Parasitic Momentum Flux in the Tokamak Core

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Careful geometric analysis shows that energy transfer from the electrostatic potential to ion parallel flows breaks symmetry in the fully nonlinear toroidal momentum transport equation, causing countercurrent rotation peaking without applied torque.

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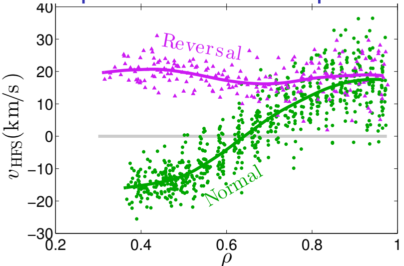
Outline

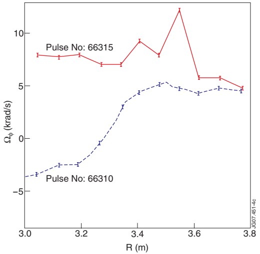
- Background
  - Experiment:
    - Intrinsic rotation and rotation reversals
  - Theory:
    - Intrinsic rotation: Vanishing momentum flux
    - Symmetry restrictions on spin-up
    - Symmetry-breaking mechanisms in the literature
- Rotation model
  - Intuitive cartoon of simple example
  - Model Equations
  - Axisymmetric case (pedagogical example)
    - GAM damping
  - Nonaxisymmetric case (dominant rotation drive)
    - Fluxtube coordinates and $v_{E,r}$
    - Free-energy flow in phase space and momentum flux
  - Predictions
    - Core rotation peaking: scaling, behavior, and experimental comparison

Tokamak plasmas rotate spontaneously without applied torque.



TCV Ohmic shots ($I_p \approx$155, 195kA)

Stoltzfus-Dueck et al PoP '15



JET ICRH shots ($I_p \approx$1.5, 2.6MA)

Eriksson et al PPCF '09

Important for stability against resistive wall modes at low torque (ITER).

Typical intrinsic rotation profiles have three regions:

- Edge: Co-rotating due to ion orbit shifts
- Mid-radius "gradient region": Countercurrent peaking or ∼flat
  - Gradient exhibits sudden 'reversals' at critical parameter values.
- Sawtoothing region inside $q = 1$: Flat or weak cocurrent peaking

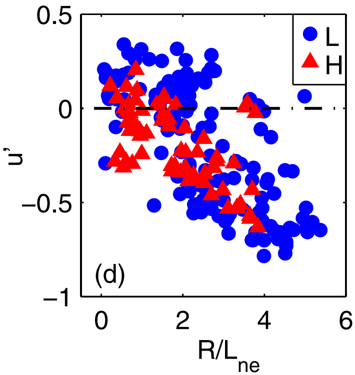
In axisymmetric geometry, neoclassical momentum transport is negligible.

AUG measurements indicate correlation of density peaking and counter-current momentum peaking.

AUG intrinsic rotation database
(Angioni PRL '11⇒)

- >200 intrinsic rotation profiles
- Ohmic, L-, and H-modes
- With & without ECH & ICH
- $u' \doteq -(R/v_{ti}) dv_\zeta / dr$

Across all discharge types, density peaking correlates with countercurrent rotation peaking.



Today's model offers a simple explanation for this. Stay tuned...

Intrinsic rotation profiles result from vanishing momentum flux.

Axisymmetric steady state with no torque⇒zero momentum outflux:

$$0 = \Pi = -\underbrace{\nu \nabla L}_{\sim O(\rho_*^2 v_\zeta / v_{ti})} + \underbrace{v_{\mathrm{pinch}} L}_{\sim O(\rho_*^3)} + \underbrace{\Pi^{\mathrm{res}}}_{\sim O(\rho_*^3)} \implies \nabla L = (v_{\mathrm{pinch}} L + \Pi^{\mathrm{res}})/\nu$$

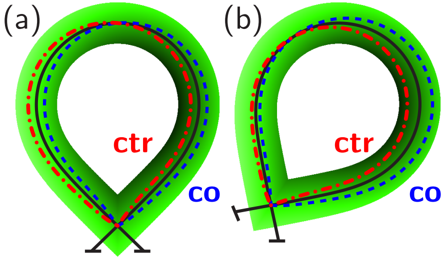
Toroidal momentum gradient ∇L is set by balancing

