M3D-C^1 Progress on JA-2

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32nd meeting of ITPA MHD Topical Group
October 1-3

Naples, Italy

Status of JA-2 Code Benchmark

Outline

1. Single unstable mode case
2. Multiple unstable mode cases
3. Problems with extending these cases
4. New Stationary Equilibrium with Sources (NSES)
5. Initial Results with NSES
6. Summary and Future Studies

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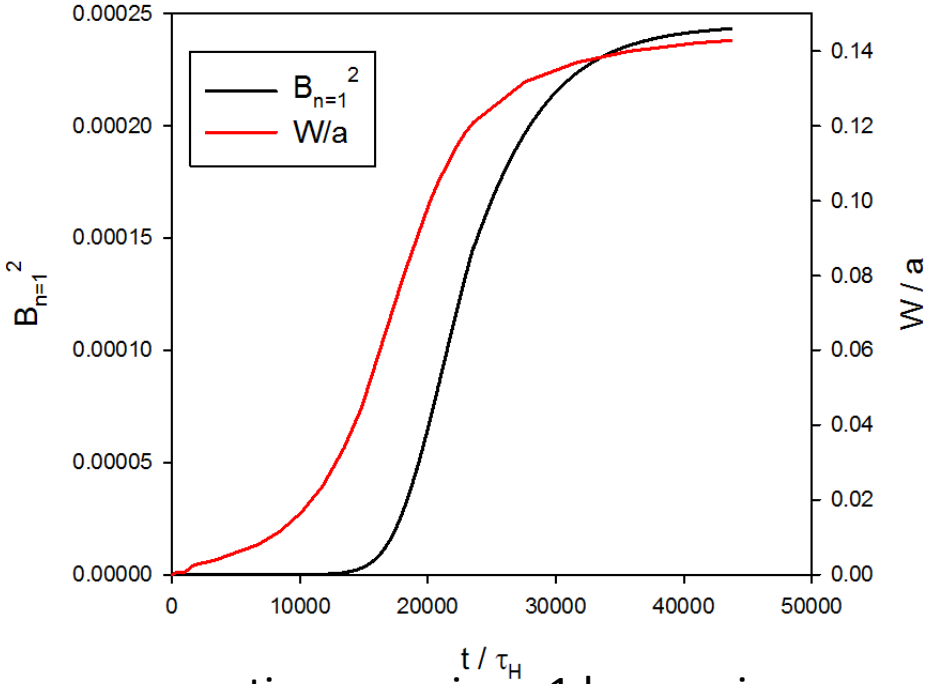
Single Unstable Mode Case

$$q(r) = 1.15\left[1 + \left(\frac{r}{0.81}\right)^2\right]$$

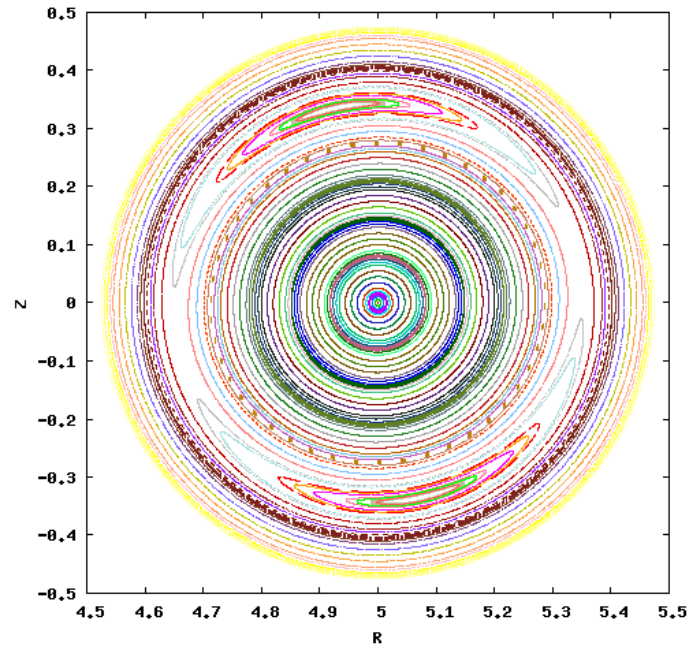
Nonlinear Comparison of:

1. Reduced MHD in a cylinder
2. Full MHD in a cylinder
3. Full MHD in a torus

Single Unstable Mode Case in a cylinder



magnetic energy in n=1 harmonic and island width vs time



Poincare plot at final time.

SI Units: R=5, a=0.5, B_T = 4.2, $n_0 = 10^{20}$

$$q(r) = 1.15\left[1+\left(\frac{r}{0.81}\right)^2\right]$$

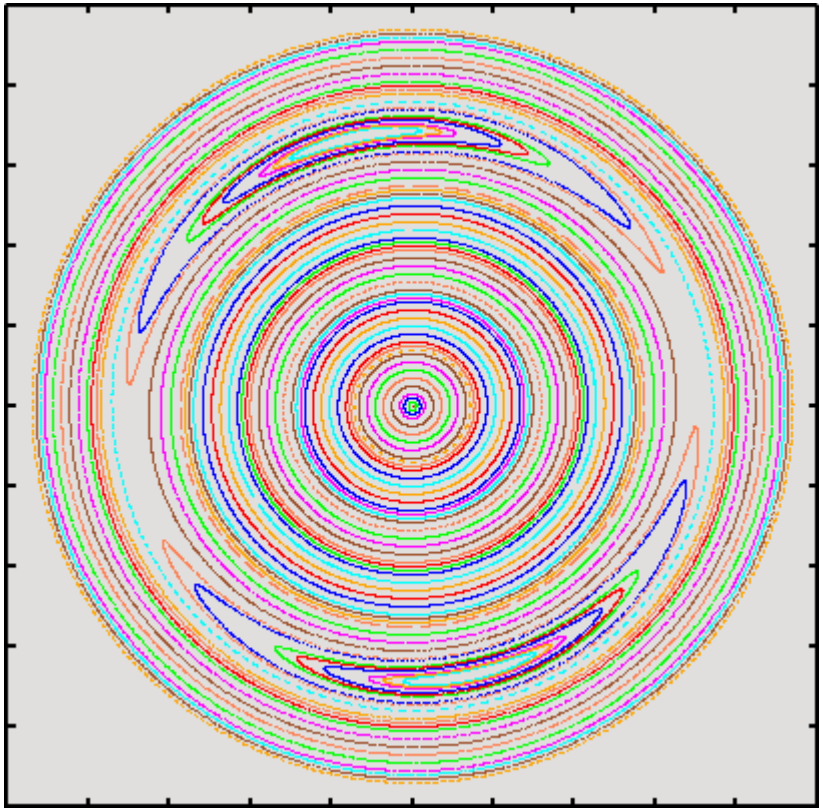
$$S = \frac{a^2 B_T}{\eta R}\left[\frac{\mu_0}{n_0 M_i}\right]^{1/2} = 10^5, \quad \tau_H \equiv \frac{R}{B_T}\left[\mu_0 n_0 M_i\right]^{1/2}, \quad \nu/\eta = .047$$

Reduced MHD (numvar=1). n=0 (axisymmetric) equilibrium not advanced in time. Low $\beta < 10^{-7}$

