Disintegration of Langmuir solitons in inhomogeneous plasmas

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Outline

- Behavior of Langmuir solitons in inhomogeneous plasmas. Employ Zakharov equations (fluid equations).[a]
- Acceleration of Langmuir solitons in an inhomogeneous density background[b] Resultant emission of density cavities [c] and self-adjustment of the soliton.
- Disintegration threshold of Langmuir solitons at steep density gradients.
- Formation of high energy electron tails in the presence of Langmuir solitons.[d] Electron distribution function in the presence of nonlinear waves.

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[a] V.E.Zakharov, Sov. Phys. JETP **35**, 908 (1972).
[b] H.H.Chen and C.S.Liu, Phys. Rev. Lett. **37**, 693 (1976).
[c] P. K. Kaw, N. L. Tsintsadze, and D. D. Tskhakaya, Soviet Phys. JETP **55**, 839 (1982).
[d] G.J.Morales and Y.C.Lee, Phys. Rev. Lett. **33**, 1534 (1974).

Zakharov Eq. is revisited for Langmuir soliton studies

- Nonlinear Schrödinger equation is derived by V.E.Zakharov by a fluid approach.[a] For low frequency E (envelope for the high frequency part)

$$i\partial_t E + \partial_x^2 E = nE$$

- Ion density equation in the presence of ponderomotive force

$$\partial_t^2 n - \partial_x^2 n = \partial_x^2 |E^2|$$

- By letting $n = -|E|^2$, we obtain a nonlinear Schrödinger equation.

$$i\partial_t E + \partial_x^2 E + |E^2|E = 0$$

- More general solution has the form of a soliton [b] (here, $V_g = 2K_1$)

$$E(x,t) = E_0 \cdot sech\left[K_0\left(x - V_g t\right)\right] e^{-i\left[K_1 x - \left({K_1}^2 - {K_0}^2\right)t\right]}$$

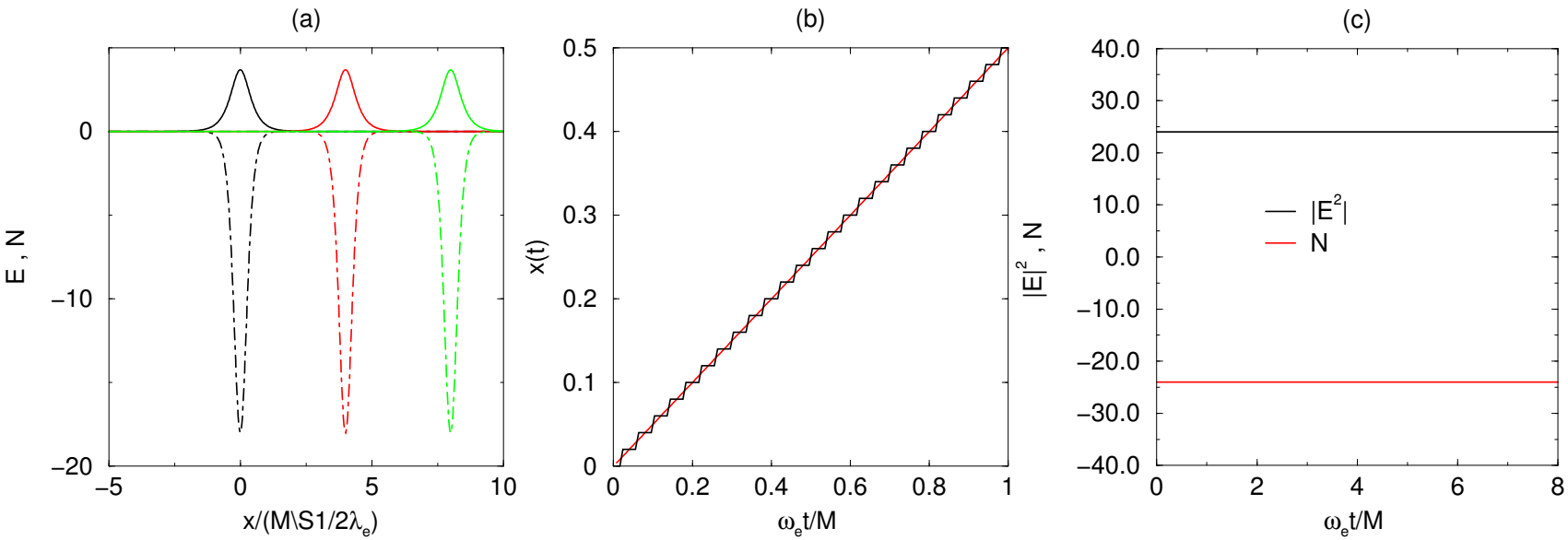
$$n(x,t) = -2{K_0}^2 \cdot sech^2\left[K_0\left(x - V_g t\right)\right].$$

[a] V.E.Zakharov, Sov. Phys. JETP **35**, 908 (1972). Initial condition parameterized by K_0 and K_1. Also, ${E_0}^2 = 2{K_0}^2\left(1 - V_g^2\right)$.

[b] N.R.Pereira, R.N.Sudan, J. Denavit, Phys. Fluids **20**, 271 (1977).

Standard test results in homogeneous plasma : Langmuir soliton propagates at a constant velocity

- Initial condition with $K_0 = 3.0$ and $K_1 = 0.25$ ($V_g = 0.5$) is given.



- (a) Propagation of density cavity (dashed) and the electric field soliton (solid), (b) The positions are estimated by quadratic fitting of the peak values, and (c) Conservation of the *area* $\int |E^2| dx$ and $\int N dx$ vs time.

Zakharov Eq. in the presence of background inhomogeneity is revisited

- The density gradient enters through change in background plasma frequency (in a dimensional form[a])

$$\partial_t^2 E_h - 3v_e^2 \partial_x^2 E_h + \omega_e^2 \left[\frac{n(x)}{n_0}\right] E_h = -\omega_e^2 \left[\frac{n_{el}}{n_0}\right] E_h$$

  where a linear density profile of the form $n(x) = n_0(1 + x/L)$ and a parabolic profile are employed. For the linear case, we then have [b]

$$i\partial_t E + \partial_x^2 E = \alpha x E + nE.$$

- Ion density equation as before

$$\partial_t^2 n - \partial_x^2 n = \partial_x^2 |E^2|$$

- Finite difference method (leapfrog) [c] is employed to time advace the equations to incorporate Dirchlet and Neuman boundary condition.

[a] D.R.Nicholson, Introduction to Plasma Theory, P.177 (1992). We set $E_h = (1/2)Ee^{-i\omega_e t}$.

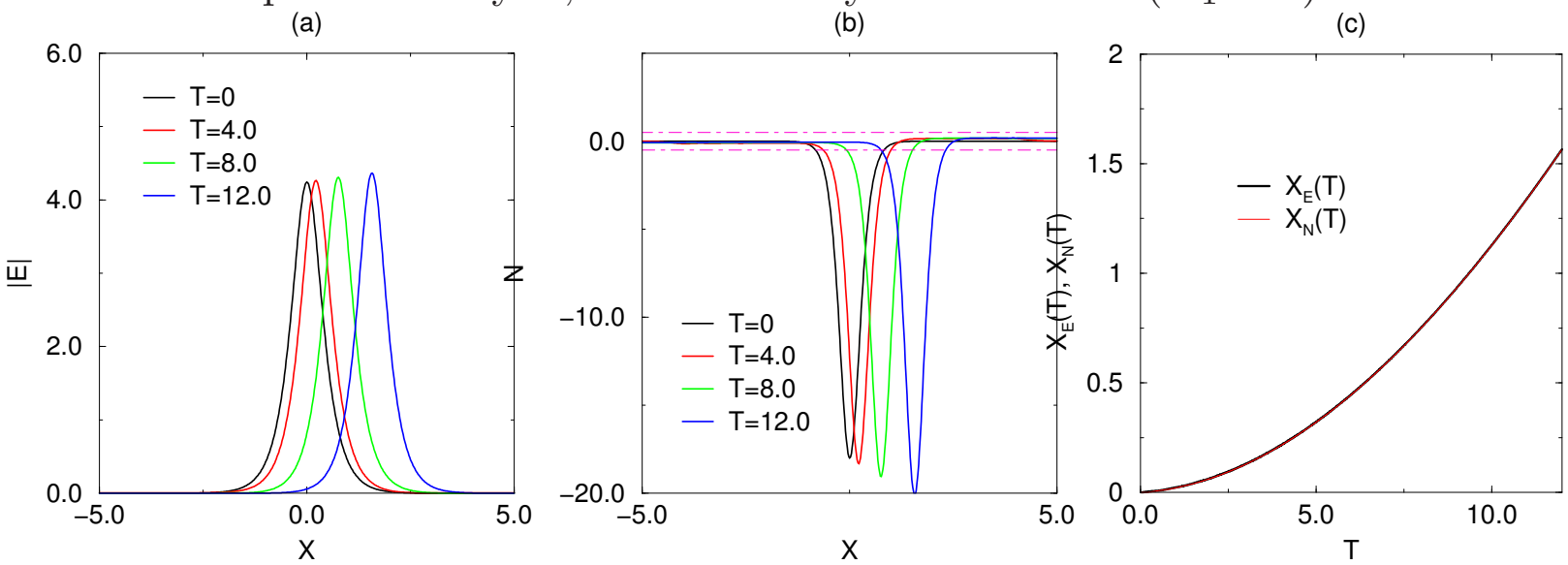
[b] Here, $\alpha = 4M/3L$ where $M = m_i/m_e = 1836$. Assumed "$L \gg W \gg \lambda$".

