Parallel Electron Force Balance and the L-H Transition

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In a popular description of the L-H transition, energy transfer to the mean flows directly depletes turbulence fluctuation energy, resulting in suppression of the turbulence and a corresponding transport bifurcation. However, electron parallel force balance couples nonzonal velocity fluctuations with electron pressure fluctuations on rapid timescales, comparable with the electron transit time. For this reason, energy in the nonzonal velocity stays in a fairly fixed ratio to electron thermal free energy, at least for frequency scales much slower than electron transit. In order for direct depletion of the energy in turbulent fluctuations to cause the L-H transition, energy transfer via Reynolds stress must therefore drain enough energy to significantly reduce the sum of the free energy in nonzonal velocities and electron pressure fluctuations. At low $k_\perp$, the electron thermal free energy is much larger than the energy in nonzonal velocities, posing a stark challenge for this model of the L-H transition.

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Overview

- Background: paradigms for the L-H transition
- Model equations: Hasegawa-Wakatani plus curvature terms
- Parallel electron physics and adiabatic response
- Free energy balance equations
- Implications for rapid L-H transition
- Implications for slow L-H transition
- Conclusions

This work is part of a collaboration with Ahmed Diallo, Stewart Zweben, and Santanu Banerjee, doing experimental investigation of the L-H transition on NSTX with gas puff imaging (GPI).

Background: Paradigms for the L-H transition

What is the L-H transition?

- sudden transition to a state of good energy confinement in the edge
- appears as heating power *increases* past some threshold

Most models of L-H transition focus on $\boldsymbol{E} \times \boldsymbol{B}$ shear and have two parts:

- Something drives sheared zonal $\boldsymbol{E} \times \boldsymbol{B}$ flows
  - Nonlinear energy transfer from turbulence to flows via Reynolds stress.
  - 'Diamagnetic' flow shear due to ∇p_i contribution to E_r.
- Suppression of turbulence by flow shear (two possibilities):
  1. Energy transfer to flows directly depletes turbulent fluctuations.
  2. Shearing of eddies destroys turbulence in other ways, e.g.
     - Reduce effective growth rate
     - Increase damping

Some experimental investigations consider an energy balance between $E_z \doteq \int \mathrm{d}\mathcal{V}\, \frac{1}{2} n_0 m_i \langle v_E^y \rangle^2$ and $E_\sim \doteq \int \mathrm{d}\mathcal{V}\, \frac{1}{2} n_0 m_i [(\tilde{v}_E^x)^2 + (\tilde{v}_E^y)^2]$:

$$\partial_t E_\sim = \gamma E_\sim - n_0 m_i \int \mathrm{d}\mathcal{V}\, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \left\langle v_E^y \right\rangle$$
$$\partial_t E_z = -\mu E_z + n_0 m_i \int \mathrm{d}\mathcal{V}\, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \left\langle v_E^y \right\rangle$$

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Hasegawa-Wakatani plus curvature is enough for this problem.

$$\partial_t \overbrace{n_e}^{\text{fluctuating part}} + \overbrace{\boldsymbol{v}_E \cdot \nabla}^{\boldsymbol{E}\times\boldsymbol{B}\,\text{adv}} (n_e + n_0) = \frac{1}{e}\nabla_{\|} j_{\|} + \frac{1}{e}\mathcal{K}(\overbrace{n_0 e\phi}^{n_0\nabla\cdot\boldsymbol{v}_E} - \overbrace{n_e T_{e0}}^{\text{curv. drift}})$$

$$\underbrace{\frac{n_0 m_i}{B^2}(\partial_t + \boldsymbol{v}_E \cdot \nabla)\nabla_\perp^2 \phi}_{\text{ion polarization current}} = \nabla_{\|} j_{\|} - \underbrace{\mathcal{K}(n_e T_{e0})}_{\text{curv. current}}$$

$$\eta j_{\|} = \frac{T_{e0}}{n_0 e}\nabla_{\|} n_e - \nabla_{\|}\phi$$

Assumptions:

$$\mathcal{K} \doteq -\frac{2}{B^2}\hat{\boldsymbol{b}} \times \nabla B \cdot \nabla$$

- isothermal electrons
- single species of cold singly-ionized ions
- purely resistive parallel dynamics
- shearless, simple-circular, large-aspect-ratio magnetic geometry
- frequencies fast relative to ion transit $\omega \gg c_s/qR$ (justified)

None of the assumptions (except the last one) are generally justified for edge turbulence.
All of the assumptions are OK for the upcoming analysis.