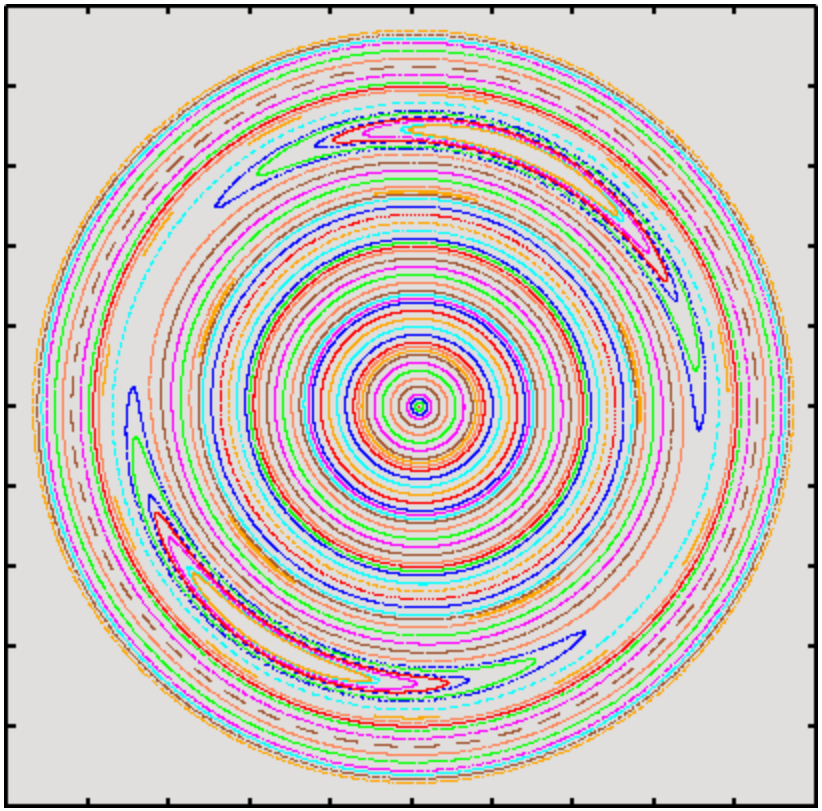
Δt increased by 2 at $t \cong 25000$

/tigress/sjardin/JA-2/One_Mode/Cylinder

Model Comparison—final island size



Reduced MHD, cylindrical geometry



Full MHD, toroidal geometry A=10

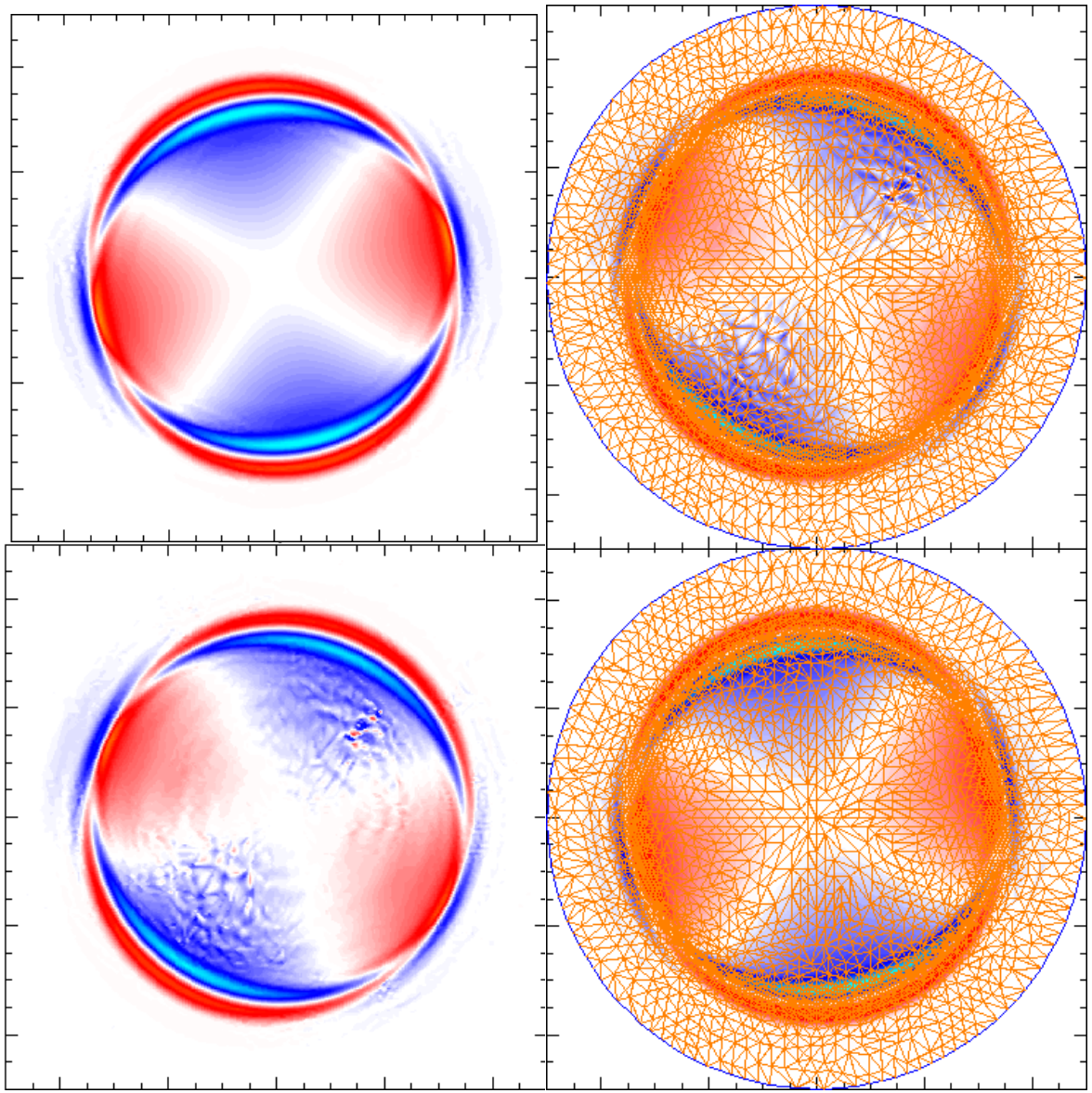
/tigress/sjardin/JA-2/One_Mode/A=10Torus
/tigress/sjardin/JA-2/One_Mode/Cylinder

Perturbed Jφ

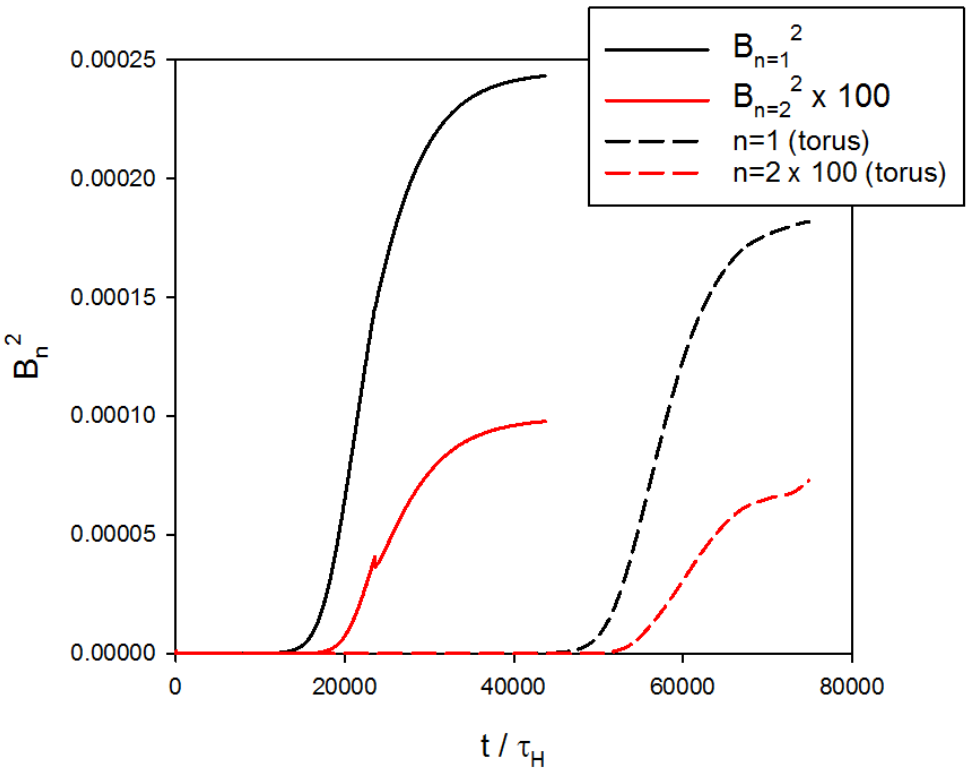
With adapted mesh

Cylinder →

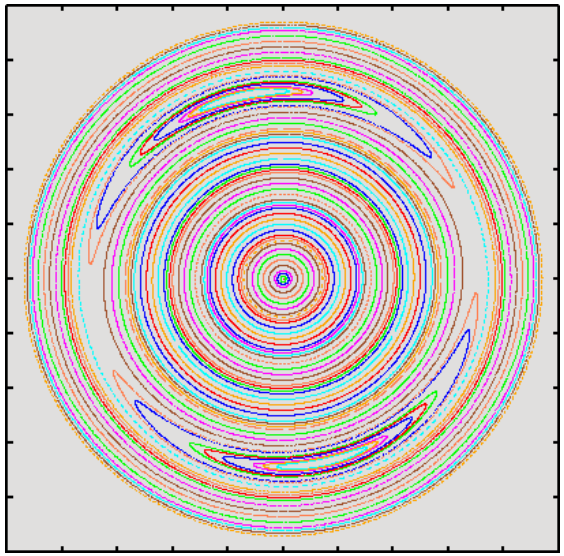
Torus →



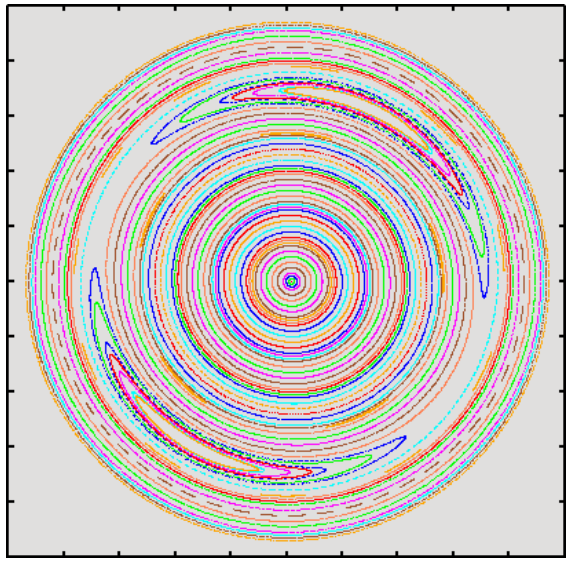
Comparison of B_n^2 between cylinder and torus



