Equilibrium Potential Well due to Finite Larmor Radius Effects at the Tokamak Edge

W. W. Lee

June 16, 2016

We present a novel mechanism for producing the equilibrium potential well near the edge of a tokamak. Briefly, because of the difference in gyroradii between electrons and ions, an equilibrium electrostatic potential is generated in the presence of spatial inhomogeneity of the background plasma, which, in turn, produces a well associated with the radial electric field, Er, as observed at the edge of many tokamak experiments. We will show that this theoretically predicted Er field, which can be regarded as producing a long radial wave length zonal flow, agrees well with recent experimental measurements. The work is in collaboration with R. B. White [PPPL Report No. 5254 (2016)].

Phys. Fluids 23(10), October 1980

Ion temperature drift instabilities in a sheared magnetic field

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(Received 13 February 1980; accepted 26 June 1980)

Results from the first particle code simulations of the ion-temperature-gradient-driven instabilities in a sheared slab geometry are reported. In the linear stage of the instability, the results are in very good agreement with the theoretical calculations of the mode frequency, growth rate, and radial mode structure. Ion energy transport caused by the instability is found to be the process primarily responsible for nonlinear saturation. Enhanced fluctuations associated with marginally stable eigenmodes have been observed.

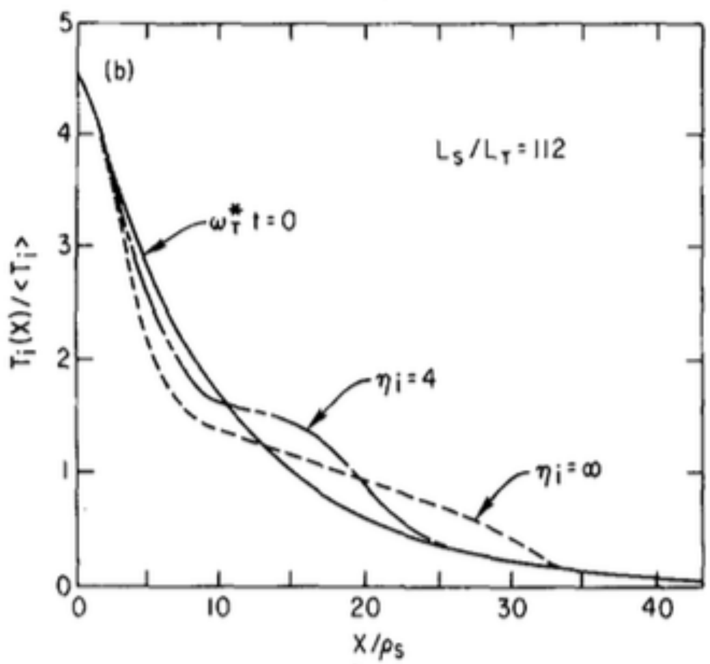

FIG. 4. Simulation results of (a) the saturation amplitude and (b) the resultant ion temperature profiles.

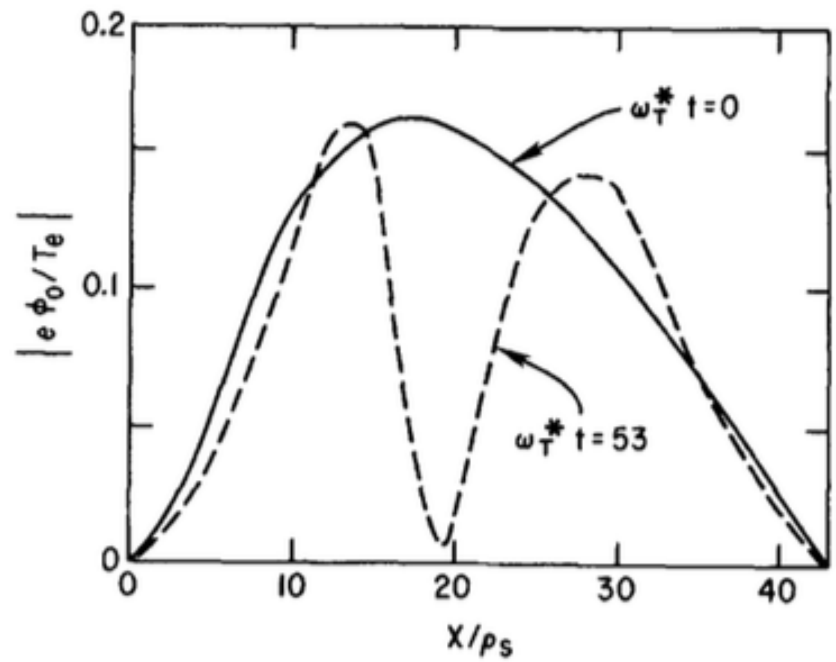

FIG. 5. Ambipolar ($k_y = 0$) mode structures at the onset of simulation and at saturation for $\eta_i = \infty$, $L_S/L_T = 112$.

Naitou, Tokuda and Kamimura, JCP **38**, 265 (1980): attempted to eliminate this extra charge density.

556 Phys. Fluids 26 (2), February 1983

Gyrokinetic approach in particle simulation

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(Received 16 October 1981; accepted 20 October 1982)

A new scheme for particle simulation based on the gyrophase-averaged Vlasov equation has been developed. It is suitable for studying linear and nonlinear low-frequency microinstabilities and the associated anomalous transport in magnetically confined plasmas. The scheme retains the gyroradius effects but not the gyromotion; it is, therefore, far more efficient than conventional ones. Furthermore, the reduced Vlasov equation is also amenable to analytical studies.

- Origin of this extra ion charge density due to spatial inhomogeneity was first discussed from the gyrokinetic point of view.

