$$

- From the on-shell conditions we obtain the relations ($n = s' - s$):

$$s = \frac{1}{(2R-1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2), \quad s' = \frac{1}{(2R+1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2)$$

- To include quantum processes into a PIC code, the initial conditions for the classical propagation of the created particles must be known
- Approach so far:
  - Ignore the transverse degree of freedom
  - All particles move initially into the forward direction
- From first principles:
  - We need to provide initial values for R, t_1 and t_2
  - Constant-crossed field rate: distribution for R and t_2
- **Question: which initial value for $\mathbf{t_1}$?** Our answer: $t_1 = 0$

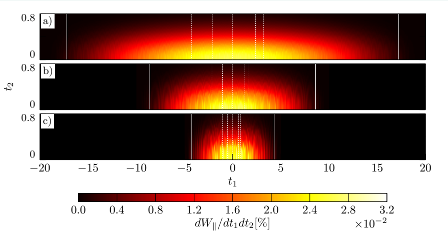
Momentum distribution of the created pairs



- Both R and t_2 are constants of motion for a plane-wave (constant-crossed) field
- The corresponding distributions are not changed by the classical propagation

Parameters:
$\chi = 1$, $\xi = 10$, $N = 5$, $\phi_0 = \pi/2$

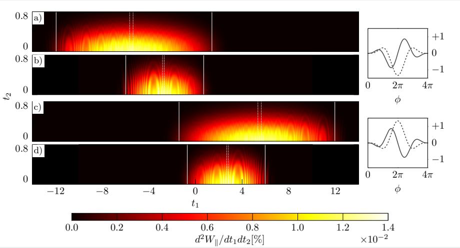
Scaling of the transverse momentum distribution



- The extend of the spectrum is determined by classical physics
- $t_1 \sim \xi$ (classical acceleration)
- $t_2 \sim 1$ (quantum distribution)

Parameters:
$N = 5,\ \phi_0 = \pi/2,\ \chi = 1,$
$\xi_1 = \xi = 20, 10, 5$ (a,b,c)

CEP dependence of the spectrum



- The spectrum exhibits a strong CEP-dependence [K. Krajewska et al, PRA 86, 052104 (2012)]
- Can be completely understood from classical electrodynamics

Parameters:
$N = 2,\ \chi = 1,\ \phi_0 = 0/\pi,\ \xi = 5/10$

Laser pulse shape: $\psi_1'(\phi) = \sin^2[\phi/(2N)]\ \sin(\phi + \phi_0)$, $\psi_2'(\phi) = 0$ (linear polarization)

Total/Differential probability

$$W(q,\epsilon) = \frac{m^2}{(kq)^2} \sum_{\text{spin}} \int_{-1/2}^{+1/2} dR \int_{-\infty}^{+\infty} dt_1 dt_2 \, \frac{w}{8} \, \frac{1}{(2\pi)^3} \, |\mathcal{M}(p_1,p_2;q)|^2$$

$$w = 4/(1-4R^2)$$

- Reduced S-matrix element: $\mathrm{i}\mathcal{M}(p_1,p_2;q) = \epsilon_\mu \, \bar{u}_{p_1} \mathcal{G}^\mu(p_1,q,-p_2) v_{p_2}$

$$\mathcal{G}^\rho = (-\mathrm{i}e)\left\{\gamma_\mu\left[\mathfrak{G}_0 g^{\mu\rho} + \sum_{j=1,2}\left(G_1 \mathfrak{G}_{j,1} f_j^{\mu\rho} + G_2 \mathfrak{G}_{j,2} f_j^{2\mu\rho}\right)\right] + \mathrm{i}\gamma_\mu\gamma^5 \sum_{j=1,2} G_3 \mathfrak{G}_{j,1} f_j^{*\mu\rho}\right\},$$