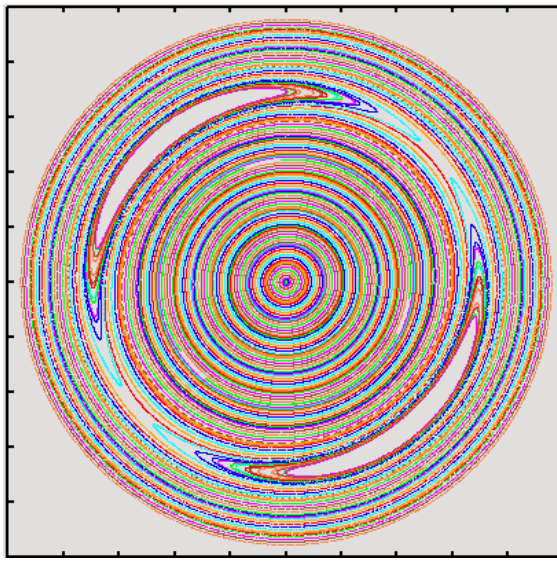
Model Comparison—final island size



Reduced MHD,
cylindrical geometry



Full MHD,
toroidal geometry A=10

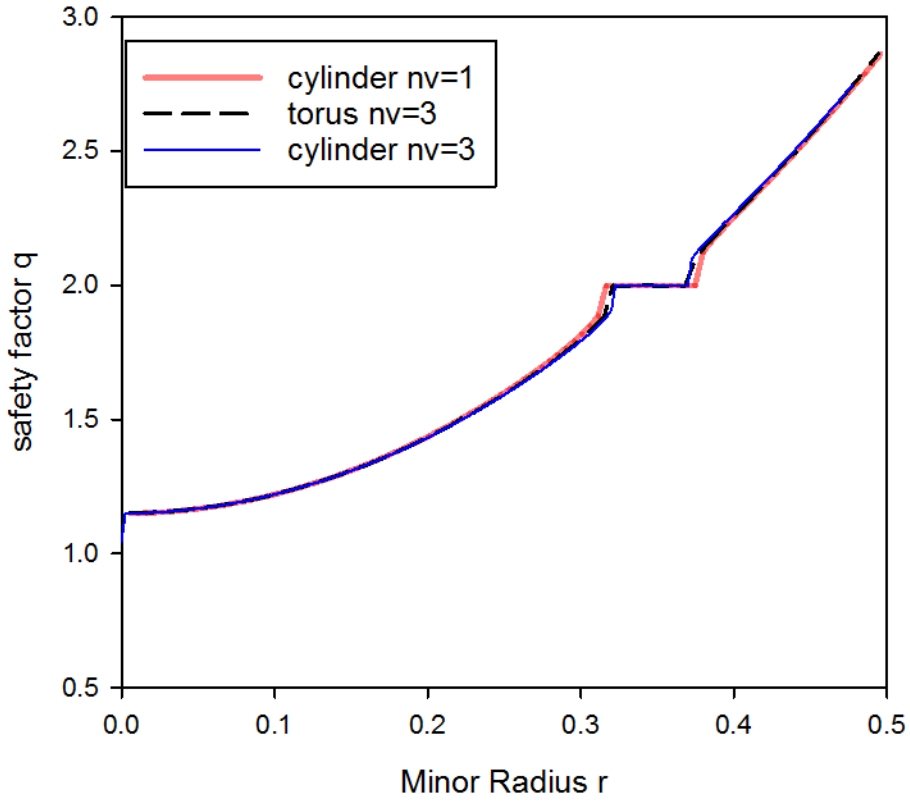


Full MHD,
cylindrical geometry

➔ Very similar final island size for the 3 cases

Final configuration in each case is a 3D equilibrium

Final q-profiles are very similar for the 3 cases



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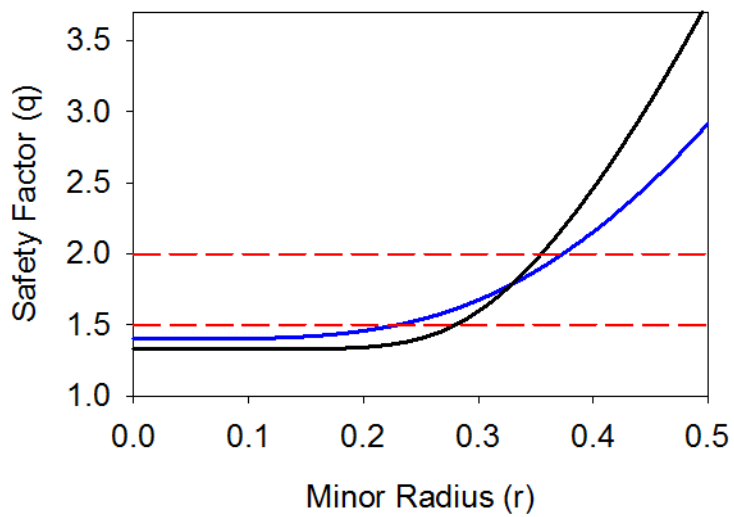
Equilibrium Unstable to 2 Modes

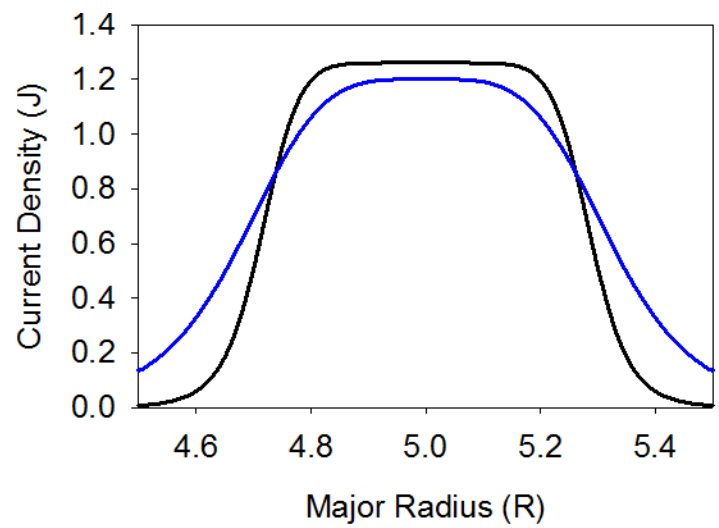
$$q(r) = 1.33\left[1 + \left(\frac{(r/a)^2}{.354}\right)^4\right]^{1/4}$$

- **Two-Mode** Case Proposed 10/28/2015 by Hender
- n=1 and n=2 unstable
- Cylinder, A=3, A=10 torus
- Linear and Nonlinear

$$q(r) = 1.4\left[1 + \left(\frac{(r/a)^2}{0.5476}\right)^2\right]^{1/2}$$

- **Two-Mode** Case Proposed 6/17/2016 by Hender
- n=1 and n=2 unstable





$$q(r) = 1.33\left[1 + \left(\frac{(r/a)^2}{.354}\right)^4\right]^{1/4}$$

$$q(r) = 1.33\left[1 + \left(\frac{(r/a)^2}{.354}\right)^4\right]^{1/4}$$

Test of linear growth rate with different options for cylinder

p0 = 1.E-4, dt = 2.0, amu = 1.e-5, eta=1.e-5 (S = 2.1 x 10^4), uniform mesh 0.03

	isplitstep=1						isplitstep=0
	impmod=0			impmod=1			
	16	32	linear	16	32	linear	linear
n=1	.0063	.0063	.0063	.0070	.0070	.0071	.0072
n=2	.0048	.0048	.0048	.0064	.0064	.0064	.0066