- Gyrokinetic Poisson's Equation

$$\nabla^2\Phi - k_{Di}^2\frac{n_i}{n_0}(\Phi-\tilde{\Phi}) = -4\pi e(n_i + \frac{1}{2}\rho_i^2\nabla_\perp^2 n_i^{inho} - n_e)$$

- For $k_\perp\rho_i \ll 1$ $\Downarrow$

$$-k_{Di}^2(\Phi-\tilde{\Phi}) \sim \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\Phi$$

PHYSICS OF PLASMAS **16**, 044506 (2009)

On higher order corrections to gyrokinetic Vlasov–Poisson equations in the long wavelength limit

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$$\bar{n}(\mathbf{x}) = \int \left(1 + \frac{1}{4}\frac{v_\perp^2}{\Omega^2}\nabla_\perp^2\right) F_{gc}(\mathbf{R})\, dv_\parallel d\mu,$$

$$\nabla_\perp^2 F_{gc}^M = \left[\frac{\nabla_\perp^2 n}{n} + \left(-\frac{3}{2} + \frac{mv^2}{2T}\right)\left(\frac{\nabla_\perp^2 T}{T} + 2\frac{\nabla_\perp T}{T}\cdot\frac{\nabla_\perp n}{n}\right) + \left(\frac{15}{4} - \frac{5mv^2}{2T} + \frac{m^2v^4}{4T^2}\right)\frac{\nabla_\perp T\cdot\nabla_\perp T}{T^2}\right] F_{gc}^M.$$

$$\bar{n}(\mathbf{x}) = n + \frac{1}{2}\rho_t^2 \frac{1}{T}\nabla_\perp^2 nT,$$

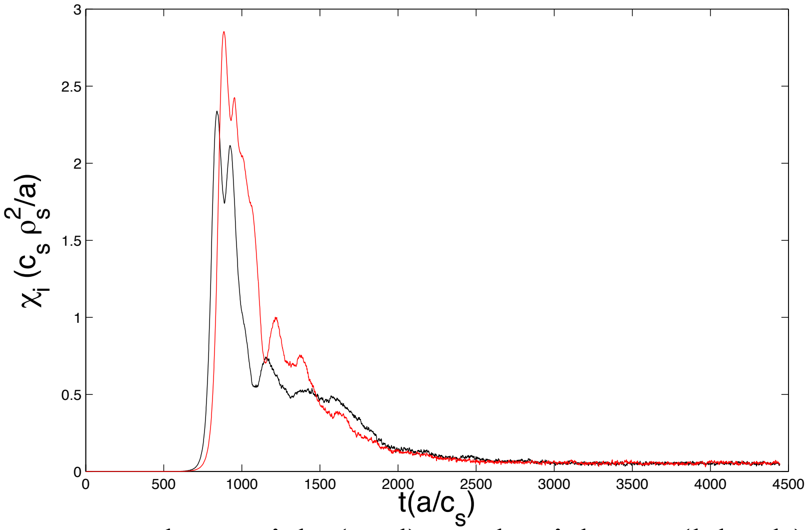
$$\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e} = -\frac{\delta n_i}{n_0}, \qquad \frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2}{k_\perp^2}, \sim O(1) \qquad \frac{V_{E\times B}}{c_s} \equiv k_\perp\rho_s\frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2\rho_s^2}{k_\perp\rho_s}.$$

Linear Upshift of ITG Gradient due to Equilibrium Ion Density

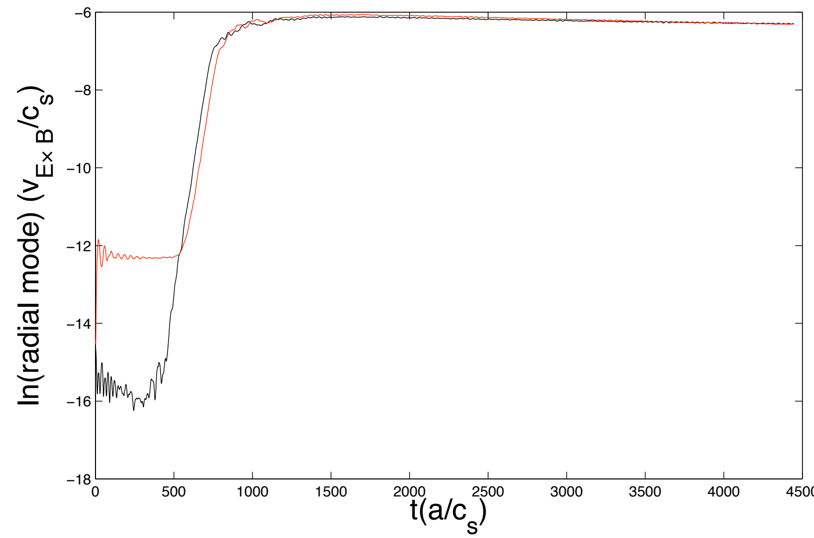
- Gyrophase-Averaged Equilibrium Ion Number Density

$$\bar{n}(\mathbf{x}) = n + \underline{\frac{1}{2}\rho_t^2 \frac{1}{T}\nabla_\perp^2 nT}$$

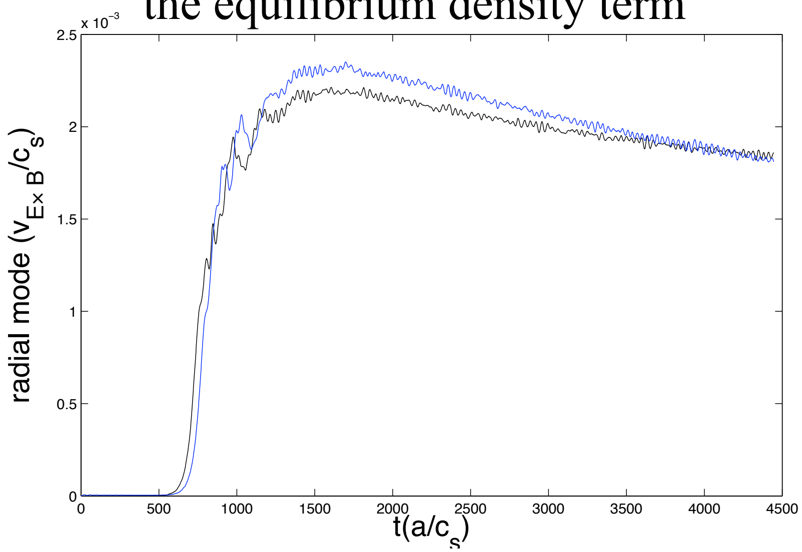
- ITG simulation using GTC* (a/rho =125)



Heat Flux with (red) and without (black) the equilibrium density term



Zonal flow amplitude with (red) and without (black) the equilibrium density term



-- Zonal flow amplitude with (blue) and without (black) the equilibrium density term

-- Similar to the Dimits Shift, which is nonlinear

-- This shift is linear

[W. W. Lee, Sherwood Conference 2014]

Magnetohydrodynamics for Collisionless Plasmas from the Gyrokinetic Perspective

PPPL Report No. 5236 (2016)

