Realistic characterization of chirping instabilities in tokamaks

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PPPL Theory Seminar, March 31, 2016

Outline

- Introduction to frequency chirping
- Berk-Breizman model: cubic equation for mode amplitude evolution at early times
- Bump-on-tail modeling
- Generalization to multi-dimensional resonances in $(P_\varphi, \mathcal{E}, \mu)$ space and realistic mode structure
- Inclusion of microturbulence in the model
- Analysis of modes in TFTR, DIII-D and NSTX
- Conclusions

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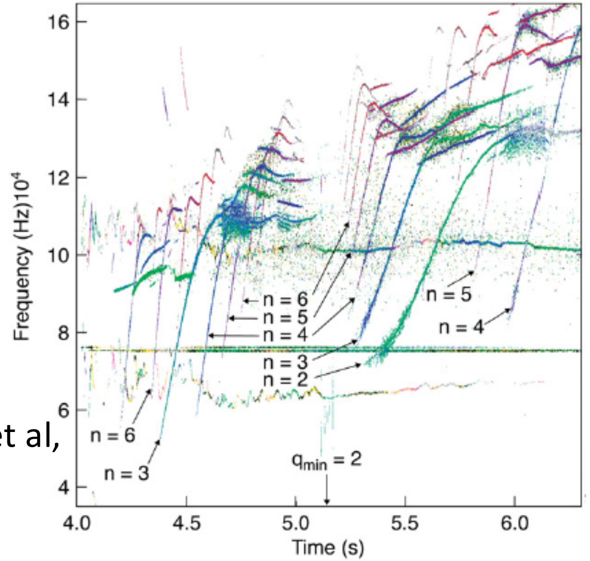
Two types of frequency shift observed experimentally

Frequency sweeping

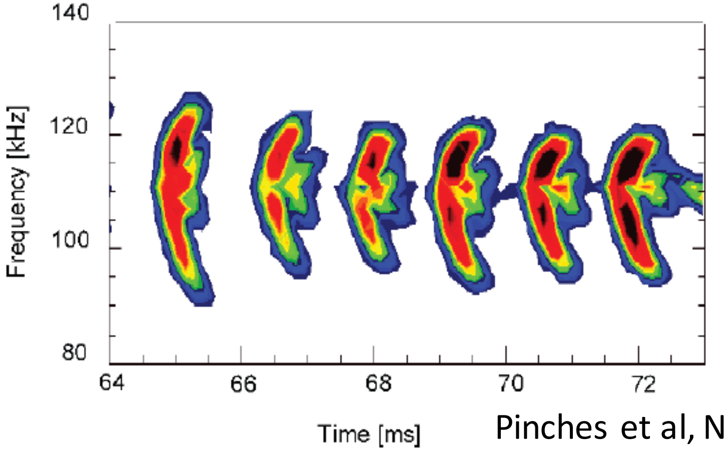
- frequency shift due to time-dependent background equilibrium
- exists without resonant particles
- timescale: ~100ms



Sharapov et al, PoP 2006

Frequency chirping

- frequency shift due to trapped particles
- does not exist without resonant particles
- timescale: ~1ms



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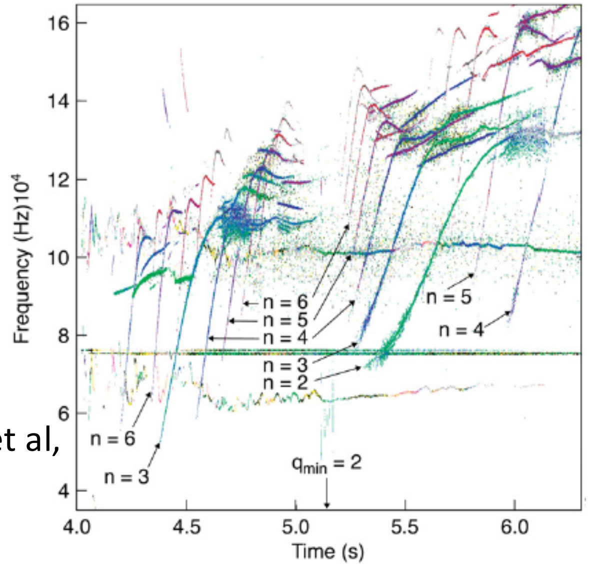

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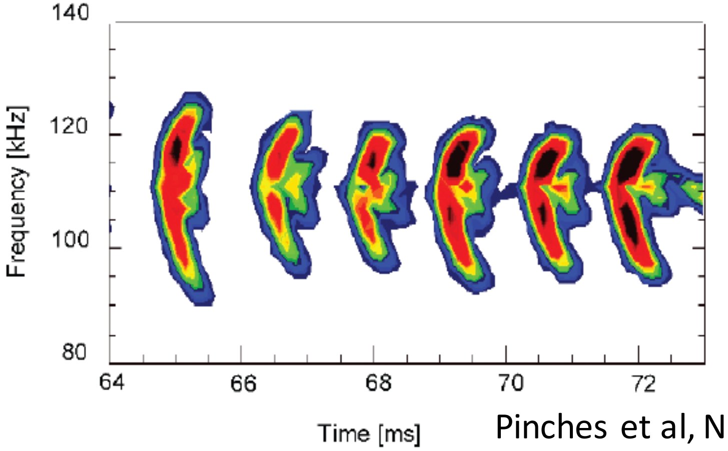
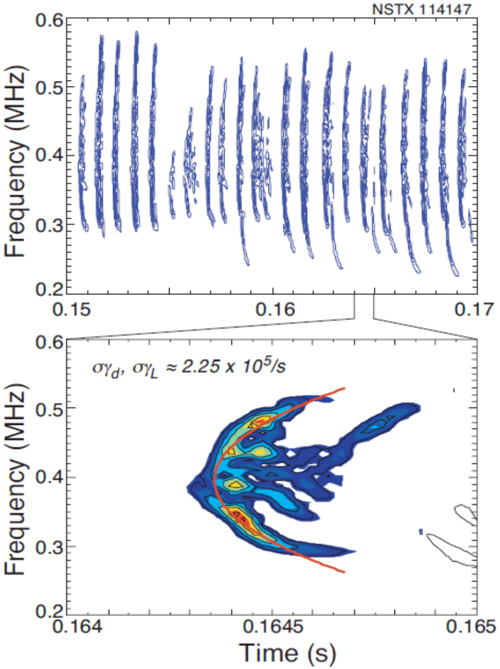


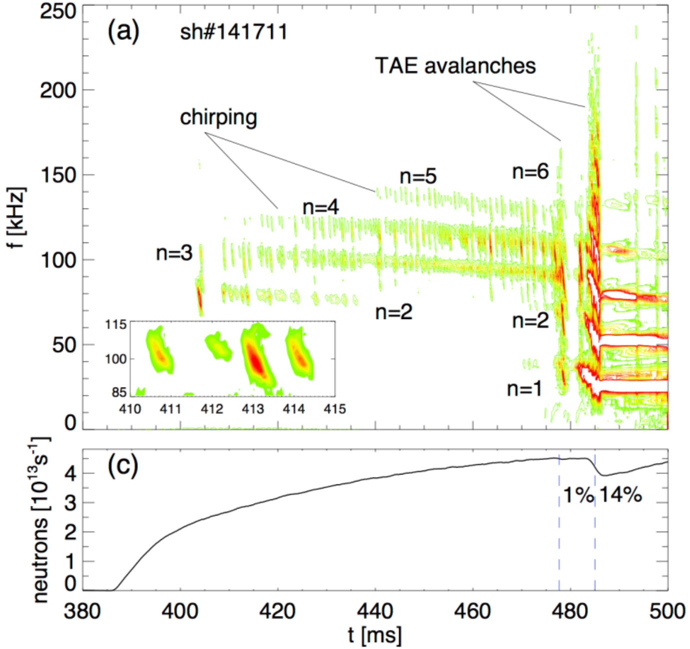
Pinches et al, NF 2004

Chirping modes can degrade the confinement of energetic particles

GAE/CAE
Fredrickson et al, PoP 2006.



