Modelling magnetotail fields and particle injections associated with dipolarizations

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In collaboration with

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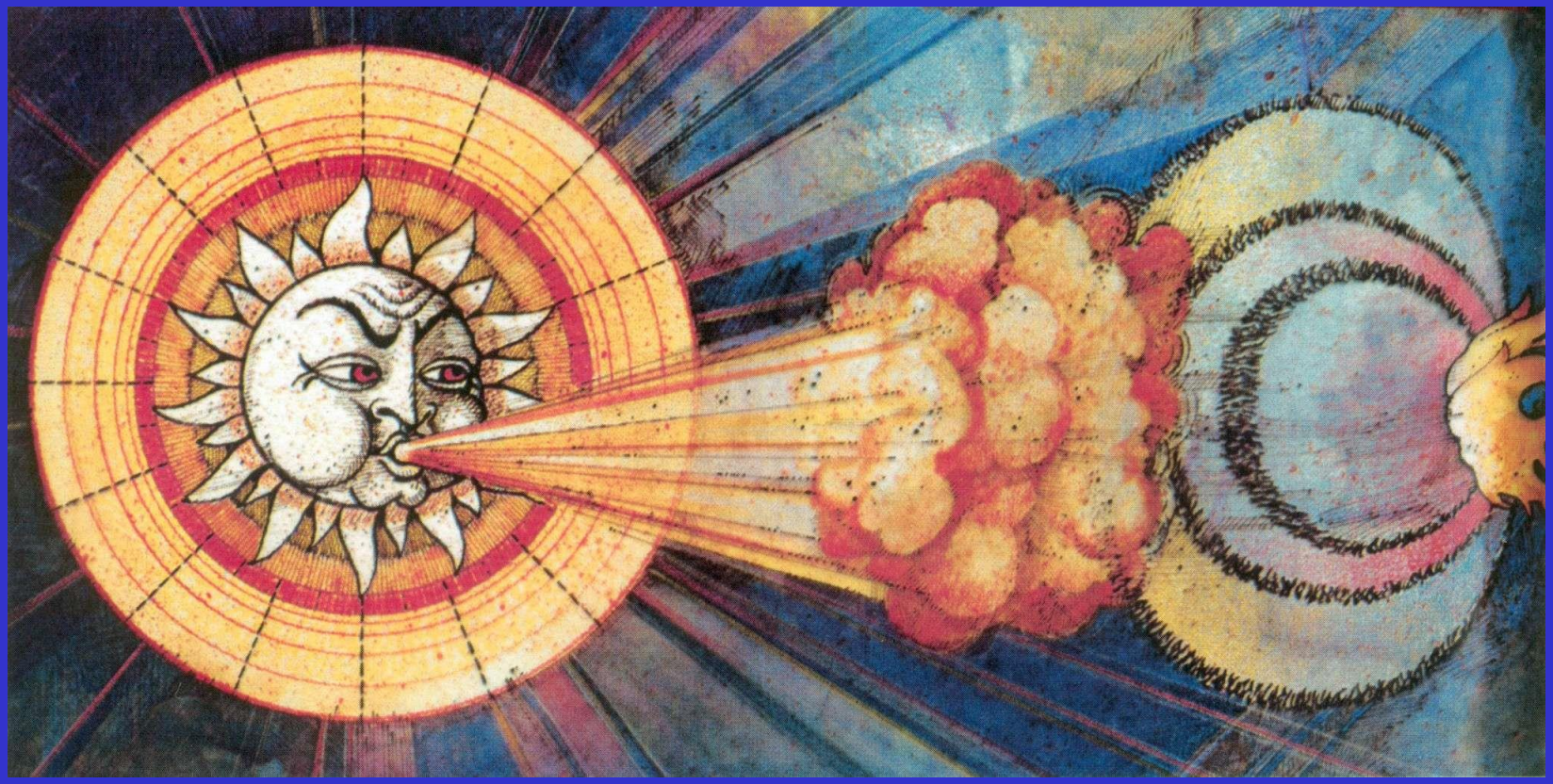
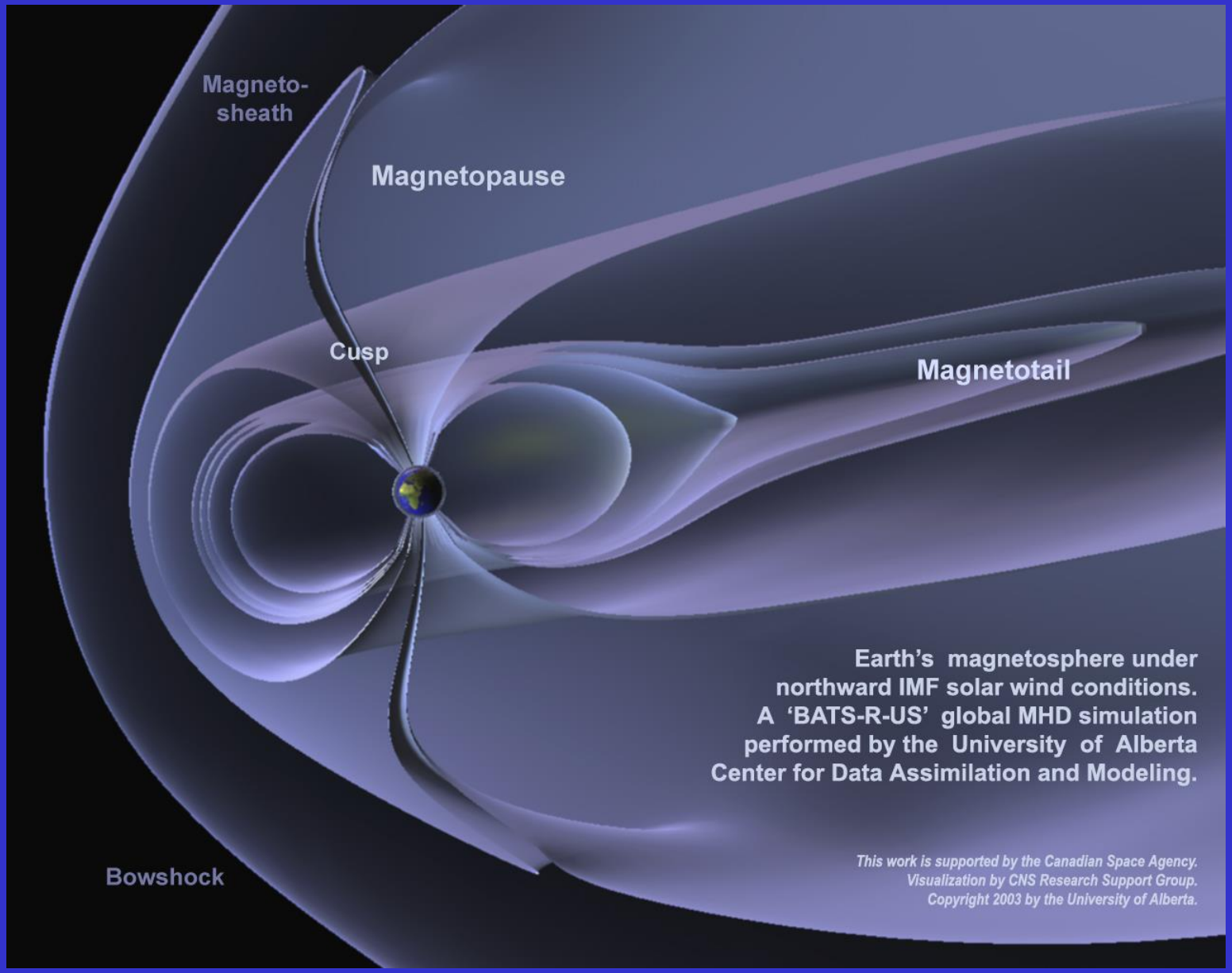
Origin of Space Weather and Magnetospheric Physics




Magneto-
sheath
Magnetopause
Cusp
Magnetotail
Bowshock
Earth's magnetosphere under
northward IMF solar wind conditions.
A 'BATS-R-US' global MHD simulation
performed by the University of Alberta
Center for Data Assimilation and Modeling.
This work is supported by the Canadian Space Agency.
Visualization by CNS Research Support Group.
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Introduction

- Substorm – a reconfiguration of the magnetotail which, among other things, “creates” a population of electrons with energies of tens to hundreds of keV during the substorm expansion phase.

- Increasing evidence from auroras points to injection beginning at a sharp transition between very stretched and less stretched field lines around 8 RE.

- We attempt to model evolution of the magnetotail which leads to the electron injections.

Motivation for a New Model

- There are numerous magnetotail models. We invented yet another one (Kabin et al., JGR 2011).
- To model the energization process, it is desirable to have a magnetotail model with few adjustable parameters, directly controlling magnetotail thickness and transition from dipole to tail-like fields.
- The model should be simple.

Model Description

Start with an "auxiliary" field: dipole + cut-off tail

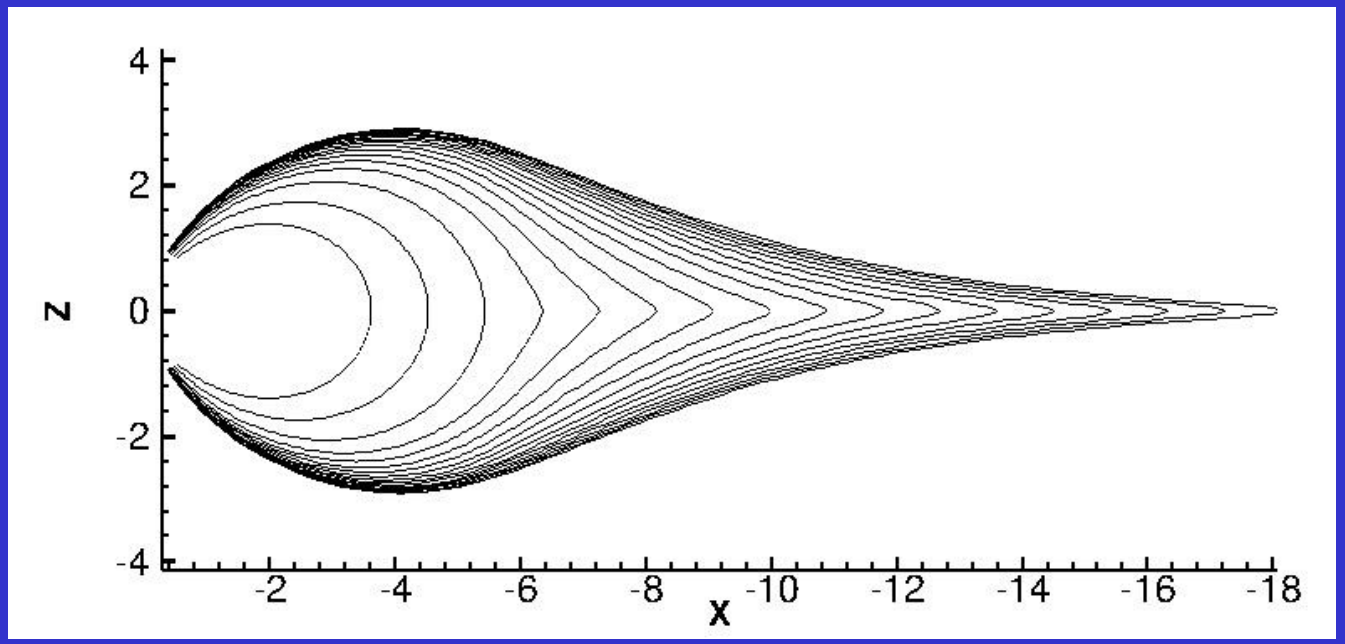
$$\mathbf{b} = \mathbf{B}_{dip} + \mathbf{e_x} f(x) B_0 \tanh(z/L_z)$$