- Viscous flux ($-\nu \nabla L$) (saturation) against both
- Momentum pinch ($v_{\mathrm{pinch}} L$) due to
  - 'Turbulent equipartition' due to ∇B (Hahm et al PoP '07)
  - Coriolis force (Peeters et al PoP '09)
- Residual stress (Π^{res}, independent of L)
  - Only explanation for peaked profiles that cross $L = 0$

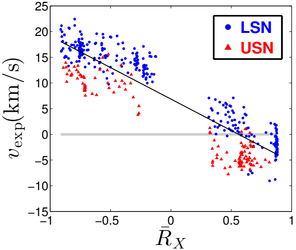
More than one mechanism may be important for a given discharge.

Like most other models, my Π^{res} is $O(\rho_*^3)$, causing $v_\xi \sim \rho_*^1 v_{ti}$, consistent with the implications of standard orderings, as pointed out by Parra & Catto PPCF (2008). Not all ρ_*^3 terms are yet known, but the unknown terms are small in B_p/B.

Edge intrinsic rotation follows from interaction of ion drift orbits with spatially varying turbulent diffusivity.



Stoltzfus-Dueck PRL '12



TCV Ohmic L-modes

Stoltzfus-Dueck et al PRL '15

Theoretical Model based on edge-specific orderings:

- Orbit shifts plus diffusive transport with edge & SOL.
- Concrete rotation prediction depends on X-point major radius $\bar{R}_X$.
- Edge intrinsic rotation on TCV exhibits the expected $\bar{R}_X$ dependence.

Symmetry restricts contributions to residual stress.

In the simplest radially local fluxtube limit with

- up-down symmetric magnetic geometry,
- no background rotation or rotation shear, and
- no background $\boldsymbol{E} \times \boldsymbol{B}$ shear,

the delta-f gyrokinetic equations satisfy a symmetry:

If $f(\rho, \vartheta, \xi, v_{\|}, \mu, t)$, $\phi(\rho, \vartheta, \xi, t)$, $A_{\|}(\rho, \vartheta, \xi, t)$ is a solution
so is $-f(-\rho, -\vartheta, \xi, -v_{\|}, \mu, t)$, $-\phi(-\rho, -\vartheta, \xi, t)$, $A_{\|}(-\rho, -\vartheta, \xi, t)$,

with opposite sign of the dominant toroidal momentum flux.
(Peeters and Angioni PoP '05, Parra et al PoP '11)

This implies: toroidal momentum flux should vanish for terms that flip sign, but does *not* imply that invariant terms *must* drive momentum flux.

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What drives symmetry-breaking and momentum flux,
in the absence of rotation and of rotation shear?

Symmetry-breaking mechanisms to drive residual stress include:

- Background $\boldsymbol{E} \times \boldsymbol{B}$ shear (Dominguez and Staebler Phys. Fluids B '93)
- Up-down asymmetric magnetic geometry (Camenen et al PRL '09)
- Quasilinear: assume phase between $\tilde{v}_r$ and $\tilde{v}_\parallel$ from a linear eigenmode
  - Drift waves (Coppi NF '02)
  - With intensity gradient (Gürcan PoP '10)
- Radially global effects via gyrokinetic simulation
  - GTS: magnetic & $\boldsymbol{E} \times \boldsymbol{B}$ shear, intensity gradients, neoclassical effects (Wang et al PRL '09, '11)
  - XGC1: avalanche momentum & heat transport (Ku et al NF '12)
- Corrections to fluxtube gyrokinetics (Parra and Barnes PPCF '15)
  - Neoclassical perturbation to turb mom transport
  - Turbulence inhomogeneity & finite orbit widths