[c] Instead of conventional split step Fourier methods, T.R.Taha J. Comput. Phys. 55, 203 (1984), for example.

Langmuir solitons with background density change gives rise to acceleration

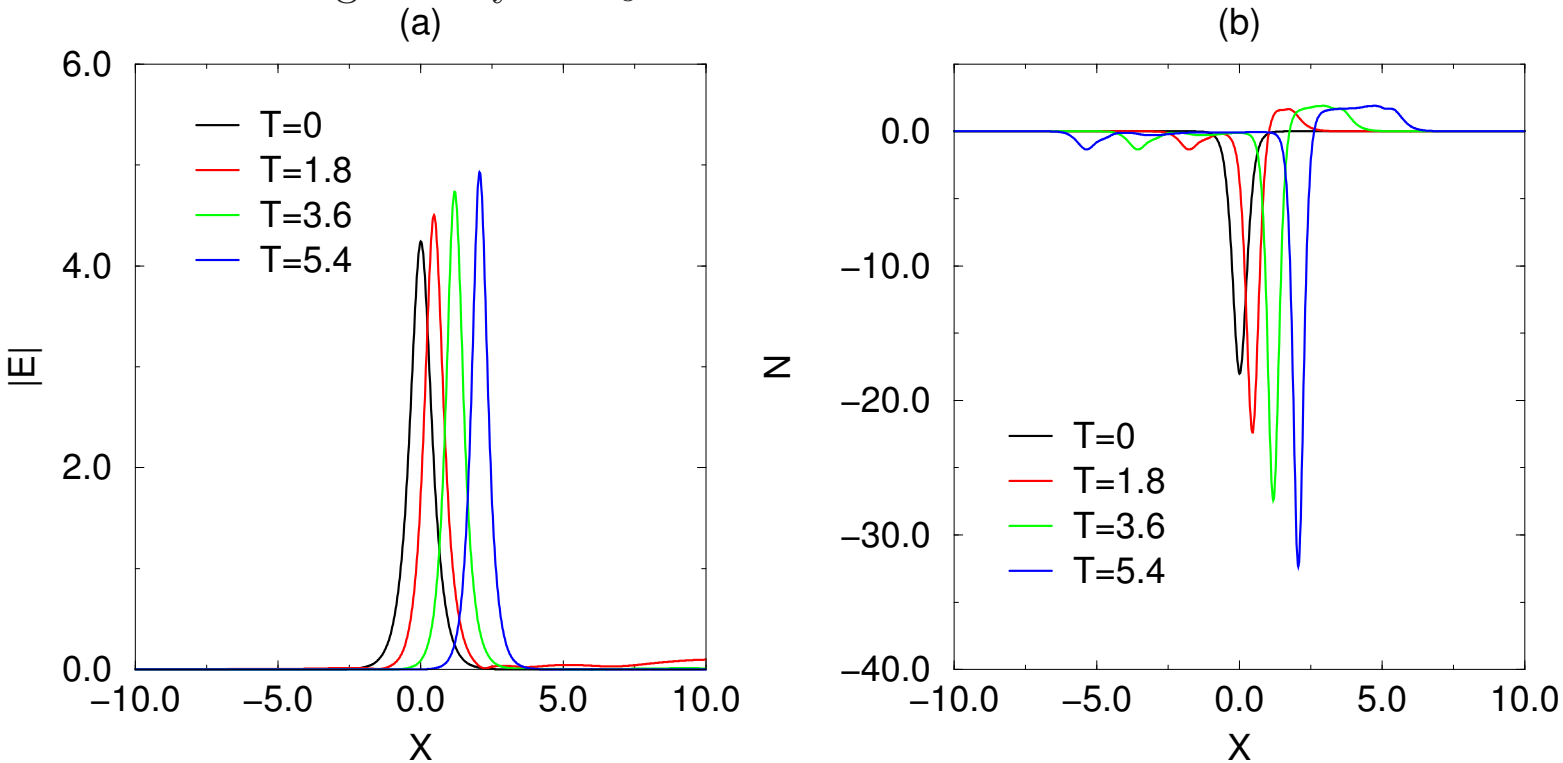
- In case with linear density profile. Here, $L = 5 \times 10^3$.

  For this part of analysis, initial velocity is set to zero ($K_1 = 0$).



- Measured acceleration in (c) is much smaller than 2α of NLS limit, because the density cavity drags. By zooming in, we can see peaks of X_E and X_N oscillates and leap on each other alternatively.

- Shorter density scale length of $L = 5 \times 10^2$ is taken. Emission of density cavities moving exactly at C_s is observed.[a]

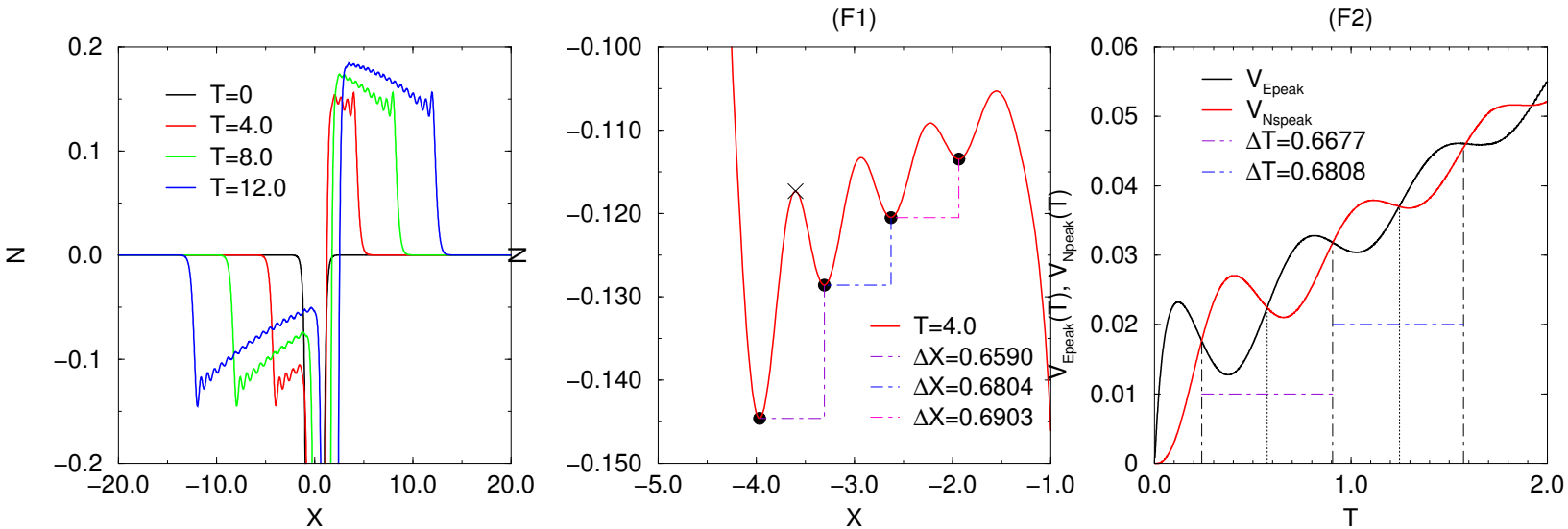


- The values of $\int |E^2| dx$ and $\int N dx$ still conserve.