$$f(x) = \frac{1}{2}\left(1 - \tanh\frac{x + R_t}{\delta}\right)$$

The cut-off function f(x) is used to control the location and sharpness of the transition from dipole-like to tail-like fields

Although the divergence of "**b**" is nonzero, it is possible to compute a divergence-free magnetic field with identical field lines

Field Lines in the Model



The field line topology has desirable features in that the amount of stretching, pressure gradients in the tail, and the transition region can be constrained by satellite and ground-based data.

Making the Field Divergence-free

- Define a divergence-free magnetic field via flux function

$$\mathbf{B} = \nabla\Psi \times \nabla\phi = \frac{\nabla\Psi \times \mathbf{e}_\phi}{r\sin\theta}$$

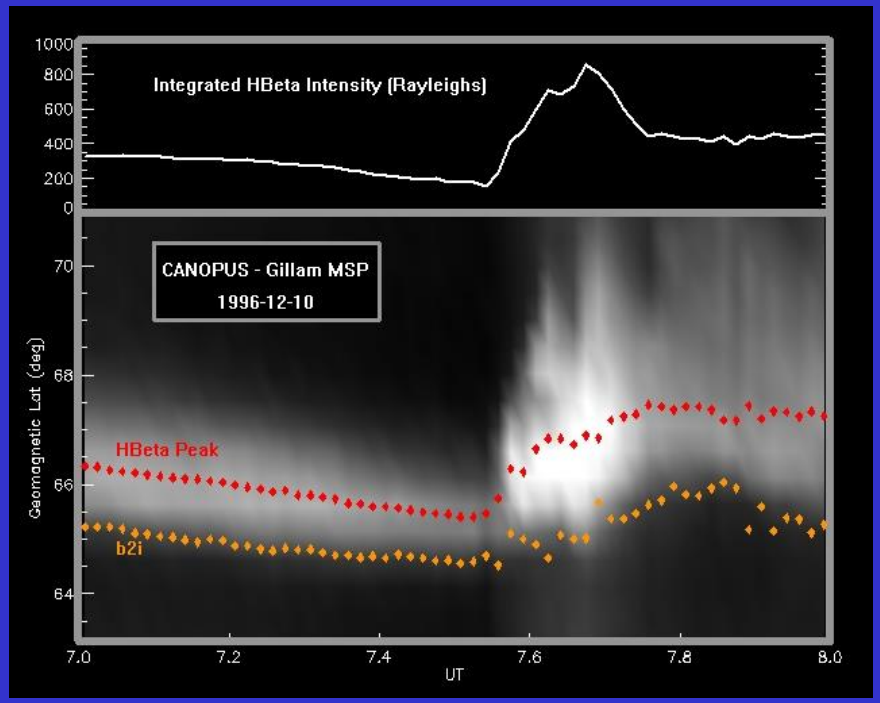
- Define the flux function in the dipole field region close to the Earth, and extend it into the magnetosphere along field lines of the auxiliary field:

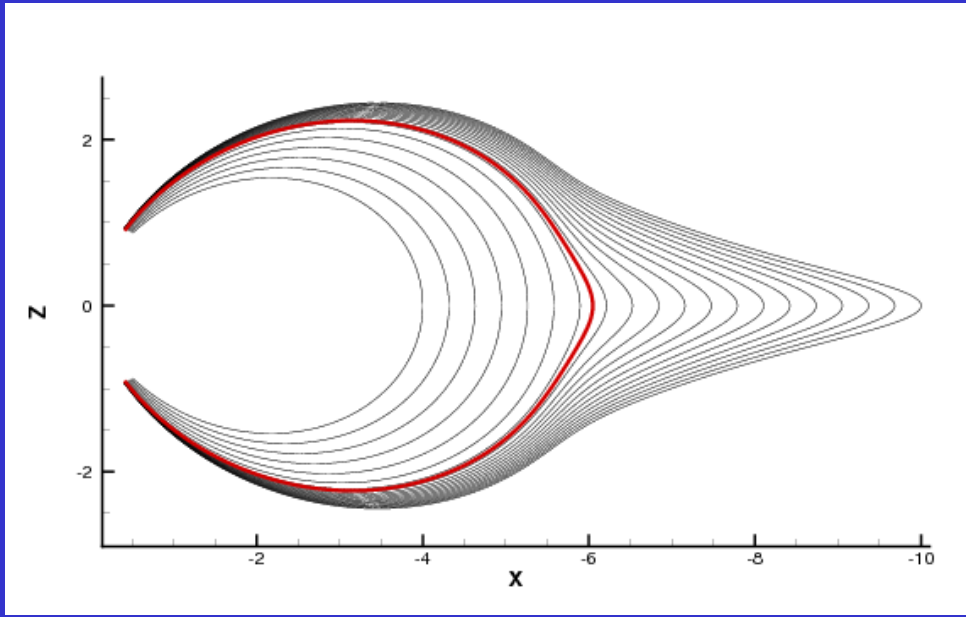
$$\Psi_I = \frac{\sin^2\theta_I}{r_I}$$

- The magnetic field **B** now has the same field line topology (and the same desirable properties) as "**b**"

Observations of the b2i-Boundary

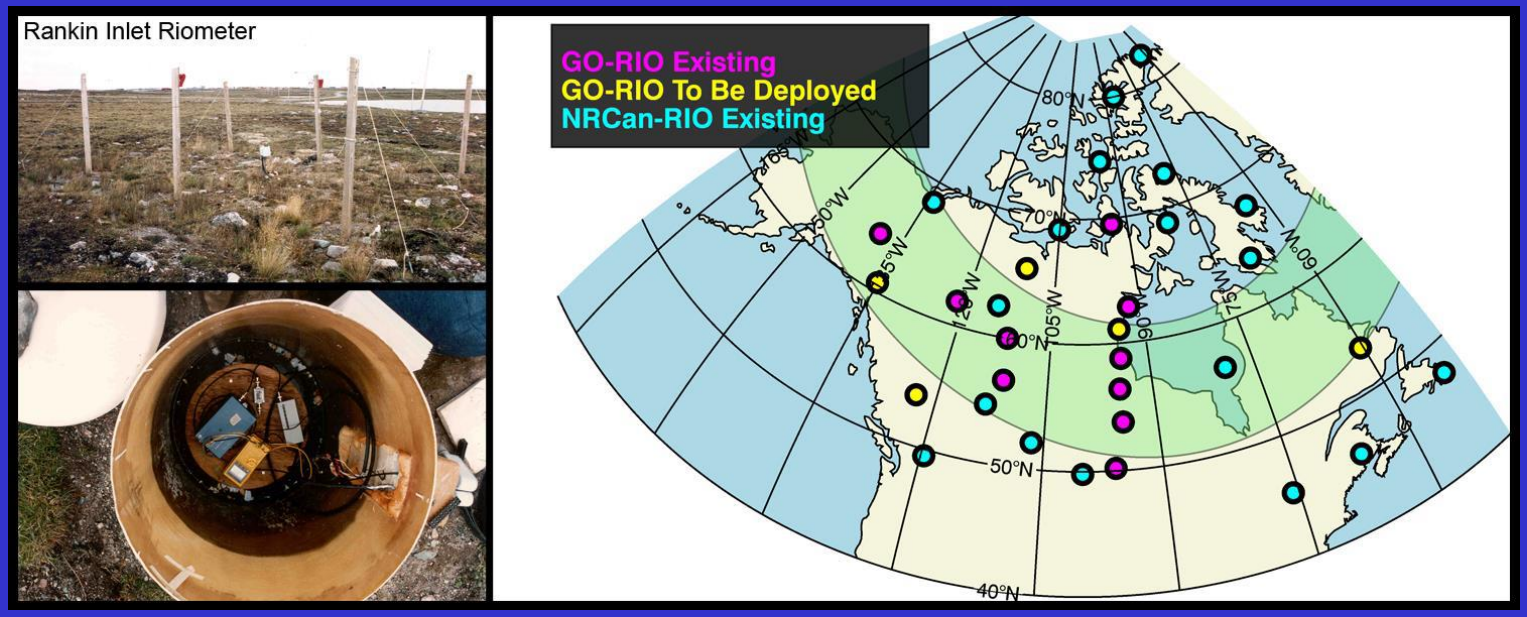
Proton isotropy boundary is associated with scattering on field line curvature