- The nontrivial information is contained in the master integrals

$$\mathfrak{G}_0 = \int_{-\infty}^{+\infty} d\phi \, e^{\mathrm{i}\tilde{S}_\Gamma(t_1,t_2;\phi)}, \qquad \mathfrak{G}_{j,l} = \int_{-\infty}^{+\infty} d\phi \, [\psi_j(\phi)]^l e^{\mathrm{i}\tilde{S}_\Gamma(t_1,t_2;\phi)}.$$

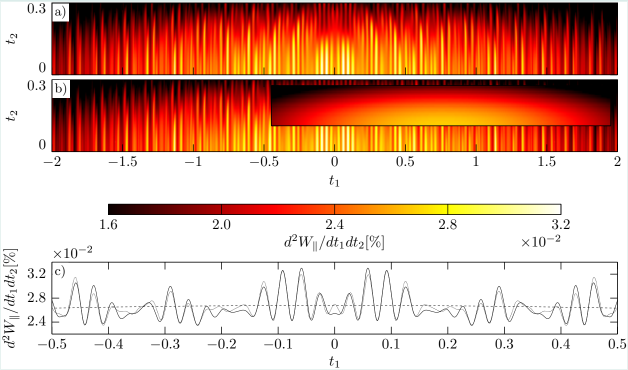
- For strong fields ($\xi \gg 1$): stationary-phase approximation

$$\mathfrak{G}_0 \approx \frac{kq}{m^2}\frac{2}{w}\left[\frac{w/2}{|\chi(\phi_k)|}\right]^{2/3} 2\pi \, \mathrm{Ai}(\rho) \, e^{\mathrm{i}\widetilde{S}_\Gamma(\phi_k)}, \qquad \rho = \left\{\frac{w}{[2\,|\chi(\phi_k)|]}\right\}^{2/3}(1+t_2^2)$$

- The Airy functions typical for processes inside constant-crossed fields are obtained but also a phase factor (interference effects!)

Pair production: Importance of interference effects

Local constant-crossed field approximation

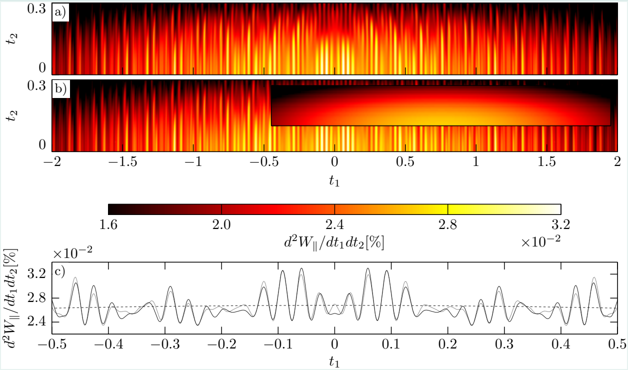


$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17\,\mathrm{GeV}$ and $10^{20}\,\mathrm{W/cm^2}$

- Stationary-phase approximation possible for $\xi \gg 1$
- Location of the stationary points: classical equation of motion
- Probability amplitude: pair-creation inside a constant-crossed field
- However: interference between different formation regions important

SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)

Local constant-crossed field approximation



$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17\,\mathrm{GeV}$ and $10^{20}\ \mathrm{W/cm^2}$

- Stationary-phase approximation possible for $\xi \gg 1$
- Loca … n
- Prob … field
- How … portant

SM, C. H.

Applied on the probability level, the local constant-crossed field approximation cannot reproduce the substructure!
This was observed for Compton scattering in:
Harvey, Ilderton, King, PRA **91** 013822 (2015)

Summary: main topics of the talk

- Why should we study Strong-Field QED?
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
- Lasers as a tool to study the critical field
  - Nonlinear Compton scattering
  - Nonlinear Breit-Wheeler pair production
- From a single vertex to a QED cascade
  - QED-PIC approach
  - Formation region and hierachy of scales
- Radiative corrections
  - Quantum dressing: exact wave functions
  - Fully nonperturbative regime of QED
- Nonlinear Breit-Wheeler process
  - Semiclassical description
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
  - Momentum distribution of the created pairs
  - Importance of interference effects

Thank you for your attention and your questions!