Quasilinear relaxation of energetic particles interacting with Alfvénic modes

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Transport Task Force, Williamsburg, VA, April 27th 2017
Alfvén waves can exhibit a range of bifurcations upon their interaction with fast ions

Typical scenarios:
- fixed frequency and frequency splitting -> frequency is determined by the equilibrium
- chirping and avalanches -> frequency is highly affected by the fast ions response

Fasoli, PRL 1998
Fredrickson, PoP 2006
Podestà, NF 2011
Prediction of character of energetic-particle-driven transport in tokamaks

What tools can be used to model each type of transport?

**Diffusive transport (typical for fixed-frequency modes)**
- can be modelled using reduced theories, such as quasilinear
- typical in conventional tokamaks

**Convective transport (typical for chirping frequency modes)**
- needs to retain full nonlinear features of the wave, is sustained by nonlinear phase-space structures
- typical in spherical tokamaks
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Both can lead to similar fast ion loss levels, up to 40%
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In this talk:
- development of a criterion for the likelihood of each nonlinear scenario and its comparison with experiments
- quasilinear diffusion approach and perspectives for whole device modeling

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Weak nonlinear dynamics of driven kinetic systems can be used to distinguish between fixed-frequency and chirping responses

Starting point: kinetic equation plus wave power balance

Assumptions:
• Perturbative procedure for $\omega_b \ll \gamma$
• Truncation at third order due to closeness to marginal stability
• Bump-on-tail modal problem, uniform mode structure

Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude $A$:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^2\tau^2(2\tau/3+\tau_1)+i\nu_{drag}^2\tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

Berk, Breizman and Pekker, PRL 1996
Lilley, Breizman and Sharapov, PRL 2009
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stabilizing
destabilizing (makes integral sign flip)

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Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude \( A \):

\[
\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{\text{scatt}}^2 (2\tau/3+\tau_1)} + \omega_{\text{drag}}^2 (\tau+\tau_1) A(t-\tau-\tau_1) A^* (t-2\tau-\tau_1)
\]

- If nonlinearity is weak: linear stability, solution saturates at a low level and f merely flattens (system not allowed to further evolve nonlinearly).
- If solution of cubic equation explodes: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (precursor of chirping modes).

Berk, Breizman and Pekker, PRL 1996
Lilley, Breizman and Sharapov, PRL 2009
A criterion for chirping onset in tokamaks

Using an action and angle formulation, the previous weak nonlinear theory leads to

\[
\sum_{l,\sigma} \int dP_\varphi \int d\mu \frac{\tau_b}{\nu_{\text{drag}}} |V_l|^4 \left| \frac{\partial \Omega_l}{\partial I} \right| \frac{\partial F}{\partial I} \ Int > 0
\]

\[\text{Int} \equiv Re \int_0^\infty dz \frac{\nu_{\text{stoch}}^3}{\nu_{\text{drag}}^3} \exp \left[ -\frac{2}{3} \frac{\nu_{\text{stoch}}^3}{\nu_{\text{drag}}^3} z^3 + iz^2 \right] \]

Criterion was incorporated into NOVA-K code: nonlinear prediction from linear physics elements

The criterion predicts that micro-turbulence should be key in determining the likely nonlinear character of a mode, e.g., fixed-frequency or chirping
Correlation between chirping onset and a marked reduction of the turbulent activity in DIII-D

- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport
- Experiments in DIII-D are scheduled to further test the proposed criterion
Correlation between chirping onset and a marked reduction of the turbulent activity in NSTX

- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport
- GTS code is being used as an independent calculation of fast ion diffusivity

Chirping is ubiquitous in NSTX but rare in DIII-D, which is consistent with the inferred fast ion micro-turbulent levels.
Alfvén wave chirping quantitatively agrees with the criterion \( \langle v_{\text{stoch}} \rangle / \langle v_{\text{drag}} \rangle \)
### Nonlinear chirping vs Quasilinear approach

<table>
<thead>
<tr>
<th>Nonlinear chirping</th>
<th>Quasilinear approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requires particles to remember their phases from one trapping bounce to another;</td>
<td>Requires particles to forget their phase (via collisions, turbulence or mode overlap);</td>
</tr>
<tr>
<td>Full kinetic approach necessary;</td>
<td>Assumes that the <strong>modes remain linear</strong> (therefore NOVA is suited) while the distribution function is allowed to slowly evolve nonlinearly in time;</td>
</tr>
<tr>
<td>Entropy is conserved in the absence of collisions.</td>
<td>Entropy increases due to particle memory loss.</td>
</tr>
</tbody>
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**The chirping criterion is a useful tool to make sure the quasilinear approach is applicable for a given mode**
Resonance-broadened quasilinear (RBQ) diffusion model¹