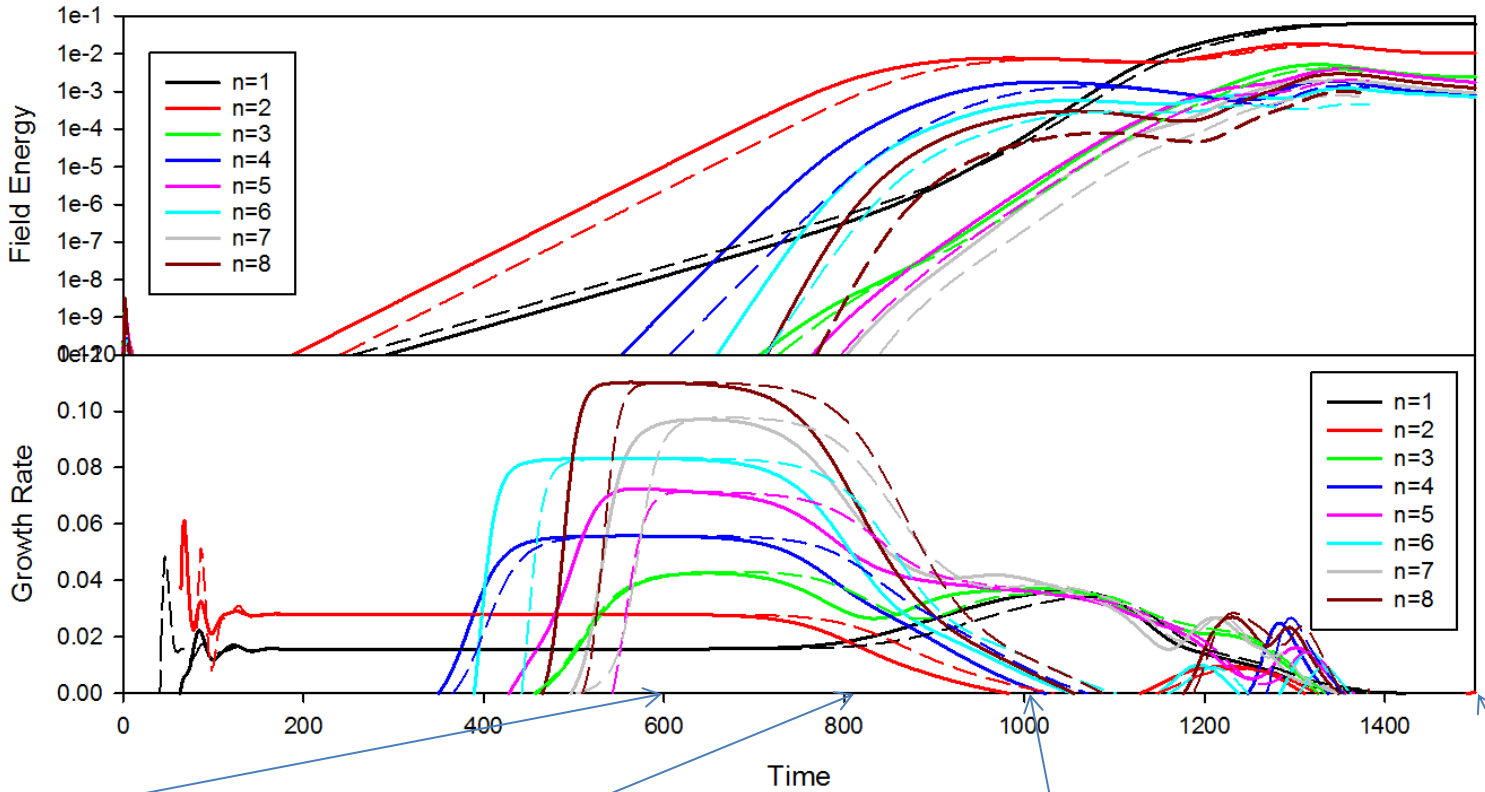
Conclusions:

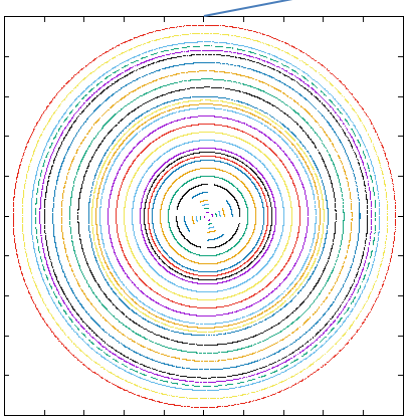
1. isplitstep=0 (unsplit) is most accurate, but only viable for linear runs
2. Isplitstep=1 (split)
    1. impmod = 1 (Caramana adv.) more accurate than impmod=0 (θ-implicit)
3. Good convergence in toroidal mode number for low-n modes (1 and 2)

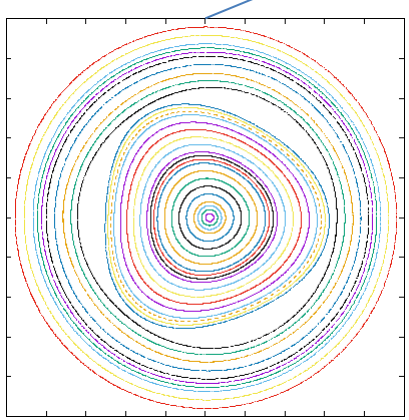
tigress/sjardin/JA-2/Two_Modes/q0=1.33,Run04-Linear

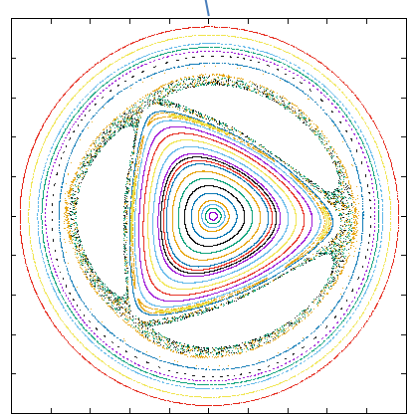
Mode Growth vs time **for toroidal q0=1.33 2 unstable mode case**

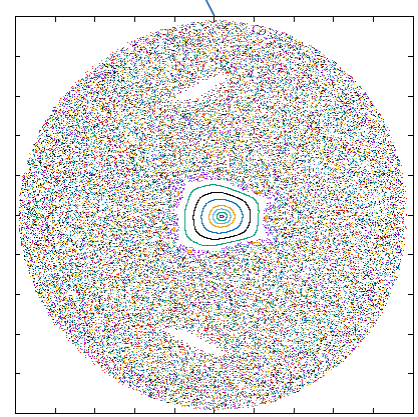
Comparison of 16 planes (solid) and 32 planes (dashed)





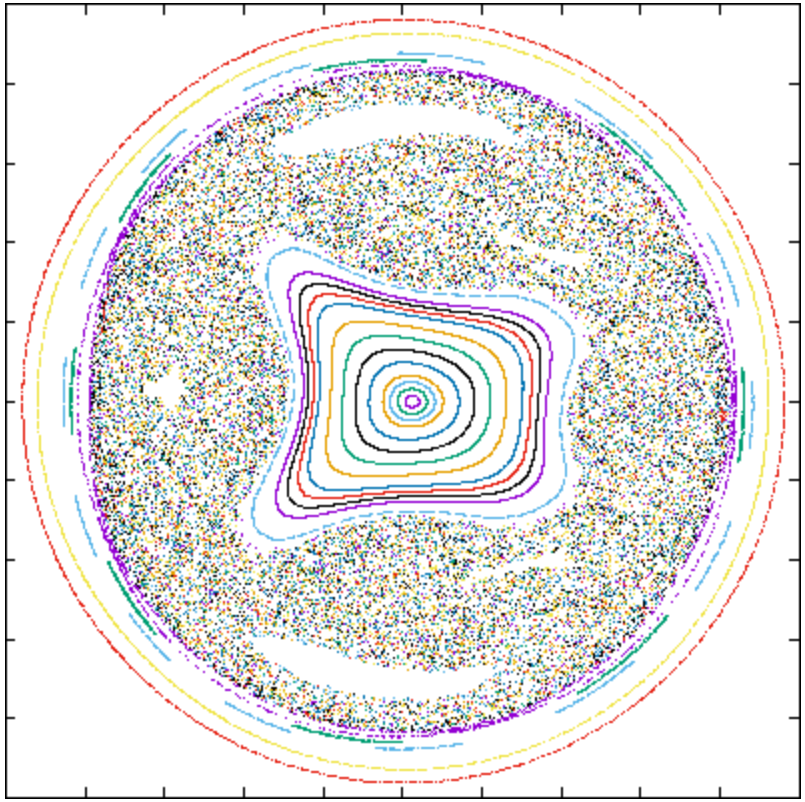




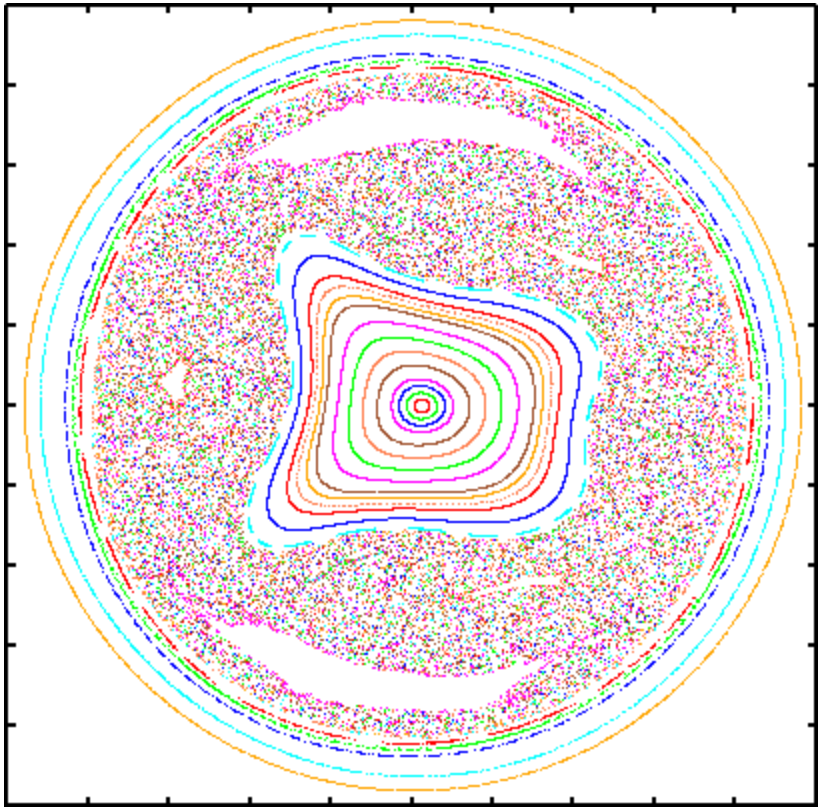


N=2 most unstable …. Very disruptive

Comparison of surfaces at time t=1200 for q0=1.33 **2-Mode case** with 16 and 32 Planes