W. W. Lee

$$\frac{n_i|_{particle}}{n_i} = 1 + \frac{1}{2}\rho_i^2 \frac{1}{p_i}\nabla_\perp^2 p_i$$

$$\mathbf{v}_{E\times B} \approx -\frac{1}{2}\hat{\mathbf{b}} \times \frac{\nabla_\perp p_i}{p_i}\frac{cT_i}{eB}$$

$$\mathbf{J}_\perp^{E\times B}(\mathbf{x}) = \sum_\alpha q_\alpha \langle \int \mathbf{v}_{E\times B}(\mathbf{R}) F_\alpha(\mathbf{R}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} d\mu dv_\parallel \rangle_\varphi$$

$$\mathbf{J}_\perp = \frac{c}{B}\hat{\mathbf{b}} \times \nabla p + e n_i \frac{\rho_i^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{p_i}\nabla_\perp^2 p_i\right]$$

-- Difference in gyroradius effects between ions and electrons

$$\boxed{\mathbf{J}_\perp \approx \frac{c}{B}\hat{\mathbf{b}} \times (\nabla p)\left[1 - \frac{1}{4}\rho_i^2 \frac{\nabla_\perp^2 p}{p}\right]}$$

-- FLR modification of pressure balance

$$\nabla\left[\frac{B^2}{8\pi} + p\left(1 - \frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right)\right] \approx 0$$

- Quasineutrality: $$n_i \approx n_e$$

- From Gyrokinetic Point of View: $$n_i^{gc} + n_i^{pol} + n_i^{inho} = n_e^{gc}$$

- Present Focus: $$n_i^{pol} + n_i^{inho} = 0 \quad \text{for} \quad n_i^{gc} = n_e^{gc}$$

- Density Profile with Monogenic Particles: $$\frac{n(r)}{n_0} = \frac{1}{2} - \frac{tanh[(r - r_0)/w]}{2} + n_s$$

- Total Ion Density: $$n = n^{gc} + \delta n = n^{gc} + \frac{1}{2T_i}\rho_i^2 \nabla_\perp^2 n^{gc} T_i = n^{gc} + \frac{\rho_i^2}{2T_i}\left(\frac{\partial^2 n^{gc} T_i}{\partial^2 r} + \frac{1}{r}\frac{\partial n^{gc} T_i}{\partial r}\right)$$

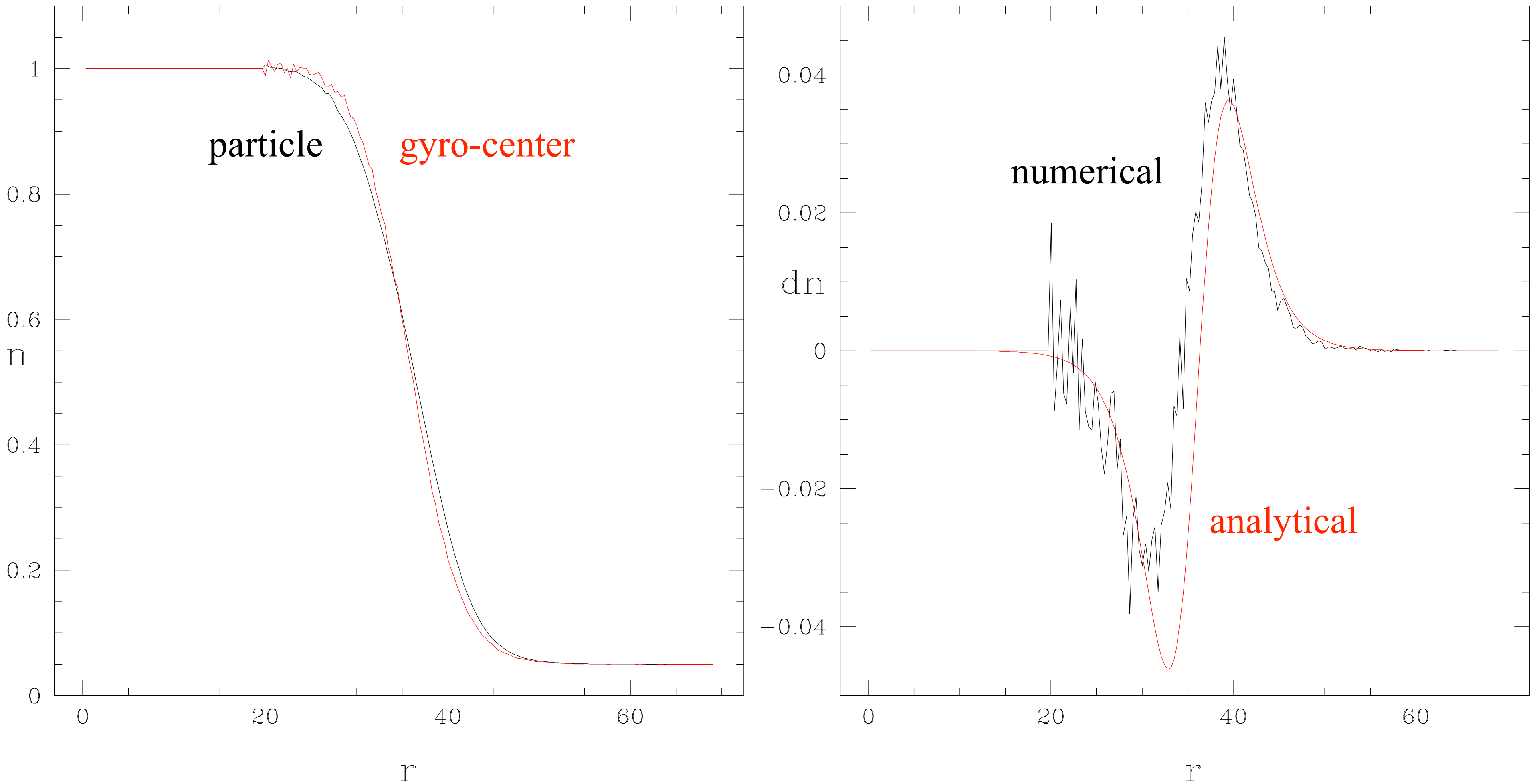
- GK Poisson's Equation: $$\rho_s^2 \nabla_\perp \cdot n \nabla_\perp \frac{e\phi}{T_e} = -\delta n$$