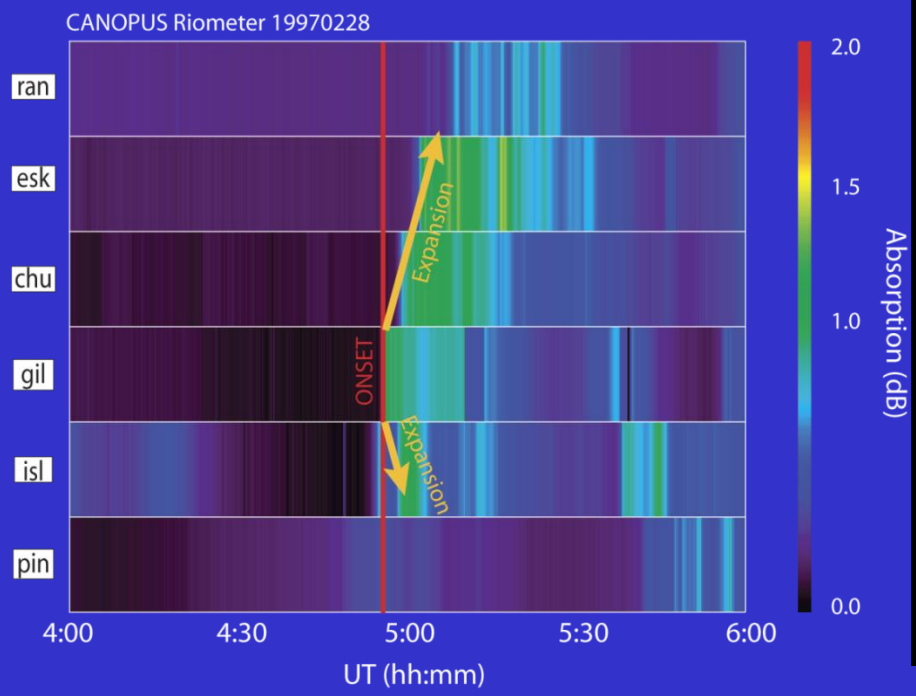
486.1 nm

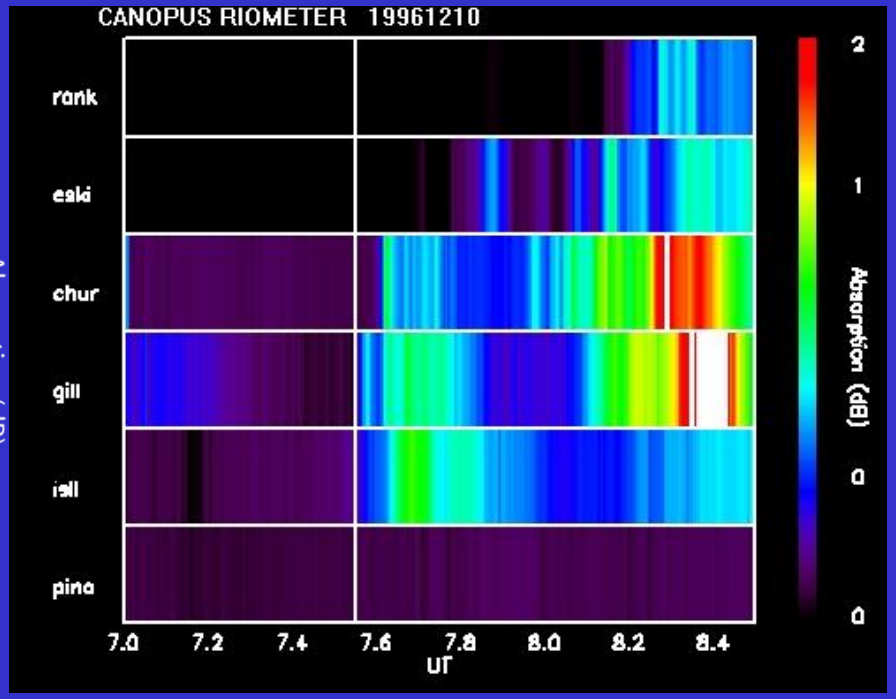
Riometers



Relative Ionospheric Opacity METERs observe radio wave propagation at 30 MHz through the ionosphere

Riometer Absorption: Substorm Injection across Churchill Line



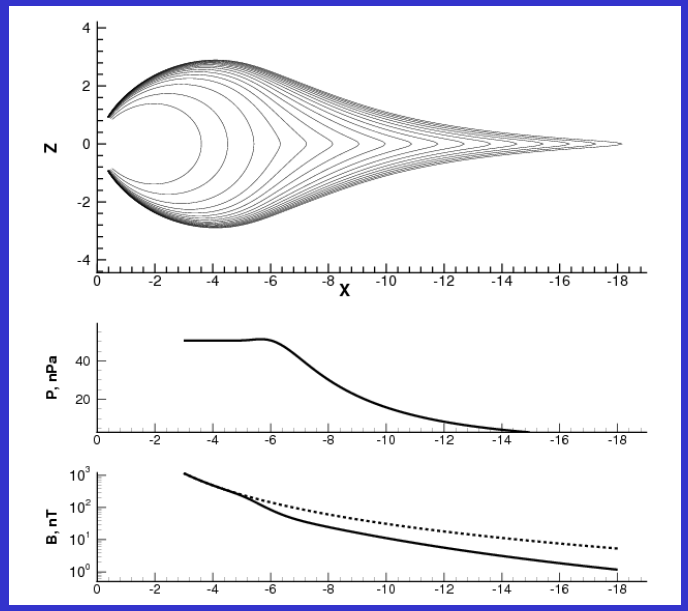


Injection expands ~5° poleward in 5-15 minutes

Not much equatorward expansion

Interpretation of Riometer Data

- A transition region that retreats into the tail, increasing the local magnetic field
- Electrons gain energy to conserve μ
- Electrons pitch-angle scatter to the loss cone, and are observed by riometers if their energy is above ~30 keV



Model parameters

Riometer, auroral and other observations allow us to constrain the parameters of the model to fit a typical dipolarization event

$$f(x) = \frac{1}{2}\left(1 - \tanh\frac{x + R_t}{\delta}\right)$$

$$R_t = 7 + \tanh(1.6t - 4)$$

$$B_0 = -51840/R_t^3$$

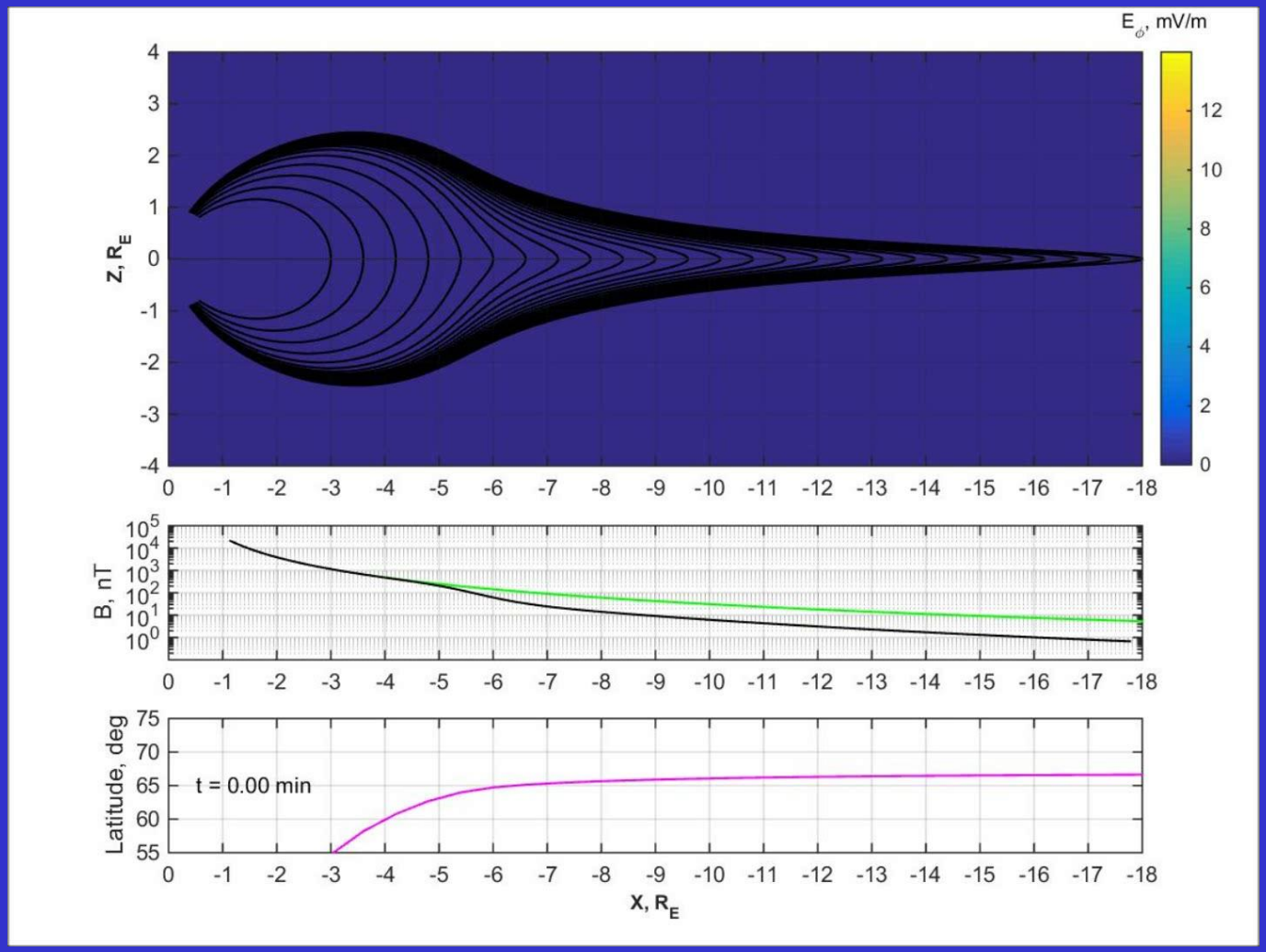
Electric field

Electric field is calculated from Faraday's law

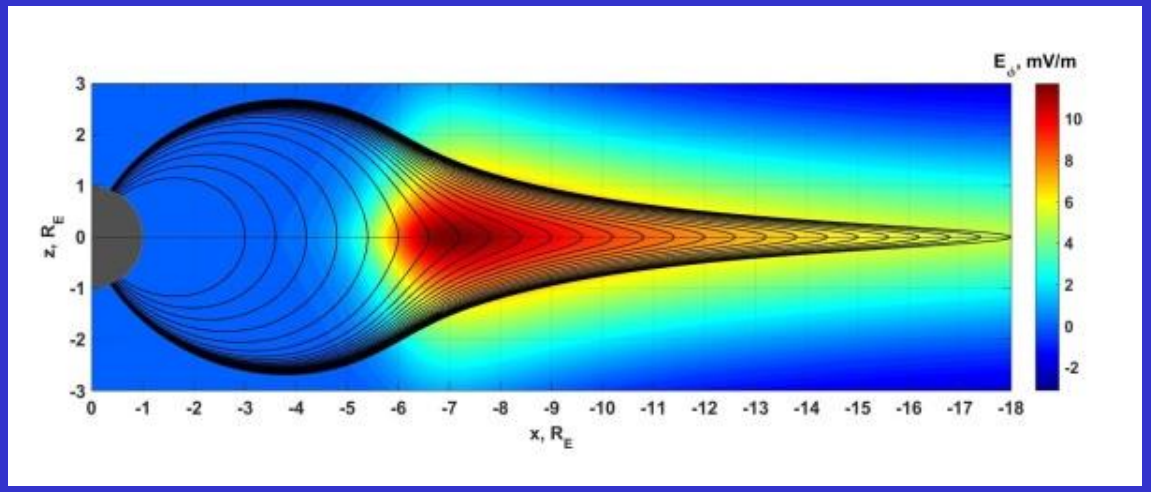
$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\phi) = -\frac{\partial B_r}{\partial t}$$