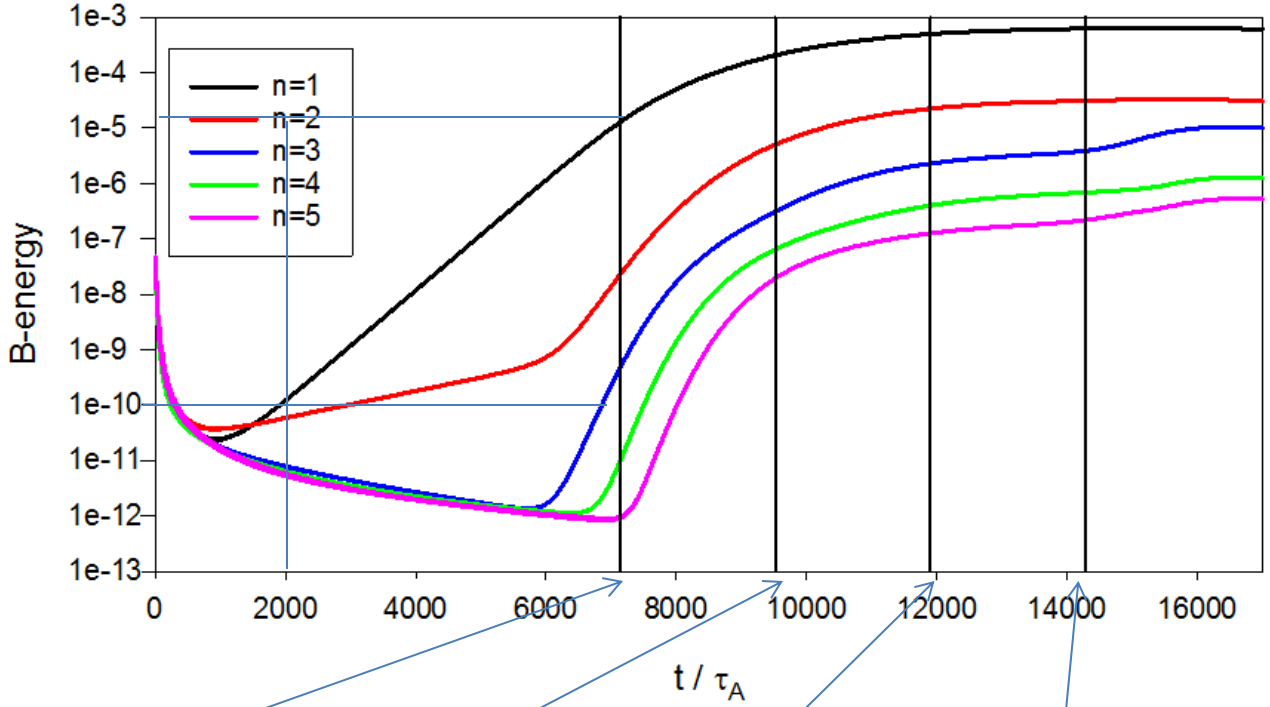
16 Planes

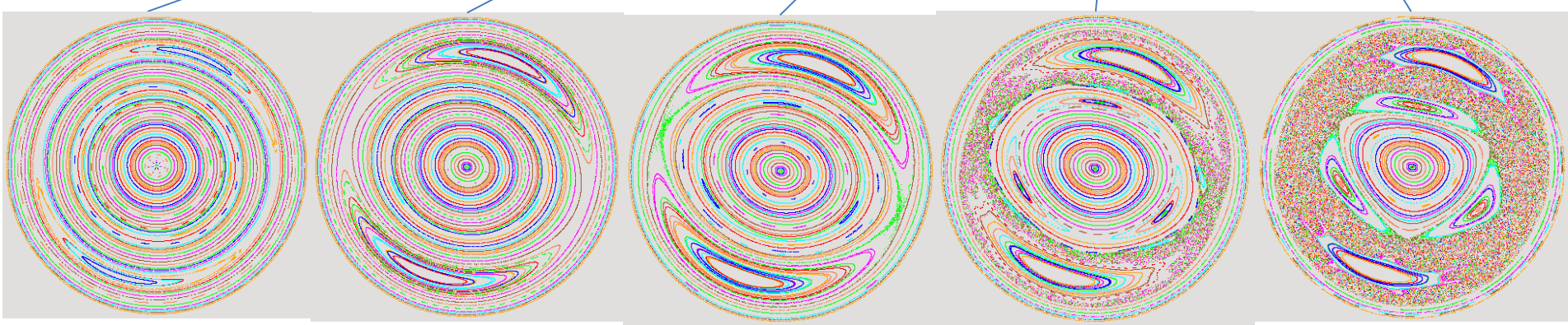


32 Planes

$$q(r) = 1.4\left[1 + \left(\frac{(r/a)^2}{0.5476}\right)^2\right]^{1/2}$$

q_0 = 1.44 case (2 unstable modes) Note n=1 most unstable

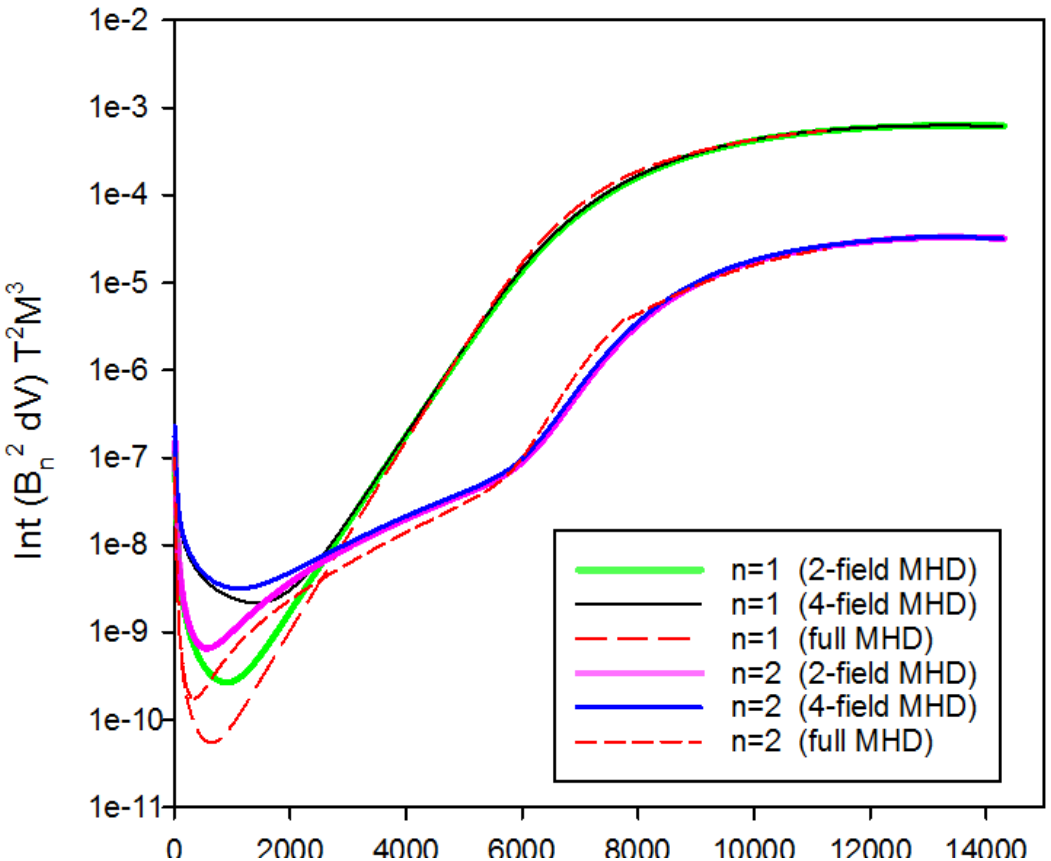




Need to redo with sheared toroidal rotation!