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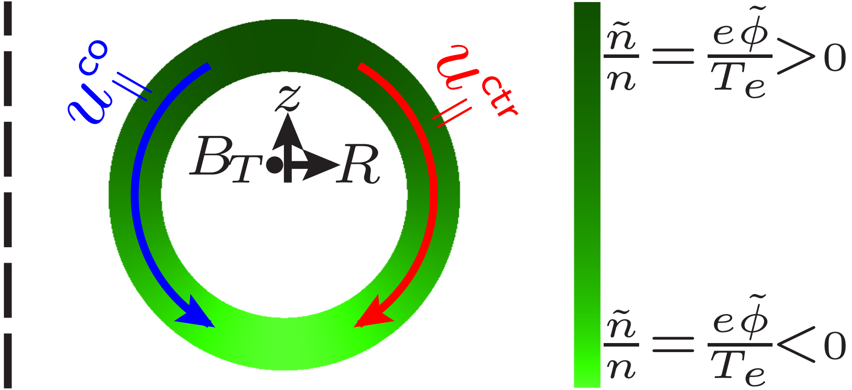
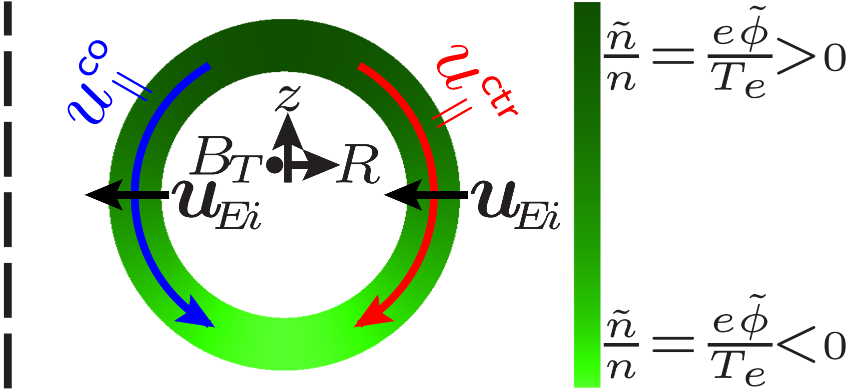
Free-energy flow in phase space + higher-order part of $\boldsymbol{E} \times \boldsymbol{B}$ drift $\Rightarrow$ residual stress

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.



I. Example: axisymmetric, $m = 1$, low-frequency density fluctuations.
$E_\parallel = -b_p(\partial_\theta \phi)/r$ accelerates ions out of density hump.
$E_\parallel u_{\parallel i} = -b_p u_{\parallel i}(\partial_\theta \phi)/r$ transfers energy to ion parallel flows.

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.

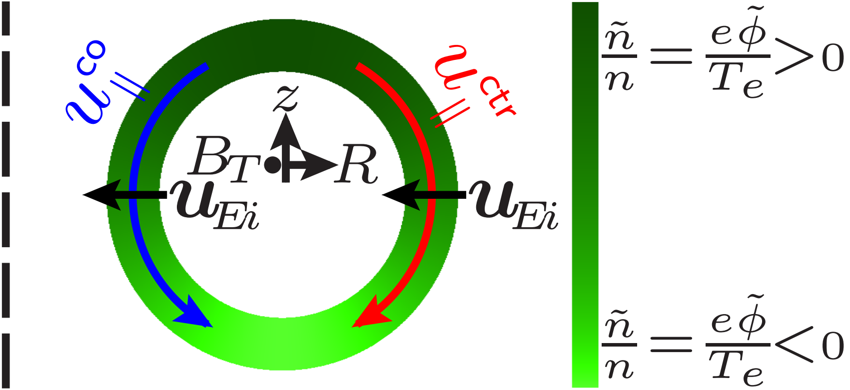


II. Weak radial $\boldsymbol{E} \times \boldsymbol{B}$ drift $v_{E,r} = -(cb_T/Br)\partial_\theta \phi$ advects ions.

Net exhaust of cocurrent momentum: $\Pi = -m_i n_{i0} R b_T^2 c(\partial_\theta \phi) u_{\parallel i}/Bb_p$

Momentum flux$\propto$energy transfer because $E_\parallel / b_p = -\partial_\theta \phi = E_\perp / b_T$.

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.



III. Slow poloidal potential variation in $\partial_\theta \phi \sim k_\parallel \phi / b_p$:

- neglected by fluxtube orderings, but
- breaks symmetry because $\hat{b}$ neither parallel nor perp to $\hat{\zeta}$.