Rapid linear physics acts to cause electron adiabatic response

Linear equations, with nonadiabatic electron response $h_e \doteq n_e/n_0 - e\phi/T_{e0}$

$$\partial_t n_e + \boldsymbol{v}_E \cdot \nabla n_0 = \frac{1}{e}\nabla_\parallel j_\parallel + \frac{1}{e}\mathcal{K}\left(n_0 e\phi - n_e T_{e0}\right)$$

$$\frac{n_0 m_i}{B^2}\partial_t \nabla_\perp^2 \phi = \quad \nabla_\parallel j_\parallel - \mathcal{K}\left(n_e T_{e0}\right)$$

$$\eta j_\parallel = \frac{T_{e0}}{n_0 e}\nabla_\parallel n_e - \nabla_\parallel \phi = \frac{T_{e0}}{e}\nabla_\parallel h_e$$

If resistivity is small enough that parallel electron diffusion is rapid relative to drift wave frequencies, then we may neglect $\boldsymbol{v}_E \cdot \nabla n_0$ and $\mathcal{K}(\cdots)$. Also take a single perpendicular mode $\nabla_\perp^2 \to -k_\perp^2$, then

$$\partial_t h_e \approx \frac{1}{n_0 e}\left(1 + \frac{1}{k_\perp^2 \rho_s^2}\right)\nabla_\parallel j_\parallel = \frac{T_{e0}}{\eta n_0 e^2}\left(1 + \frac{1}{k_\perp^2 \rho_s^2}\right)\nabla_\parallel^2 h_e$$

where $T_{e0}/\eta n_0 e^2 = v_{te}^2/\nu_{ei}$ is a parallel electron diffusion coefficient.

At frequencies lower than $(k_\parallel^2 v_{te}^2/\nu_{ei} k_\perp^2 \rho_s^2)$, we expect $\tilde{n}_e/n_0 \approx e\tilde{\phi}/T_{e0}$.

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Parallel electron physics restricts free energy balance.

Our equations nonlinearly conserve a free energy $(E_n + E_\sim + E_z)$ for

$$E_n \doteq \frac{T_{e0}}{2n_0}\int \mathrm{d}\mathcal{V}\, n_e^2; \quad E_\sim \doteq \frac{1}{2}n_0 m_i \int \mathrm{d}\mathcal{V}\left[(\tilde{v}_E^x)^2 + (\tilde{v}_E^y)^2\right]; \quad E_z \doteq \frac{1}{2}n_0 m_i \int \mathrm{d}\mathcal{V}\langle v_E^y\rangle^2$$

which evolve as

$$\partial_t E_n = \int \mathrm{d}\mathcal{V}\Big[\frac{T_{e0}}{L_n} n_e v_E^x - \frac{T_{e0}}{n_0 e} j_\parallel \nabla_\parallel n_e - T_{e0}\phi\mathcal{K}(n_e)\Big],$$

$$\partial_t E_\sim = \int \mathrm{d}\mathcal{V}\Big[j_\parallel \nabla_\parallel \phi + T_{e0}\tilde{\phi}\mathcal{K}(n_e) - n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y)\partial_x \langle v_E^y\rangle\Big],$$

$$\partial_t E_z = \int \mathrm{d}\mathcal{V}\Big[T_{e0}\langle\phi\rangle \mathcal{K}(n_e) + n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y)\partial_x \langle v_E^y\rangle\Big],$$

$$\partial_t (E_n + E_\sim + E_z) = \int \mathrm{d}\mathcal{V}\Big[\frac{T_{e0}}{L_n} n_e v_E^x - \frac{T_{e0}}{e} j_\parallel \nabla_\parallel h_e\Big].$$

Note that $(T_{e0}/e) j_\parallel \nabla_\parallel h_e = \eta j_\parallel^2 = (T_{e0}/e)^2 (\nabla_\parallel h_e)^2/\eta > 0$.

When $\tilde{h}_e$ becomes small ($\tilde{h}_e \ll \tilde{n}_e/n_0, e\tilde{\phi}/T_{e0}$), then $(T_{e0}/n_0 e) j_\parallel \nabla_\parallel n_e \approx j_\parallel \nabla_\parallel \phi$, so the red terms are approximately a conservative energy transfer.

Implications for energy transfer in rapid L-H transition:

$$\partial_t E_n = \int \mathrm{d}\mathcal{V} \Big[\frac{T_{e0}}{L_n} n_e v_E^x - \frac{T_{e0}}{n_0 e} j_\parallel \nabla_\parallel n_e - T_{e0} \phi \mathcal{K}(n_e) \Big]$$

$$\partial_t E_\sim = \int \mathrm{d}\mathcal{V} \Big[j_\parallel \nabla_\parallel \phi + T_{e0} \tilde{\phi} \mathcal{K}(n_e) - n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \Big]$$

$$\partial_t E_z = \int \mathrm{d}\mathcal{V} \Big[T_{e0} \langle \phi \rangle \mathcal{K}(n_e) + n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \Big]$$