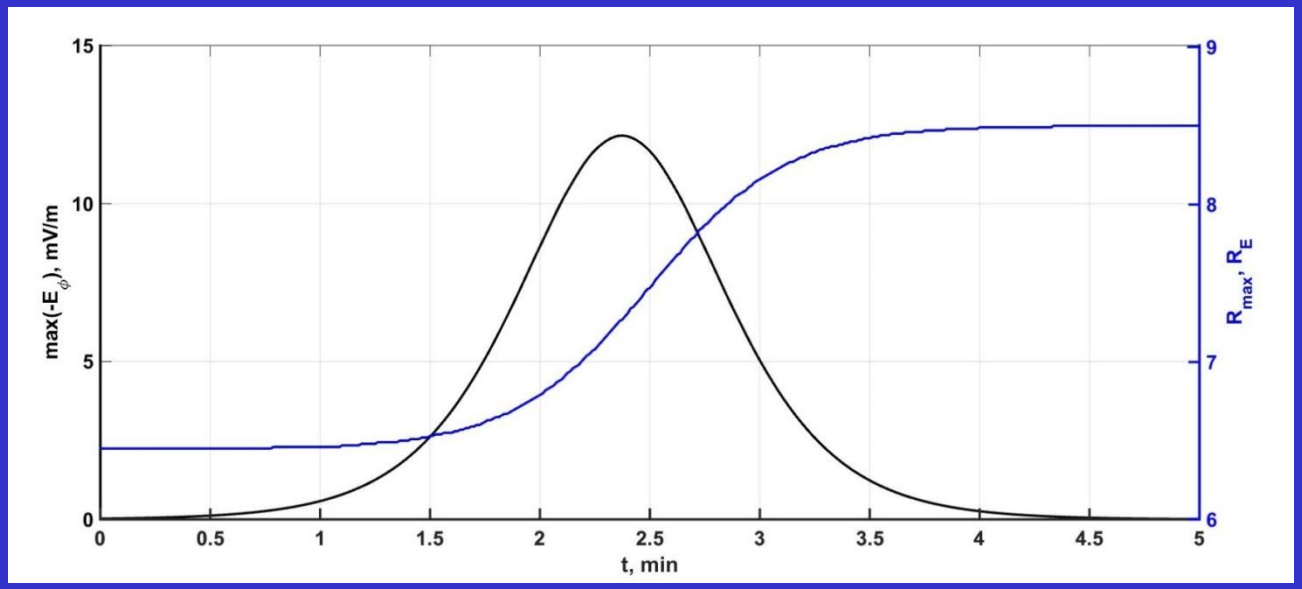
$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) = \frac{\partial B_\theta}{\partial t}.$$

Example of dipolarization



Max E during the event





Particle trajectories

Next, we trace electrons and protons in the calculated dipolarization fields.

Guiding center equations

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu_m}{\gamma q}\frac{\mathbf{B}\times\nabla B}{B^2}$$

$$\frac{dr}{dt} = -\frac{E_\phi}{B_\theta}$$

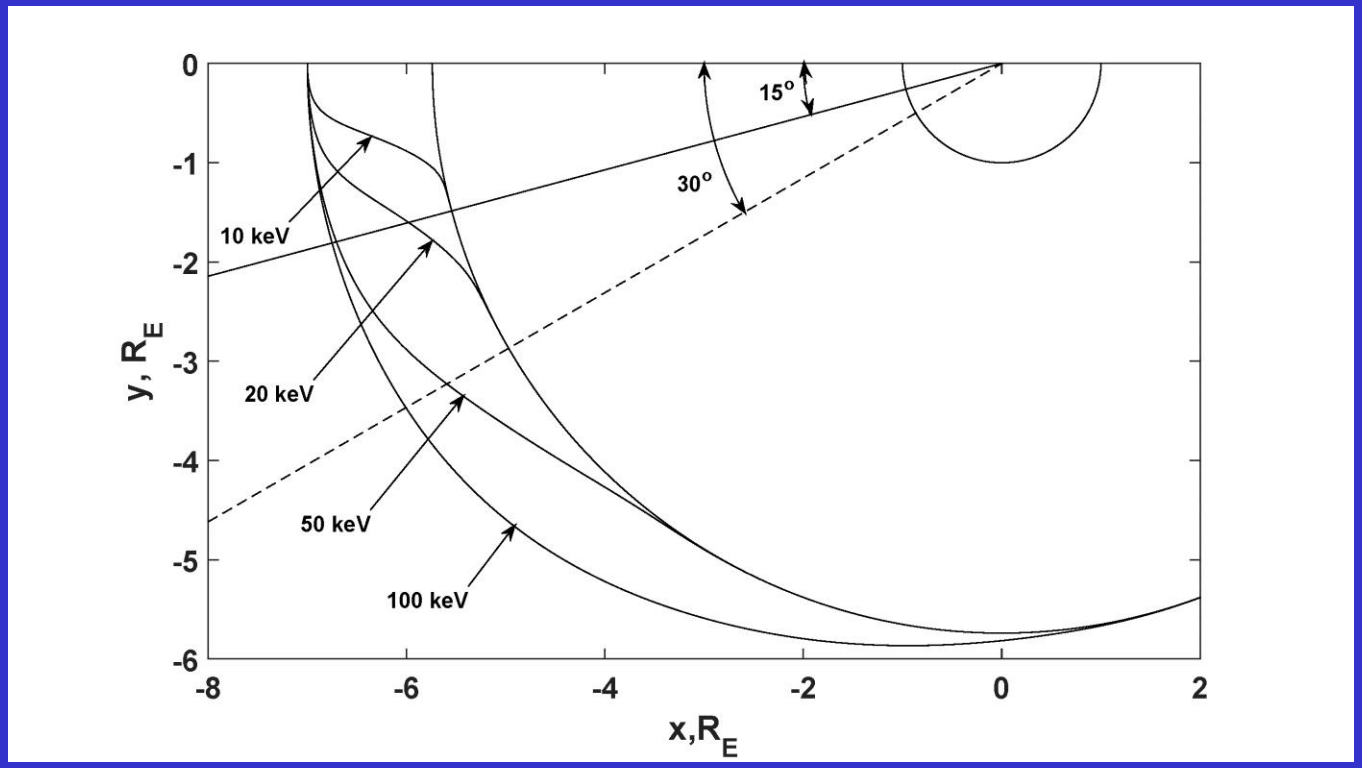
$$r\frac{d\phi}{dt} = -\frac{\mu_m}{\gamma q B_\theta}\frac{\partial B_\theta}{\partial r}$$

Lorentz equations

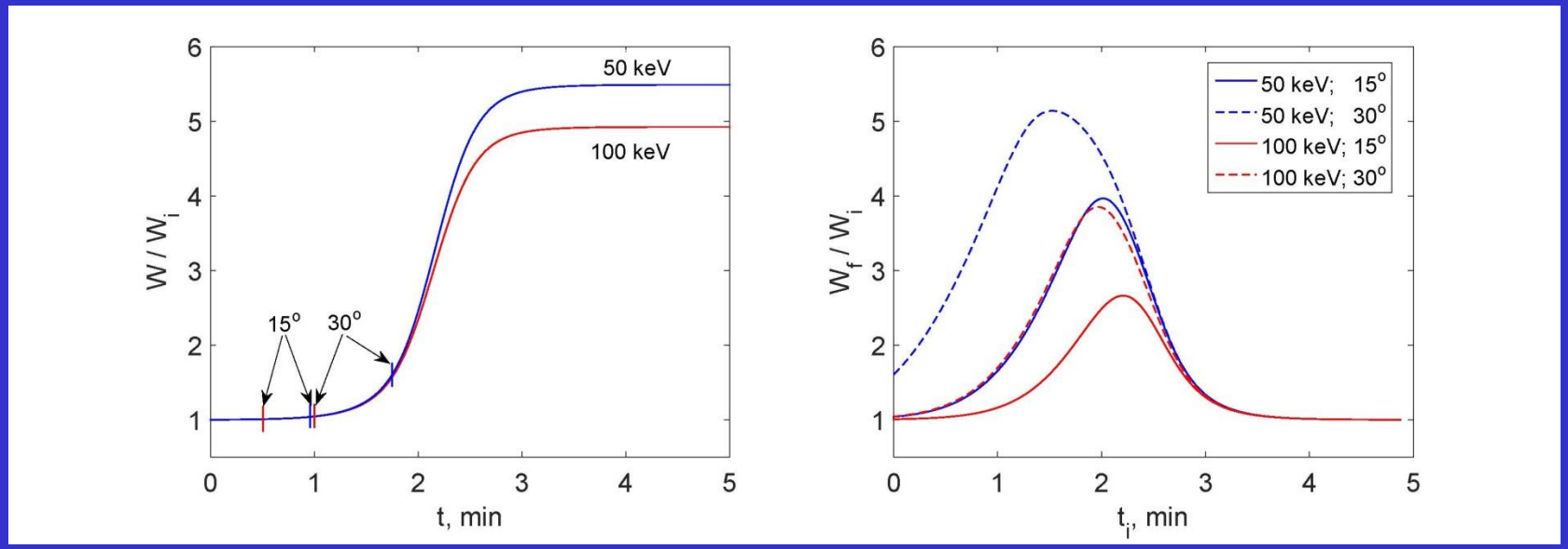
$$\frac{d\mathbf{r}}{dt} = \mathbf{v}\ ,$$