Comparison of Reduced MHD and Full MHD In cylindrical geometry

M3D-C1



$$q(r) = 1.4\left[1 + \left(\frac{(r/a)^2}{0.5476}\right)^2\right]^{1/2}$$

Circular cylinder
a = 0.5, R = 5.0

$S = 10^5$, $\nu/\eta = 1$

(R,φ,Z) cylindrical coordinates

2-field MHD

$$\mathbf{A} = \psi\nabla\varphi$$
$$\mathbf{V} = R^2\nabla U \times \nabla\varphi$$
$$p = 0$$

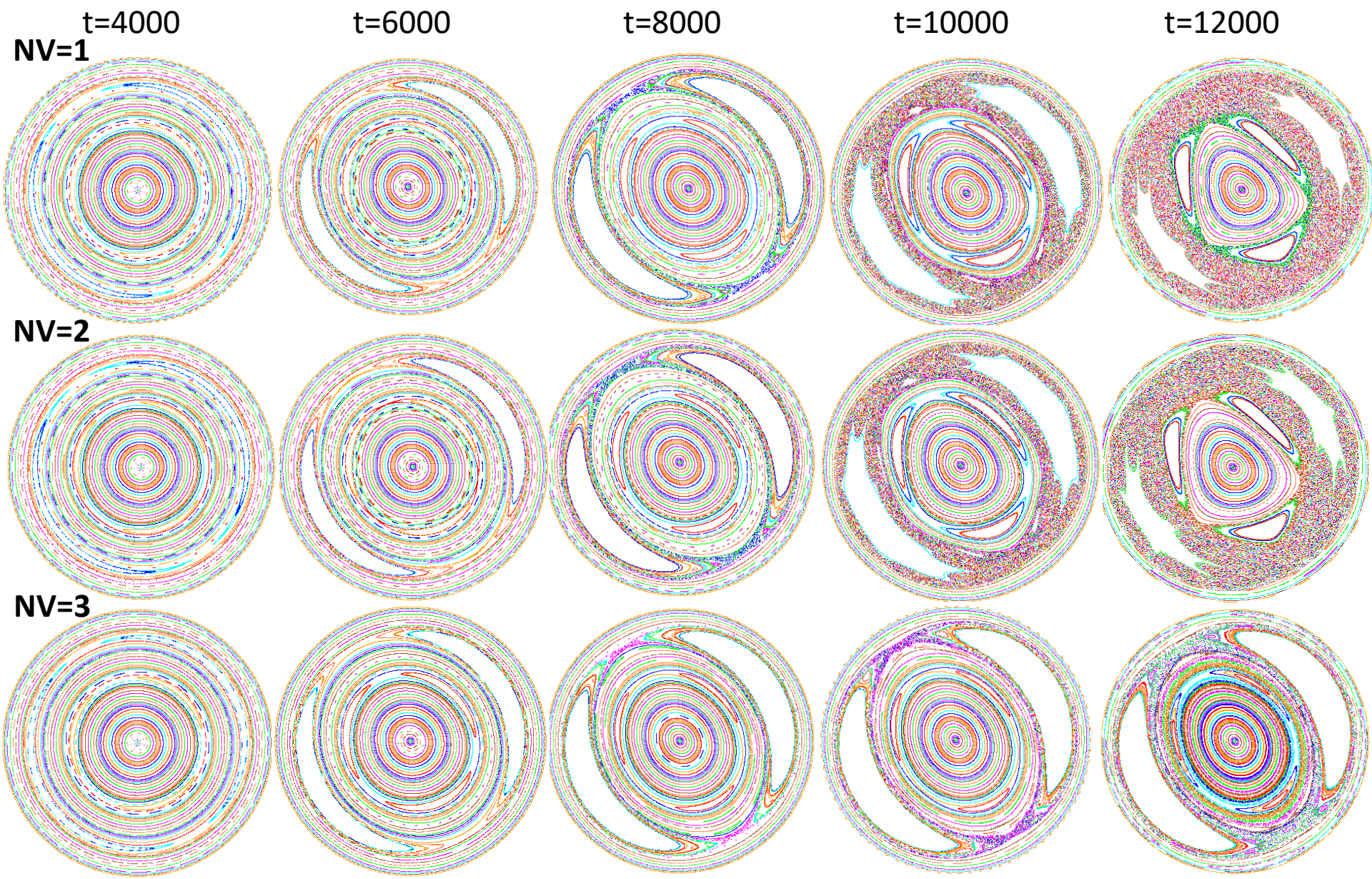
4-field MHD

$$\mathbf{A} = R^2\nabla\varphi \times \nabla f + \psi\nabla\varphi$$
$$\mathbf{V} = R^2\nabla U \times \nabla\varphi + \omega R^2\nabla\varphi$$
$$p = 0$$

Full MHD

$$\mathbf{A} = R^2\nabla\varphi \times \nabla f + \psi\nabla\varphi$$
$$\mathbf{V} = R^2\nabla U \times \nabla\varphi + \omega R^2\nabla\varphi + R^{-2}\nabla_\perp\chi$$
$$p \neq 0$$

NOTE: in Circular cylinder, $R=R_0$ (constant) in above and $\nabla\varphi$ is a unit vector


t=4000
t=6000
t=8000
t=10000
t=12000
NV=1
NV=2
NV=3

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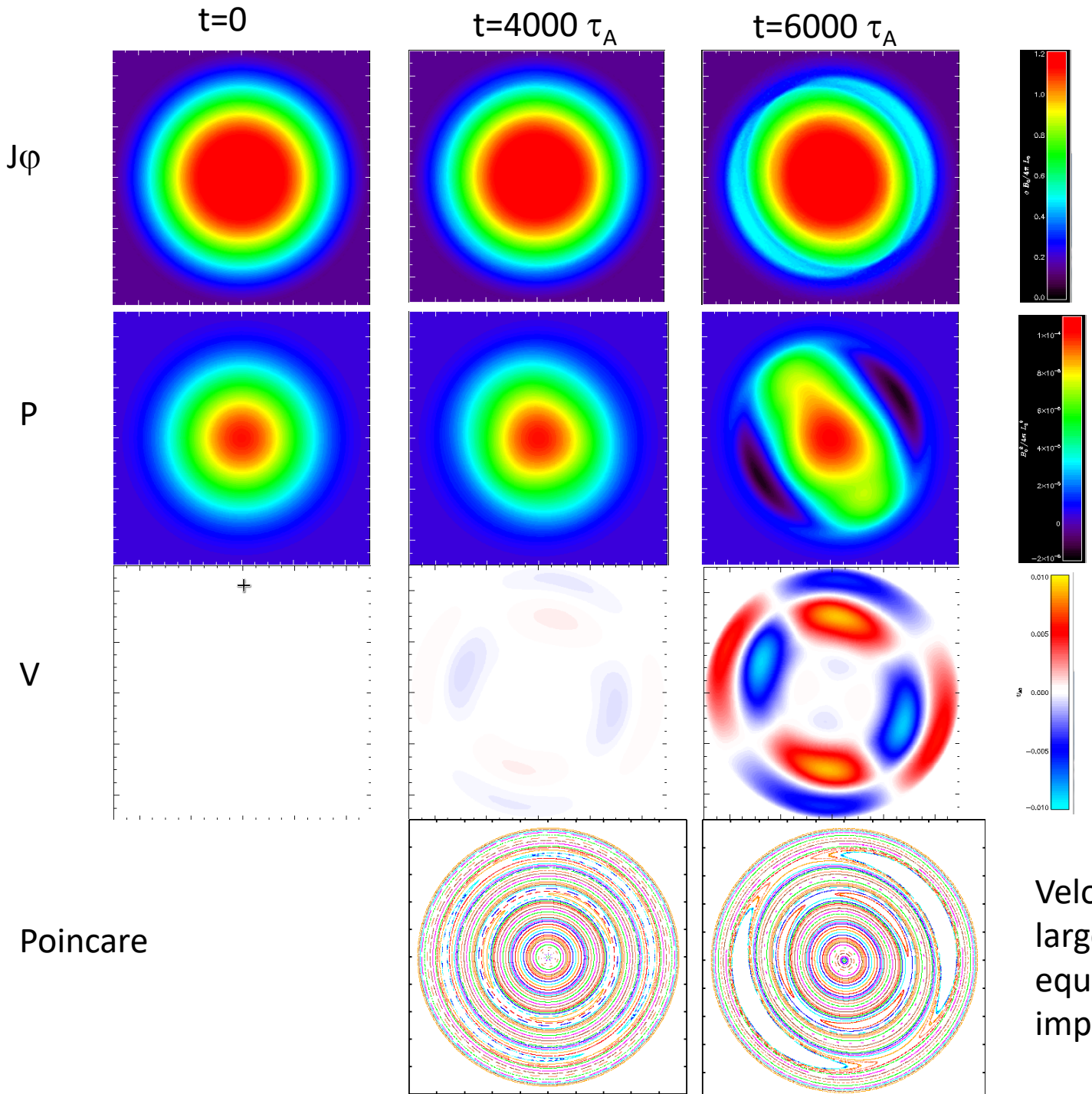
Problems with extending these cases

- All the equilibrium with 2 unstable modes led to large island overlap (presumably disruption)
- Next step was to add sheared toroidal rotation to these
- However, this was not easily done, since no sources were specified: implies eqsubtract = 1 in modeling
- eqsubtract = 1 $p = p_0 + p_1(t)$
  - p_0 stays fixed…..implies source term is present
- eqsubtract = 0 $p = p_1(t)$
  - Sources must be added explicitly. Need to do this

eqsubtract = 1



Current density flattens in Island

Pressure goes negative!!

Large toroidal velocity develops.

$v^2 \sim p$

Velocity that develops is larger than any equilibrium velocity imposed!

Why does the pressure go negative?

Because the sources are not specified, we are assuming that the equilibrium pressure is in balance with the equilibrium Ohmic heating.