Start with the simplest model capturing the relevant physics:

Electrostatic, isothermal gyrofluid in a radially thin geometry:

$$\partial_t n_s + \boldsymbol{u}_{Es}\cdot\nabla(n_s + n_{s0}) = \mathscr{K}\left(n_s T_{s0}/Ze + n_{s0}\phi_G\right) - n_{s0}\nabla_\parallel u_{\parallel s},$$

$$m_s n_{s0}(\partial_t + \boldsymbol{u}_{Es}\cdot\nabla) u_{\parallel s} = -\nabla_\parallel\left(n_s T_{s0} + Zen_{s0}\phi_G\right) + m_s n_{s0}\left[\frac{2}{Ze} T_{s0}\mathscr{K}(u_{\parallel s}) - D_{\parallel s}\right]$$

$$\sum_s n_{s0} Z^2 e^2 \frac{1-\Gamma_{0s}}{T_{s0}}\phi = \sum_s Ze\Gamma_{1s} n_s,$$

Take large aspect ratio but $q = 1$, so $b_p \doteq B_p/B \sim r/R \ll 1$.
Assuming $\langle D_{\parallel s}\rangle = 0$, conserve a simplified toroidal angular momentum:

$$\partial_t\left\langle L_\zeta\right\rangle = -\partial_x\left\langle \Pi_\zeta\right\rangle$$

$$L_\zeta \doteq m_i n_{i0} b_T R_0 u_{\parallel i}; \qquad \Pi_\zeta = m_i n_{i0} b_T R_0\left(u_{Ei}^x - \frac{2T_{i0}}{Ze}\mathscr{K}^x\right) u_{\parallel i}$$

Derivation ∼identical in delta-f or full-F gyrokinetics.

Axisymmetric sidebands capture GAM oscillations.

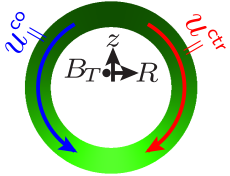
Specialize to shearless simple-circular geometry $\mathscr{K}^x \to (2cb_T/B_0R_0)\sin\theta$.
Evolve axisymmetric sidebands $n_i^{\mathrm{s}} \doteq \langle n_i \sin\theta\rangle$, $u_\parallel^{\mathrm{c}} \doteq \langle u_{\parallel i}\cos\theta\rangle$, $u_E^{\mathrm{z}} \doteq \langle u_{Ei\theta}\rangle$:

$$\partial_t n_i^{\mathrm{s}} = \frac{n_{i0}}{R_0} u_E^{\mathrm{z}} + \frac{b_p n_{i0}}{r} u_\parallel^{\mathrm{c}} - \partial_x \langle \Gamma_i \sin\theta \rangle ,$$

$$m_i n_{i0} \partial_t u_\parallel^{\mathrm{c}} = -\frac{b_p}{r} T_a n_i^{\mathrm{s}} - \nu_\parallel m_i n_{i0} u_\parallel^{\mathrm{c}} - \partial_x \big\langle \Pi_\parallel \cos\theta \big\rangle ,$$

$$n_{i0} m_i \partial_t u_E^{\mathrm{z}} = -\frac{2}{R_0} T_a n_i^{\mathrm{s}} - \partial_x \langle \Pi_E \rangle ,$$

Linearization recovers GAM dispersion:
$\omega^2 - 2\frac{T_a}{m_i R_0^2} = \frac{\omega}{\omega + i\nu_\parallel} \frac{T_a}{m_i q^2 R_0^2}$
Although damped, GAMs driven nonlinearly by poloidal $\boldsymbol{E} \times \boldsymbol{B}$ Reynolds stress $-\partial_x \langle \Pi_E \rangle$.



GAMs' ion Landau damping drives cocurrent momentum outflux.