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

Electron trajectories

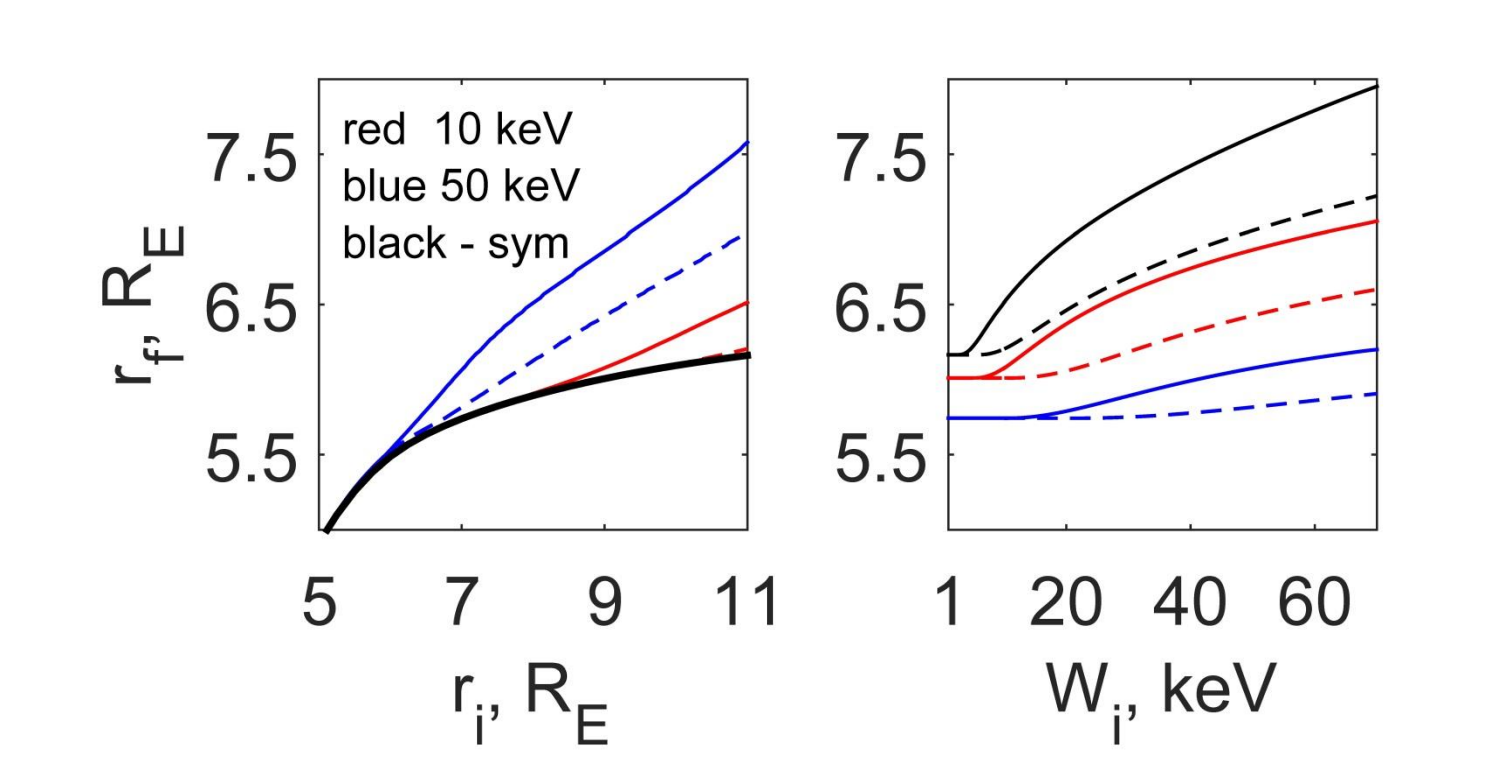


Energy gain by electrons

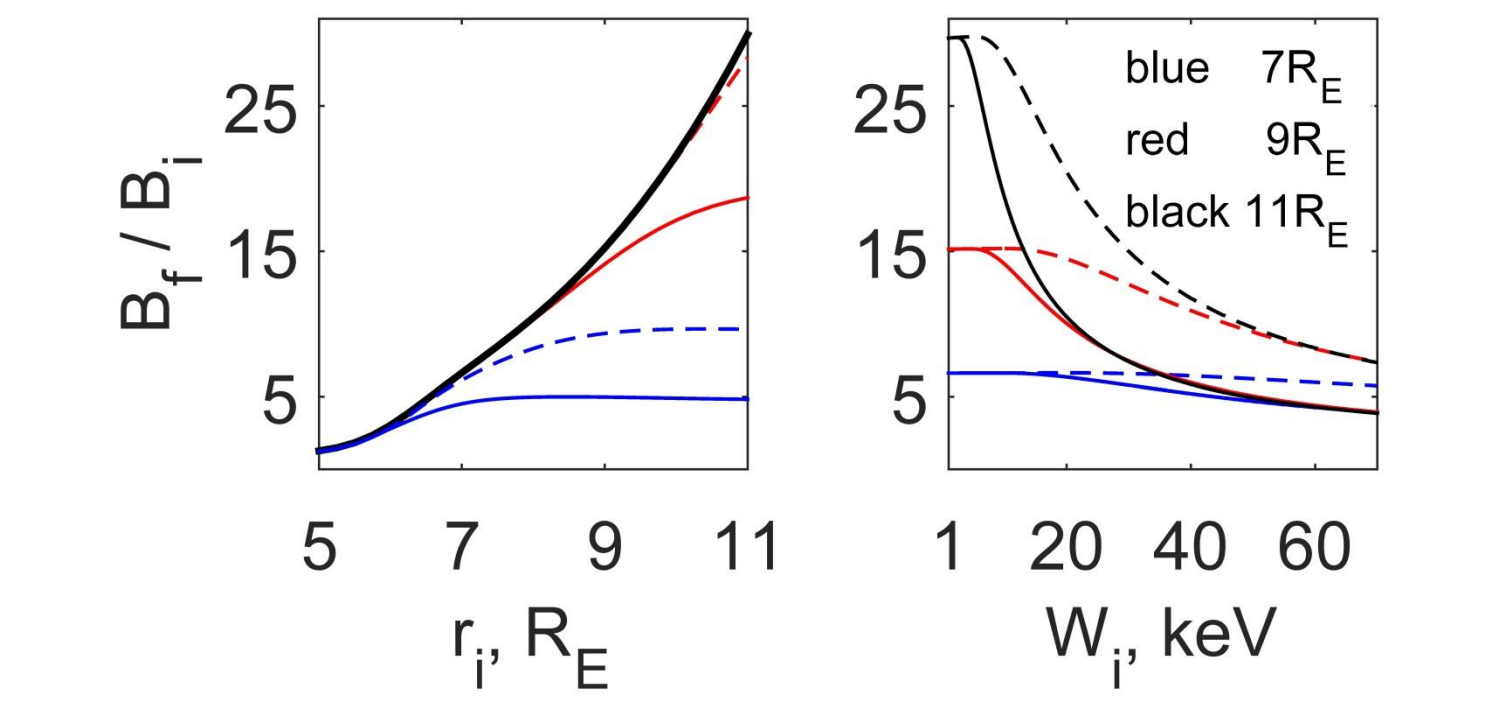


Initialized at 7 R_E

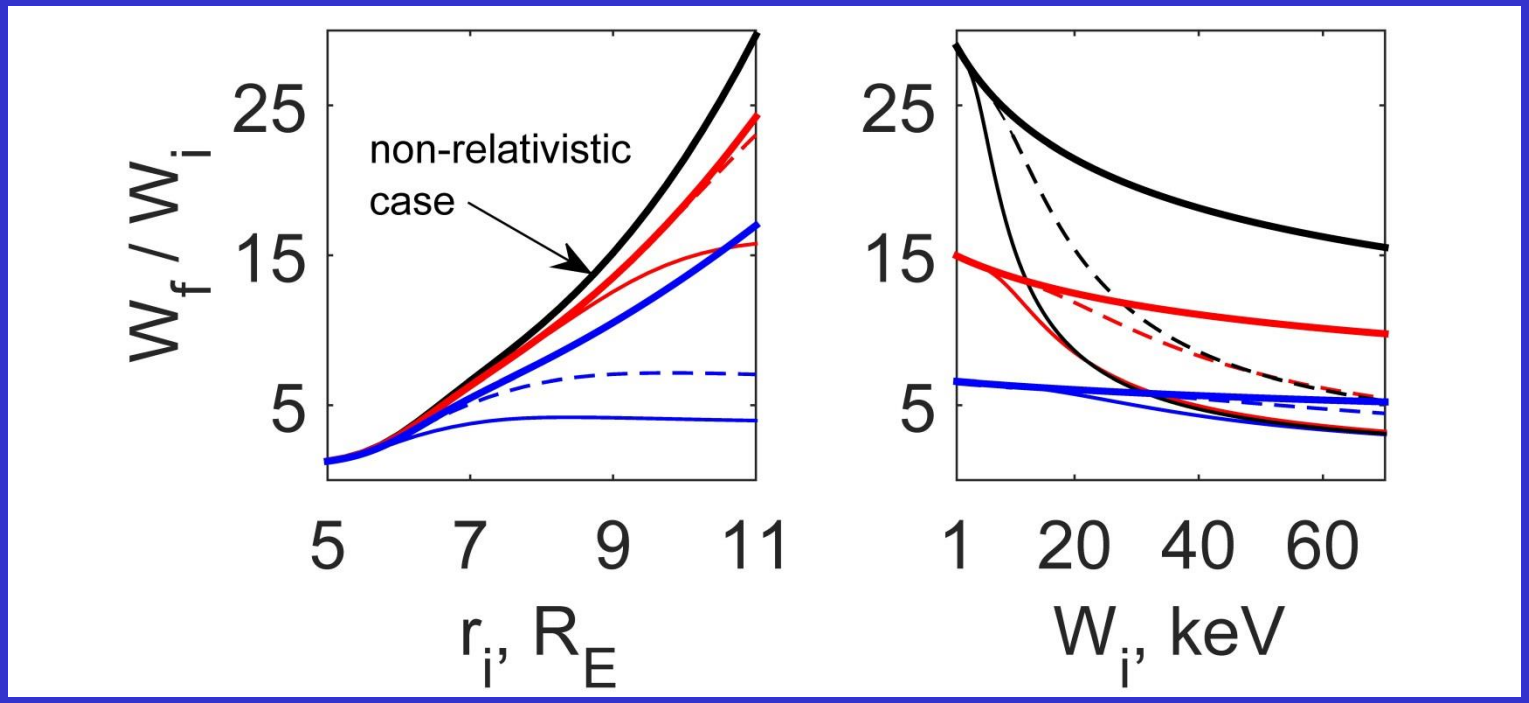
Final position of electrons



Magnetic field increase



Energy gain by electrons



