Parasitic Momentum Flux in the Tokamak Core

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Careful geometric analysis shows that energy transfer from the electrostatic potential to ion parallel flows breaks symmetry in the fully nonlinear toroidal momentum transport equation, causing countercurrent rotation peaking without applied torque.

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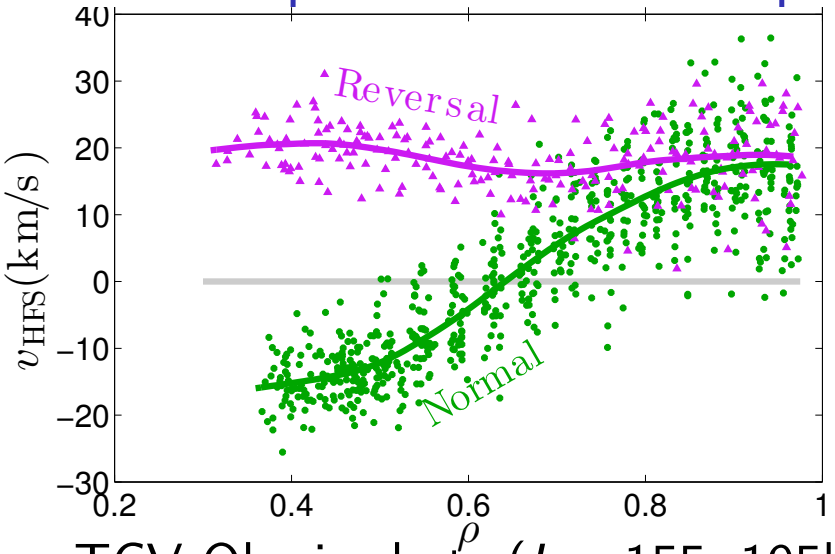
Outline

- Background
  - Experiment:
    - Intrinsic rotation and rotation reversals
  - Theory:
    - Intrinsic rotation: Vanishing momentum flux
- Rotation model
  - Intuitive cartoon of axisymmetric example
  - Symmetry constraints on nonaxisymmetric equations
  - Free-energy flow in phase space⇒momentum flux
  - Predicted core rotation peaking:
    scaling, behavior, and rough magnitude

Tokamak plasmas rotate spontaneously without applied torque.



TCV Ohmic shots ($I_p \approx$155, 195kA)
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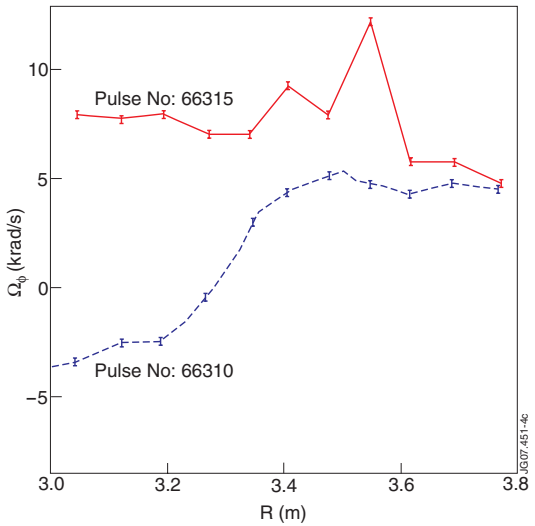

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Important for stability against resistive wall modes at low torque (ITER).

Typical intrinsic rotation profiles have three regions:

- Edge: Co-rotating (due to ion orbit shifts)
- Mid-radius "gradient region": Countercurrent peaking or ∼flat
  - Gradient exhibits sudden 'reversals' at critical parameter values.
  - Rotation profiles often pass through zero.
- Sawtoothing region inside $q = 1$: Flat or weak cocurrent peaking