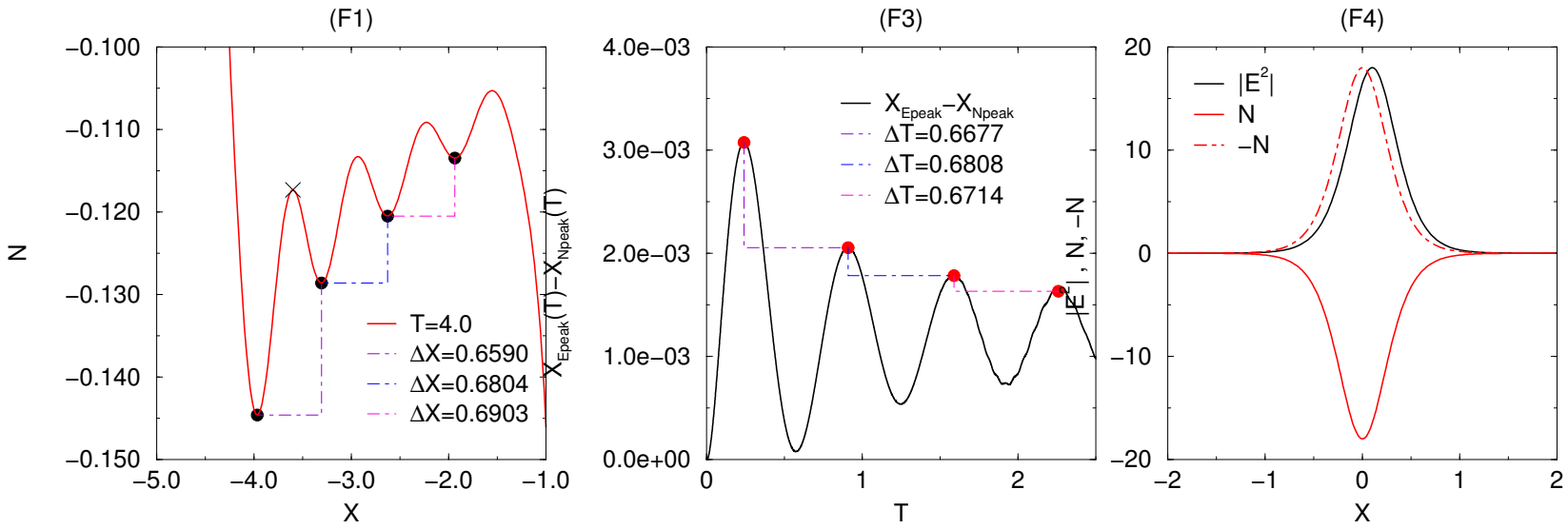
[a] P. K. Kaw, N. L. Tsintsadze, and D. D. Tskhakaya, Soviet Phys. JETP **55**, 839 (1982).

Finite cavity emission expanded for the small $1/L$ case

- Emission of cavity at C_s observed as long as acceleration is finite (for the previous $L = 5000$ case).



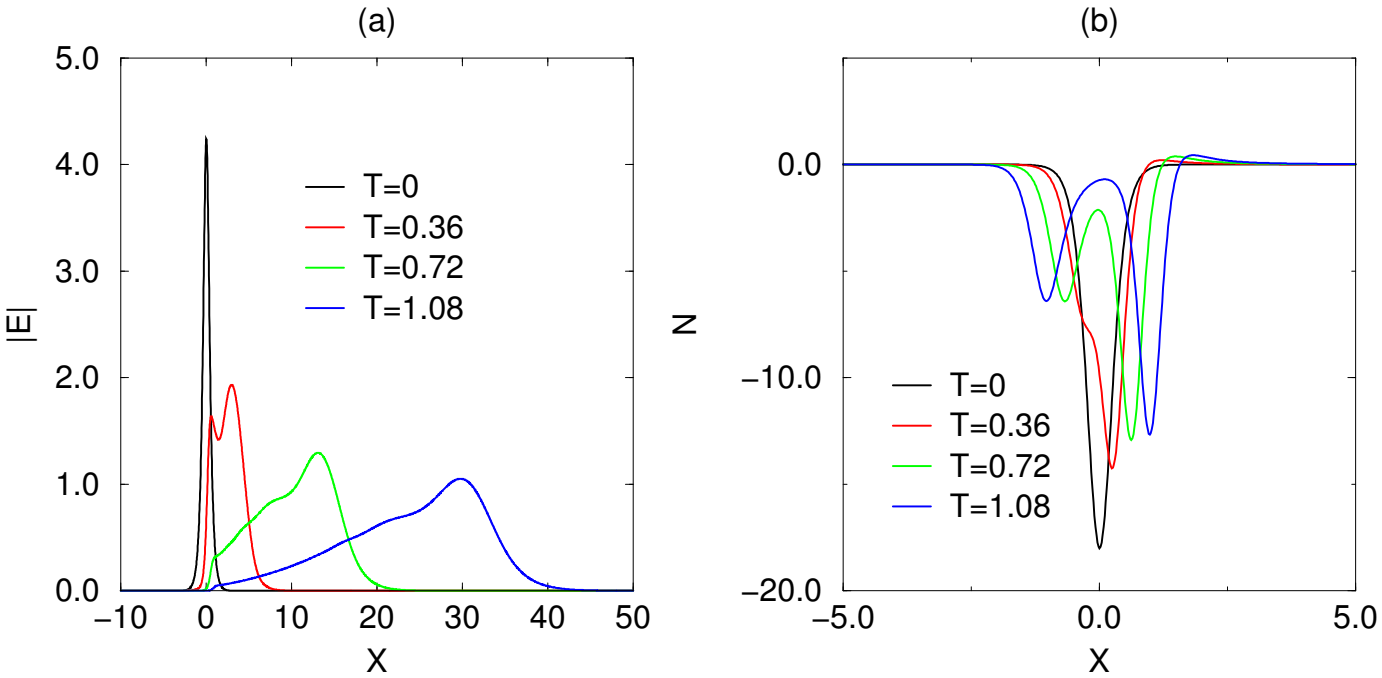
- (F1) Further expansion of the red curve. (F2) shows velocities of the peak positions. Small ripples correspond to bounce motions of the soliton within the cavity.



- (F3) Difference between the peak positions. (F4) Conceptual figure of electric field soliton (black solid curve) preceding (to the right) of the density cavity (red solid curve; red dashed curve for $-N$).

With further increase in density gradient, we observe disintegration of soliton

- Here, $L = 50$. In the absence of sustaining electric field (ponderomotive force), the original density cavity splits into two (which then propagate in the opposite directions at C_s).

