Parallel Electron Force Balance and the L-H Transition

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In a popular description of the L-H transition, energy transfer to the mean flows directly depletes turbulence fluctuation energy, resulting in suppression of the turbulence and a corresponding transport bifurcation. However, electron parallel force balance couples nonzonal velocity fluctuations with electron pressure fluctuations on rapid timescales, comparable with the electron transit time. For this reason, energy in the nonzonal velocity stays in a fairly fixed ratio to electron thermal free energy, at least for frequency scales much slower than electron transit. In order for direct depletion of the energy in turbulent fluctuations to cause the L-H transition, energy transfer via Reynolds stress must therefore drain enough energy to significantly reduce the sum of the free energy in nonzonal velocities and electron pressure fluctuations. At low $k_\perp$, the electron thermal free energy is much larger than the energy in nonzonal velocities, posing a stark challenge for this model of the L-H transition.

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Overview

- Background: paradigms for the L-H transition
- Parallel electron physics and adiabatic response
- Free energy balance
  - for rapid L-H transition
  - for slow L-H transition
- Conclusions

This work is part of a collaboration with Ahmed Diallo, Stewart Zweben, and Santanu Banerjee, doing experimental investigation of the L-H transition on NSTX with gas puff imaging (GPI).

Background: Paradigms for the L-H transition

What is the L-H transition?

- sudden transition to a state of good energy confinement in the edge
- appears as heating power *increases* past some threshold

Most models of L-H transition focus on $\boldsymbol{E} \times \boldsymbol{B}$ shear and have two parts:

- Something drives sheared zonal $\boldsymbol{E} \times \boldsymbol{B}$ flows
  - Nonlinear energy transfer from turbulence to flows via Reynolds stress.
  - 'Diamagnetic' flow shear due to ∇p_i contribution to E_r.
- Suppression of turbulence by flow shear (two possibilities):
  1. Energy transfer to flows directly depletes turbulent fluctuations.
  2. Shearing of eddies destroys turbulence in other ways, e.g.
     - Reduce effective growth rate
     - Increase damping

Some experimental investigations consider the following energy balance:

$$\boxed{E_\sim \doteq \int \mathrm{d}\mathcal{V}\, \tfrac{1}{2} n_0 m_i \tilde{v}_E^2} \quad \underset{\Longleftrightarrow}{n_0 m_i \int \mathrm{d}\mathcal{V}\, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E \rangle} \quad \boxed{E_z \doteq \int \mathrm{d}\mathcal{V}\, \tfrac{1}{2} n_0 m_i \langle v_E \rangle^2}$$

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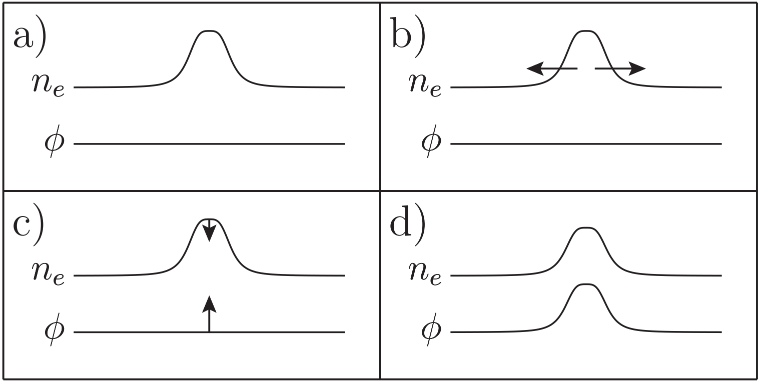
Rapid linear physics acts to cause electron adiabatic response