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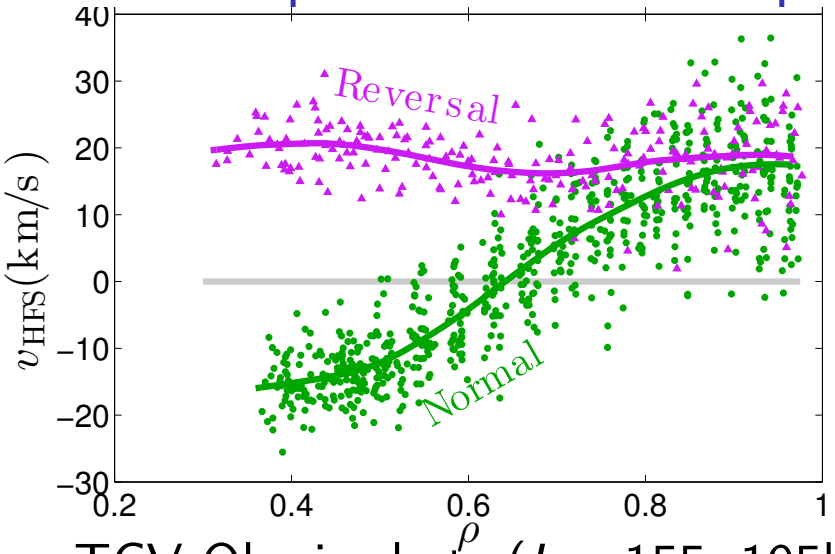

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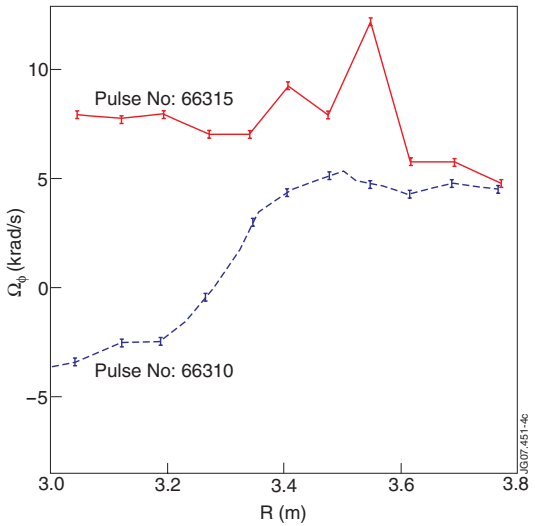

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Intrinsic rotation profiles result from vanishing momentum flux.

Axisymmetric steady state with no torque⇒zero momentum outflux:

$$0 = \Pi = -\nu\nabla L + v_{\text{pinch}} L + \Pi^{\text{res}} \Longrightarrow \nabla L = (v_{\text{pinch}} L + \Pi^{\text{res}})/\nu$$

Toroidal momentum gradient ∇L is set by balancing

- Viscous flux ($-\nu\nabla L$) (saturation) against both
- Momentum pinch ($v_{\text{pinch}} L$) due to
  - 'Turbulent equipartition' due to ∇B (Hahm et al PoP '07)
  - Coriolis force (Peeters et al PoP '09)
- Residual stress (Π^{res}, independent of L)
  - Only explanation for peaked profiles that cross $L = 0$.

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What drives symmetry-breaking momentum flux Π^{res}, in the absence of rotation and of rotation shear?

Symmetry-breaking mechanisms to drive residual stress include:

- Background $\boldsymbol{E} \times \boldsymbol{B}$ shear (Dominguez and Staebler Phys. Fluids B '93)
- Up-down asymmetric magnetic geometry (Camenen et al PRL '09)
- Quasilinear: assume phase between $\tilde{v}_r$ and $\tilde{v}_{\|}$ from a linear eigenmode
  - Drift waves (Coppi NF '02)
  - With intensity gradient (Gürcan PoP '10)
- Radially global effects via gyrokinetic simulation
  - GTS: magnetic & $\boldsymbol{E} \times \boldsymbol{B}$ shear, intensity gradients, neoclassical effects (Wang et al PRL '09, '11)
  - XGC1: avalanche momentum & heat transport (Ku et al NF '12)
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Free-energy flow in phase space + higher-order part of $\boldsymbol{E} \times \boldsymbol{B}$ drift $\Rightarrow$ residual stress

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.

$$\frac{\tilde{n}}{n} = \frac{e\tilde{\phi}}{T_e} > 0$$

$$\frac{\tilde{n}}{n} = \frac{e\tilde{\phi}}{T_e} < 0$$

1. Example: axisymmetric ($n = 0$), low-frequency density fluctuations.
   $E_\parallel = -b_p(\partial_\theta \phi)/r$ accelerates ions out of density hump.
   $E_\parallel u_{\parallel i} = -b_p u_{\parallel i}(\partial_\theta \phi)/r$ transfers energy to ion parallel flows.