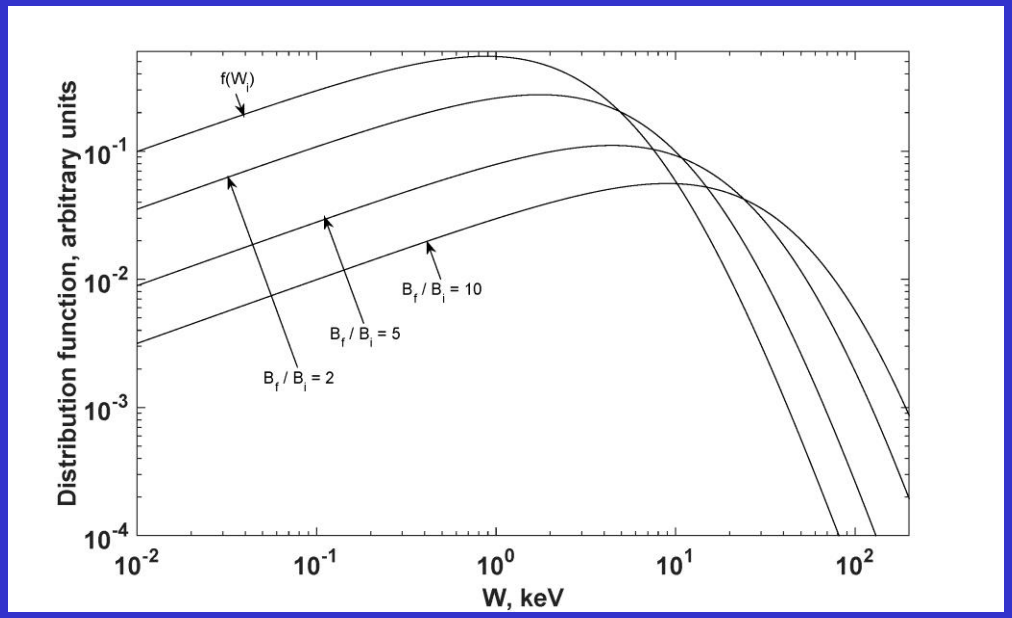
Energizing an electron population

Initial – kappa distribution with k=3.5, and W_0=2keV

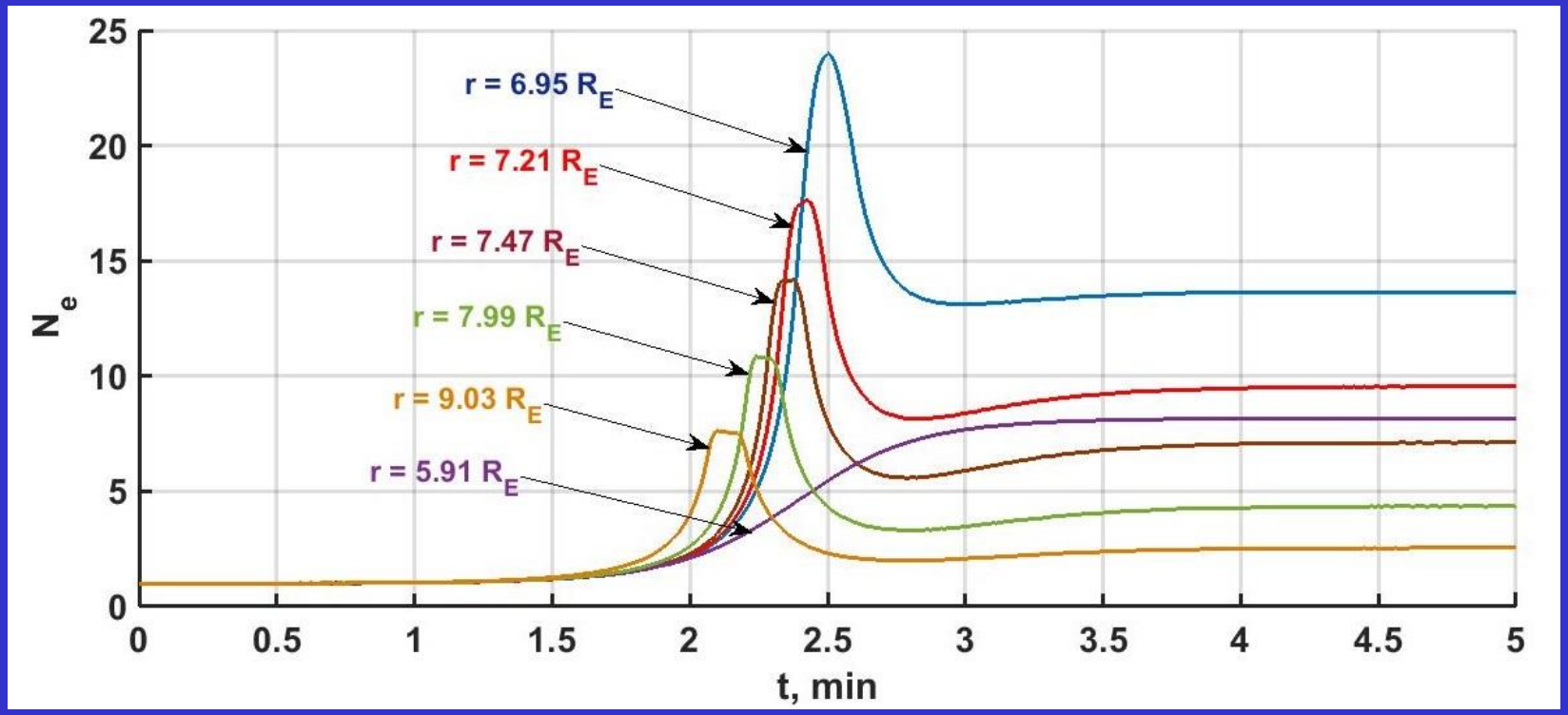
$$f(W_i) \sim \sqrt{W_i}\left(1+\frac{W_i}{\kappa W_0}\right)^{-\kappa-1}$$

Final energy:

$$\frac{W_f}{mc^2} = \sqrt{1+\frac{B_f}{B_i}\left(\left(\frac{W_i}{mc^2}+1\right)^2-1\right)}-1$$



