Towards 2D Resonance Broadened Quasi-linear (QL) model (RBQ) for fast ion relaxation

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PRINCETON PLASMA PHYSICS LABORATORY

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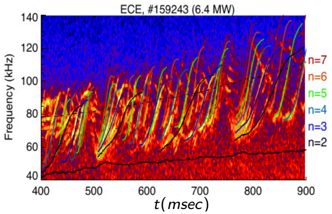
Advance RBQ1D to 2D using well diagnosed DIII-D discharge #159243



- steady state regimes for AE
- good example studied by many codes
- understood with the help of kick model in the interpretive regimes

1. RBQ methodology needs to be extended to 2D by coupling with energy transport:
   - model realistic transport by evolving EP energy, i.e. energy of fast ions need to be accounted for to simulate accurately the fusion cross section for neutron source.
   - RBQ targets numerically efficient resonance broadened quasi-linear model.

- **ISEP** with its gyro-kinetic tools can help to resolve the outstanding problem: what is the role of zonal flow in AE induced transport?
  GEM shows: Zonal Flow is not important near threshold, Y.Chen et al., PoP'18.

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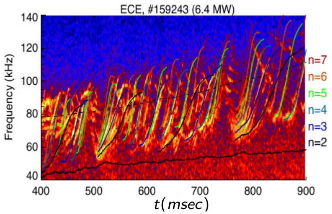

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We are in 2^{nd} year of ISEP SciDAC

RBQ plans

~~Y1: Finalize RBQ1D model with NOVA-K interface for Alfvenic eigenmodes. Provide full DF and PDFs for subsequent use by WDM project with the phase space resolution.~~
Y2: Formulate RBQ2D approach for full resolution in velocity. Use NOVA-K interface for single modes. Implement slanted phase space diffusion. Apply RBQ1D for VV with velocity space resolution.
We are learning the alternating direction implicit method (ADI) for 2D diffusion.
Y3: Continue to work on RBQ2D code. Consult on and explore RBQ with baseline structures given by couplets of m/m+1 harmonics for 1D and 2D RBQ versions.
Y4: Optimize RBQ1D/2D for applications in BP conditions (larger major radius) with high to medium n number modes and velocity space resolved structures. Y5: Continue optimization and application of RBQ in BP conditions. Explore automatic (using baseline couplet structures) RBQ computations with full predictive capabilities to treat AE relaxation.

Resonance Broadened QL diffusion through pitch angle scattering in 2D

Action-angle formalism through flux variables results in a set of equations for fast ion DF: *(Kaufman,PhFl'72, Berk, Breizman, NF'95)* and adapted for RBQ *(Duarte, PhD'17, Gorelenkov, NF'18)*

$$\frac{\partial}{\partial t} f = \pi \sum_{\mathbf{l},k} \frac{\partial}{\partial I} C_k^2 \mathscr{E}^2 \frac{G^*_{m'p} G_{mp}}{|\partial \Omega_{\mathbf{l}} / \partial I|_{res}} \mathscr{F}_{\mathbf{l}} \frac{\partial}{\partial I} f + \nu^3_{eff} \left| \frac{\partial \Omega_{\mathbf{l}}}{\partial I} \right|^{-2} \frac{\partial^2}{\partial I^2} (f - f_0) + S,$$

where EP distribution is evolved due to scattering terms on RHS amended by the scattering "source" operator, $I = P_\varphi / n - \mathscr{E}/\omega$, and S is the source.

AE amplitude satisfies

$$C_k(t) \sim e^{(\gamma_L + \gamma_d)t} \Rightarrow \frac{dC_k^2}{dt} = 2(\gamma_L + \gamma_d) C_k^2.$$

RBQ does not evolve *AE mode structure, only amplitudes.