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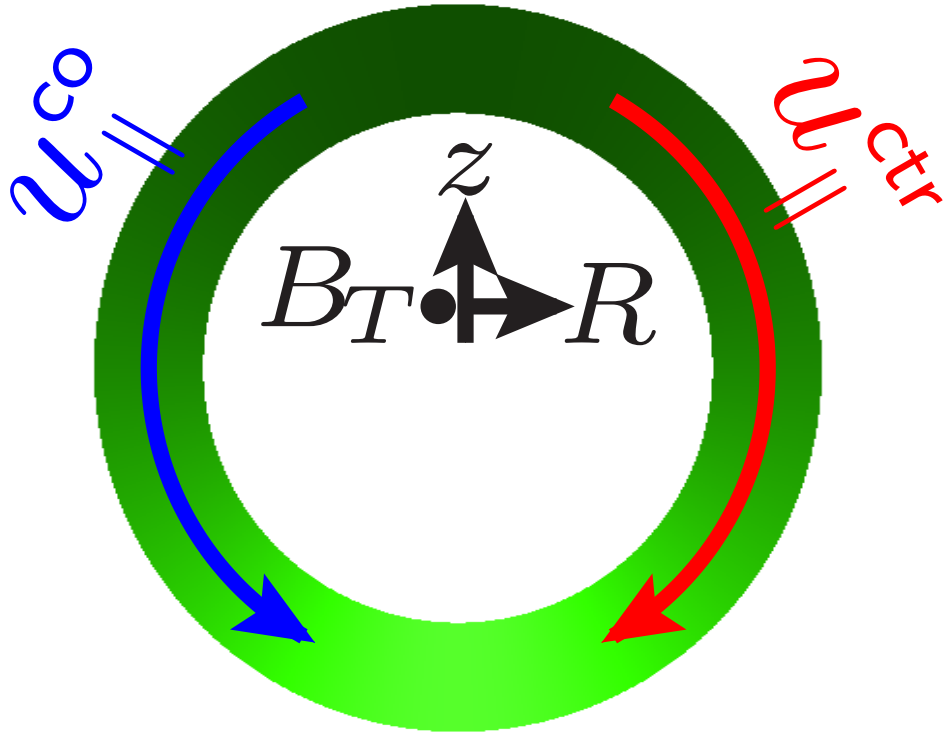

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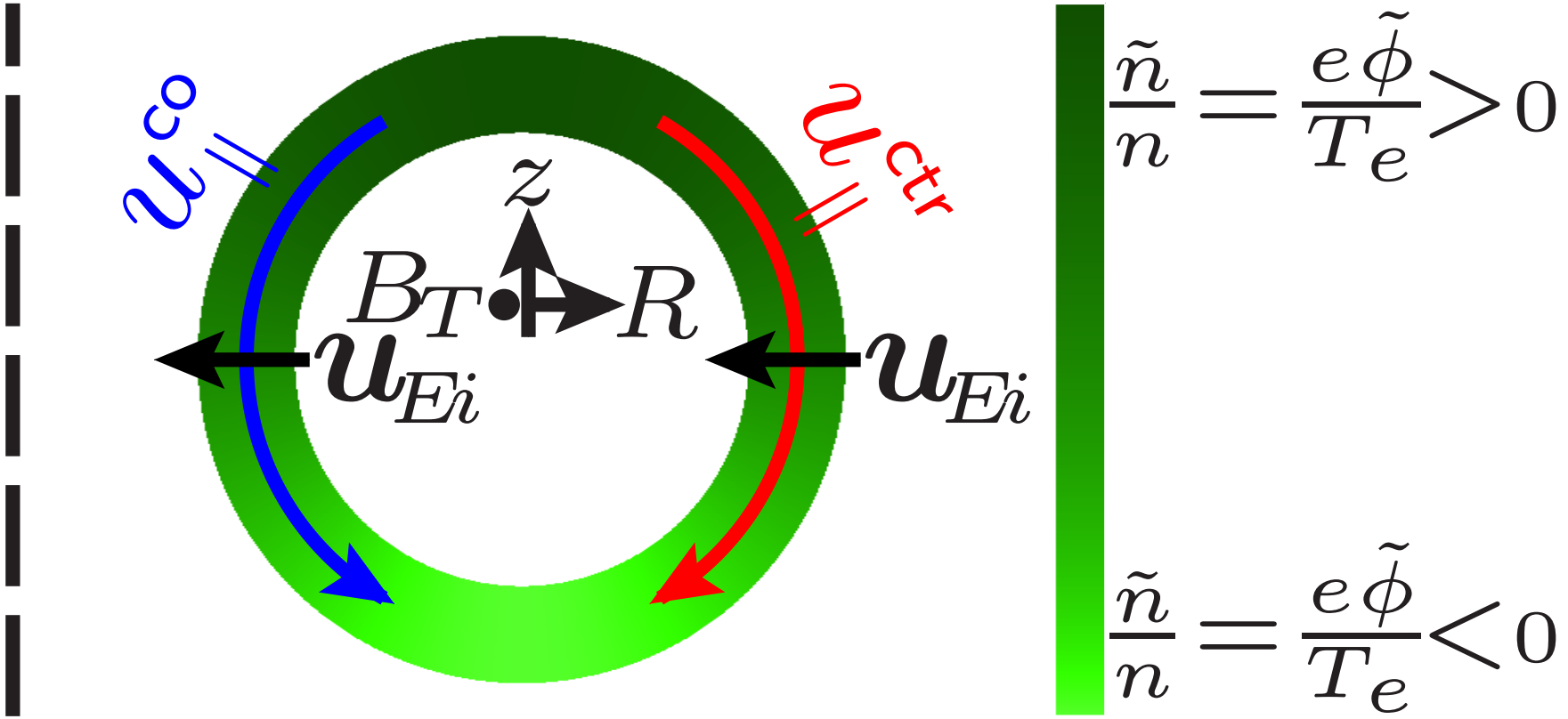