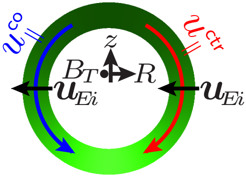
$$\partial_t E_E^{\mathrm{z}} \doteq \frac{1}{2} n_{i0} m_i (u_E^{\mathrm{z}})^2 = -\frac{2}{R_0} T_a n_i^{\mathrm{s}} u_E^{\mathrm{z}} - \overbrace{u_E^{\mathrm{z}} \partial_x \langle \Pi_E \rangle}^{\Downarrow},$$

$$\partial_t E_i^{\mathrm{s}} \doteq T_a (n_i^{\mathrm{s}})^2 / n_{i0} = 2 T_a n_i^{\mathrm{s}} \frac{u_E^{\mathrm{z}}}{R_0} + 2 T_a n_i^{\mathrm{s}} \frac{b_p}{r} u_\parallel^{\mathrm{c}} - 2 T_a n_i^{\mathrm{s}} \frac{\partial_x \langle \Gamma_i \sin\theta \rangle}{n_{i0}},$$

$$\partial_t E_\parallel^{\mathrm{c}} \doteq m_i n_{i0} (u_\parallel^{\mathrm{c}})^2 = -2 u_\parallel^{\mathrm{c}} \frac{b_p}{r} T_a n_i^{\mathrm{s}} - 2 u_\parallel^{\mathrm{c}} \nu_\parallel m_i n_{i0} u_\parallel^{\mathrm{c}} - 2 u_\parallel^{\mathrm{c}} \partial_x \langle \Pi_\parallel \cos\theta \rangle,$$

GAM energy flow

- Turbulent poloidal rotation drive
- Geodesic coupling ($\nabla \cdot \boldsymbol{u}_{Ei}$)
- Ion parallel acceleration
- Parallel phase mixing



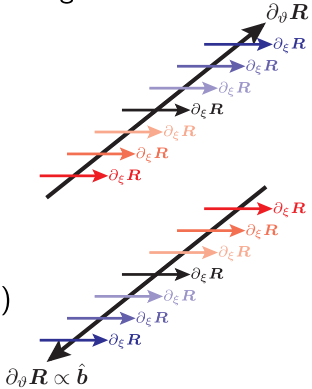
$\langle u_{Ei}^x \cos\theta \rangle = -c b_T \langle \phi \sin\theta \rangle / Br = -c b_T T_{e0} n_i^{\mathrm{s}} / e n_{i0} Br$ transports momentum:

$$\Pi_\zeta^{(\mathrm{G})} = -2 \frac{Z T_{e0}}{T_{i0}} \frac{\rho_i}{r} m_i R_0 v_{ti} n_i^{\mathrm{s}} u_\parallel^{\mathrm{c}} \gtrsim 2 \frac{\nu_\parallel}{b_p \Omega_{ci}} \frac{Z T_{e0}}{T_a} n_{i0} m_i R_0 \overline{(u_\parallel^{\mathrm{c}})^2}$$

Field-aligned coordinates enable the symmetry argument.

Most gyrokinetic models use a variant of the following coordinates:

- Flux label (radial coordinate) ρ
  - Axisymmetric & satisfies $\hat{b}\cdot\nabla\rho = 0$
- Poloidal angle (parallel coordinate) ϑ
  - Axisymmetric
- "Binormal" coordinate ξ
  - Satisfies $\hat{b}\cdot\nabla\xi = 0$



These choices have the key properties that:

- $\hat{b}\cdot\nabla = (\hat{b}\cdot\nabla\vartheta)\partial_\vartheta$ (from definitions of ρ, ξ)
  - Only slow variation in ϑ
- $\partial_\xi \propto \hat{\zeta}\cdot\nabla$ (from definitions of ρ, ϑ)
  - Retains simple periodicity in ξ
  - ∂_ξ vanishes for all equilibrium quantities

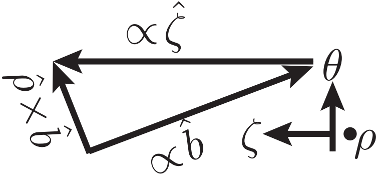
These properties underlie the Peeters/Parra symmetry argument, which is seen to be a somewhat "skew" transform, giving rise to symmetry breaking.

The radial $\boldsymbol{E} \times \boldsymbol{B}$ drift with true $\nabla_\perp \phi$ breaks the symmetry.

Define convenient directions

$$\hat{\rho} \doteq \frac{\nabla \rho}{|\nabla \rho|}, \quad \hat{p} \doteq \hat{\zeta} \times \hat{\rho}$$

and decompose $\hat{b} = b_T \hat{\zeta} + b_p \hat{p}$.