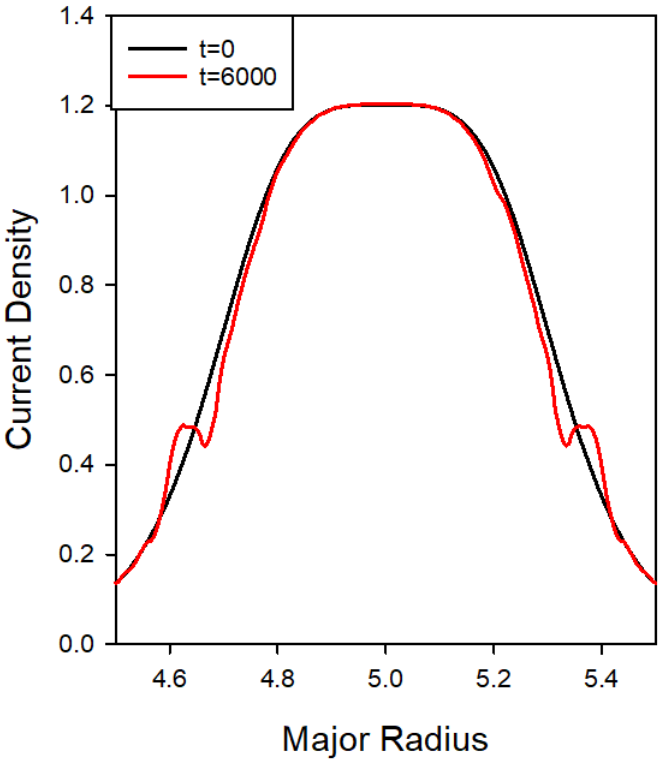
$$\frac{3}{2}\frac{\partial p}{\partial t} = \cdots + \eta J^2$$

$$p = p_0 + p_1(t)$$

$$J = J_0 + J_1(t)$$

$$\frac{3}{2}\frac{\partial p_0}{\partial t} = \cdots + \eta J_0^2$$

$$\frac{3}{2}\frac{\partial p_1}{\partial t} = \cdots + \eta\left[2J_1J_0 + J_1^2\right]$$



➔Pressure will continuously decrease where J_1 <0…will eventually go negative

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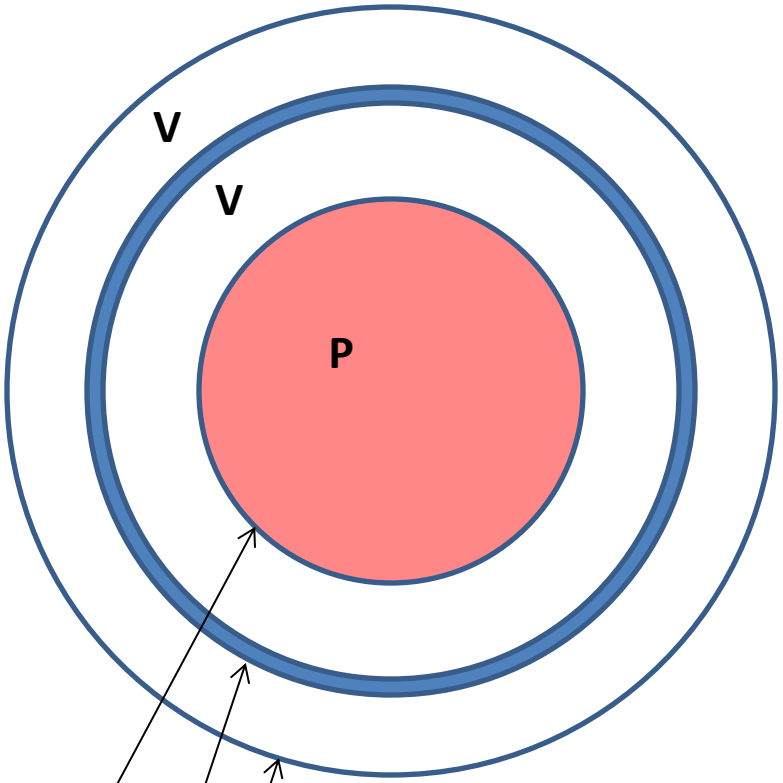
Model Cylindrical Equilibrium in true stationary state



Computational boundary

Finite thickness wall

Plasma boundary (r=a)

- Poloidal flux low order polynomial in r^2
- Current density vanishes at a
- $q_0 < q < q_a$

$$\Psi(r)=\begin{cases}\dfrac{B_T}{2q_0R}\left[1-(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+\dfrac{1}{3}(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right] & r<a\\ \dfrac{a^2B_T}{6q_0R}\left[1+\dfrac{5}{2}\dfrac{q_0}{q_a}+6\dfrac{q_0}{q_a}\ln(r/a)\right] & r\geq a\end{cases}$$

$$J(r)=\begin{cases}\dfrac{B_T}{6q_0R}\left[1-4(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+3(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right] & r<a\\ 0 & r\geq a\end{cases}$$

$$q(r)=\begin{cases}q_0\left[1-2(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right]^{-1} & r<a\\ q_a(r/a)^2 & r\geq a\end{cases}$$

Stationary State Stationary Equilibrium equations

Uniform density: $T \sim p$; Spitzer resistivity: $\eta \sim p^{-3/2}$

Constant loop voltage: $\eta J \sim (\eta_0 p_0^{3/2}) p^{-3/2} J \sim V_L$

$$==> p = p_0 \left[1 - (r/a)^2 \left(4 - 6\frac{q_0}{q_a} \right) + (r/a)^4 \left(3 - 6\frac{q_0}{q_a} \right) \right]^{2/3}, \qquad V_L = B_T \eta_0 p_0^{3/2} / 6 q_0 R$$

$$\nabla p = \mathbf{J} \times \mathbf{B} \qquad ==> \frac{1}{2}\frac{d}{dr^2} B_z^2 = \frac{-\frac{dp}{dr^2} + \frac{B_z^2}{(Rq)^2}\left(1 - \frac{r^2}{q}\frac{dq}{dr^2}\right)}{1 + r^2/(Rq)^2}, \qquad B_z(a) = B_T$$

Stationary pressure profile: $\frac{1}{r}\frac{d}{dr}\left(r\kappa \frac{dp}{dr} \right) = -\eta J^2 = -V_L J$

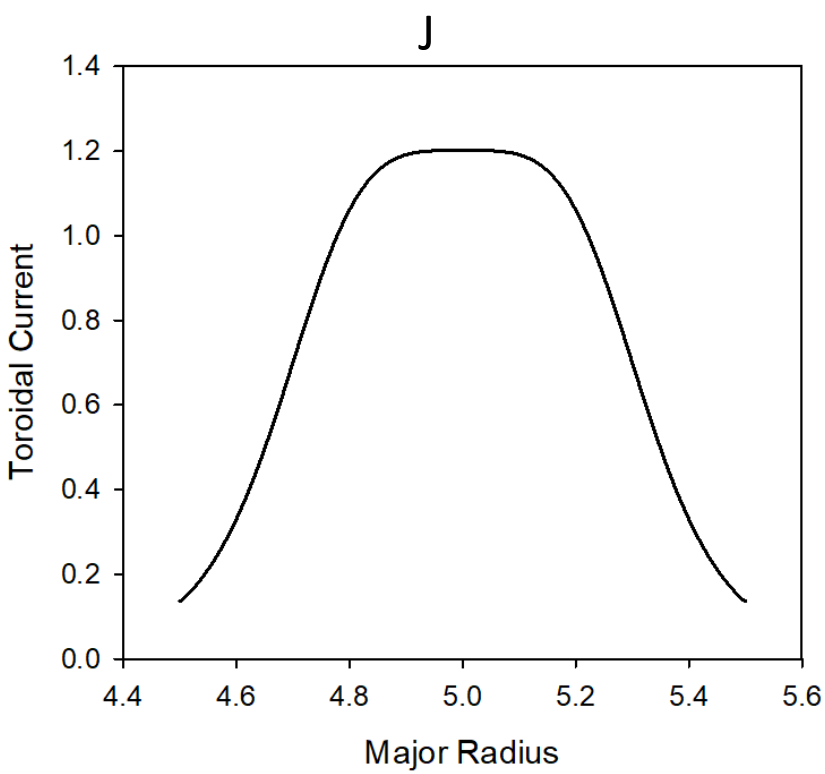
$$==> \quad \kappa \frac{dp}{dr} = -\frac{V_L B_T}{6 q_0 R} \frac{1}{r} \int_0^r r \left[1 + (r/a)^2 \left(-4 + 6\frac{q_0}{q_a} \right) + (r/a)^4 \left(3 - 6\frac{q_0}{q_a} \right) \right] dr$$