II. Weak radial $\boldsymbol{E} \times \boldsymbol{B}$ drift $u_{Ei}^r = -(cb_T/Br)\partial_\theta \phi$ advects ions.
Outflow of cocurrent momentum: $\Pi = [-cb_T(\partial_\theta \phi)/Bb_p] m_i n_{i0} R b_T u_{\parallel i}$
Momentum flux$\propto$energy transfer because $E_\parallel / b_p = -\partial_\theta \phi = E_\perp / b_T$.

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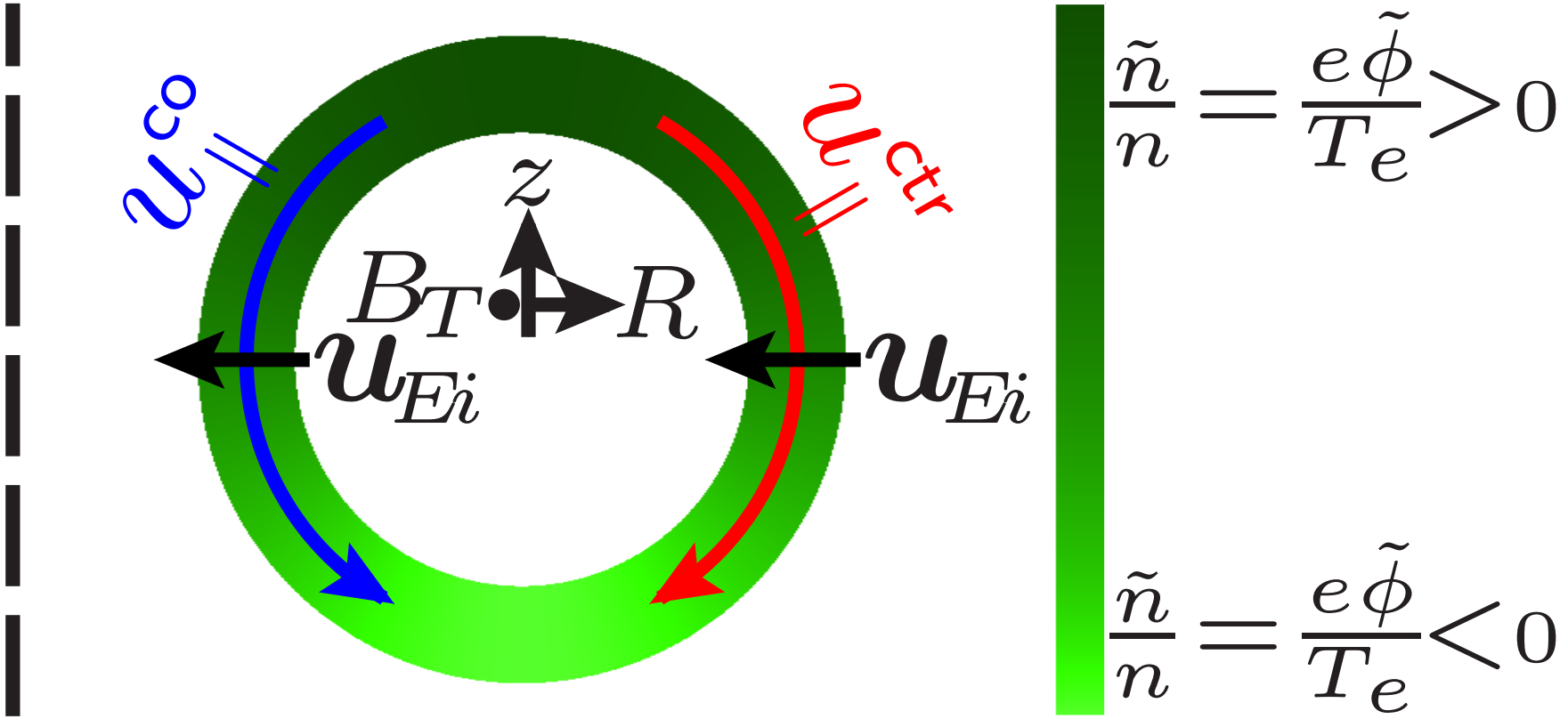