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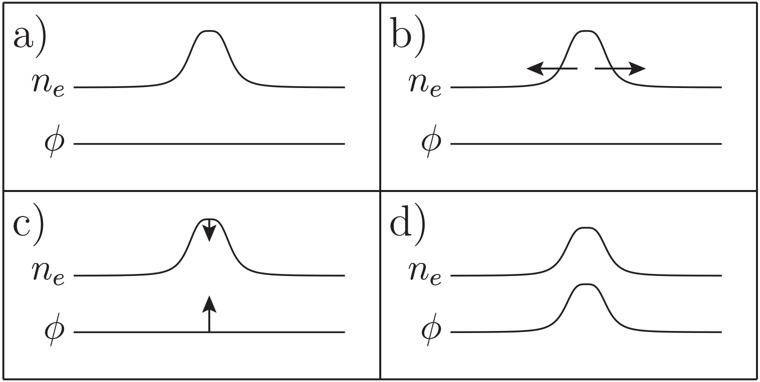

At frequencies lower than $\sim$parallel electron transit, we expect electron parallel force balance:

$$\nabla_{\parallel} n_e T_{e0} = n_0 e \nabla_{\parallel} \phi \Longrightarrow \tilde{n}_e / n_0 \approx e\tilde{\phi} / T_{e0}.$$

Parallel electron physics restricts free energy balance.

$$\gamma E_{\tilde{n}}$$
$$\Downarrow$$

$$\boxed{E_n \doteq \frac{T_{e0}}{2n_0} n_e^2} \Leftrightarrow \boxed{E_\sim = \frac{1}{2} n_0 m_i \tilde{v}_E^2} \Longleftrightarrow \boxed{E_z = \frac{1}{2} n_0 m_i \langle v_E \rangle^2}$$

$$\overbrace{j_\parallel \nabla_\parallel \phi = j_\parallel \nabla_\parallel \tilde{\phi}}^{\Updownarrow} \qquad \overbrace{n_0 m_i \int \mathrm{d}\mathcal{V}\, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E \rangle}^{\Updownarrow}$$

$$\Downarrow$$
Resistive dissipation
(weak for small η)

$$\Downarrow$$
Poloidal rotation
damping

$$\frac{E_\sim}{E_{\tilde{n}}} = \frac{\int \mathrm{d}\mathcal{V}\, \tilde{v}_E^2 / c_s^2}{\int \mathrm{d}\mathcal{V}\, (\tilde{n}_e^2 / n_0^2)} = \frac{\int \mathrm{d}\mathcal{V}\, \left|\nabla_\perp \tilde{\phi}\right|^2 / c_s^2 B^2}{\int \mathrm{d}\mathcal{V}\, (\tilde{n}_e^2 / n_0^2)} \xRightarrow{\frac{\tilde{n}_e}{n_0} \approx \frac{e\tilde{\phi}}{T_{e0}}} k_\perp^2 \rho_s^2$$

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Reynolds stress must drain energy from $(E_{\tilde{n}} + E_\sim) \gg E_\sim$.

For slow transition, energy transfer too weak for Waltz rule.

$$\gamma E_{\tilde{n}} \Downarrow$$

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$$\overbrace{j_\| \nabla_\| \phi = j_\| \nabla_\| \tilde{\phi}}^{\Updownarrow} \qquad \overbrace{n_0 m_i \int \mathrm{d}\mathcal{V} \, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E \rangle}^{\Updownarrow} \qquad \Downarrow$$

$\Downarrow$

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Turbulence suppression by energy transfer only for:

$$\gamma E_{\tilde{n}} < n_0 m_i \int \mathrm{d}\mathcal{V} \, (\tilde{v}_E^x \tilde{v}_E^y) \partial_x \langle v_E \rangle \leq \max |\partial_x \langle v_E \rangle| \, E_\sim, \text{ so for}$$

$$\max |\partial_x \langle v_E \rangle| > \gamma \frac{E_{\tilde{n}}}{E_\sim} \sim \frac{\gamma}{k_\perp^2 \rho_s^2} \gg \gamma$$

Of course, flow shear may play other roles.

$$\gamma E_{\tilde n}$$
$$\Downarrow$$
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$$\Downarrow$$

Resistive dissipation (weak for small η)

Poloidal rotation damping

Assume now that energy transfer via Reynolds work is negligible, flow shear could still distort the spatial variation of the turbulence and thereby:

- Decrease the effective growth rate γ by decorrelating growing modes,
- Increase dissipation due to transfer to high-$k_\perp$ by eddy shearing

Or maybe the transition is even triggered by something else entirely?

Conclusions

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