TAE Podestà et al, NF 2012



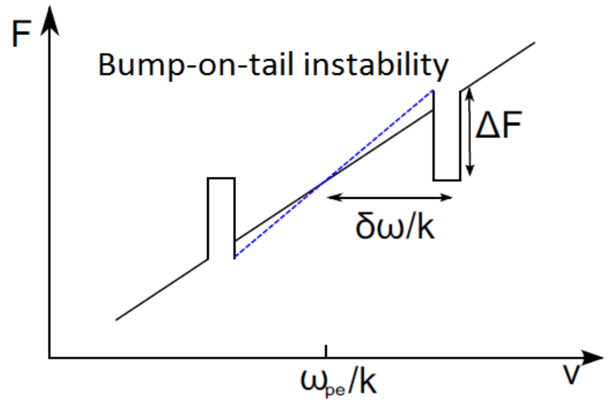
Up to 40% of injected beam is observed to be lost in DIII-D and NSTX

Chirping is ubiquitous in NSTX but rare in DIII-D. Why??

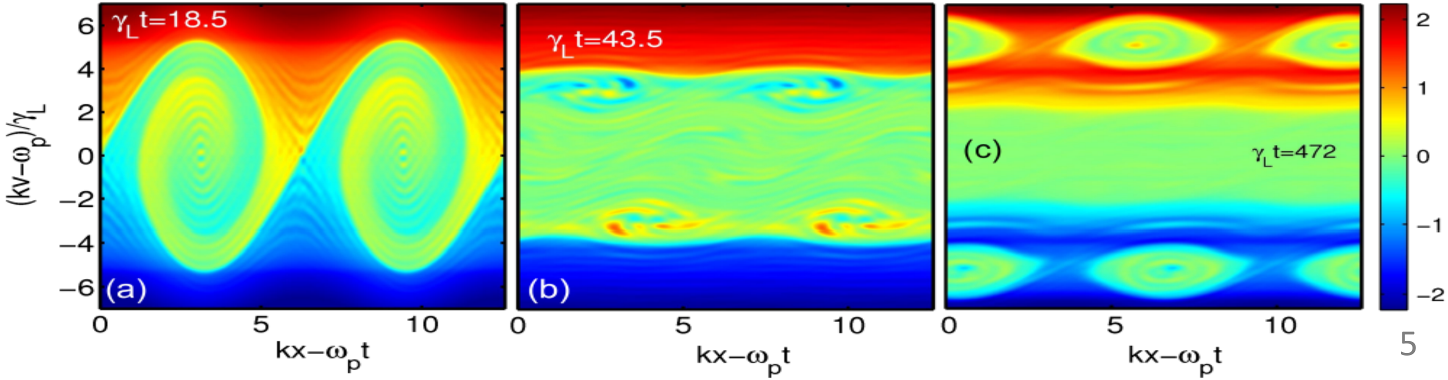
This presentation focuses on the conditions for chirping onset rather than their long-term evolution

Phase space holes and clumps in kinetically driven, dissipative systems – reduced bump-on-tail model

- Nonlinear Landau damping perspective: incomplete phase mixing leads to small sideband oscillations that may tap free energy at the edges of the plateau
- Chirping in frequency may allow for a continuous interplay between the free energy from the distribution function and the wave dissipation
- Collisions eventually degrade the resonant island plateau, and the process restarts



Vlasov simulations by Lilley and Nyqvist, PRL 2014



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Nonlinear dynamics of driven kinetic systems close to threshold

Starting point: **kinetic equation** plus **wave power balance**

Assumptions:

- Perturbative procedure for $\omega_b \ll \gamma$
- Truncation at third order due to closeness to marginal stability
- Bump-on-tail modal problem, uniform mode structure

Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude A:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^3 \tau^2 (2\tau/3+\tau_1) + i\nu_{drag}^2 \tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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stabilizing

destabilizing (makes integral sign flip)

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destabilizing (makes integral sign flip)

- If nonlinearity is weak: linear stability, solution saturates at a low level and f merely flattens (system not allowed to further evolve nonlinearly).
- If solution of cubic equation explodes: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (precursor of chirping modes).

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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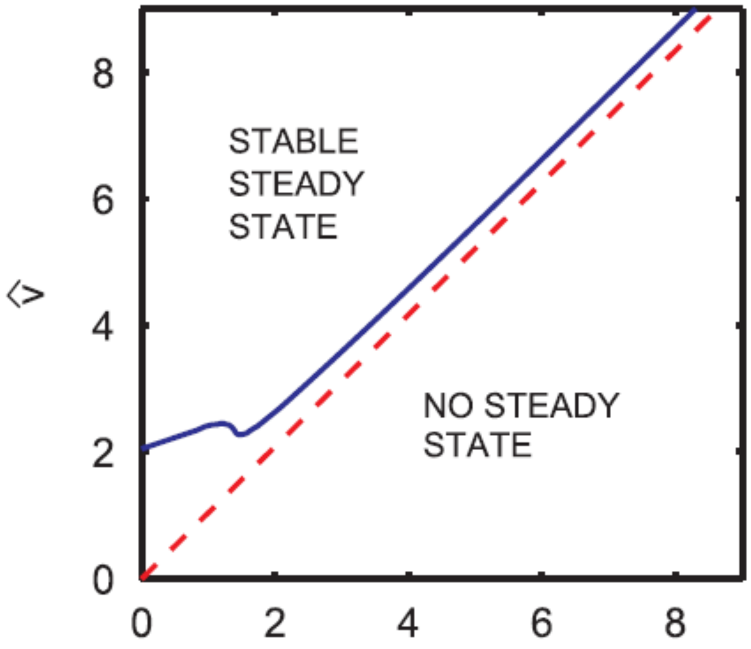
Existence and stability boundaries of solutions of the cubic equation – bump-on-tail case