III. Slow poloidal potential variation in $\partial_\theta \phi \sim k_\parallel \phi / b_p \sim \phi / r$:

- neglected by fluxtube orderings, but
- breaks symmetry because $\hat{b}$ neither parallel nor perp to $\hat{\zeta}$.

Symmetry restricts contributions to residual stress.

In the simplest radially local fluxtube limit with

- up-down symmetric magnetic geometry,
- no background rotation or rotation shear, and
- no background $\boldsymbol{E} \times \boldsymbol{B}$ shear,

the delta-f gyrokinetic equations satisfy a symmetry:

If $f(\rho, \vartheta, \xi, v_\parallel, \mu, t),\ \phi(\rho, \vartheta, \xi, t)$ is a solution
so is $-f(-\rho, -\vartheta, \xi, -v_\parallel, \mu, t),\ -\phi(-\rho, -\vartheta, \xi, t),$

with opposite sign of the dominant toroidal momentum flux.
(Peeters and Angioni PoP '05, Peeters et al NF '11)

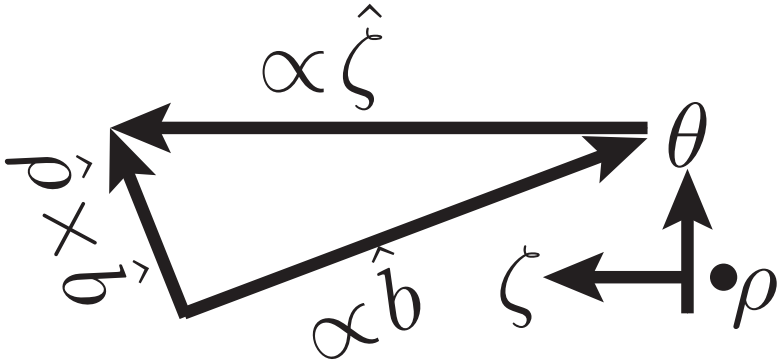
This implies: toroidal momentum flux should vanish for terms that flip sign (but does *not* imply that invariant terms *must* drive momentum flux).

The radial $\boldsymbol{E}\times\boldsymbol{B}$ drift with true $\nabla_\perp\phi$ breaks the symmetry.

Define convenient directions

$$\hat{\rho} \doteq \frac{\nabla\rho}{|\nabla\rho|},\quad \hat{\rho} \doteq \hat{\zeta}\times\hat{\rho}$$

and decompose $\hat{b} = b_T\hat{\zeta} + b_p\hat{\rho}$.



Use $\hat{\rho}\times\hat{b} = (\hat{\zeta} - b_T\hat{b})/b_p$ to evaluate

$$\boldsymbol{u}_{Ei}\cdot\hat{\rho} = \frac{c}{B}\hat{b}\times\nabla\phi\cdot\hat{\rho} = \frac{c}{B}\hat{\rho}\times\hat{b}\cdot\nabla\phi = \frac{c}{b_pB}\hat{\zeta}\cdot\nabla\phi \overbrace{-\frac{cb_T}{b_pB}\hat{b}\cdot\nabla\phi}^{u_{Ei}^{(2)}}.$$

Symmetry prevents first term $\propto \hat{\zeta}\cdot\nabla\phi \propto \partial_\xi\phi$ from driving residual stress.