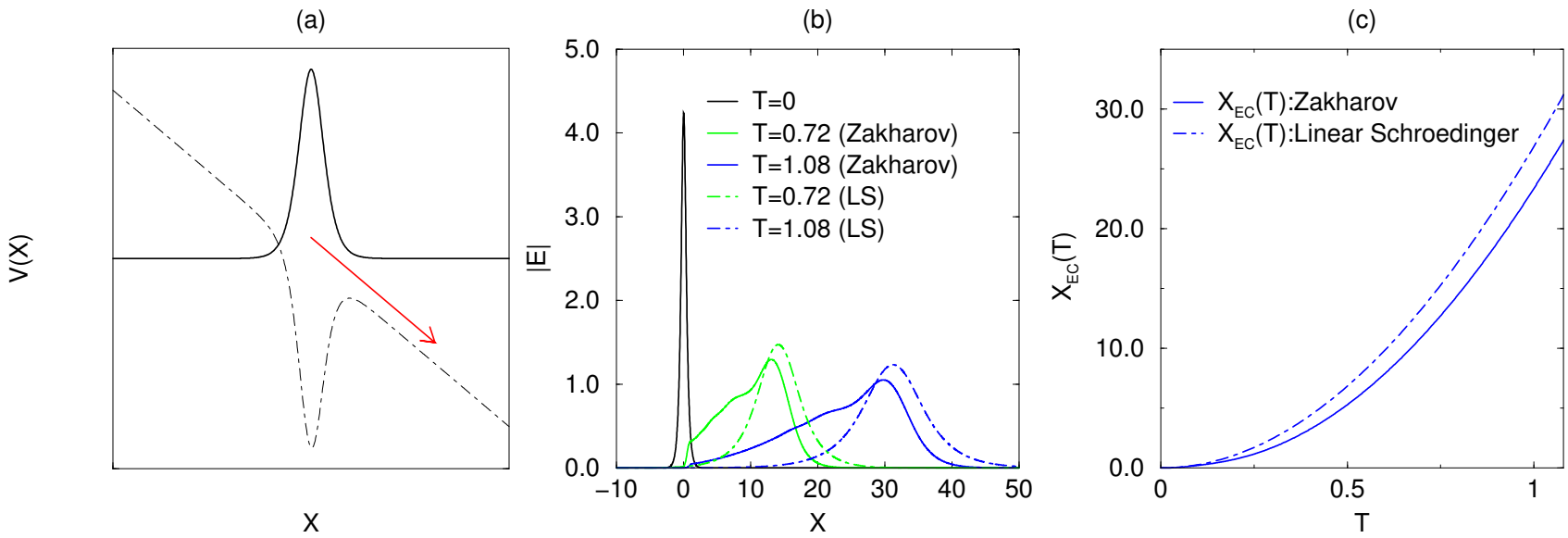


The mechanism of disintegration can be understood by regarding the soliton as a quasi-particle

- Schrödinger equation as a reminder

$$i\hbar\frac{\partial\psi}{\partial T} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial X^2} + V(X)\psi, \quad H = P^2/2m + V(X), \quad V(X) \to N - \alpha X,$$

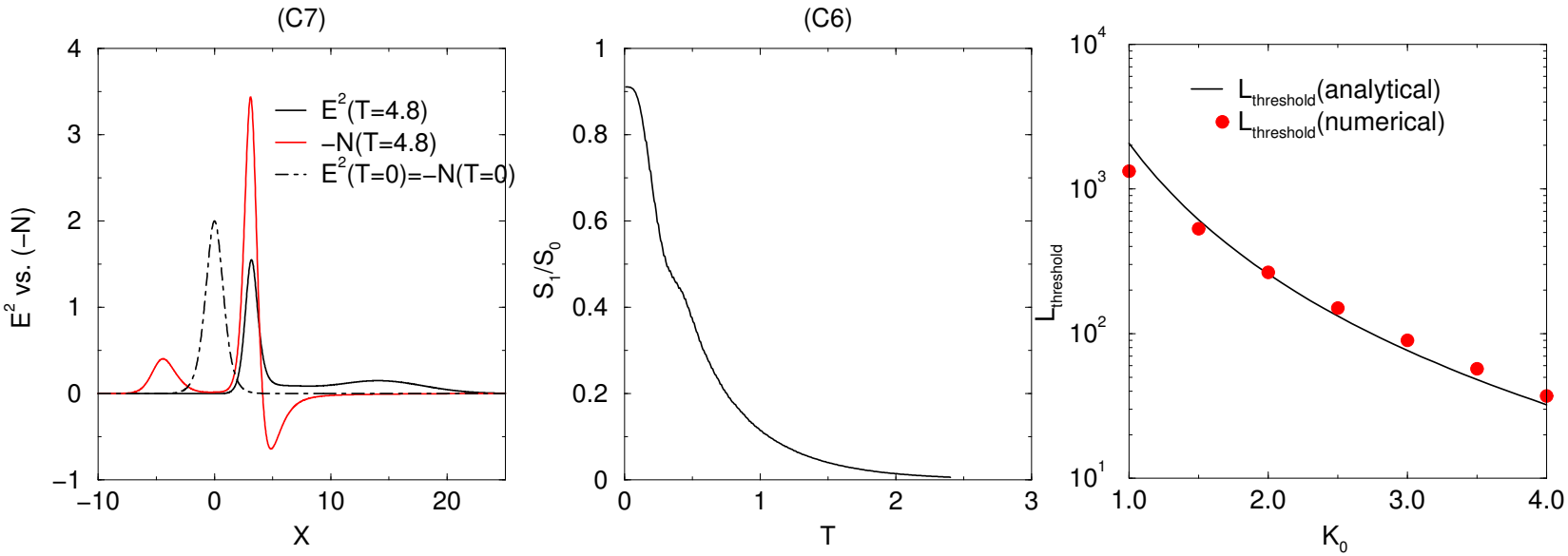
- Classical picture of point mass falling down the hill. Wave packets falling down the potential. Restoring force is small toward high density side.



Soliton's disintegration threshold is estimated based on energy conservation of the quasi-particles

- For a complete escape, $AT^2/2 = W$ (the soliton width W).
- If *kinetic energy* converted into *potential energy*, $V^2/2 = N = 2K_0^2$.
- Because $V = AT$ and $A = 3M/2L$, [a] Movie

$$L_{threshold} = (3MW)/(2K_0)^2.$$



[a] Y.A.Chen, Y.Nishimura(*), Y.Nishida, C.Z.Cheng, Phys. Rev. E, **95**, 033205 (2017).

Kinetic simulation : Electrostatic Vlasov simulation is employed to investigate soliton dynamics

- Let us consider 1d 1v phase space. A Vlasov-Poisson system normalized by "λ_d and ω_e " (with uniform ions) reads

$$\partial_t f_s + v\partial_x f_s \pm E\partial_v f_s = 0$$
$$\partial_x E = \int f_i dv - \int f_e dv$$

- To time advance Vlasov eq. numerically, we employ splitting scheme.

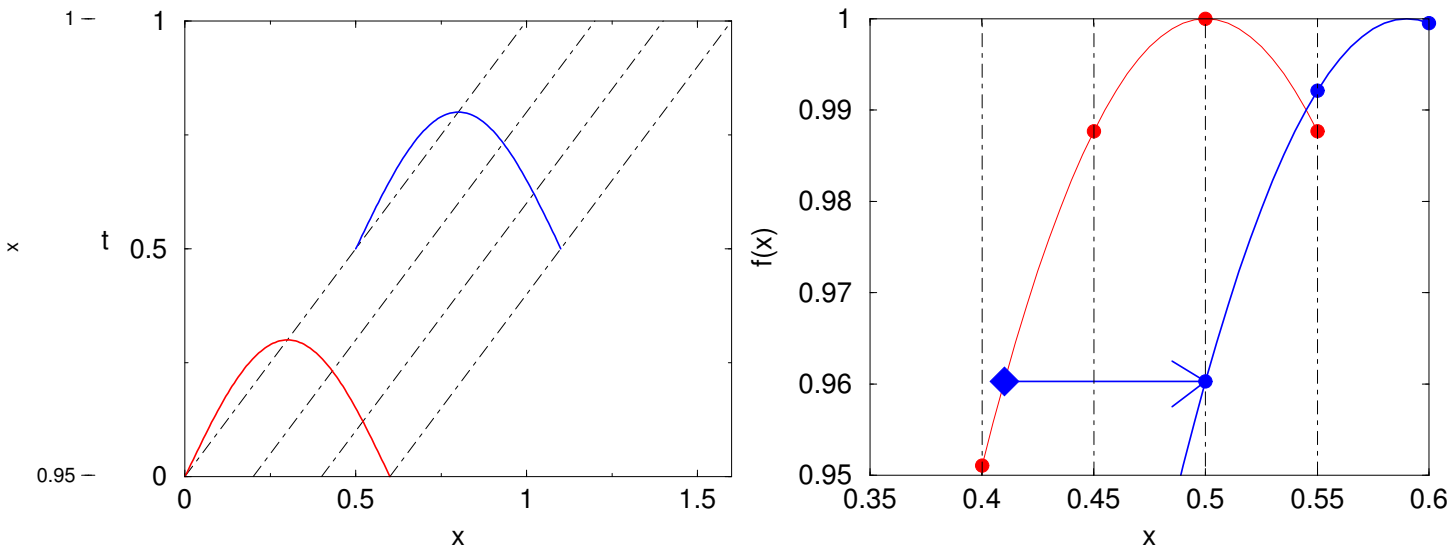
$$f_s^* (x, v) = f_s^n (x - v\Delta t/2, v)$$
$$f_s^{**} (x, v) = f_s^* (x, v \mp E(x)\Delta t)$$
$$f_s^{n+1} (x, v) = f_s^{**} (x - v\Delta t/2, v)$$

which is equivalent to leap-frog in PIC simulation, possessing a symplectic (a phase volume conserving) nature.[a]

[a] C.Z.Cheng and G.Knorr, J. Comput. Phys. **22**, 330 (1976). Fourth order symplectic schemes: Forest E. 2006 J. Phys. A: Math. Gen. **39** 5321.