Lilley, Breizman and Sharapov, PRL 2009

Scattering



Drag $\hat{\alpha}$

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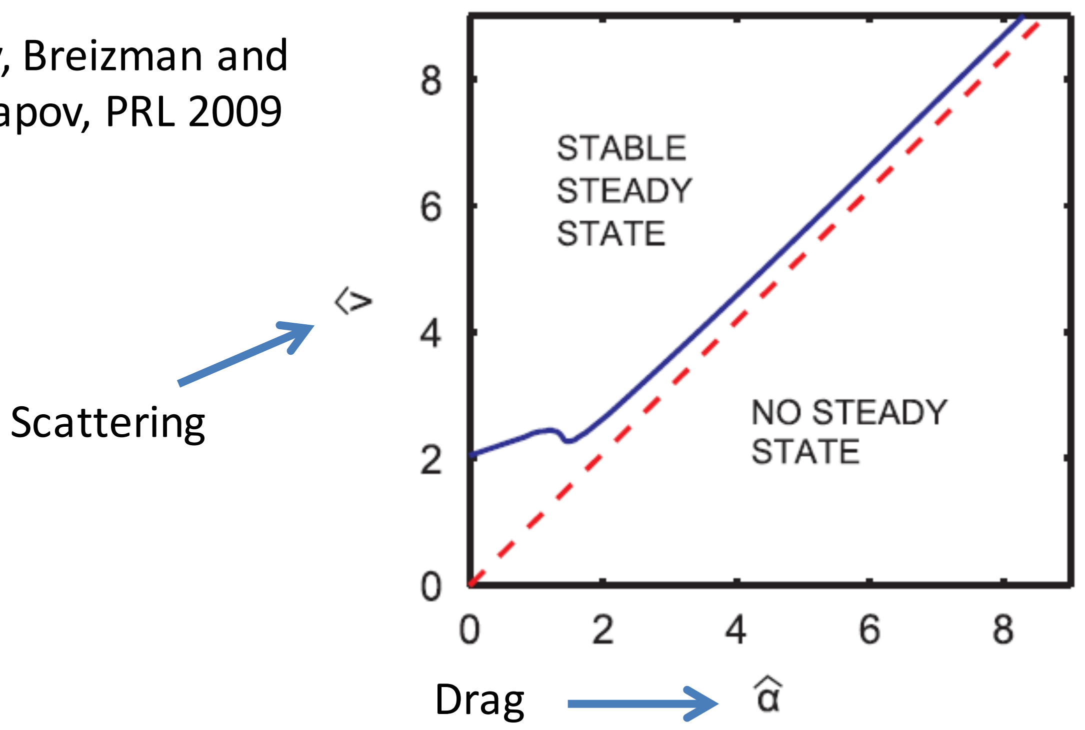
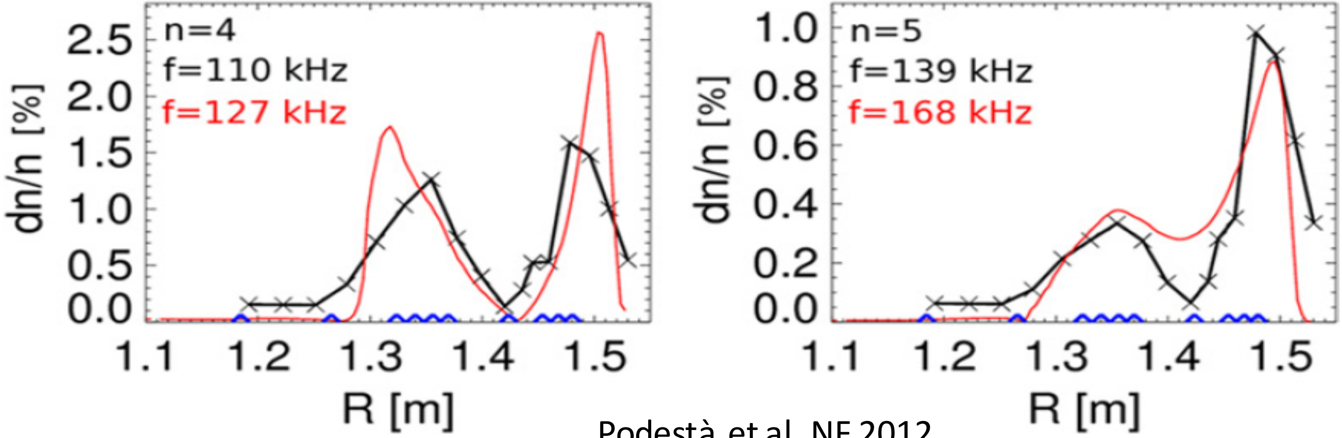


Stable steady solution cannot exist without scattering

We want to compare this prediction with modes observed in real discharges. How?

Mode structure identification

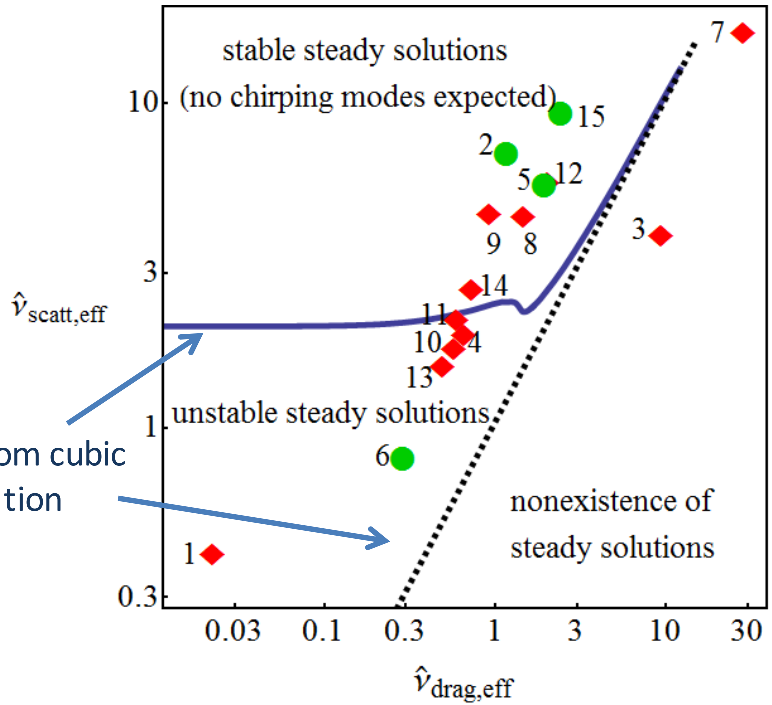
- NOVA code: finds linear, ideal mode structures
- Its kinetic postprocessor NOVA-K computes resonance surfaces and provides damping and linear growth rates. Phase space and bounce averages are necessary to calculate effective collisional coefficients
- NOVA's mode structures are compared with NSTX reflectometer measurements (fluid displacement times the local density gradient is equivalent to the density fluctuation). In DIII-D ECE is used



Podestà et al, NF 2012

Chirping in terms of effective collisional coefficients for realistic resonances and mode structures

Experimental observations:
Red diamonds: chirping was observed
Green dots: no chirping observed



Follow from cubic equation

Pitch-angle scattering: leads to loss of correlation (loss of phase information from one bounce to another)

Drag (slowing down): coherently moves structures down in velocity

Bump-on-tail modeling is not enough to resolve the regions in collisional space that allows for chirping modes

Missing physics in the simplified theoretical prediction: mode structure, (multiple) resonance surfaces and phase-space and bounce averages

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Generalization: cubic equation with collisional coefficients varying along resonances and particle orbits

Action-angle formalism for the general problem, with a similar perturbative approach employed before, leads to the **generalized criterion for existence of steady-state solutions** (no chirping):

$$\sum_{l,\sigma_\parallel} \int dP_\varphi \int d\mu \frac{\tau_b}{\nu_{drag}^4} |V_l|^4 \left| \frac{\partial \Omega_l}{\partial I} \right| \frac{\partial F}{\partial I} Int > 0$$

$$Int \equiv -Re \int_0^\infty dz \frac{z}{-\frac{\nu_{diff}^3}{\nu_{drag}^3} z + i} exp \left[-2 \frac{\nu_{diff}^3}{\nu_{drag}^3} z^3/3 + iz^2 \right]$$

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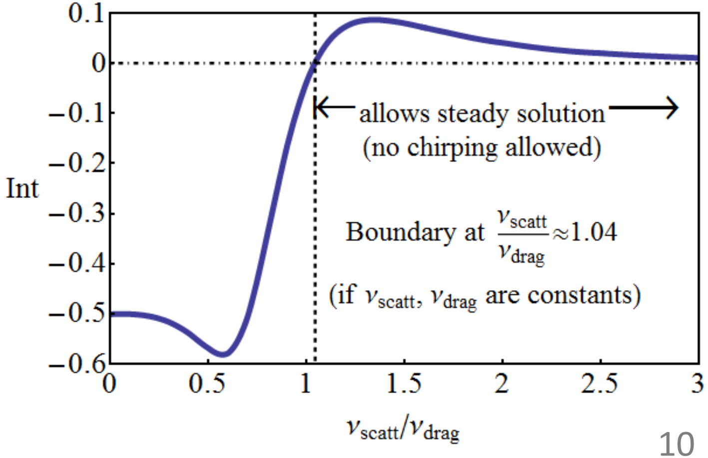