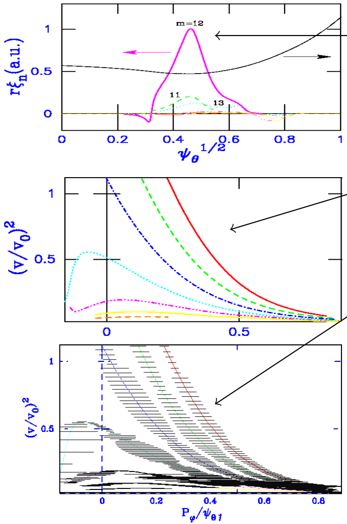
Critical for RBQ platform *for multiple mode cases (Dupree'66, Berk'95, White'18) is* resonant frequency and its broadening by nonlinear bounce ω_{bNL} and effective scattering ν_{eff}:

$$\delta\left(\Omega = \omega + n\dot{\varphi} - m\dot{\theta} - l\omega_b\right) \to window\ \ function\ \mathscr{F}(I).$$

RBQ2D promises to be:

- Time efficient.
- Conforming grid is implemented for resonance island dynamics: 6 times advantage in RBQ1D already.

RBQ workflow remains as in 1D for $n=4$ RSAE (at q_{min})



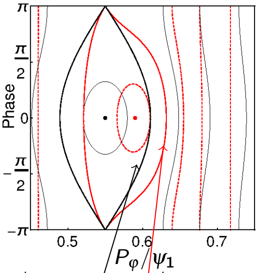
- Ideal MHD NOVA finds Reversed Shear Alfven Eigenmode structure $f=84kHz$ *(Collins, PRL'16)*.
  - This mode provides a channel for ion diffusion and an inverted fast ion pressure profiles: resonant particles are close to the injected pitch angle near the axis.
- NOVA-K code computes resonancies for particle interactions with the mode and $\langle \boldsymbol{v}\cdot\boldsymbol{E}\rangle$ matrices.
- RBQ broadens those resonancies along $P_\varphi/n-\mathscr{E}/\omega$ direction

  using

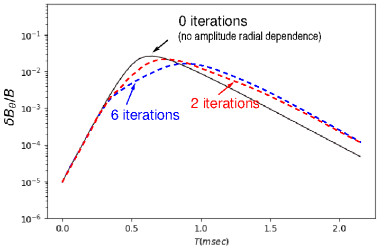
  QL presciptions for each mode. Shown is the broadening at measured amplitude $\delta B_\theta/B=7\times10^{-3}$.
- Monte-Carlo TRANSP package post-processes RBQ diffusion to compute the fast ion distribution function evolution.
  - The Probability Density Function for ion diffusion in the velocity space for further processing within TRANSP is evaluated.
- RBQ2D is to find the 2D diffusion in the constant of motion space.
  - Employ kick model probability density function to describe QL diffusion.

Resonant ion island dynamics is accounted for using Hamiltonian technique

EP islands for "Gaussian" mode



RBQ needs ~2 iterations to converge well. Lowers saturation ampl.



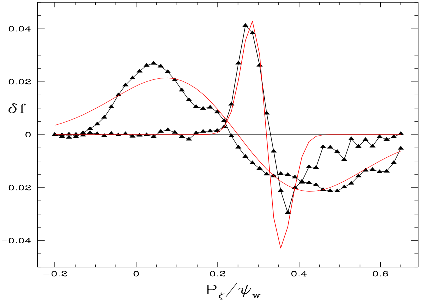
0^{th} iteration (Berk-Breizman approach)

6^{th} iteration ("new" island) accounting for RSAE radial structure

- Low amplitude $\Delta P_\varphi \sim \Delta\Omega = 4\omega_b$ at $\delta B_\theta/B \lesssim (1\div 5)\times 10^{-4}$ *(via ORBIT modeling, G.Meng, NF'18).* Supports resonant frequency approach for nonlinear wave particle interaction.
- Radial amplitude structure limits NL resonance frequency *(R.White et al., PoP'18).*
- Within RBQ easily generalizable to 2D case. By specifying the slantness the broadening is in $I = \mathcal{E}/\omega - P_\varphi/n$,direction.
- R. White has submitted a paper with the generalization of the res. broadening due to scattering. The pitch angle scattering induced res. broadening? Window function?