Splitting scheme is based on the method of characteristics

- It <u>is not</u> a finite difference method. We **trace** the distribution function along the characteristic curves (red for past and blue for future in example below).

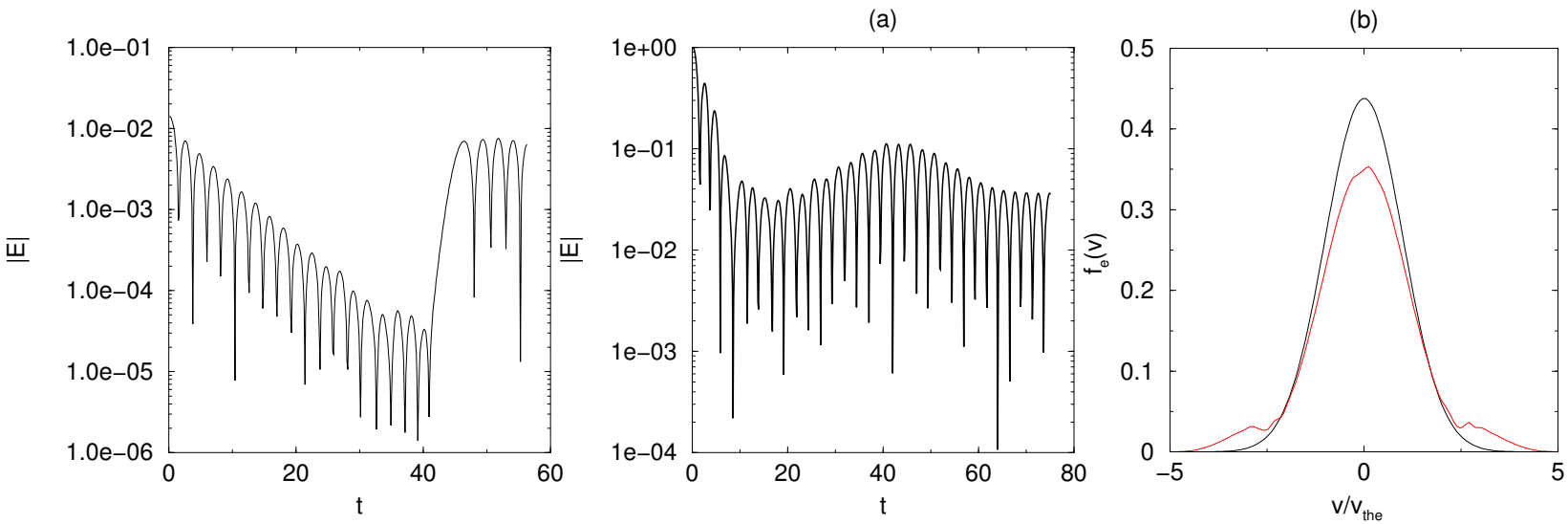


- In *tracing*, if the reference points along the characteristic curves "$x - v\Delta t$" are exactly on the mesh points, the method is quite trivial.
  $\rightarrow$ In general, the reference points ($\diamond$) are located in between mesh points.
  $\rightarrow$ <u>Need an interpolation scheme.</u>[a]

[a] C.Z.Cheng and G.Knorr, J. Comput. Phys. **22**, 330 (1976).

Linear and non-linear Landau damping of Cheng and Knorr '76 are reproduced

- Electric field is dynamically evolved (solving Poisson self-consistently). **(Left)** Linear damping **(Middle)** Non-linear **(Right)** Profile flattening.

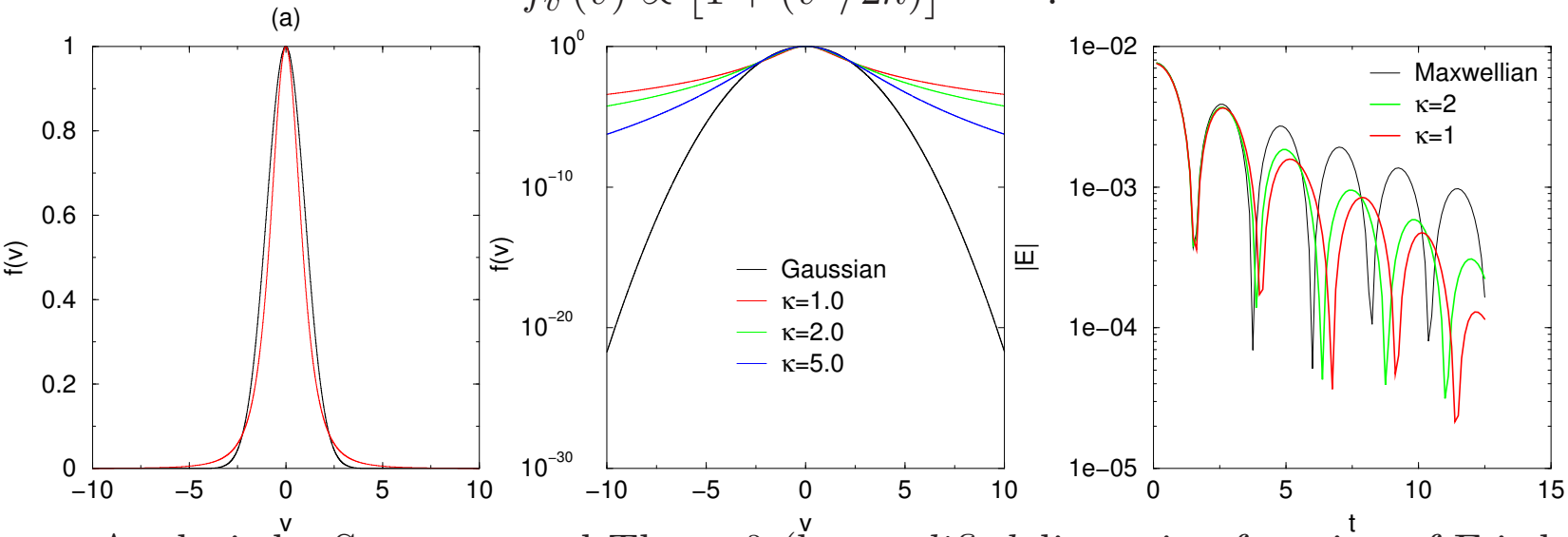


- Recurrence occurs ($t = 2\pi/k_x\Delta v \sim 48$) due to finite number of mesh points.
- Parameters: cut-off velocities $v_{max} = 4.0v_{the}$ (linear) / $v_{max} = 8.0v_{the}$ (nl), and $0 \leq x/\lambda_e \leq 4\pi$. For linear, $\omega = 1.41$ and $\gamma = -0.155$ match with the theory. Initial condition $f(x, v, 0) = [1 + A\cos(kx)]e^{-v^2/2}$ with $k = 0.5$.

Kappa distribution with larger high energy population gives rise to higher damping rate

- A "κ distribution function" is given by (note the exponent "$-\kappa-1$")

$$f_v(v) \propto \left[1+(v^2/2\kappa)\right]^{-\kappa-1}.$$



- Analysis by Summers and Thorne[a] (by *modified* dispersion function of Fried and Conte type) is reproduced.[b]

$$Z_\kappa^*(\xi) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa)}{\Gamma(\kappa-1/2)\kappa^{3/2}}\int_{-\infty}^{\infty}\frac{\left(1+s^2/\kappa\right)^{-\kappa-1}}{s-\xi}ds.$$

[a] D. Summers and R.M.Thorne, Phys. Fluids, B **3**, 1835 (1991).

