Variational methods for modeling linear and nonlinear waves in fusion plasmas: fundamental physics and applications

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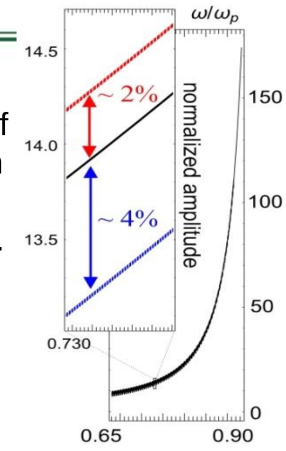
Objectives

- To advance mathematical foundations of plasma wave theory through application of first-principle variational methods;
- To develop techniques for modeling MFE plasma waves more efficiently and robustly using the natural advantages of variational methods ("hard-wired" conservation laws);
- To find new ways for manipulating waves in MFE plasmas, discover new basic physics, and simplify understanding of known complex wave phenomena.

Accomplishments

- The general axiomatic theory that was developed earlier under the NNSA funding is applied to MFE plasma waves, including RF waves and nonlinear energetic particle modes (EPM).
- By building on universal theorems, a basic explanation is found for the momentum balance at plasma rotation driven by resonant RF waves [Guan, Dodin, Qin, Liu, and Fisch, *Physics of Plasmas* (2013)]. The key effect is akin to α channeling.
- The frequency sweeping due to the clump/hole dynamics is explained in terms of commonly known nonlinear dispersion relations for trapped-particle waves. Application of VM further improves matching between theory and simulations.
- The "negative-mass instability" (NMI) theory (developed under non-MFE grant) is used to explain the so-called KEEN waves.

Fig.1: The relation between the nonlinear frequency and the amplitude (at fixed *k*) of a typical EPM clump mode: exact solution (black), lowest-order analytic theory (blue), and extended analytic theory (red). The close-up in the inset demonstrates the curves almost coincide. The figure shows that the clump/hole dynamics well fits the established analytic theory of trapped-particle waves.



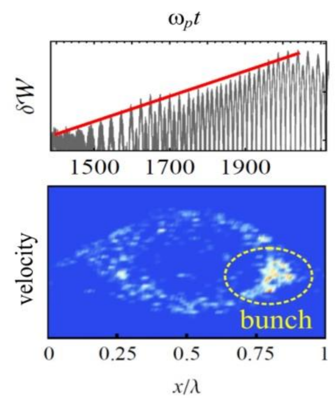


Fig.2: The exponential growth of sidebands in a nonlinear plasma wave and the formation of a trapped-electron phase space bunch, constituting the NMI. The so-called KEEN waves, which were previously reported in literature, are understood as certain saturated states of the NMI.

Impact

- The results provide better understanding of the basic mechanisms behind the EPM dynamics, stability of nonlinear WTP, and wave-driven plasma rotation.
- The advances in the fundamental theory open the path toward improving modeling of RF waves in tokamaks.

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