ORBIT confirms single resonance broadening with and without scattering

R. White, V. Duarte, N. Gorelenkov, G. Meng, submitted to Phys. Plasmas



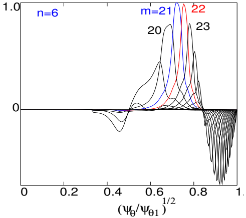
- New prescription $\Delta\Omega = 2.925\omega_b + 3.2\nu_{eff}$ at low AE amplitudes $\xi/R_0 \sim 10^{-4}$.
- This is to be compared with the model theory collisional: $\Delta\Omega = 2.7\omega_b + 3.05\nu_{eff}$ (K.Ghantous PoP'14),
  and collisionless $\Delta\Omega = 3.1\omega_b$ numbers (Fried et al., UCLA Preprint'72).

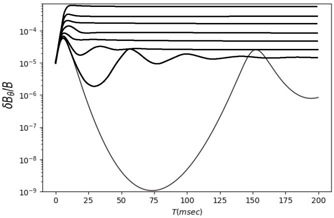
Status and plans for Quasi-Linear model development

1. RBQ model is formulated for *realistic numerically efficient simulations* based on earlier Quasi-Linear theories *(Berk, NF'95,PoP'96, Dupree, PhFl'66, Kaufman, PhFl'72).*
2. Broadening technique is included for non-slanted 1D mesh in P_φ.
3. RBQ1D employs full diffusion solver to resolve island dynamics near the resonance.
4. RBQ1D is applied to DIII-D cases for V&V *(Gorelenkov, et al., IAEA TCM'17, NF'18)*. Low rotation is critical for present RBQ applicability by ignoring energy change.
   Can we apply RBQ to chirping frequency scenarios? Zonal flow, convective transport?
5. 2D extension will be developed within ISEP SciDAC in FY19,20.
   - 1D problem requires direct information about the diffusion coefficients in whole device modeling simulations.
   - Conservative estimates for RBQ approach imply $(0.1 \div 1)\,PFlop$ computer power needed for burning plasma devices to allow a reasonable time to run.

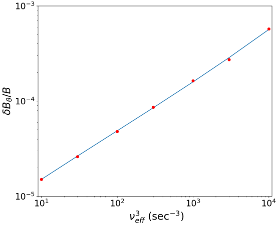
RBQ verification via Coulomb collisions

Global $n = 6$ TAE saturates over $\sim$ *msec*





RBQ shows $\delta B_\theta/B \propto \nu_{eff}^{\mathbf{1.65}}$



- TAE amplitude scales with fast ion Coulomb scattering frequency, $\delta B_\theta/B \sim \nu_{eff}^{\mathbf{2}} \sim \nu_\perp^{\mathbf{2/3}}$, where $\nu_{eff}^{\mathbf{3}} = \nu_\perp \left|\frac{\partial\Omega}{\partial\chi}\right|^{\mathbf{2}}$ *(Berk et al.,Phys. Fluids B'90).*
- Dirichlet boundary conditions, $f_h(\bar{\psi}_\theta \to 0) = const$ and $f_h(\bar{\psi}_\theta \to 1) = 0$, are required to account for Coulomb scattering.

- Intermittency (fluctuations in losses) is expected in predictive RBQ simulations!!

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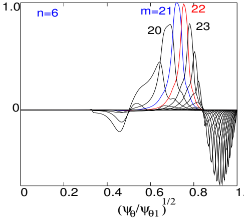

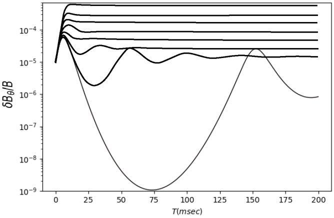

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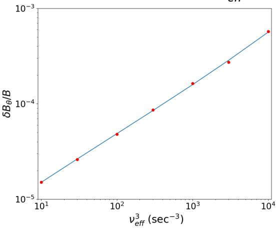