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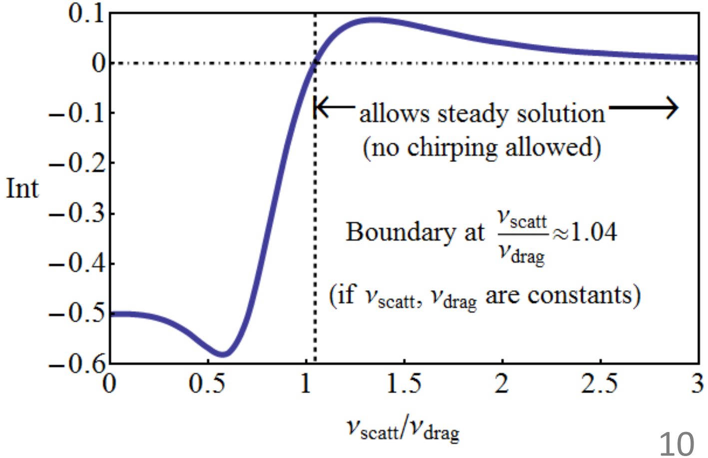
Phase space integration



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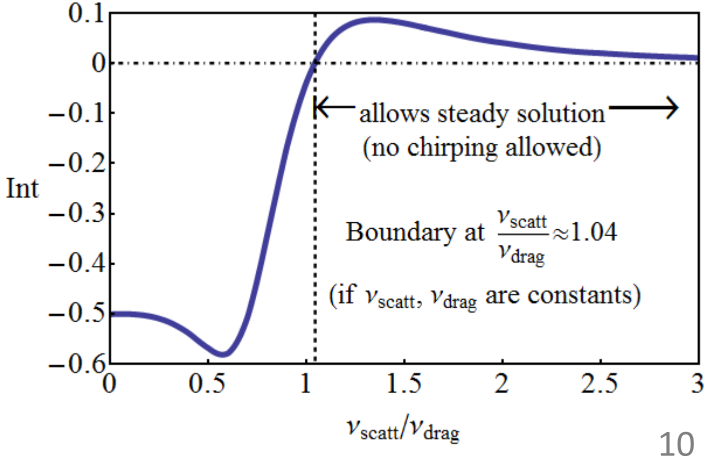
$$q\int dt \mathbf{v}_{dr} \cdot \delta \mathbf{E} e^{i\omega t}$$



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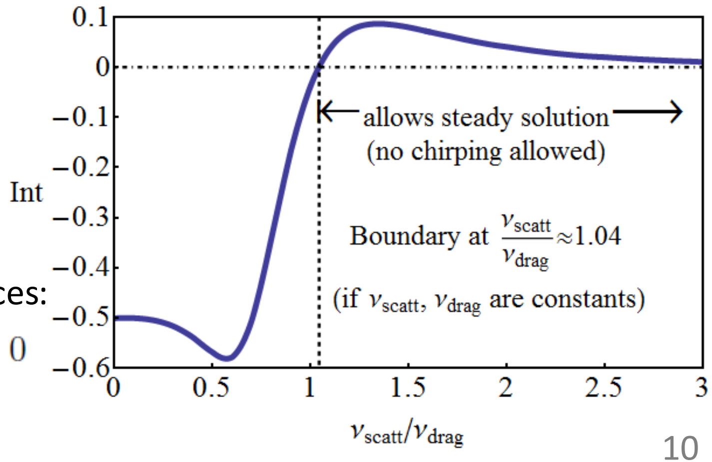

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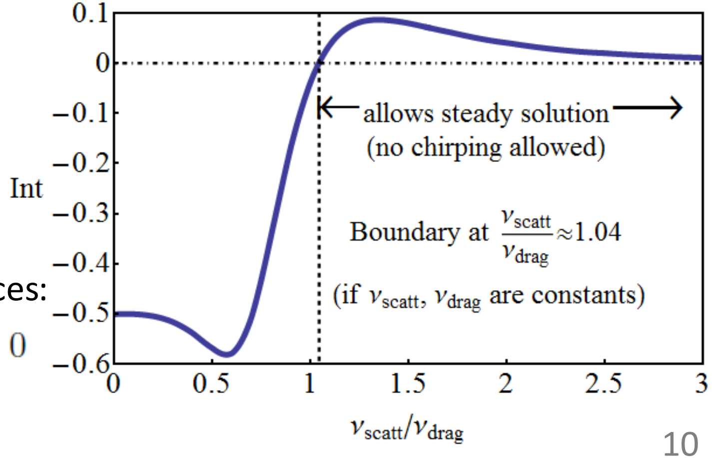

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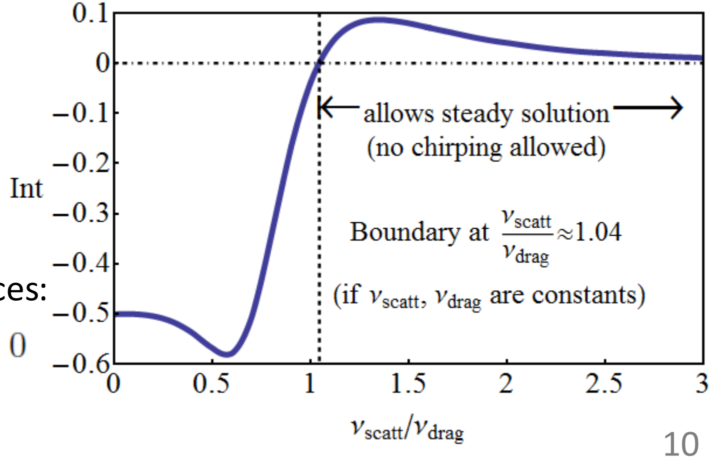

Criterion was incorporated into NOVA-K

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Correction to the diffusion coefficient: the inclusion of electrostatic microturbulence

- Microturbulence can well exceed pitch-angle scattering at the resonance[1]
- From GTC gyrokinetic simulations for passing particles:[2]

$$D_{EP}(E) \approx D_{th,i} \frac{5T_e}{E}$$

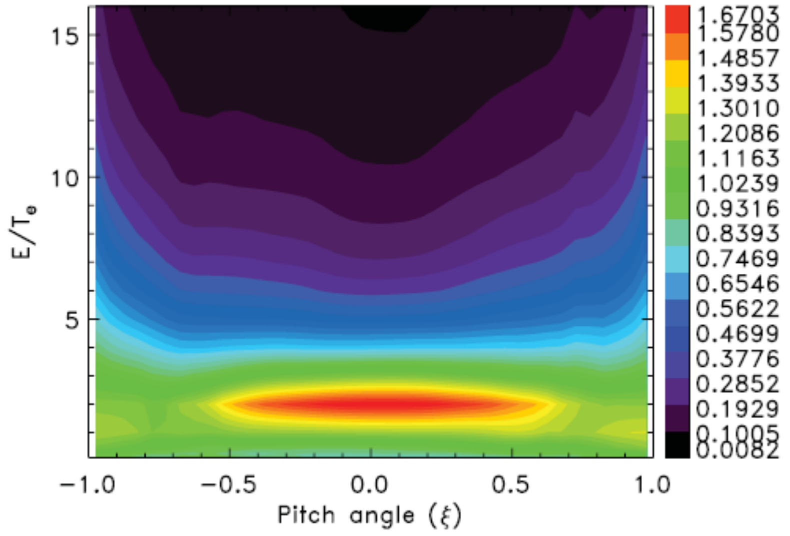
- As pitch-angle scattering, microturbulence acts to destroy phase-space holes and clumps
- Unlike DIII-D and TFTR, transport in NSTX in mostly neoclassical
- Complex interplay between gyroaveraging, field anisotropy and poloidal drift effects leads to non-zero EP diffusivity[3]