Use $\hat{\rho} \times \hat{b} = (\hat{\zeta} - b_T \hat{b})/b_p$ to evaluate

$$\boldsymbol{u}_{Ei} \cdot \hat{\rho} = \frac{c}{B} \hat{b} \times \nabla \phi_G \cdot \hat{\rho} = \frac{c}{B} \hat{\rho} \times \hat{b} \cdot \nabla \phi_G = \frac{c}{b_p B} \left(\hat{\zeta} \cdot \nabla \phi_G - b_T \hat{b} \cdot \nabla \phi_G \right).$$

Symmetry prevents first term $\propto \hat{\zeta} \cdot \nabla \phi_G \propto \partial_\xi \phi_G$ from driving residual stress.

Second term cancels the parallel gradient included in $\hat{\zeta} \cdot \nabla \phi_G \neq \rho \times \hat{b} \cdot \nabla \phi_G$:

- Nominally smaller than the first term, by $k_\parallel / k_\perp b_p$, but
- Contributes a symmetry-breaking term to momentum flux $m_i n_{i0} b_T R_0 u_{Ei}^x u_{\parallel i}$:

$$\Pi_\zeta^{(2)} = -(c m_i n_{i0} R_0 / b_p B_0) u_{\parallel i} \nabla_\parallel \phi_G$$

If ion parallel flows are excited, co-current momentum flows out.

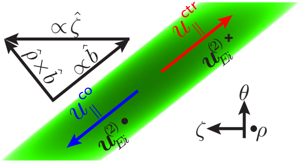
Turbulence fluctuation amplitude is regulated by free-energy balance:

$$\partial_t E_{ns} = T_{s0}\int dV \left[u_{\parallel s}\nabla_\parallel n_s + n_s \mathscr{K}(\phi_G) - n_s \frac{\boldsymbol{u}_{Es}\cdot\nabla n_{s0}}{n_{s0}} \right]$$

$$\partial_t E_{\parallel s} = -\int dV\, u_{\parallel s}\left[T_{s0}\nabla_\parallel n_s + Zen_{s0}\nabla_\parallel \phi_G + m_s n_{s0} D_{\parallel s}\right]$$

$$\partial_t E_E = \sum_s \int dV \left[-T_{s0} n_s \mathscr{K}(\phi_G) + Zen_{s0} u_{\parallel s}\nabla_\parallel \phi_G\right],$$

- Source: flux down the pressure gradient
- Sink: parallel ion viscosity
- Transfers:
  - parallel pressure gradient
  - parallel electric force
  - for low-$k_\perp$ adiabatic electrons, $Zen_{i0}\nabla_\parallel \phi_G = ZT_{e0}\nabla_\parallel n_i$



When energy is transferred to ion parallel flows, definite sign for

$$\Pi_\zeta^{(2)} = -(cm_i n_{i0} R_0/b_p B_0)u_{\parallel i}\nabla_\parallel \phi_G$$

$E_{ns} \doteq \int dV\, \frac{1}{2} T_{s0} n_s^2/n_{s0}$, $E_{\parallel s} \doteq \int dV\, \frac{1}{2} m_s n_{s0} u_{\parallel s}^2$, $E_E \doteq \sum_s \int dV\, \frac{1}{2} n_{s0} Z^2 e^2 \phi\, \frac{1-\Gamma_{0s}}{T_{s0}}\phi$

When ion Landau damping is significant, one obtains counter-current rotation peaking with a simple scaling.

$$\partial_t E_{ns} = T_{s0} \int dV \left[u_{\parallel s} \nabla_\parallel n_s + n_s \mathscr{K}(\phi_G) - n_s \frac{\boldsymbol{u}_{Es} \cdot \nabla n_{s0}}{n_{s0}} \right]$$

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$$\partial_t E_E = \sum_s \int dV \left[-T_{s0} n_s \mathscr{K}(\phi_G) + Zen_{s0} u_{\parallel s} \nabla_\parallel \phi_G \right],$$