Second term cancels the parallel gradient included in $\hat{\zeta}\cdot\nabla\phi \neq \rho\times\hat{b}\cdot\nabla\phi$:

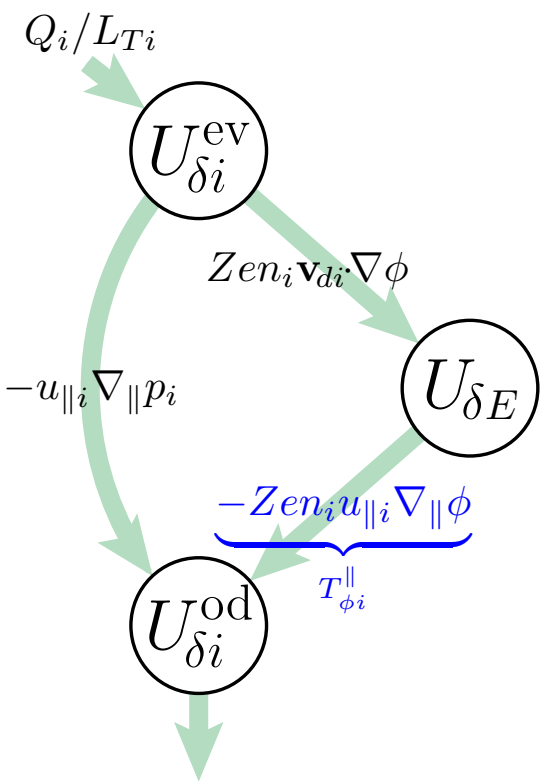
- Nominally smaller than the first term, by $k_\parallel/k_\perp b_p$, but
- Contributes a symmetry-breaking term to momentum flux $m_i n_{i0} b_T R_0 u_{Ei}^x u_{\parallel i}$:

$$\Pi_\zeta^{(2)} = n_{i0}m_iR_0b_Tu_{\parallel i}u_{Ei}^{(2)} = -(cm_in_{i0}R_0/b_pB_0)u_{\parallel i}\nabla_\parallel\phi$$

*T. Sung et al, Phys. Plasmas **20**, 042506 (2013).

If ion parallel flows are excited, co-current momentum flows out.

Turbulence fluctuation amplitude is regulated by free-energy balance:



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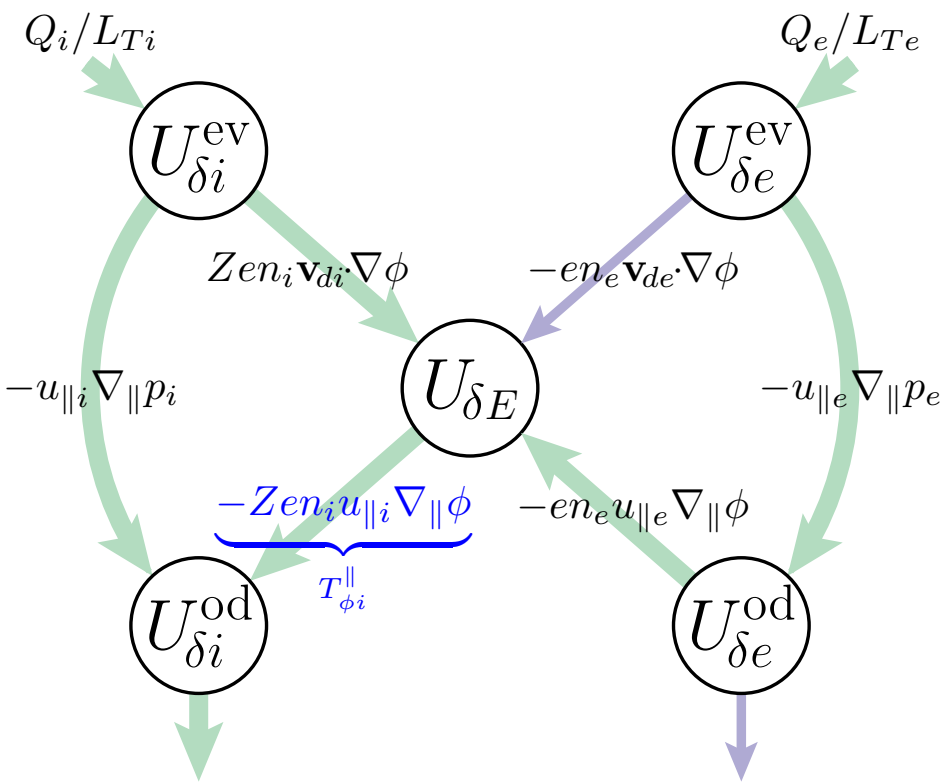