- GK Fields: $$E(r) = \frac{T_i}{2nre}\int_0^r sds[n''(s) + n'(s)/s]$$

$$\frac{e\phi(r)}{T_i} = -\int_0^r dr' \frac{1}{2nr'}\int_0^{r'} sds[n''(s) + n'(s)/s], \qquad enE_\perp = (1/2)(T_e/T_i)\nabla_\perp p_i$$

$$p_i \equiv n^{gc} T_i$$

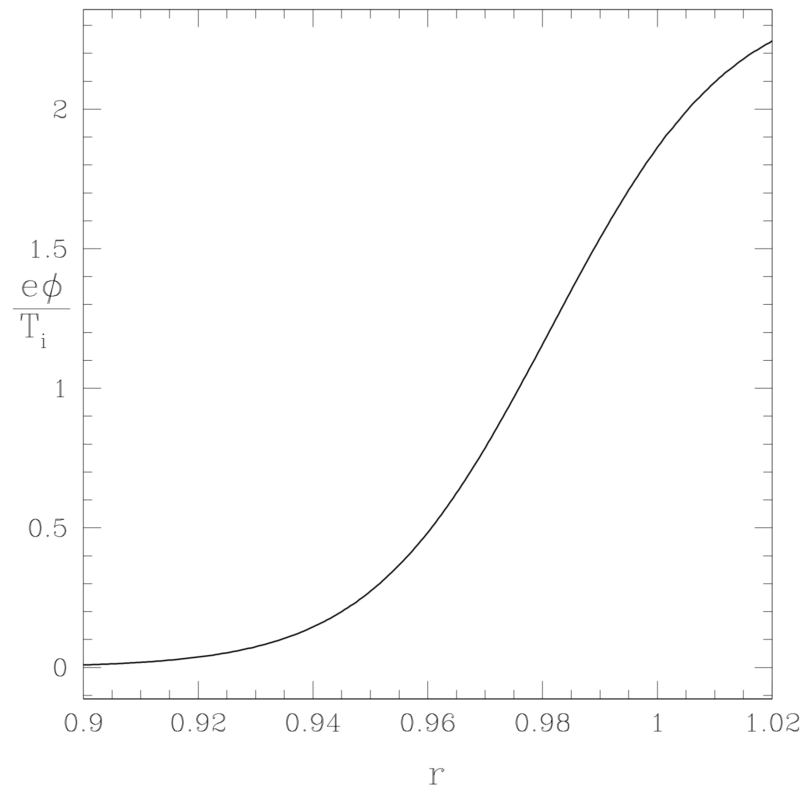
Particle and Gyro-Center Densities

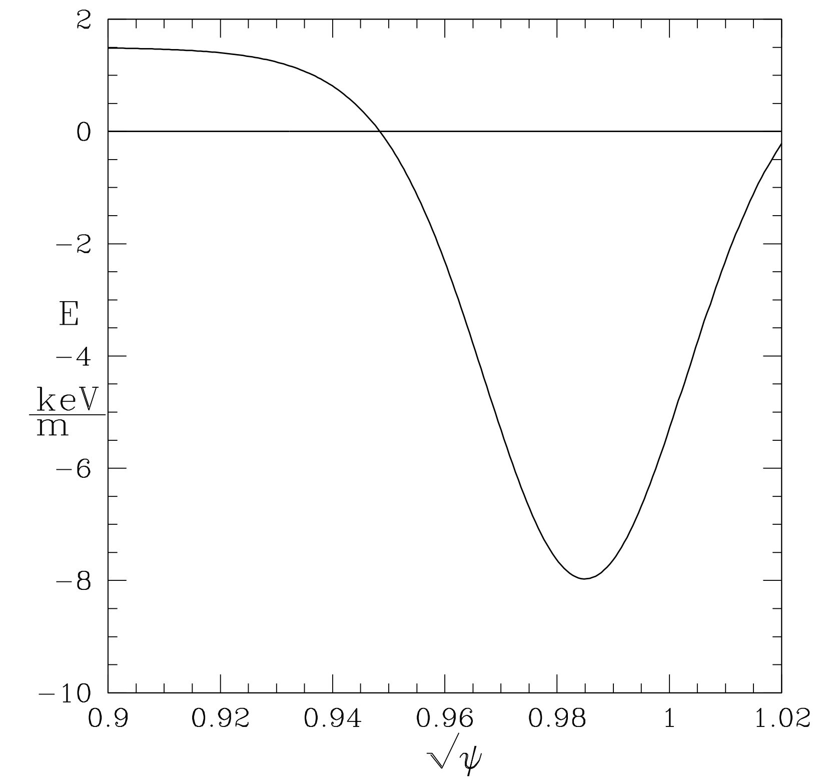


Plots of particle and gyro center densities as a function of minor radius (cm) with particle density in black and gyro-center density in red at left, and at the right the difference, shown in black, with the analytic expression, shown in red. Gyro radius $\rho_i = 2.3$ cm, $w = 5$ cm, and $r_0 = 36$ cm.

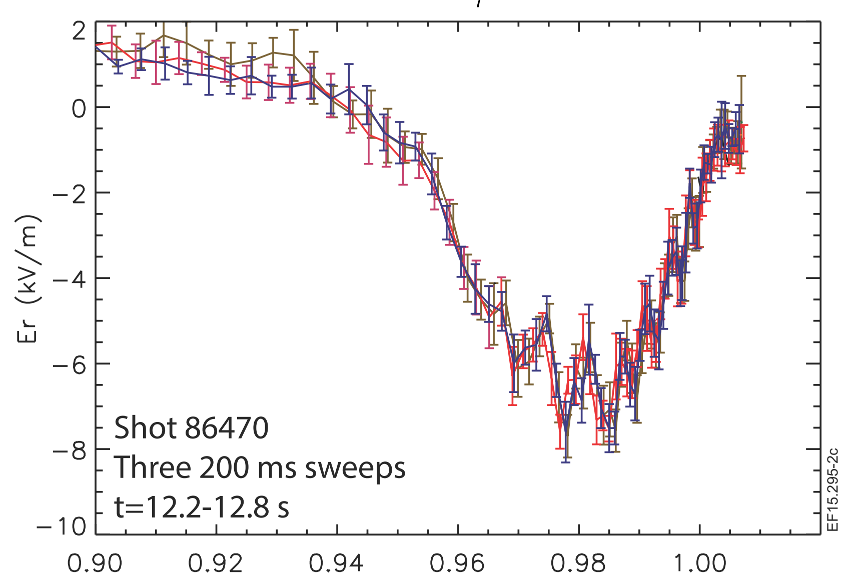
JET Ohmic Discharge

.C.Hillesheim, E.Delabie, H.Meyer,C.F.Maggi, L.Meneses,E.Poli, and JETContributors, Phys. Rev. Letters, 116, 065002 (2016).