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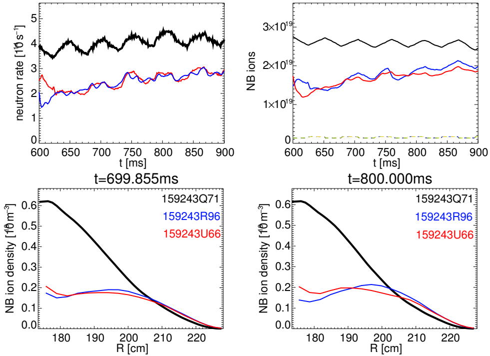
Outline

11 observed modes are employed similar to kick model ***interpretive*** *version*

Color-coded evolution/profiles

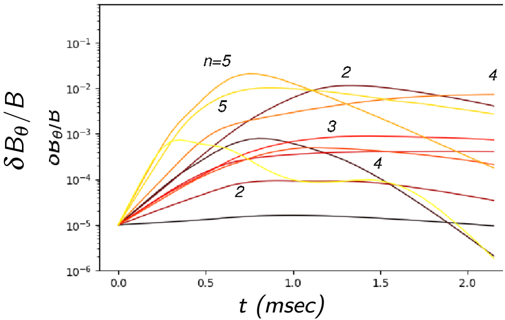


- Black curve is TRANSP calculations without AE effects
- Blue: kick model with neutron flux constraint
- Red: RBQ with the same constraint
- AE amplitudes are used in RBQ1D as inferred at $t = 805msec$ time.

RBQ applications in the interpretive mode are consistent with neutron rate data.

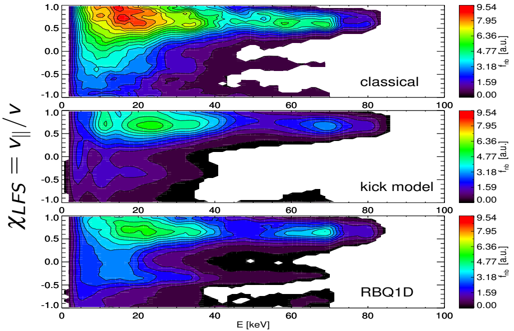
Used probability density function will be substituted with actual diffusion in the COM phase space.

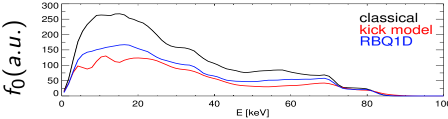
Predictive capability results from selfconsistent evolution of all 11 modes



- RBQ1D computes selfconsistent Alfvén Eigenmode amplitudes consistent with measured values $O\left(10^{-4}-10^{-2}\right)$ *(Collins, PRL'16)*.
- Shown are AE evolutions with modified prescriptions for broadening *(White et al., PoP'18)*.
- Amplitudes (diffusion coefficients) at saturation are sensitive to growth rate values: growth rates need to be robustly computed!

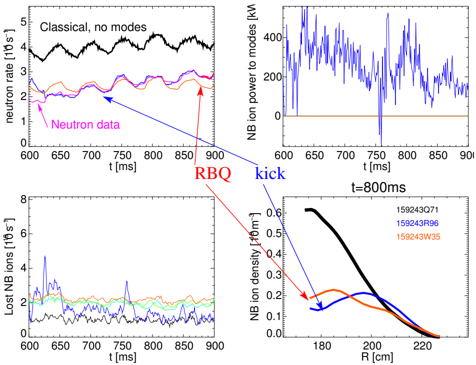
Distribution function has similar properties with the kick model distribution





- Co-going passing ions are strongly redistributed.
- Amplitudes are kept constant throughout observed times.
- Neutron rate includes radial and energy dependence within TRANSP simulations.
- (Near) hollow EP density is due COM location sensitive diffusion.
- Rotation is ignored!! It can be significant and could lead to EP energy shift $\sim E_0/2$ in DIII-D.

Compare RBQ1D, kick simulations with neutron deficit using TRANSP



- Distributions are evolved by TRANSP Monte-Carlo package.
- Kick model agrees with FIDA data over the velocity space region.
- RBQ1D and kick model simulations are consistent.

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