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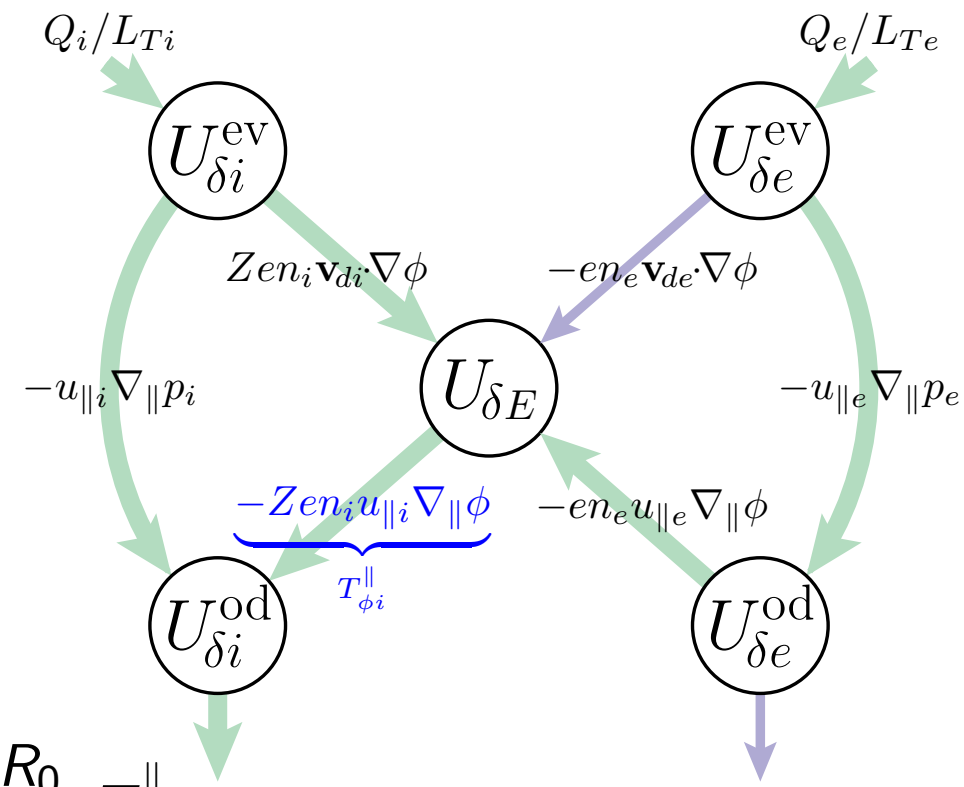
Counter-current peaking due to ion Landau damping $T^{\parallel}_{\phi i} > 0$, if

$$\omega \lesssim v_{ti}/qR.$$



$$\Pi^{(2)}_{\zeta} = -\frac{cm_i n_{i0} R_0}{b_p B_0} u_{\parallel i}\nabla_{\parallel}\phi = \frac{R_0}{\Omega_{ci\theta}} T^{\parallel}_{\phi i}$$

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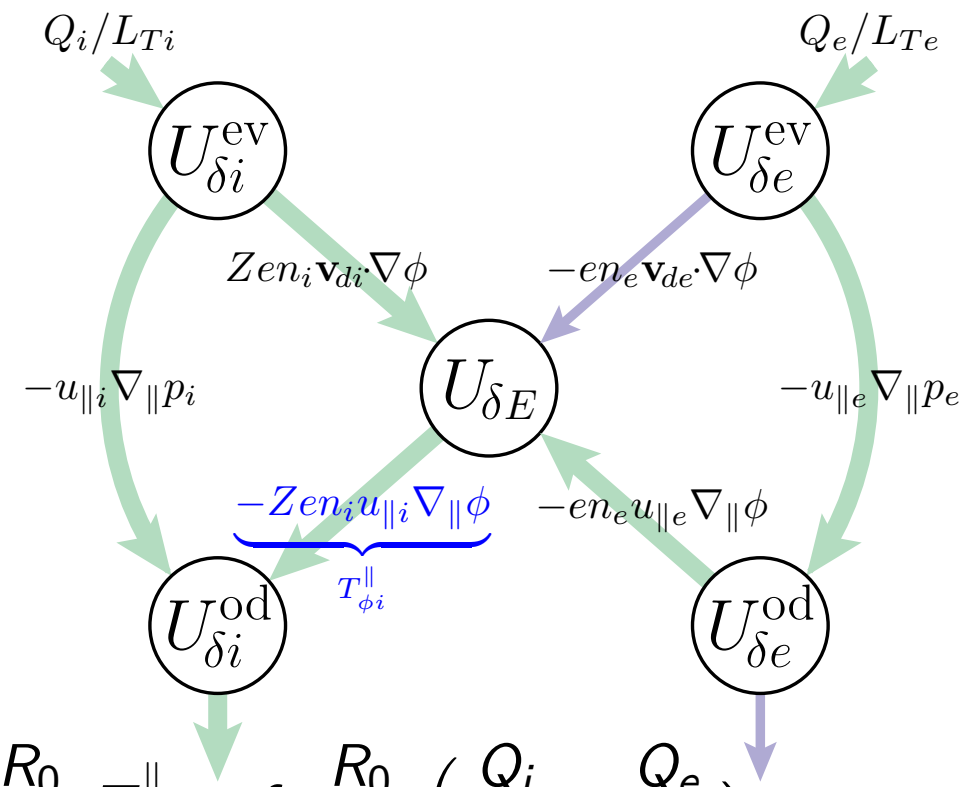