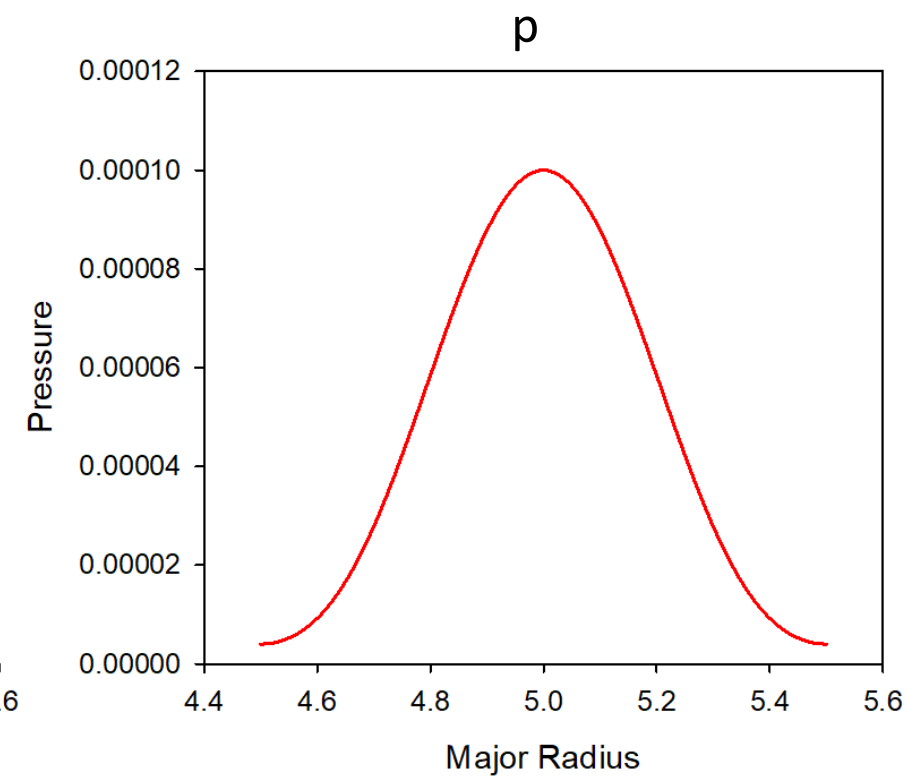
$$\kappa = \kappa_0 \frac{\left[1 - 2(r/a)^2 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) + (r/a)^4 \left(1 - 2\frac{q_0}{q_a} \right) \right] \left[1 - 4(r/a)^2 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) + 3(r/a)^4 \left(1 - 2\frac{q_0}{q_a} \right) \right]^{1/3}}{\left[\left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) - \frac{3}{2}(r/a)^2 \left(1 - 2\frac{q_0}{q_a} \right) \right]}$$

$$\kappa_0 = \frac{3}{192} \frac{V_L B_T a^2}{q_0 R p_0 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right)}$$

$$q_0, q_a, B_T, a, R, \eta_0, p_0 \quad ==> \quad V_L, \kappa_0$$

Typical stationary-state profiles: q_0=1.11, q_a = 2.6





This class of equilibrium and sources are stationary on all timescales (in 2D)

q_0

q_a

	1.01	1.11	1.33	1.67	1.80	2.01
2.4						
2.6						
2.8						
3.0						
3.2						
3.4						
3.6						
3.8						
4.0						
4.2						
4.4						
4.6						
4.8						
5.0						
5.2						
5.4						
5.6						
5.8						

This is unstable to a 2/1 and 3/2 mode with similar growth rates

	2/1 most unstable
	3/1 most unstable
	4/1 most unstable
	Stable to all modes

$$q(r) = \begin{cases} q_0 \left[1 - 2(r/a)^2 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) + (r/a)^4 \left(1 - 2\frac{q_0}{q_a} \right) \right]^{-1} & r < a \\ q_a (r/a)^2 & r \geq a \end{cases}$$

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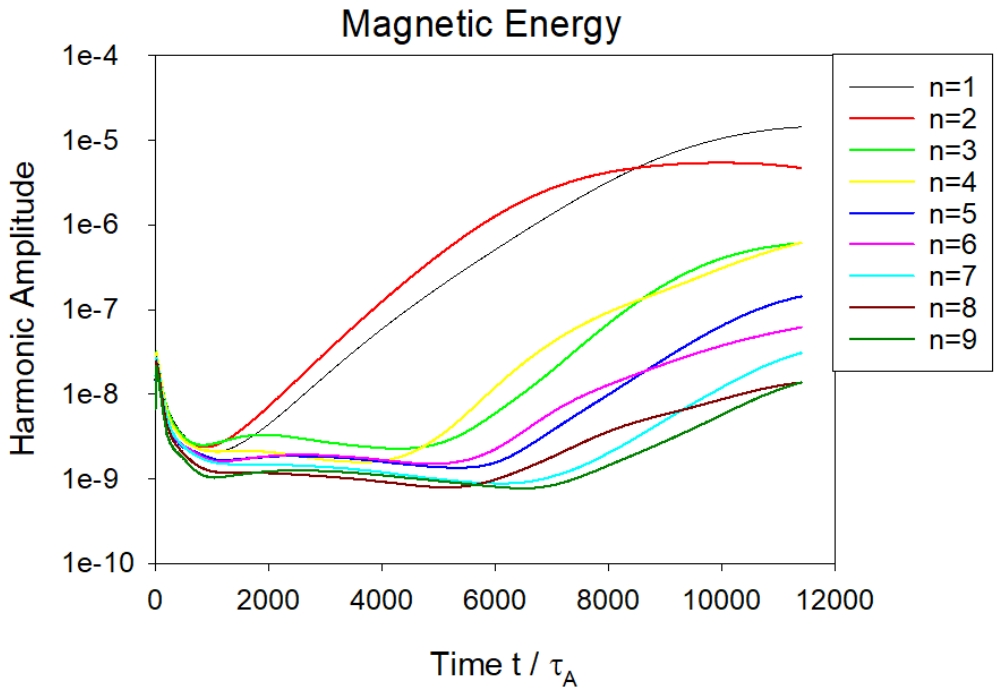
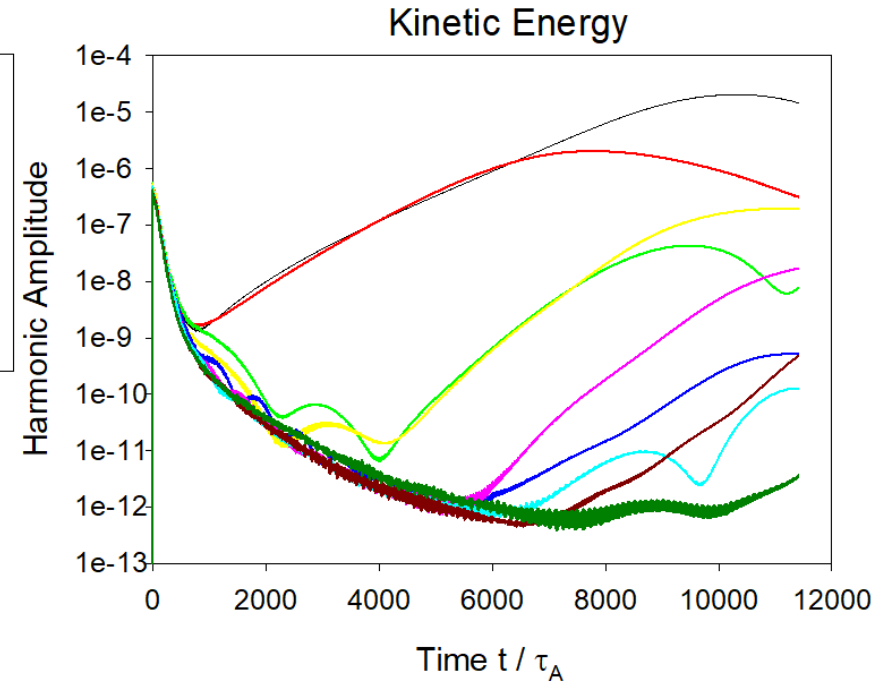
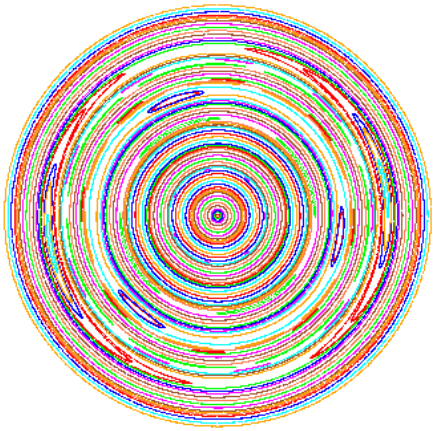
New stationary equilibrium has unstable 2/1 and 3/2 modes with similar growth rates. Saturate at modest size. Next will add sheared toroidal rotation





Final Time Poincare plot:

3/2 + 2/1 saturated islands



S=100 t=2000, p=64

Status of JA-2 Code Benchmark

Outline

1. Single unstable mode case
2. Multiple unstable mode cases
3. Problems with extending these cases
4. New Stationary Equilibrium with Sources (NSES)
5. Initial Results with NSES
6. Summary and Future Studies

Summary and Future Studies

- Original JA-2 study (single unstable mode) was useful for code benchmarking and comparing results from different physics models
- Extending this to 2 unstable modes led to very unstable configurations that led to island overlap (large stochastic regions)
- These could not be easily extended to add sheared toroidal rotation
- New class of equilibrium and sources proposed that lead to stationary equilibrium on all time scales (in 2D)…much more physical
- Now in process of adding torque sources to study:
  - Island saturation size vs sheared rotation
  - Tendency for island chains to phase lock ?
  - Mode locking in presence of error fields and/or wall