[1]Lang and Fu, PoP 2011

[2]Zhang, Lin and Chen, PRL 2008

[3]Estrada-Mila et al, PoP 2005

Ratio of fast ion diffusivity to thermal ion diffusivity[2]



Pueschel et al, NF 2012 gives similar microturbulence levels

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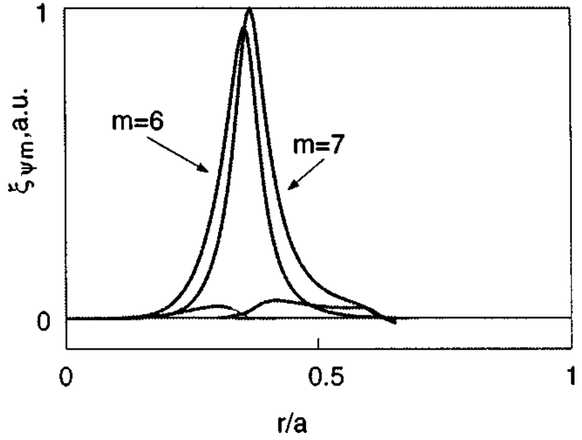
TAE in TFTR shot 103101: no chirping observed

TAE mode structure



Gorelenkov et al, PoP 1999

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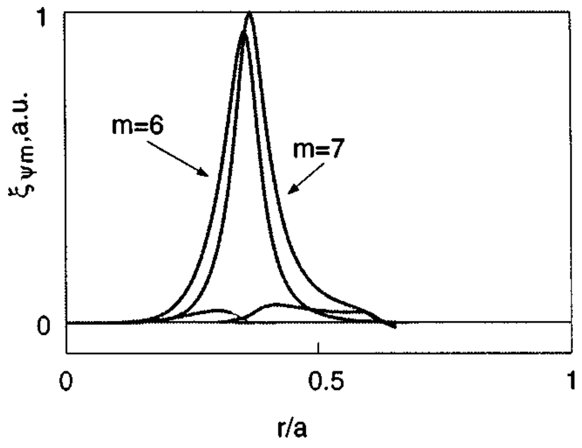

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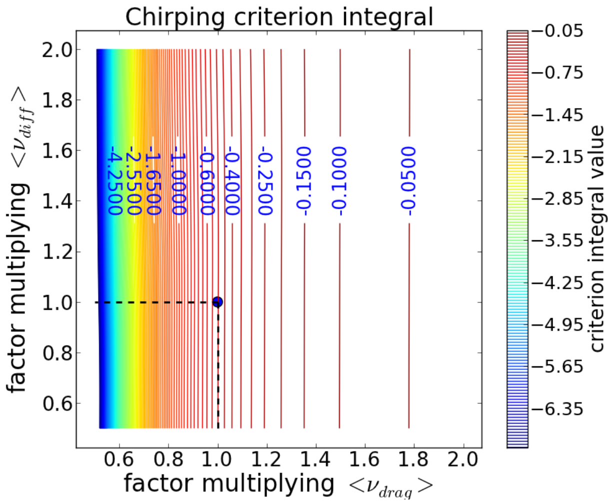
Drag vs pitch-angle scattering:



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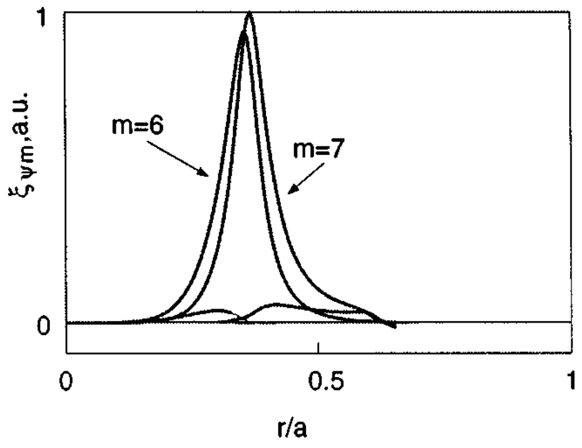

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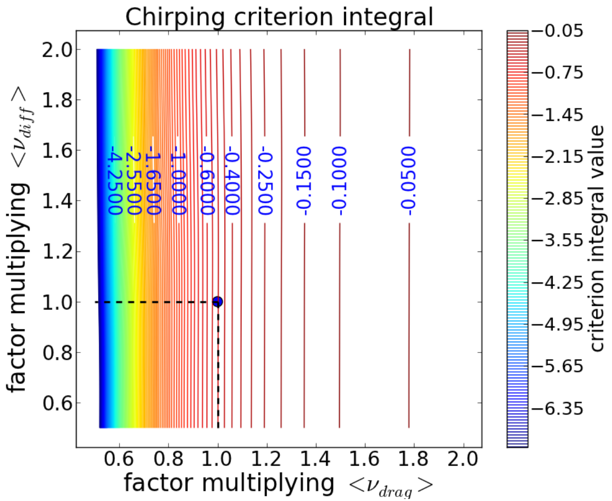
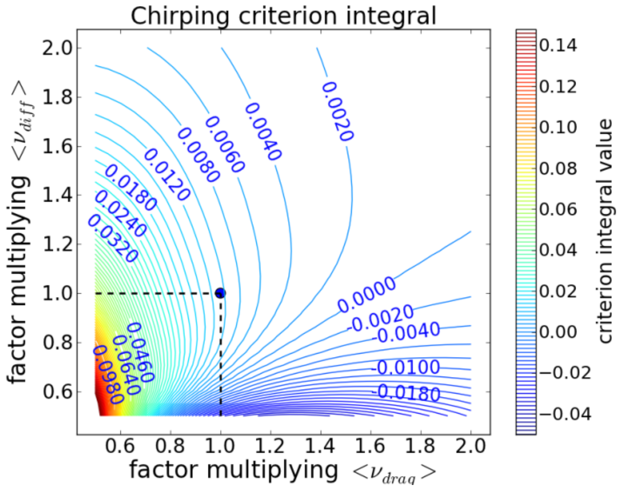


Drag vs pitch-angle scattering + microturbulence:



TAE in TFTR shot 103101: no chirping observed

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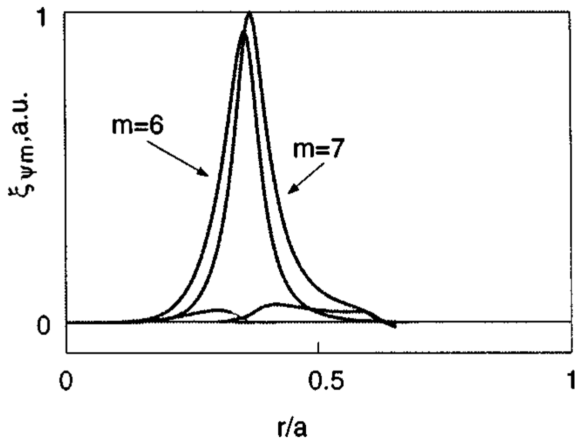

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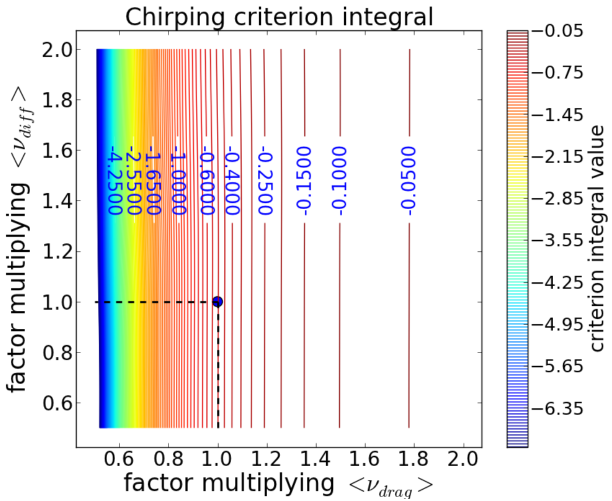