Counter-current peaking due to ion Landau damping $T_{\phi i}^{\parallel} > 0$, if

$$\omega \lesssim v_{ti}/qR.$$

Let a fraction $0 \leq f_L \leq 1$ of turbulent free energy pass through $T_{\phi i}^{\parallel}$, then residual stress may be solved for as:

$$\Pi_\zeta^{(2)} = -\frac{cm_i n_{i0} R_0}{b_p B_0} u_{\parallel i}\nabla_\parallel\phi = \frac{R_0}{\Omega_{ci\theta}} T_{\phi i}^{\parallel} = f_L \frac{R_0}{\Omega_{ci\theta}}\left(\frac{Q_i}{L_{Ti}} + \frac{Q_e}{L_{Te}}\right)$$

When ion Landau damping is significant, one obtains counter-current rotation peaking with a simple scaling.

Recall $f_L = T^{\parallel}_{\phi i}/[\sum_s Q_s/L_{Ts}]$ and $\Pi^{(2)}_{\zeta} = (R_0/\Omega_{ci\theta})T^{\parallel}_{\phi i}$

Balance viscosity against this residual stress:

$$\chi_\varphi n_{i0} m_i R_0 \partial_r u_\varphi = \Pi^{(2)}_{\zeta} \sim f_L \frac{R_0}{\Omega_{ci\theta}} \sum_s \frac{Q_s}{L_{Ts}}$$

Define Prandtl number $\mathrm{Pr} \doteq \chi_\varphi/\chi_i$, then solve for peaking:

$$\partial_r u_\varphi \sim \frac{f_L}{\mathrm{Pr}} \frac{v_{ti}}{\Omega_{ci\theta} L_{Ti}} \left[\sum_s \frac{Q_s/L_{Ts}}{Q_i/L_{Ti}}\right] \frac{v_{ti}}{L_{Ti}}.$$

Assume flat current $2\pi r B_p \sim (4\pi/c) I_p (r^2/a^2)$to get dimensional peaking

$$\boxed{a\partial_r u_\varphi \sim -5 \frac{f_L}{\mathrm{Pr}} \frac{a^3}{L_{Ti}^2 r} \frac{T_{i0}(\mathrm{keV})}{Z I_p(\mathrm{MA})} \left[\sum_s \frac{Q_s/L_{Ts}}{Q_i/L_{Ti}}\right] \mathrm{km/s},}$$

comparable with peaking measured on DIII-D, C-mod, TCV, & KSTAR, if $f_L \sim 0.1$.

Summary

A geometrically higher-order portion of the $\boldsymbol{E} \times \boldsymbol{B}$ drift causes a nondiffusive momentum flux that:

- results from symmetry-breaking by excitation of ion parallel flows
  - does not require $\langle u_\varphi \rangle$ or $\nabla \langle u_\varphi \rangle \Rightarrow$residual stress
  - a fully nonlinear mechanism, not quasilinear
- causes counter-current rotation peaking in the core
- may drive experimentally relevant rotation peaking around

$$a\partial_r u_\varphi \sim -5 \frac{f_L}{\mathrm{Pr}} \frac{a^3}{L_{Ti}^2 r} \frac{T_{i0}(\mathrm{keV})}{Z I_p(\mathrm{MA})} \left[\sum_s \frac{Q_s / L_{Ts}}{Q_i / L_{Ti}} \right] \mathrm{km/s}$$

  - Quantitative evaluation of f_L requires simulation, ongoing.
- acts only when turbulence is at low enough frequencies to excite ion parallel flows
  - allows for both hollow and flat rotation profiles