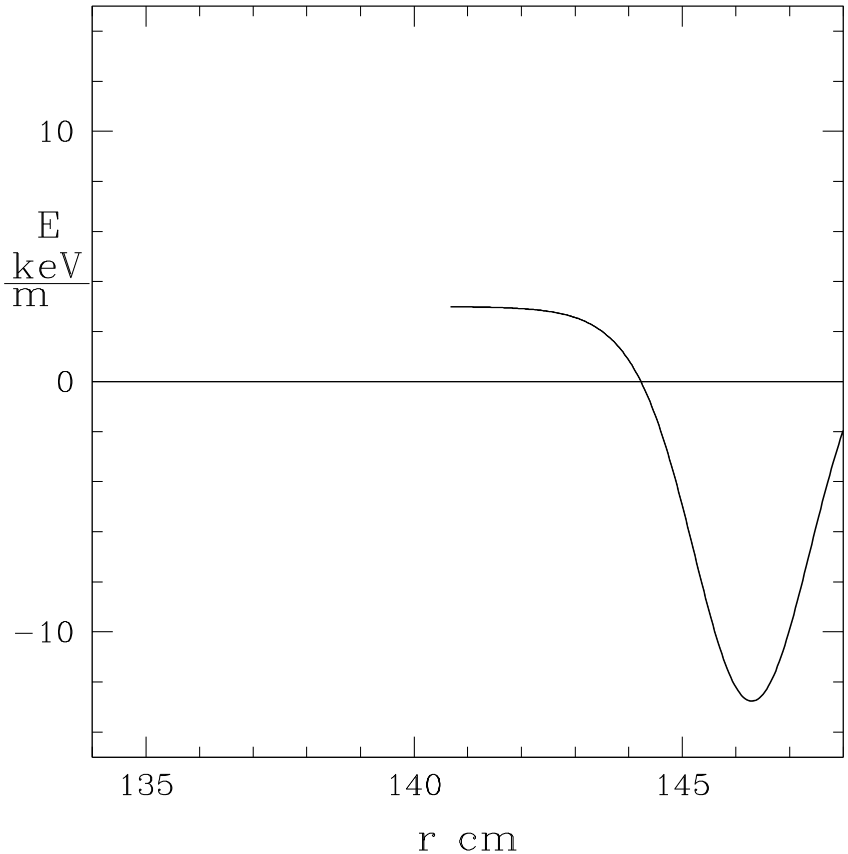


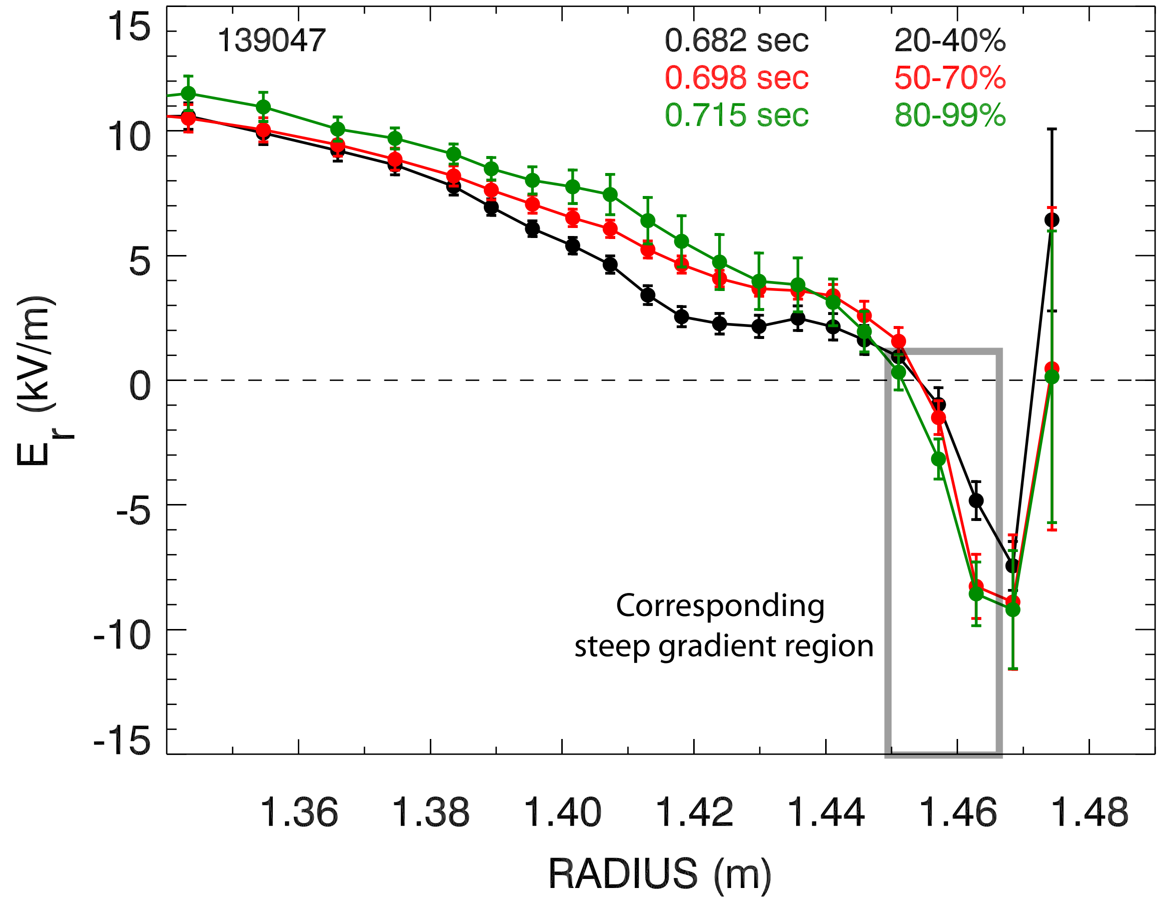
Electric potential and electric field caused by the charge density resulting from ion cyclotron motion in the density distribution and the electric field measurement from JET Ohmic discharge, shot 86470.