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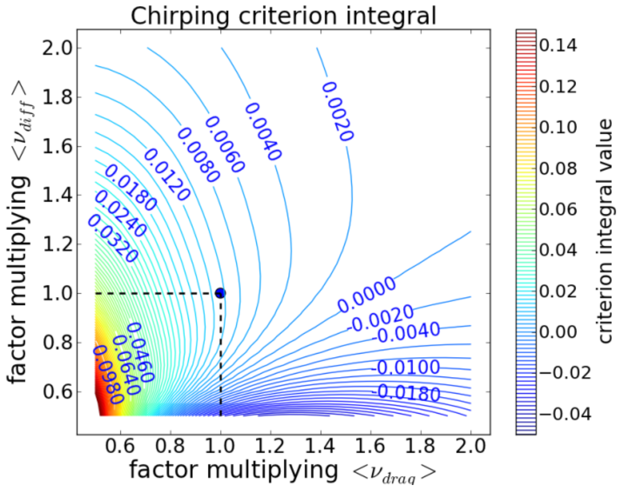
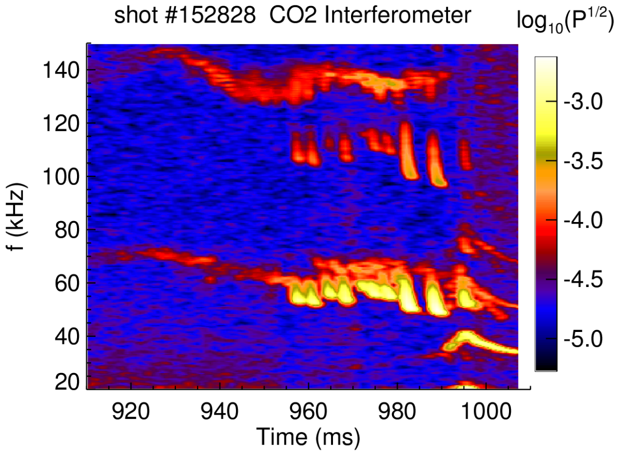


Microturbulence has a strong effect on destroying resonant, coherent phase-space structures

Characterization of a rarely observed chirping mode in DIII-D



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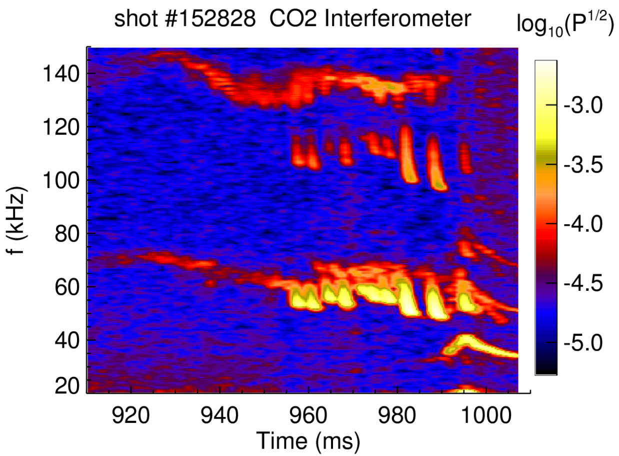

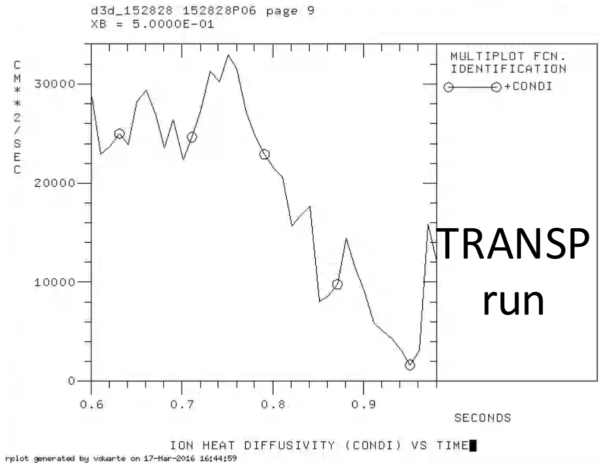

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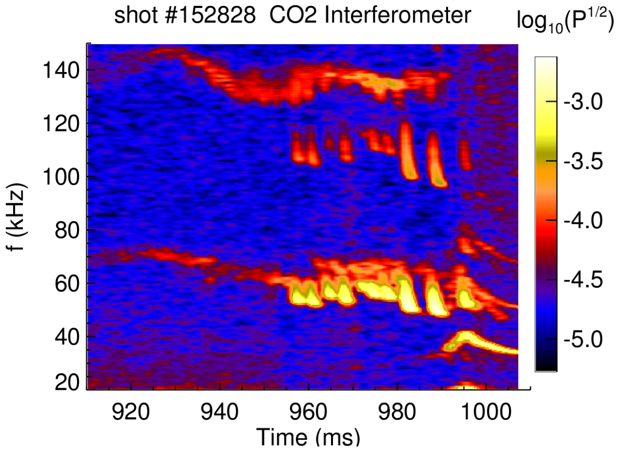
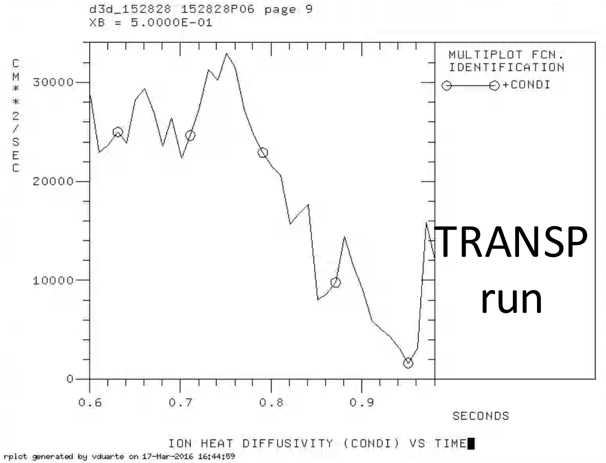
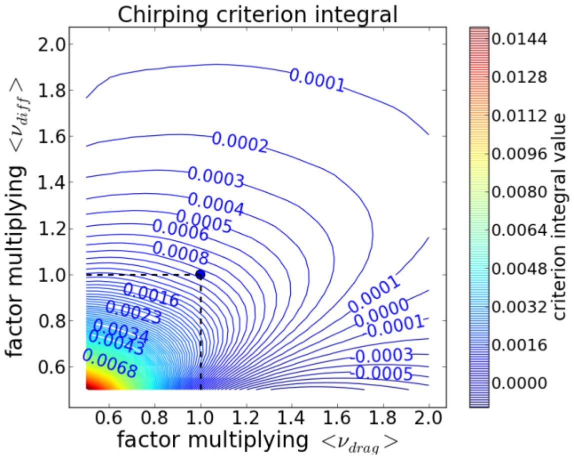