Electron density enhancements

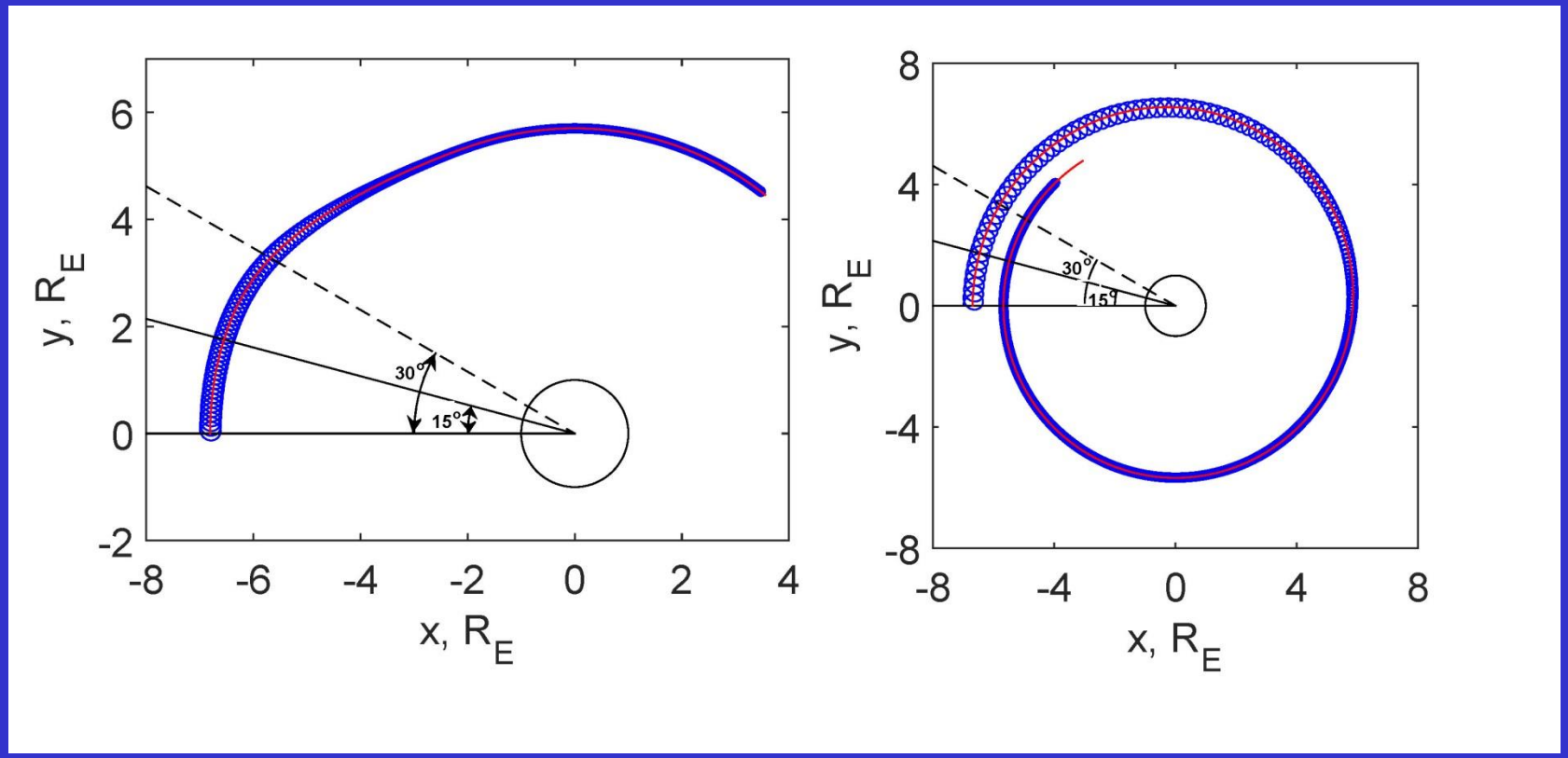


Assuming uniform initial distribution of 5 keV electrons

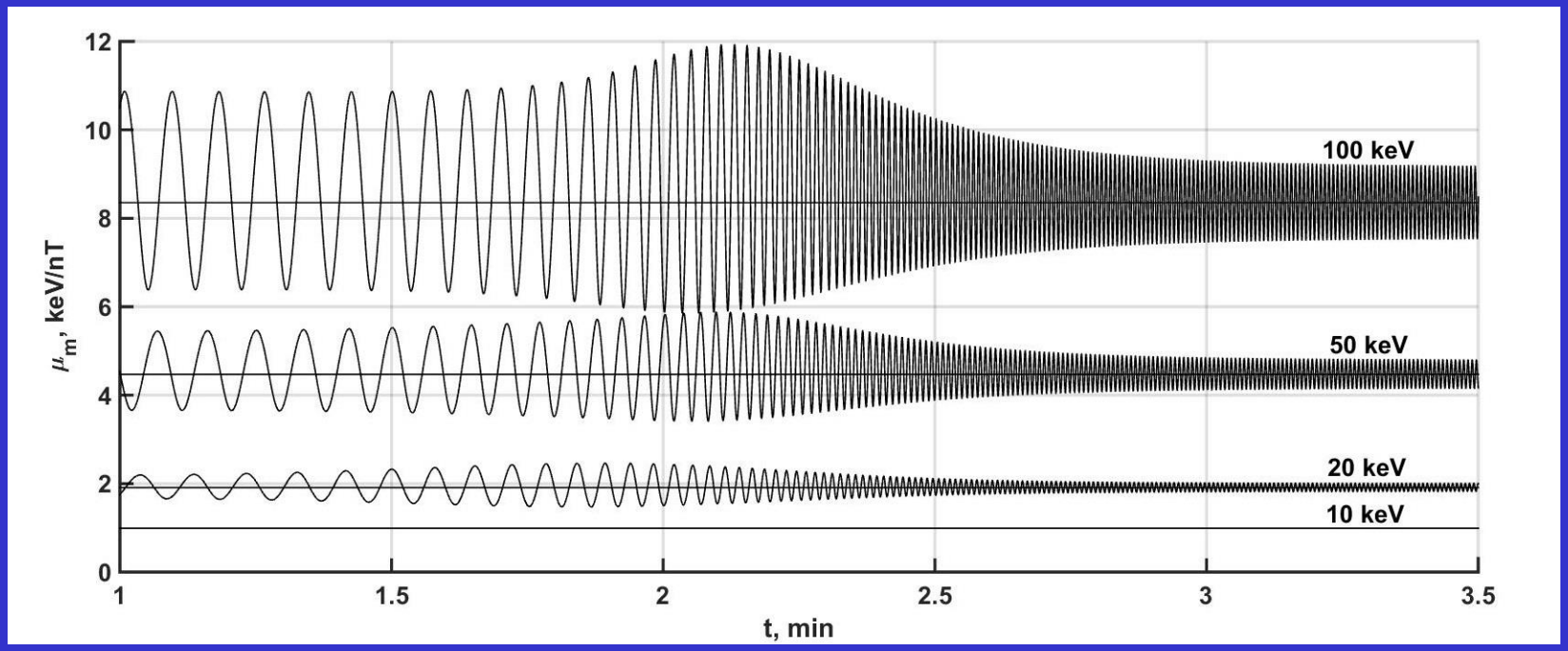
Proton trajectories

58 keV

200 keV

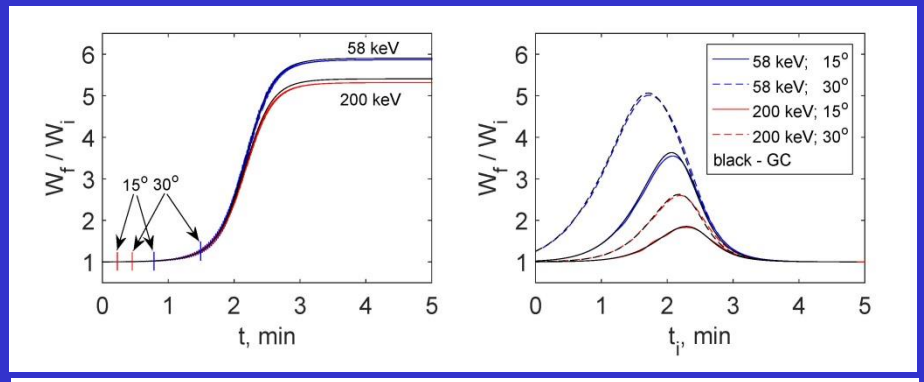


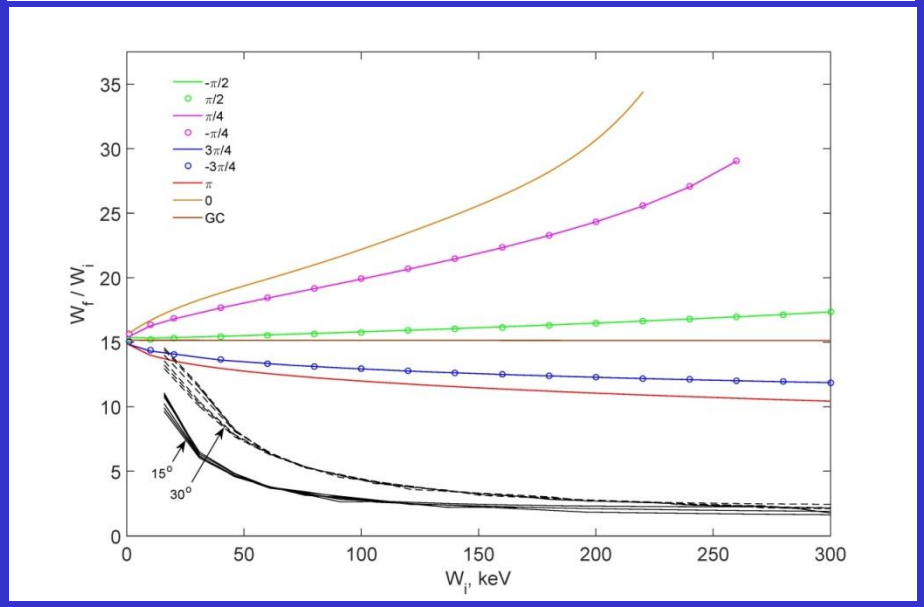
Magnetic moments of Protons



GC approximation is not very accurate even for 50 keV !