NSTX Discharge

A. Diallo, J. Canik, T. Goerler, S.H. Ku, G.J. Kramer, T. Osborne, P. Snyder, D.R. Smith, W. Guttenfelder, R.E. Bell, D.P. Boyle, C.S. Chang, B.P. LeBlanc, R. Maingi, M. Podesta, and S. Sabbagh, Nucl Fus, 53, 1 (2013).

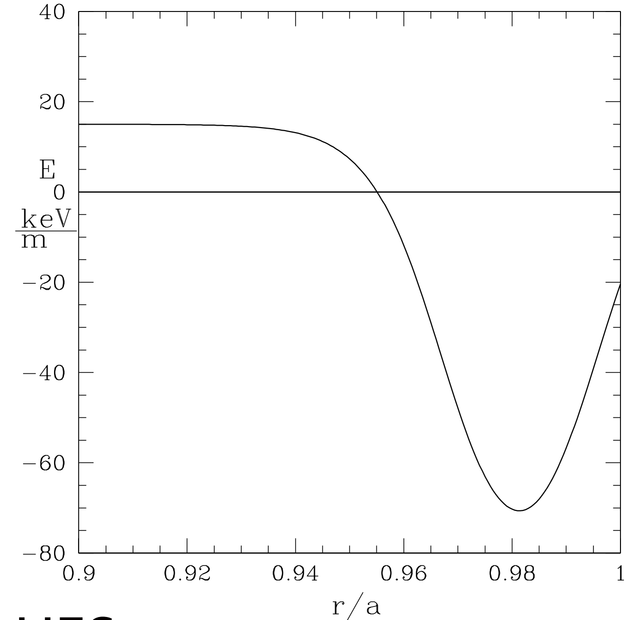


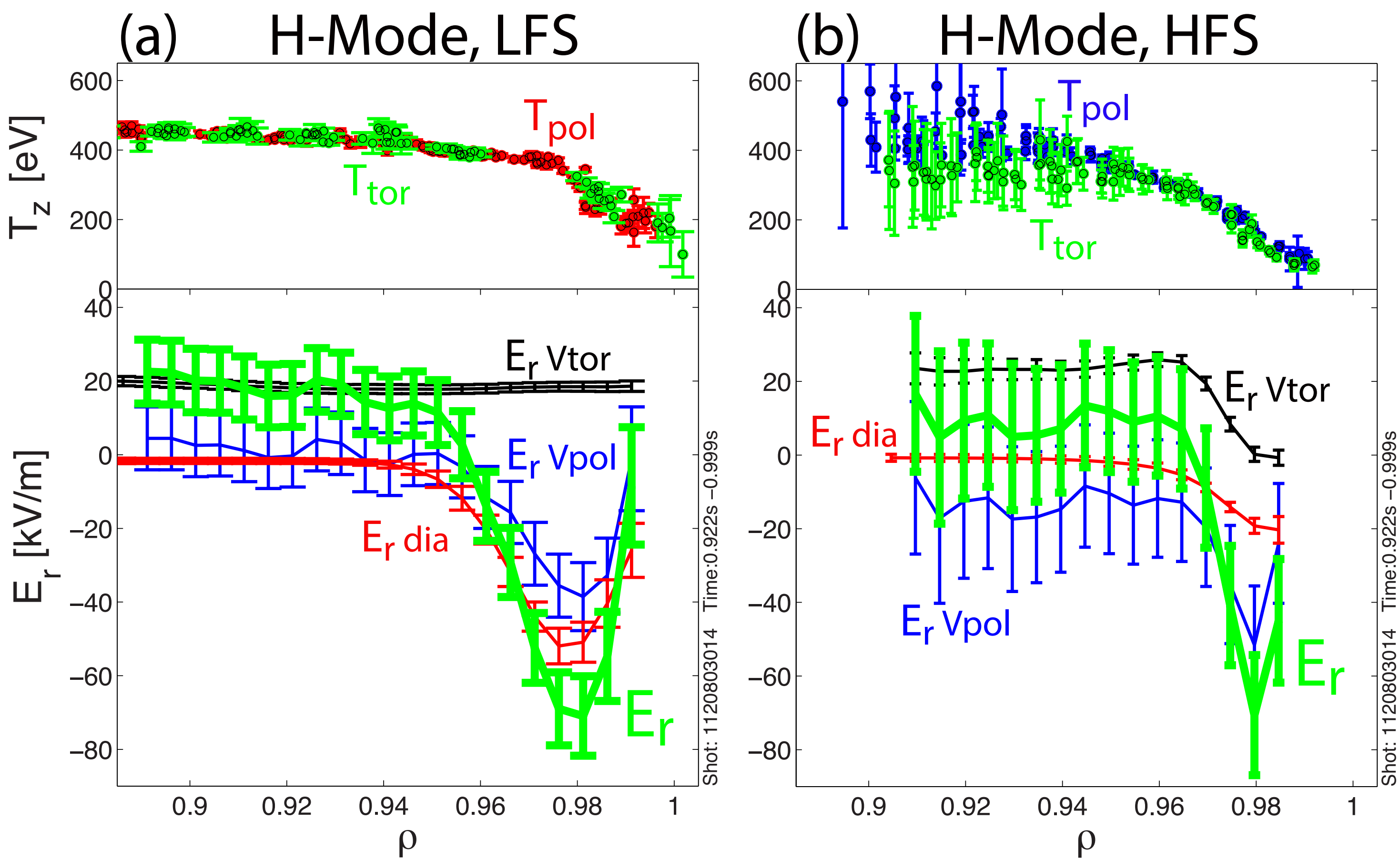


Electric field caused by the charge density resulting from ion cyclotron motion in the density distribution for a NSTX discharge, and the radial electric field observed.

C-MOD Discharge

Electric field caused by the charge density resulting from ion cylotron motion in the density distribution for a Alcator C-Mod H-mode discharge, and the observed temperature profile and radial electric field observed.