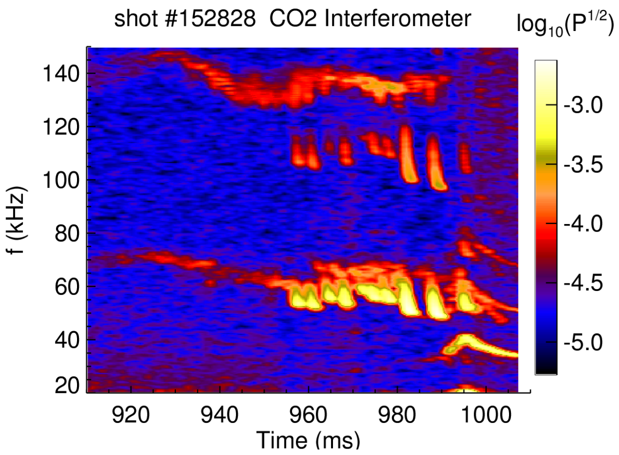


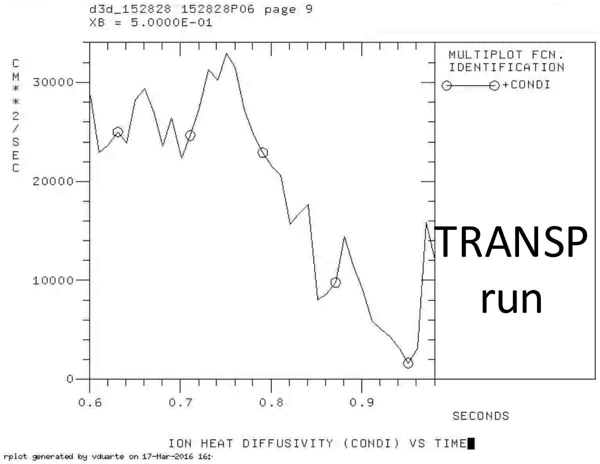
TRANSP run

Before chirping, at 900ms: D~1m^2/s



Characterization of a rarely observed chirping mode in DIII-D





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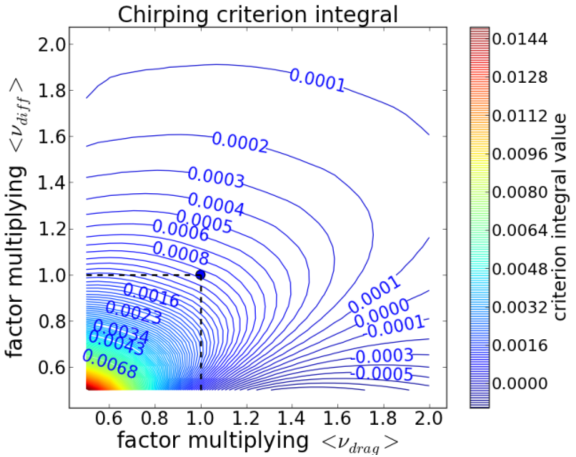
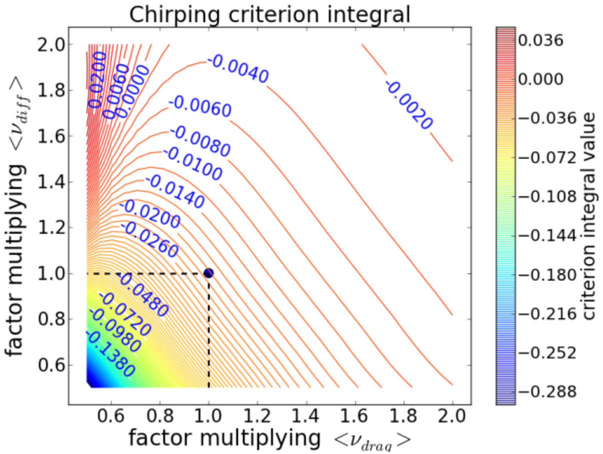


During chirping, at 960ms: D~0.2m²/s



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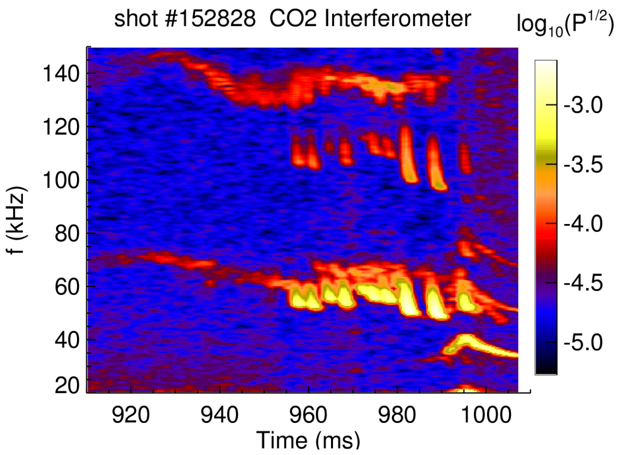
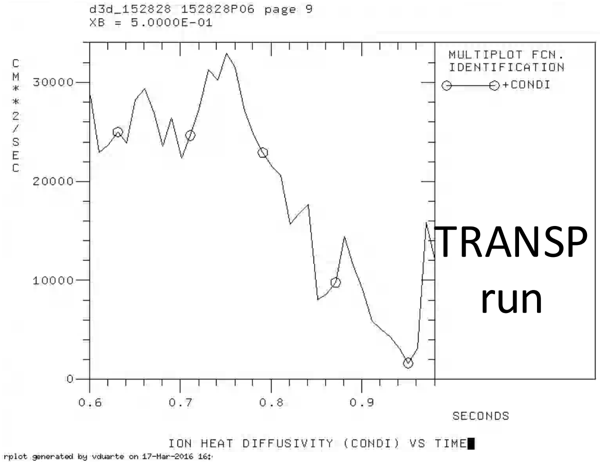
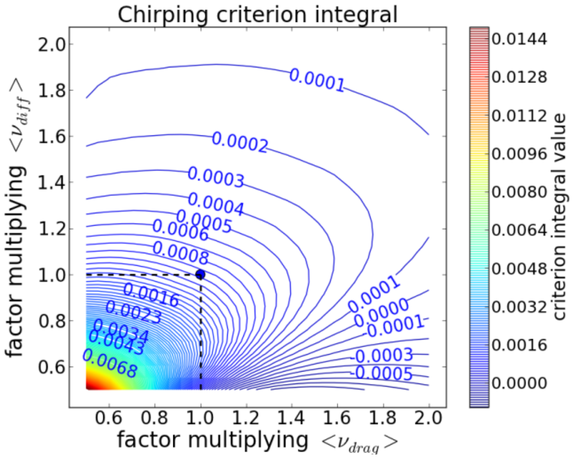