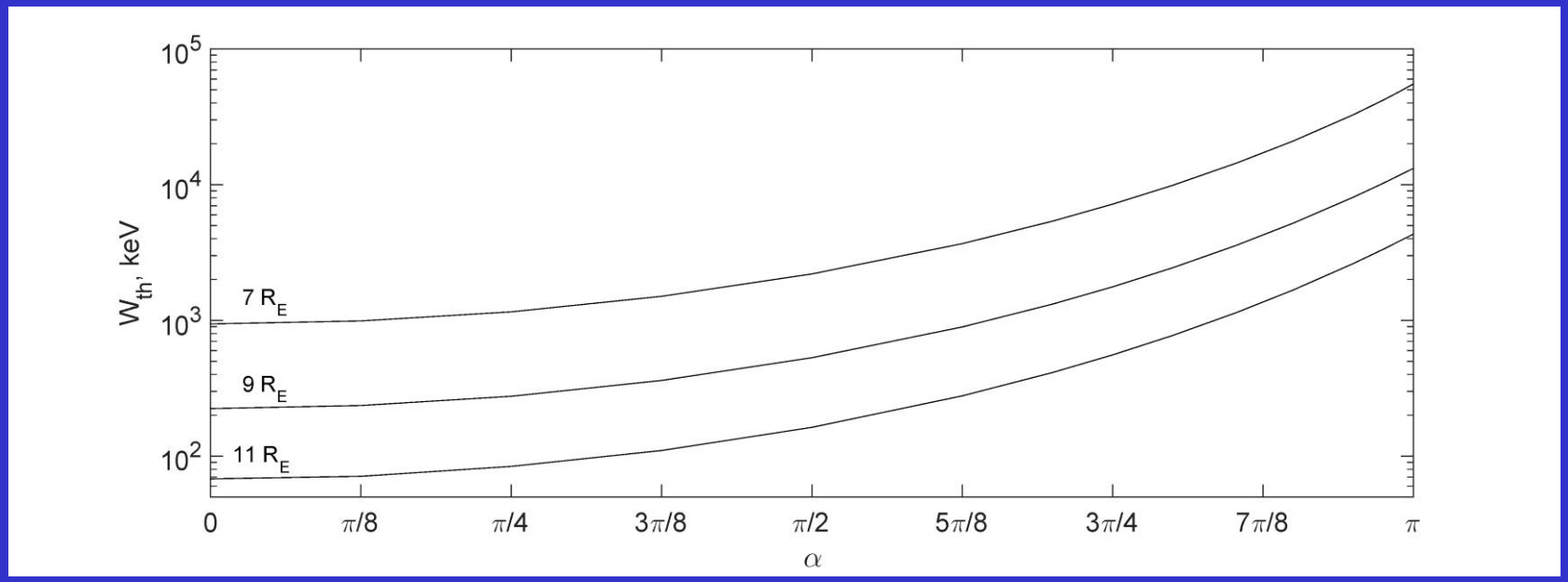
Proton energy gains





Protons initially at 7 R_E

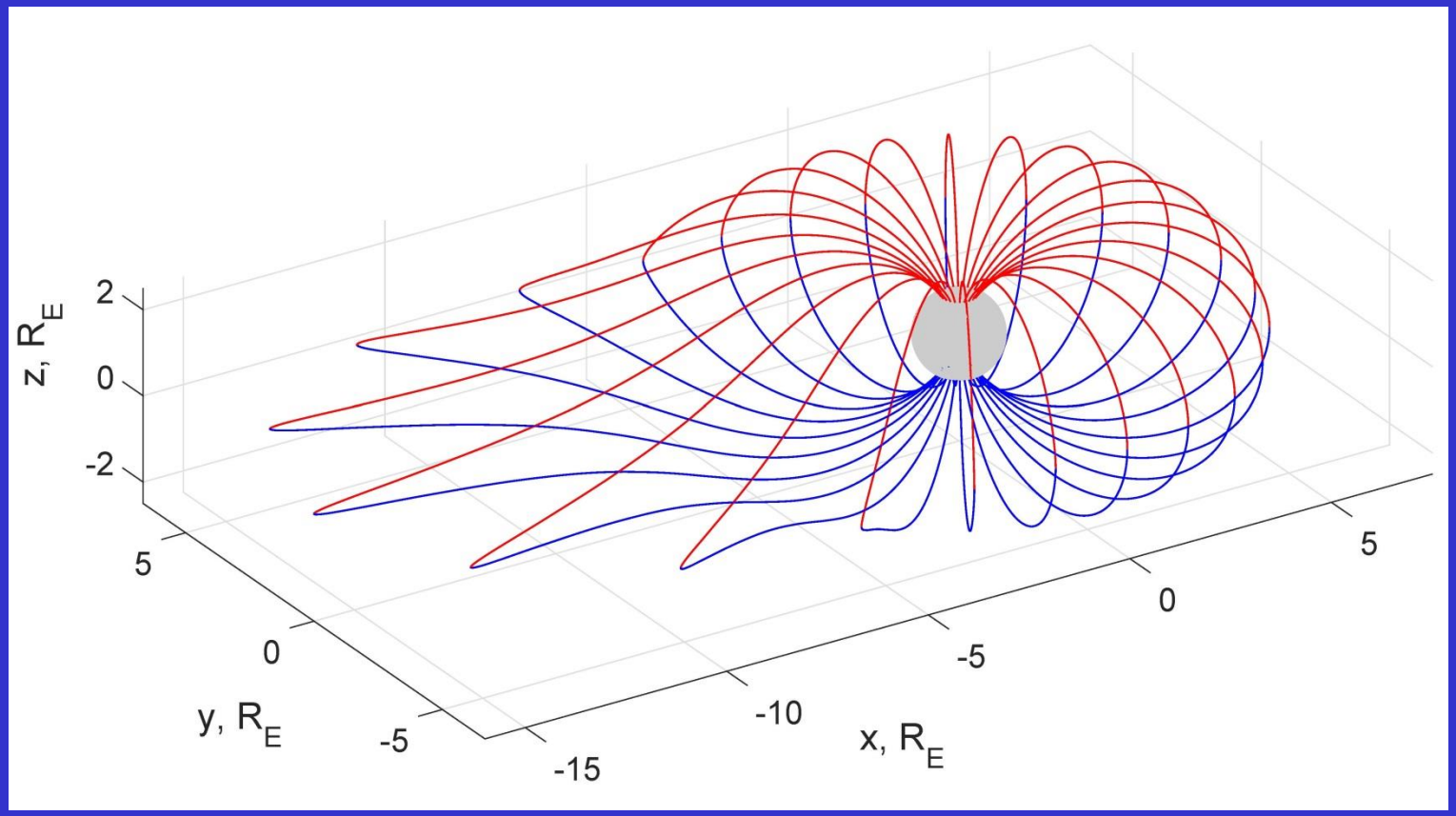
Maximum energy of trapped protons



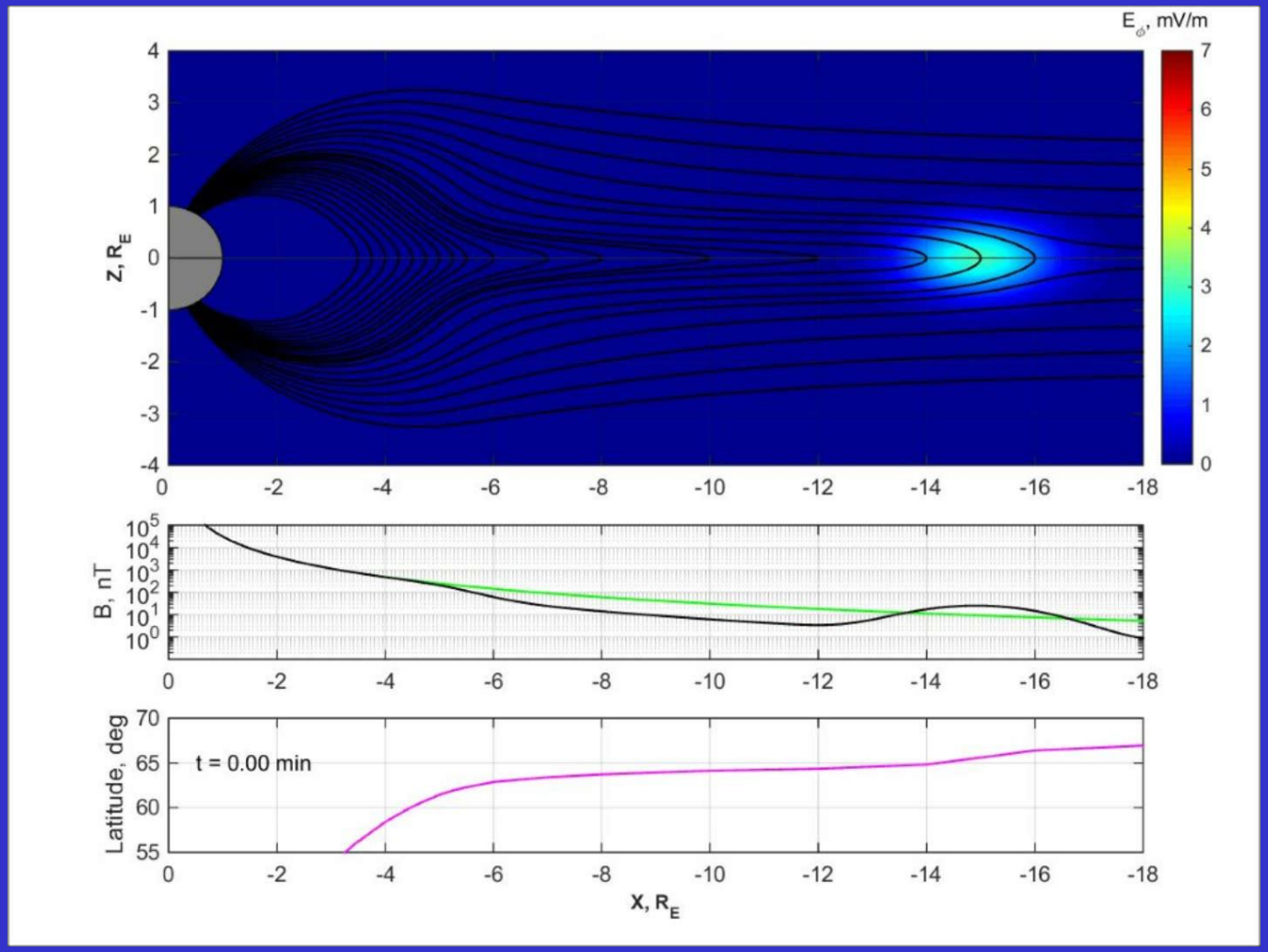
Direction of the initial velocity measured counter-clockwise from east

Future work: make the model 3D

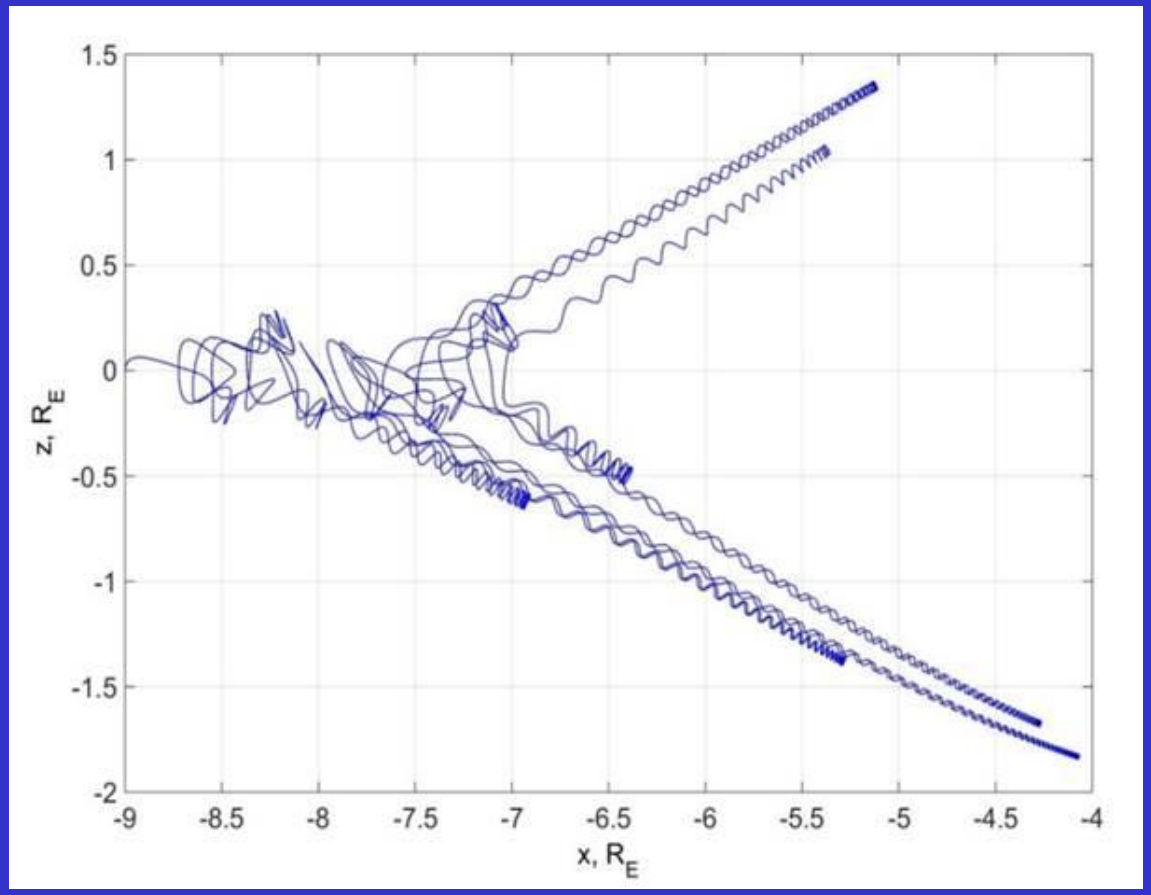
$$\mathbf{B}=\mathbf{B}_{dip}+f(\varphi)\mathbf{B}_{tail}$$



Future work: study energization by an earthward pulse



Future work: study non-equatorial particles



Conclusions

- We have developed and implemented a new parameterized model for a transition region between highly stretched and dipolar magnetic field topologies.
- We use that model to track the effects of a tailwards propagating dipolarization on electron and proton energies.
- For some electrons the energization factor is ~25.
- This provides an alternative to earthward propagating pulse models for substorm injections.
- More details in Kabin et al., JGR 2011, 2017