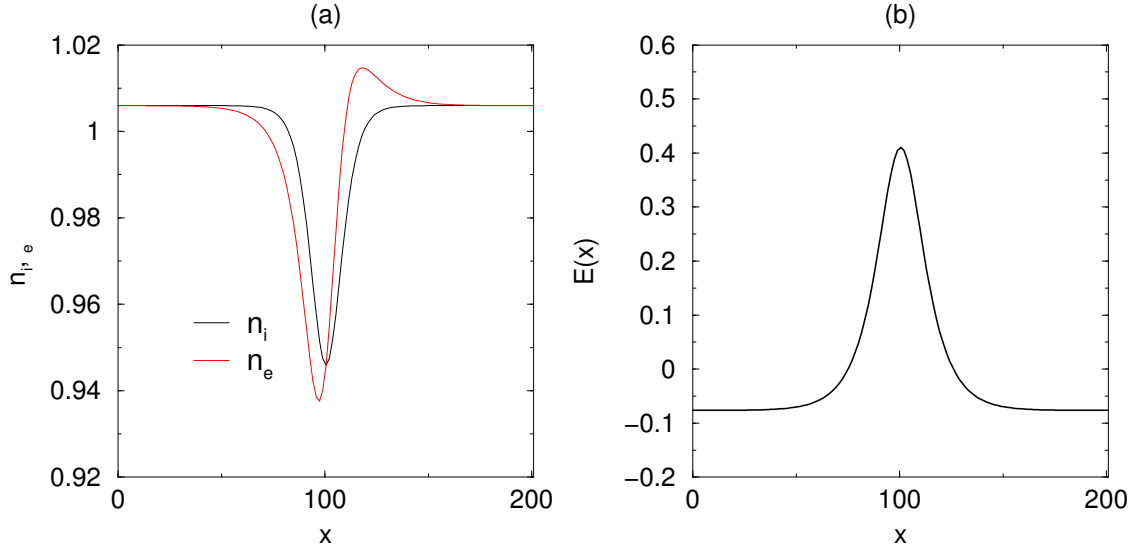
[b] Y.H.Chen, Y.Nishimura$^{(*)}$, C.Z.Cheng, Terrr. Atmos. Ocean Sci **24**, 273 (2013).

In the Vlasov simulation we incorporate Zakharov solutions as initial conditions

- As initial condition, we take $E(x,t) = E_0 \cdot sech\,(K_0 x)$ and $n_i(x,t) = -2{K_0}^2 \cdot sech^2\,(K_0 x)$.[a]

  Inverting Poisson equation, $n_e = n_i - \partial_x E$.

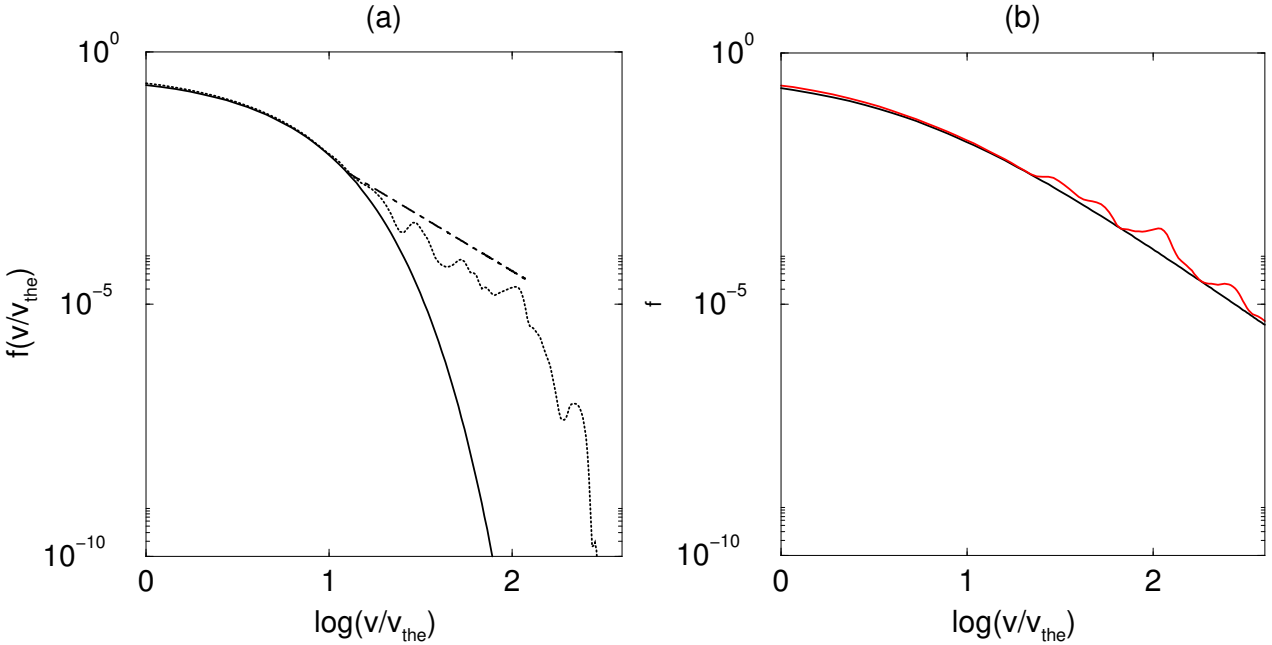
- Both the electrons and ions are time advanced. The initial f_e and f_i are given by $(n_{i,e})\times$ (Maxwellian: $e^{-v^2/2v_{e,i}^2}$) (or κ functions).Movie



[a]N.R.Pereira, R.N.Sudan, J. Denavit Phys. Fluids **20**, 271 (1977). We start from stationary soliton case by setting $V_g = 0$.

Formation of high energy tail in Vlasov simulation is demonstrated

- (a) Maxwellian and (b) $\kappa = 2.0$ distribution taken as initial conditions.



- Parameters employed are $n_v = 256$, $n_x = 128$, cut-off velocity $12.0v_{the}$ and $12.0v_{thi}$ and $0 \leq x/\lambda_e \leq 64\pi$ for (a). Followed up to $\omega_e t = 5000$. Dashed line for v^{-4}. (b) For $\omega_e t = 1000$, $16.0v_{the}$ and $16.0v_{thi}$ for $\kappa = 2$. [a]

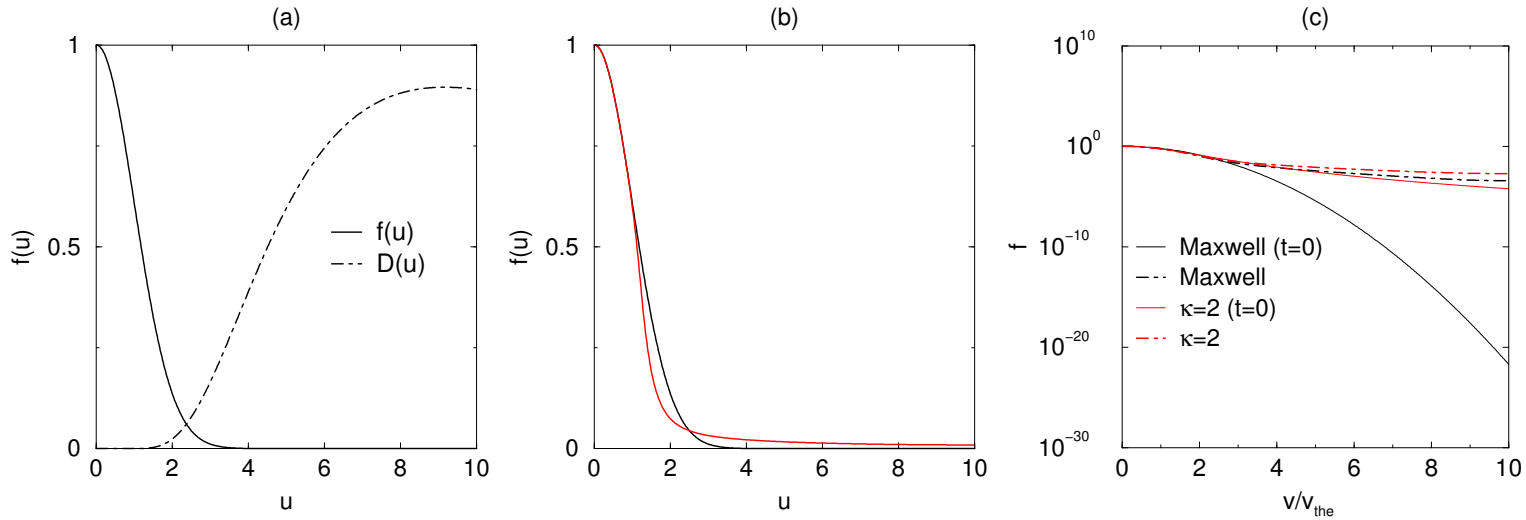
[a] C. H. Li, J. K. Chao, and C.Z.Cheng, Phys. Plasmas **2**, 4195 (1995).