Assume a fraction $0 \leq f_L \leq 1$ of turbulent free energy is dissipated via (low-$k_\perp$) ion Landau damping, then free-energy balance implies:

$$-\frac{T_{i0} + ZT_{e0}}{T_{e0}} n_{i0} e \int dV \, u_{\parallel i} \nabla_\parallel \phi = -f_L \sum_s \int dV n_s T_{s0} \frac{\boldsymbol{u}_{Es} \cdot \nabla n_{s0}}{n_{s0}} = f_L \int dV \, \frac{Q_i + Q_e}{L_p}$$

$$\int dV \, \Pi_\zeta^{(2)} = -\frac{c m_i n_{i0} R_0}{b_p B_0} \int dV \, u_{\parallel i} \nabla_\parallel \phi_G = f_L \frac{ZT_{e0}}{T_{i0} + ZT_{e0}} \frac{R_0}{b_p \Omega_{ci}} \int dV \, \frac{Q_i + Q_e}{L_p}$$

When ion Landau damping is significant, one obtains counter-current rotation peaking with a simple scaling.

Now balance viscous momentum flux against residual stress ($U_\zeta \doteq v_\zeta / v_{ti}$):

$$-\chi_\phi \frac{n_{i0} m_i R_0 U_\zeta v_{ti}}{L_{u\zeta}} = \Pi_\zeta^{(2)} = f_L \frac{Z T_{e0}}{T_{i0} + Z T_{e0}} \frac{R_0}{b_p \Omega_{ci}} \frac{n_{i0} (\chi_i T_{i0} + \chi_e Z T_{e0}) / L_p}{L_p}$$

Assume $\chi_\phi \sim \chi_i$ and also (for simplicity) $\chi_\phi \sim \chi_e$, as well as $L_{u\zeta} \sim L_p$:

$$U_\zeta = -f_L \frac{Z T_{e0}}{T_{i0}} \frac{\rho_i}{b_p L_p}$$

With Ampere's Law $2\pi r B_p \sim 4\pi I_p / c$, we get the dimensional estimate

$$U_\zeta v_{ti} \approx -f_L \frac{c^2}{e2} \frac{T_{e0}}{I_p} \frac{r}{L_p} = -5 f_L \frac{T_{e0}(\mathrm{keV})}{I_p(\mathrm{MA})} \frac{r}{L_p} \mathrm{km/s},$$

comparable with peaking measured on DIII-D, JET, C-mod, AUG, and TCV.

Density peaking and counter-current momentum peaking are both active only for low-frequency turbulence.

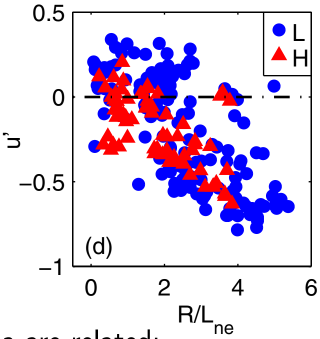
Particle pinch due to electron precession, active when frequency satisfies

$$\omega \lesssim \omega_{de} \sim k_\perp v_{te} \rho_e / R$$

(Angioni et al NF'12)

Counter-current peaking due to ion Landau damping, active when

$$\omega \lesssim v_{ti}/qR$$



Although physical mechanisms are distinct, criteria are related:

$$\frac{\omega_{de}}{v_{ti}/qR} = q \frac{Z T_{e0}}{T_{i0}} k_\perp \rho_i$$

Both will be active during low-frequency turbulence $\omega \lesssim \omega_{de} \sim v_{ti}/qR$.

Summary

A geometrically higher-order portion of the $\boldsymbol{E} \times \boldsymbol{B}$ drift causes a nondiffusive momentum flux that:

- results from symmetry-breaking by excitation of ion parallel flows
  - does not require $\langle v_\zeta \rangle$ or $\nabla \langle v_\zeta \rangle \Rightarrow$residual stress
  - a fully nonlinear mechanism, not quasilinear
- causes counter-current rotation peaking in the core
- drive experimentally relevant rotation peaking around

$$U_\zeta v_{ti} \approx -f_L \frac{ZT_{e0}}{T_{i0}} \frac{\rho_i}{b_p L_p} = -5 f_L \frac{T_{e0}(\mathrm{keV})}{I_p(\mathrm{MA})} \frac{r}{L_p} \mathrm{km/s}$$

- acts only when turbulence is at low enough frequencies to excite ion parallel flows
  - $\sim$same criterion as for density peaking, consistent with observed relation of density and rotation peaking across many discharge types