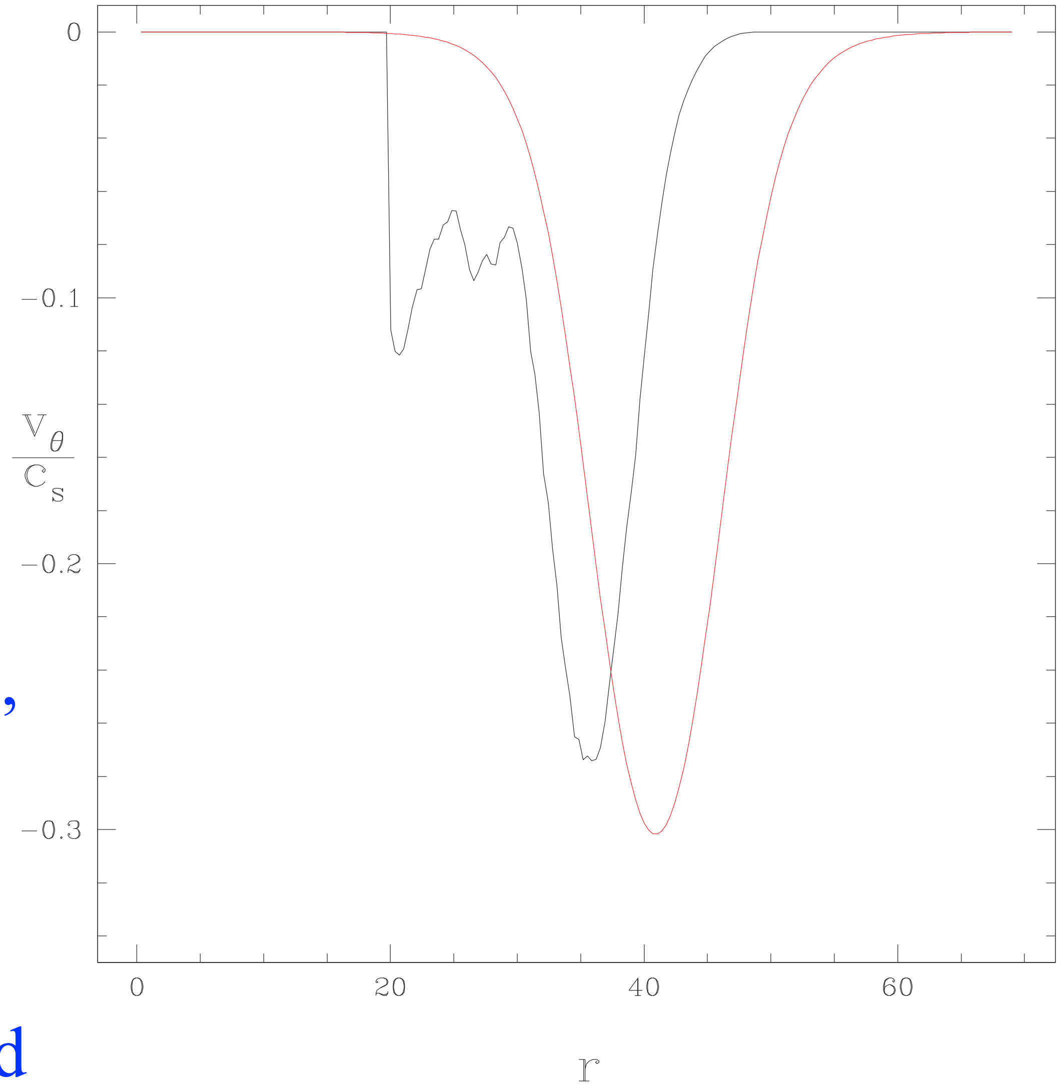


C. Theiler, R.M. Churchill, B. Lipschultz, M. Landreman, D.R. Ernst, J.W. Hughes, P.J. Catto, R.I. Parra, I.H. Hutchinson, M.L. Reinke, A.E. Hubbard, E.S. Marmar, J.T. Terry, J.R. Walk, and the Alcator C-Mod team, Nucl. Fusion 54, 083017 (2014).

Comparisons between the direct calculation of v_θ from the actual particles (black) and that from the pressure balance (red)

$$\frac{\mathbf{J}_\perp(\mathbf{x})}{enc_s} = \frac{1}{n}\int \frac{\mathbf{v}_\perp}{c_s} F(\mathbf{x},\mathbf{v})d\mathbf{v}$$

$$\frac{\mathbf{J}_\perp(\mathbf{x})}{enc_s} \approx \mathbf{b} \times \frac{\rho_s \nabla p_\perp}{nT_e}$$



- The pressure balance is calculated from the lowest order approximation in $k_\perp \rho_i$, and higher order is needed.

- It should not be confused with the poloidal velocities produced by the electric field at the tokamak edge, which comes from the charge imbalance.

Conclusions

- The present talk gives a possible theoretical explanation for the formation of observed radial electric field wells at edge pedestals through finite Larmor radius (FLR) effects of the plasma particles. The well can be regarded as producing a long radial wavelength global zonal flow.

- The surprising agreement between our model, based on equilibrium profiles with no turbulence, and the experimental measurements based on steady state profiles with turbulence, should be a topic of interest in the tokamak community.

- It is possible these two totally different states are thermodynamically related. This is the topic for the second part of the talk.

Gyrokinetic MHD

[Lee, PPPL Report 5236 (2016)]

- Fully Electromagnetic Gyrokinetic Vlasov Equation:

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\parallel \mathbf{b} - \frac{c}{B_0}\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \times \hat{\mathbf{b}}_0 \right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m}\left[\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \cdot \mathbf{b} + \frac{1}{c}\frac{\partial \overline{A}_\parallel}{\partial t} \right] \frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

- Associated Gyrokinetic Field Equations:

$$\nabla^2\phi + \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\phi = -4\pi\sum_\alpha q_\alpha \int \overline{F}_\alpha dv_\parallel d\mu \qquad \text{-- for} \qquad k_\perp^2\rho_i^2 \ll 1$$

$$\nabla^2\mathbf{A} - \underline{\frac{1}{v_A^2}\frac{\partial \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\sum_\alpha q_\alpha \int \mathbf{v}\overline{F}_\alpha dv_\parallel d\mu$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\mu_B \equiv \mu/B \approx const. \qquad \mu = v_\perp^2/2$$

$$\mathbf{v}_p^L = -(mc^2/eB^2)(\partial\nabla_\perp\phi/\partial t)$$

$$\mathbf{v}_p^T = -(mc/eB^2)(\partial^2\mathbf{A}_\perp/\partial^2 t)$$

- Energy Conservation: $\Phi \equiv \phi - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c$

$$\frac{d}{dt}\left\langle \int (\frac{1}{2}v_\parallel^2 + \mu)(m_e F_e + m_i F_i) dv_\parallel d\mu + \frac{\omega_{ci}^2}{\Omega_i^2}\frac{|\nabla_\perp\Phi|^2}{8\pi} + \frac{|\nabla A_\parallel|^2}{8\pi} \right\rangle_{\mathbf{x}} = 0$$

• Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic and Lin, PoP (2010); Startsev et al. APS (2015)]

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt} \frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0} \nabla \bar{\Phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2} \mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_\alpha}{m_\alpha} \left(\mathbf{b}^* \cdot \nabla \bar{\Phi} + \frac{1}{c} \frac{\partial \bar{A}_\parallel}{\partial t} \right)$$

Startsev et al.

$$\Omega_{\alpha 0} \equiv q_\alpha B_0 / m_\alpha c$$

$$\bar{\Phi} \equiv \bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c \qquad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} = -\frac{1}{2\pi} \frac{eB_0}{mc} \int_0^{2\pi} \int_0^{\rho} \delta B_\parallel r dr d\theta$$

Porazic and Lin

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

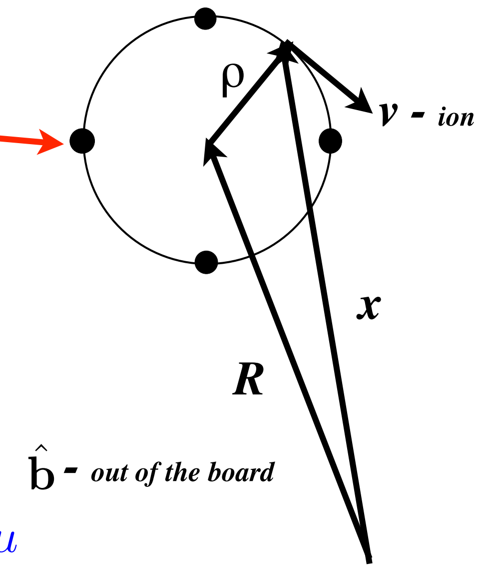
$$F_\alpha = \sum_{j=1}^{N_\alpha} \delta(\mathbf{R} - \mathbf{R}_{\alpha j}) \delta(\mu - \mu_{\alpha j}) \delta(v_\parallel - v_{\parallel \alpha j})$$

Gyrokinetic Current Densities

[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\| gc}(\mathbf{x}) + \boxed{\mathbf{J}^{M}_{\perp gc}(\mathbf{x})} + \mathbf{J}^{d}_{\perp gc}(\mathbf{x})$$

$$= \sum_{\alpha} q_{\alpha} \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_{\|} + \mathbf{v}_{\perp} + \mathbf{v}_{d})\delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_{\|} d\mu \rangle_{\varphi}$$



FLR calculation

$$\mathbf{v}_{d} = \frac{v_{\|}^{2}}{\Omega_{\alpha}} \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}}) \hat{\mathbf{b}} + \frac{v_{\perp}^{2}}{2\Omega_{\alpha}} \hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$

$$\mathbf{J}^{M}_{\perp gc}(\mathbf{x}) = -\sum_{\alpha} \nabla_{\perp} \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp} \qquad p_{\alpha\perp} = m_{\alpha} \int (v_{\perp}^{2}/2) F_{\alpha gc}(\mathbf{x}) dv_{\|} d\mu$$

$$\mathbf{J}^{d}_{\perp gc} = \frac{c}{B} \sum_{\alpha} \left[p_{\alpha\|}(\nabla \times \hat{\mathbf{b}})_{\perp} + p_{\alpha\perp} \hat{\mathbf{b}} \times (\nabla lnB) \right] \qquad p_{\alpha\|} = m_{\alpha} \int v_{\|}^{2} F_{\alpha gc}(\mathbf{x}) dv_{\|} d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^{M}_{\perp gc} + \mathbf{J}^{d}_{\perp gc} \quad = \frac{c}{B} \sum_{\alpha} \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\|} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_{\perp} \right]$$

$$p_{\alpha} = p_{\alpha\|} = p_{\alpha\perp}$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B} \sum_{\alpha} \hat{\mathbf{b}} \times \nabla p_{\alpha}}$$

• Gyrokinetic MHD Equations: a reduced set of equations but in full toroidal geometry

-- For $k_\perp^2 \rho_i^2 \ll 1$ $\quad \bar{F} \to F \quad \bar{\phi} \to \phi \quad \bar{A}_\parallel \to A_\parallel \quad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \to 0$

-- Ampere's law

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} J_\parallel$$

$$\nabla_\perp^2 \mathbf{A}_\perp - \underline{\frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\mathbf{J}_\perp$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\delta\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Pressure Driven Current: $\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b} \times \nabla p_\alpha$

-- Vorticity Equation: $\frac{d}{dt}\nabla_\perp^2\phi - 4\pi\frac{v_A^2}{c^2}\nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0 \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla$

-- Ohm's law: $E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla\phi = \eta J_\parallel \to 0$

-- Equation of State: $\frac{dp_\alpha}{dt} = 0$

-- Normal modes: $\omega = \pm k_\parallel v_A$

• MHD Equilibrium

1. For a given pressure profile, we obtain the pressure driven current from

$$\boxed{\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b}\times\nabla p_\alpha}$$

2. We then solve the coupled equations of

$$\boxed{\begin{gathered}\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\mathbf{b}\cdot\nabla)\nabla_\perp^2 A_\parallel - 4\pi\frac{v_A^2}{c^2}\nabla\cdot\mathbf{J}_\perp = 0\\ E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0\end{gathered}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\dfrac{\partial A_\parallel}{\partial t} \to 0$,

this is then the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0}$$

4. The GK vorticity equation retains all the toroidal physics, different than Strauss‘ equation [PF 77]

5. Perpendicular current is consisted of both a divergent free diamagnetic current and a magnetic drift current. Only the latter was originally included in Lee and Qin [PoP, 2003].

Summary

• This set of gyrokinetic equations can indeed be used to study steady state electromagnetic turbulence to understand the physics of radial electric field wells.

• This set of gyrokinetic equations can also recover the equilibrium MHD equations in the absence of fluctuations.

• It will be interesting to couple a 3D global EM PIC code, e.g., GTS [Wang et al., 2006] with a 3D MHD equilibrium code, e.g., SPEC [Hudson et al., 2012] for the purpose of minimizing turbulence: A proposal by Lee for opportunities in Basic Plasma Science (DoE Lab 16-1592).