Fokker-Planck solution demonstrates high energy tail formation

- Fokker-Planck equation (with $\kappa = 2$) is solved in the velocity space[a]

$$\partial_t f(u) = \partial_u \left[D\left(u, w\right) \partial_u f(u) \right]$$

with $D(u, w) = \pi(w/u) sech^2(\pi w/2u)$.[b]



- Tail formed. The bulk electrons do not change since they participate in the formation of density cavities, required to support localized electric fields.

[a] G.J.Morales and Y.C.Lee, Phys. Rev. Lett. **33**, 1534 (1974).

[b] Represents the soliton shape of the electric field envelope *of the configuration space*.

We obtain island separatrix equation in the phase space

- Given a Fourier component $\bar{E}_k \sin(kx - \omega t)$, the equation of motion reads

$$dx/dt = v$$
$$dv/dt = \bar{E}_k \sin(kx - \omega t)$$

- By going to the moving frame [letting $X = x - (\omega/k)t$ and $V = v - \omega/k$],

$$dX/dt = V$$
$$dV/dt = \bar{E}_k \sin(kX).$$

  For the corresponding Hamiltonian $H = V^2/2 - (\bar{E}_0/k)\cos(kX)$, we obtain the separatrix equation $V = \pm 2\sqrt{\bar{E}_k/k}\cos(kX/2)$ for each Fourier mode.[a]

- Extracting E_k values from the numerical Vlasov simulation, we can identify the separatrix widths and locations

$$v = \omega/k \pm 2\sqrt{\bar{E}_k/k}\cos(kX/2),$$

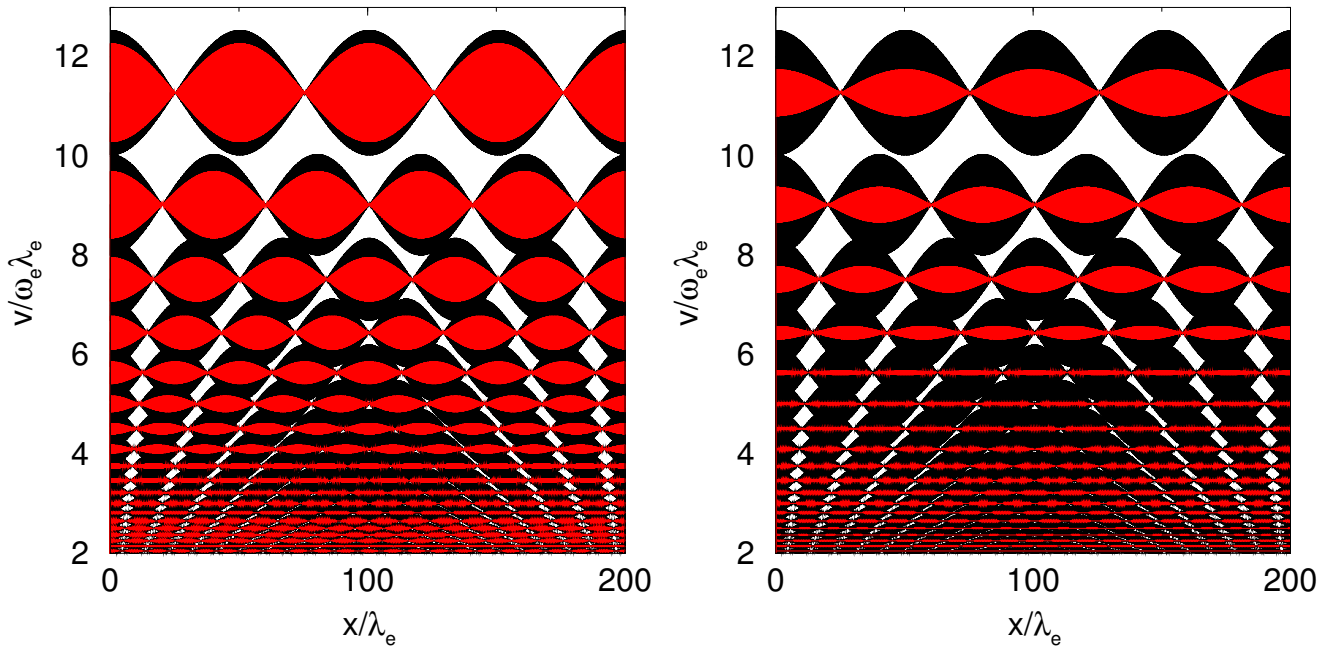
  and can apply the island overlapping criterion.

[a] Employ $\cos(\theta) = 2\cos^2(\theta/2) - 1$.

Chains of islands in velocity space is shown

- Extracting E_k values from the numerical Vlasov simulation, we can identify the separatrix widths and locations

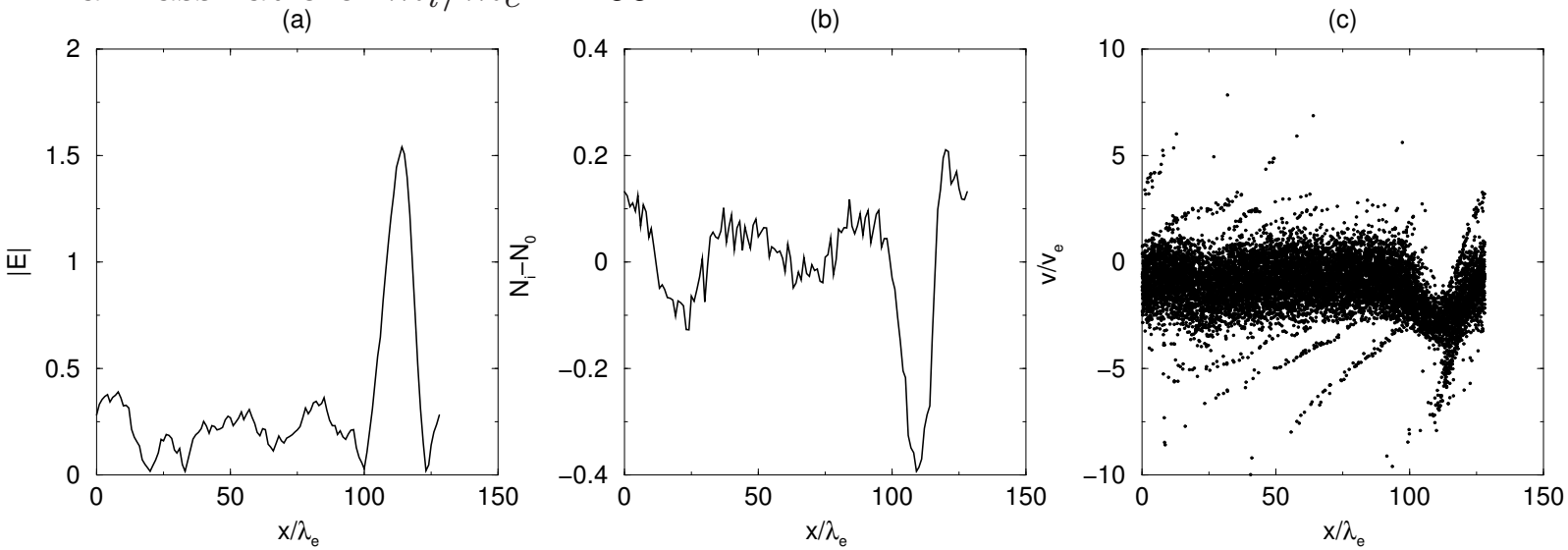
$$v = \omega/k \pm 2\sqrt{\bar{E}_k/k}\cos(kX/2).$$



- (a) Maxwellian case at $t = 0$ (black) and $t = 1000$ (red). (b) $\kappa = 2$ case at $t = 0$ (black) and $t = 125$ (red). While the island overlapping is seen (Chirikov's criterion as a reminder) at $t = 0$, the islands at $v/v_{the} \geq 2.0$ are intact at the later phase. Remind that the damping rate is large with an existing high energy tail (κ distribution).