Assume L-H transition is faster than growth rate γ but slow relative to electron parallel relaxation rate $(k_\parallel^2 v_{te}^2 / \nu_{ei} k_\perp^2 \rho_s^2)$, so $\tilde{n}_e / n_0 \approx e\tilde{\phi} / T_{e0}$ thus

$$\frac{E_\sim}{E_{\tilde{n}}} = \frac{\int \mathrm{d}\mathcal{V} \left[(\tilde{v}_E^x)^2 + (\tilde{v}_E^y)^2 \right] / c_s^2}{\int \mathrm{d}\mathcal{V} \left(\tilde{n}_e^2 / n_0^2 \right)} = \frac{\int \mathrm{d}\mathcal{V} \left| \nabla_\perp \tilde{\phi} \right|^2 / B^2 c_s^2}{\int \mathrm{d}\mathcal{V} \left(\tilde{n}_e^2 / n_0^2 \right)} \sim k_\perp^2 \rho_s^2,$$

where $k_\perp$ is roughly an rms-averaged perp wavenumber.

If

$$\frac{\int \mathrm{d}\mathcal{V}\, n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle}{E_\sim} \leq \max \left| \partial_x \langle v_E^y \rangle \right|$$

is slow relative to $k_\parallel^2 v_{te}^2 / (\nu_{ei} k_\perp^2 \rho_s^2)$, then $j_\parallel$ will keep $E_\sim / E_{\tilde{n}} \sim k_\perp^2 \rho_s^2$ so that the Reynolds work must deplete $(E_{\tilde{n}} + E_\sim)$ to extinguish the turbulence. Eddy shearing can increase $E_\sim / E_{\tilde{n}} \propto k_\perp^2$, but does not change $(E_\sim + E_{\tilde{n}})$.

Implications for energy transfer in slow L-H transition:

$$\partial_t E_n = \int \mathrm{d}\mathcal{V} \Big[\frac{T_{e0}}{L_n} n_e v_E^x - \frac{T_{e0}}{n_0 e} j_\parallel \nabla_\parallel n_e - T_{e0} \phi \mathcal{K}(n_e) \Big]$$

$$\partial_t E_\sim = \int \mathrm{d}\mathcal{V} \Big[j_\parallel \nabla_\parallel \phi + T_{e0} \tilde{\phi} \mathcal{K}(n_e) - n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \Big]$$

$$\partial_t E_z = \int \mathrm{d}\mathcal{V} \Big[T_{e0} \langle \phi \rangle \mathcal{K}(n_e) + n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \Big]$$

Assume now L-H transition is slower than both growth rate γ and electron parallel relaxation rate $(k_\parallel^2 v_{te}^2 / \nu_{ei} k_\perp^2 \rho_s^2)$.

Criterion for suppression by energy transfer to ZFs becomes stricter than

$$\gamma E_{\tilde{n}} \leq \int \mathrm{d}\mathcal{V}\, n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \leq E_\sim \max \left| \partial_x \langle v_E^y \rangle \right| ,$$

a Hahm-Burrell-like formula modified by $(k_\perp \rho_s)^2$:

$$\max \left| \partial_x \langle v_E^y \rangle \right| \geq \gamma \frac{E_{\tilde{n}}}{E_\sim} \sim \frac{\gamma}{k_\perp^2 \rho_s^2}$$

For small $k_\perp^2 \rho_s^2$, this criterion is much stricter than the usual one, implying that this mechanism is probably not the dominant one.

Of course, flow shear may play other roles.

$$\partial_t E_n = \int \mathrm{d}\mathcal{V} \Big[\frac{T_{e0}}{L_n} n_e v_E^x - \frac{T_{e0}}{n_0 e} j_\| \nabla_\| n_e - T_{e0} \phi \mathcal{K}(n_e) \Big]$$

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$$\gamma E_{\tilde{n}} \leq \int \mathrm{d}\mathcal{V}\, n_0 m_i (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E^y \rangle \leq E_\sim \max \left| \partial_x \langle v_E^y \rangle \right| ,$$

Assume now that energy transfer via Reynolds work is negligible, flow shear could still distort the spatial variation of the turbulence and thereby:

- Decrease the effective growth rate γ by decorrelating growing modes,
- Increase dissipation due to transfer to high-$k_\perp$ by eddy shearing

Or maybe the transition is even triggered by something else entirely?

Conclusions

In a popular description of the L-H transition, energy transfer to the mean flows directly depletes turbulence fluctuation energy, resulting in suppression of the turbulence and a corresponding transport bifurcation. However, electron parallel force balance couples nonzonal velocity fluctuations with electron pressure fluctuations on rapid timescales, comparable with the electron transit time. For this reason, energy in the nonzonal velocity stays in a fairly fixed ratio to electron thermal free energy, at least for frequency scales much slower than electron transit. In order for direct depletion of the energy in turbulent fluctuations to cause the L-H transition, energy transfer via Reynolds stress must therefore drain enough energy to significantly reduce the sum of the free energy in nonzonal velocities and electron pressure fluctuations. At low $k_\perp$, the electron thermal free energy is much larger than the energy in nonzonal velocities, posing a stark challenge for this model of the L-H transition.

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