Diffusivity drop due to L→H mode transition

Strong rotation shear was observed

Before chirping, at 900ms: D~1m^2/s



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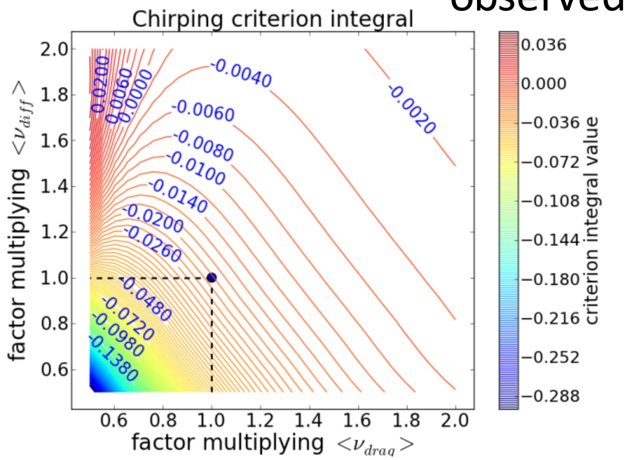
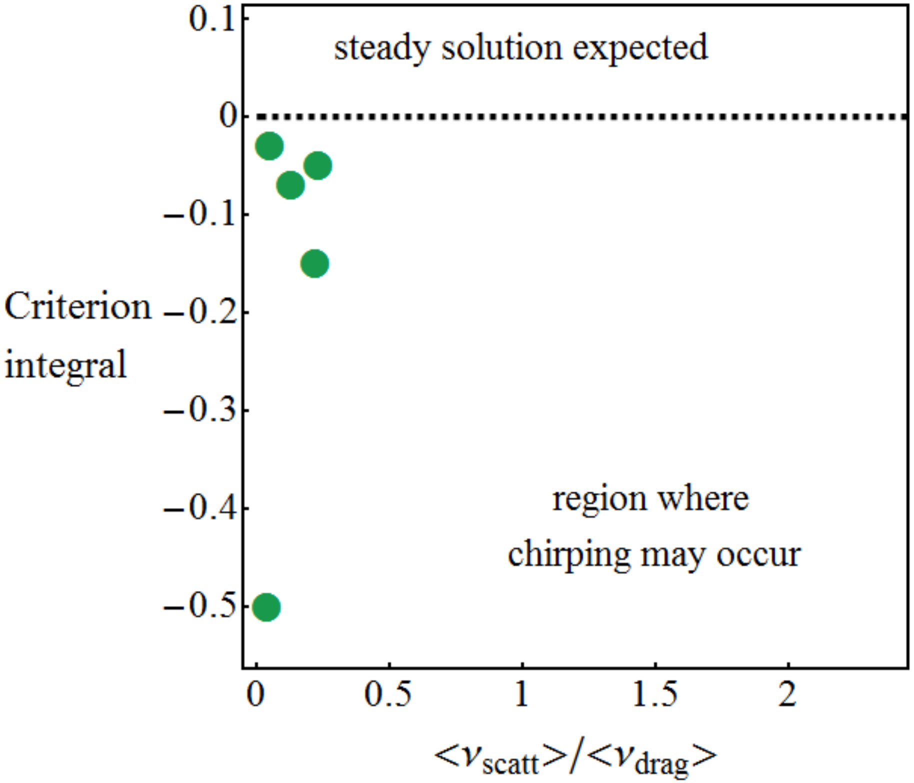
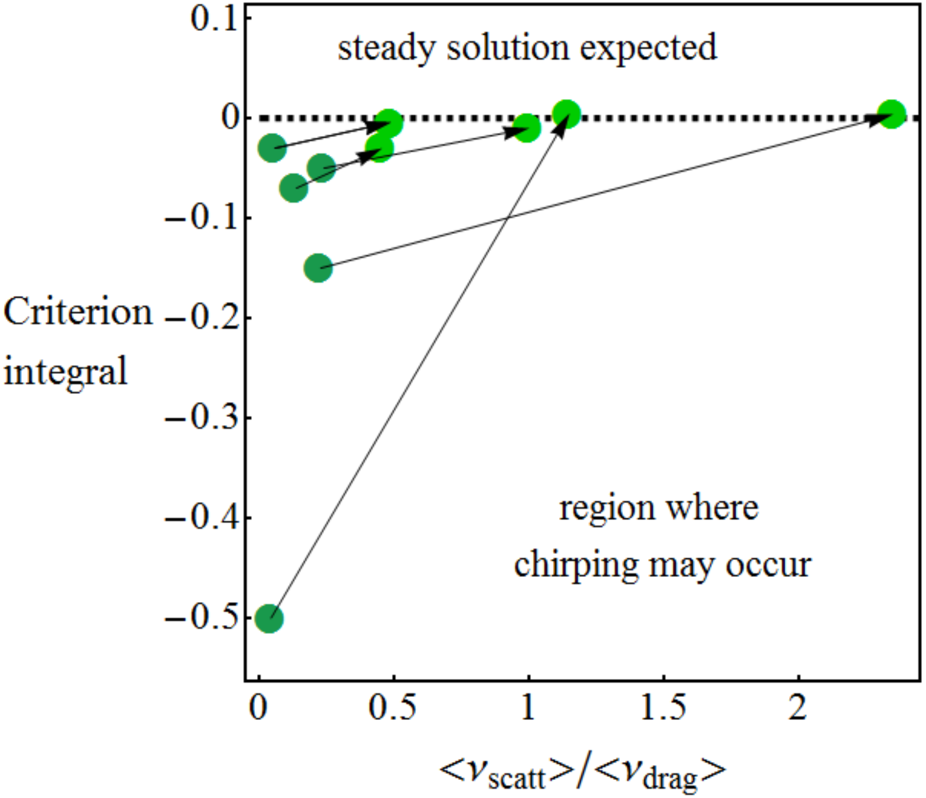


Non-chirping DIII-D shot analyzed in terms of pitch-angle scattering and drag

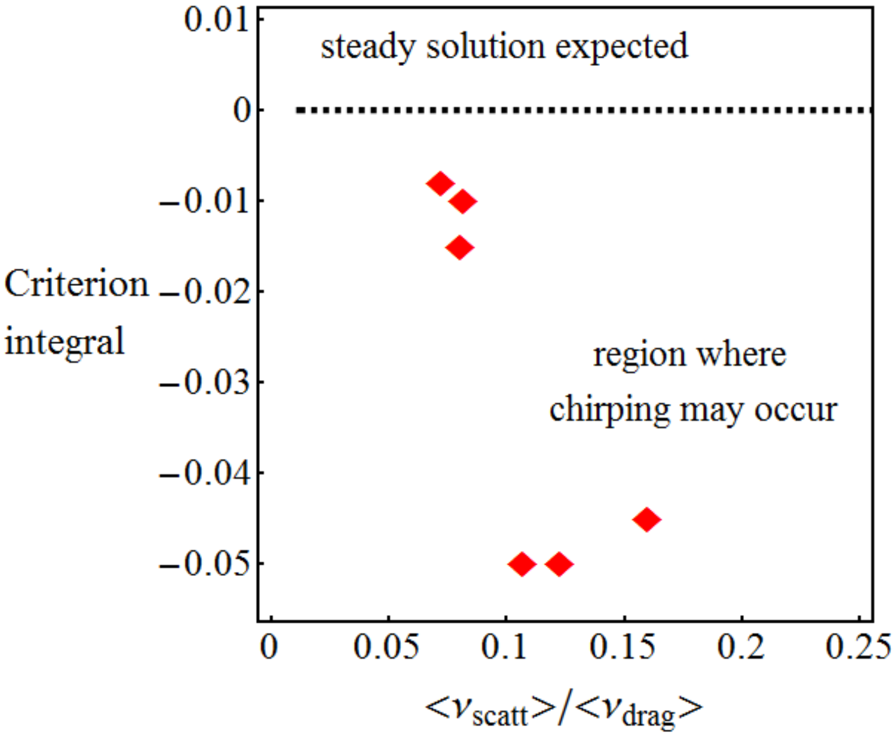


Inclusion of microturbulence in non-chirping DIII-D shots



Microturbulence has a substantial effect on bringing modes to the steady region

NSTX chirping TAEs



Microturbulence does not have appreciable effects in NSTX

Even if scattering levels is increased several times the modes cannot reach the boundary

All cases have negative values for the criterion, which is consistent with observation

Outline

- Introduction to frequency chirping
- Berk-Breizman model: cubic equation for mode amplitude evolution at early times
- Bump-on-tail modeling
- Generalization to multi-dimensional resonances in $(P_\varphi, \mathcal{E}, \mu)$ space and realistic mode structure
- Inclusion of microturbulence in the model
- Analysis of modes in TFTR, DIII-D and NSTX
- **Conclusions**

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- Refinement of turbulence contribution using Beam Emisson Spectroscopy
- Development of a line-broadened quasilinear diffusion solver coupled with NOVA and NOVA-K: chirping criterion is important for identification of parameter space for quasilinear validity

Thank you