Pump wave is applied to generate Langmuir soliton (by a Particle-in-Cell simulation)

- An external wave of $E_0 \cos(\omega_{ext} t)$ is applied with $\omega_{ext} = 0.9\omega_e$ is applied for a mass ratio of $m_i/m_e = 100$.[a]

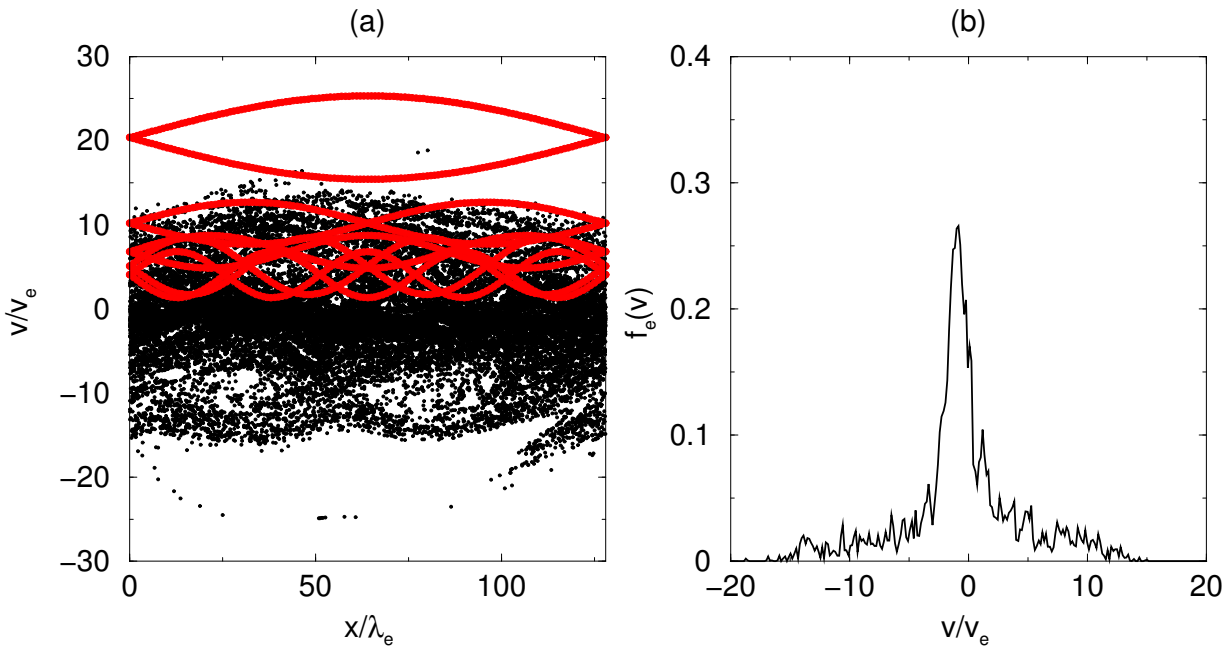


- The pump wave and ion acoustic wave couples to generate Langmuir wave.

[a] E.J.Valeo and W.L.Kruer, Phys. Rev. Lett. **33**, 750 (1974).

Interaction of multiple solitons is demonstrated

- By providing external pump wave,[a] onset of oscillating two-stream instability [b] is studied by a Particle-in-Cell method. Saturation can be understood as flattenning of distribution within island overlapping regions in phase space bounded by KAM surfaces.

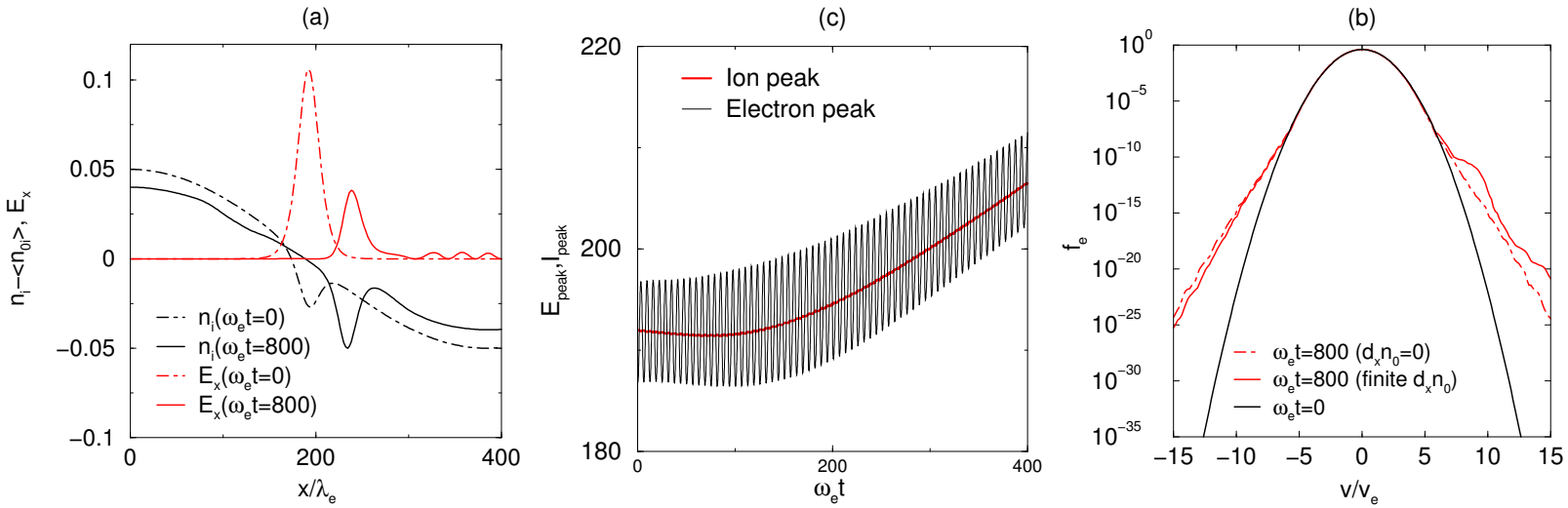


[a] E.J.Valeo and W.L.Kruer, Phys. Rev. Lett. **33**, 750 (1974).

[b] K.Nishikawa, J. Phys. Soc. Japan **24**, 916 (1968). G. J. Morales, Y. C. Lee, and R. B. White Phys. Rev. Lett. **32**, 696 (1974).

Non-uniform background density in the Vlasov-Poisson system is incorporated

- We stay in a periodic system and give a background density profile of $n_0(x) = n_0[1 + A\cos(2\pi x/x_{max})]$ $(A = 0.05)$. Corresponds to $L \sim 10^3$ of Zakharov model.



- (a) ion density cavity and the electric field soliton (b) cavity peak (minimum) positions (c) electron distribution function. (at $\omega_e t = 800$)

Summary and discussions

- Dynamics of Langmuir solitons in inhomogeneous plasmas is investigated numerically employing Zakharov equations. Have observed acceleration, emission of density and self-adjustment of the soliton.
- Disintegration threshold of Langmuir solitons at steep density gradients is discussed.
- A 1d-1v Vlasov-Poisson simulation is applied to study evolution of electron distribution function in the presence of nonlinear Langmuir waves.
- Formation of high energy electron tails in the presence of Langmuir soliton is demonstrated. For the high energy electrons tails, nonlinear waves can be playing an important role.