Formulation in action and angle variables²,³

• Diffusion equation:
  \[ \frac{\partial f}{\partial t} = \frac{\partial}{\partial I} \left( \sum_{n,l,m,m'} D(I;t) \frac{\partial}{\partial I} f + C[f] \right) \]
  \[ D(I;t) = \pi n^2 C_n^2(t) \mathcal{E}^2 \frac{\mathcal{F}(I - I_r)}{|\frac{\partial \Omega_l}{\partial I}|} G_{m'l}^{*} G_{ml} \]

• Mode amplitude evolution:
  \[ \frac{dC_n^2(t)}{dt} = 2(\gamma_L,n - \gamma_d,n) C_n^2(t) \]
  \[ \mathcal{E} : \text{unperturbed (kinetic) energy; } P_\varphi : \text{canonical toroidal momentum} \]

Broadening is the platform that allows for momentum and energy exchange between particles and waves:

\[ \Delta \Omega = a\omega_{b,n} + b|\gamma_n| + c\nu_{scatt} \]

¹Berk, Breizman, Fitzpatrick, NF 1995.
Parametric dependencies of a broadened resonance

- old idea (Dupree PoF 1966): the turbulent spectrum contributes to diffuse particles away from their original unperturbed trajectories
- broadening specification: \( \Delta \Omega = a\omega_{b,n} + b|\gamma_n| + c\nu_{scatt} \)
- analytical predictions for the simplified driven, bump-on-tail system close to and far from marginal stability lead to particular choices of coefficients \( a, b \) and \( c \).
- guiding-center code ORBIT is being used to verify the resonance width scaling for realistic mode structure and resonances calculated by NOVA.
Resonance Broadened Quasilinear (RBQ) code computation of fast ion relaxation

Upper plot: distribution function as a function of the canonical toroidal momentum
Lower plot: evolution of the nonlinear bounce frequency (~square root of mode amplitude)

Single mode saturation
Two isolated modes
Two overlapping modes

When modes overlap, there is a sudden release of stored fast ion energy, which lead to substantial mode growth.
CHAPTER 2. NONLINEAR CHIRPING AND THE QUASILINEAR APPLICABILITY

2.3 A criterion for chirping onset

It has been shown [8] that simulations employing a single representative value for the collisional coefficients is insufficient to make predictions for the nonlinear nature of a mode in a tokamak. The missing physics were shown to be the absence of non-uniform mode structures, (multiple) resonance surfaces and poloidal bounce averages that account for particle trajectories on a poloidal cross section.

An essential condition for the existence of fixed-frequency, steady-state solutions would be that the real component of the right-hand side of Eq. (2.1) be negative at late times when the response is stationary, i.e. when the nonlinear term is allowed to balance the linear growth. The delta function \( \delta(P', E, \mu) \) that restricts the integration to the resonance condition can be exploited and the following criterion for the existence of fixed-frequency oscillations is obtained [8]:

\[
Crt = \frac{1}{N} \sum_{j, k} \left| \frac{\partial}{\partial I} \right| \frac{\partial}{\partial I} \int_{V_{n,j}} \left( \sum_{\text{drag}} \right) > 0,
\]

where \( N \) is a normalization for \( Crt \), which consists in the same sums and integrations that appear in the numerator of (2.3). In eqs. (2.3) and (2.4), the quantities \( \tau_{b}, \tau_{\text{drag}}, \tau_{\text{stoch}}, V_{n,j} \) and \( \delta_{j} \) are understood to be evaluated at \( E = E_{0} \).

Criterion (2.3) was incorporated into NOVA-K in use of polynomial interpolation for \( \int \).

Crt provides a prediction for the likelihood of a fully nonlinear phenomenon obtained...
Resonance Broadened Quasilinear (RBQ) code is being interfaced with TRANSP

Whole device modeling using TRANSP is interfaced with RBQ via a Probability Distribution Function (PDF)

PDFs are used actively in Kick modeling\(^1\)

RBQ can self-consistently provide:
- mode amplitude evolution
- diffusion coefficient at any given phase space location
- Intermittency and domino behaviors

\(^1\) Podestà, PPCF 2014
Outcome

- Criterion gives confidence in the application of quasilinear modeling;
- Refinement of the fast ion diffusivity that enters the chirping criterion is being done with the gyrokinetic code GTS;
- Experiments on DIII-D to further test the proposed chirping criterion predictions;
- Although a reduced model, RBQ provides a platform for rich dynamics seen in experiments: it resolves velocity space and account for losses and intermittency;
- Whole device modeling, RBQ interfacing with TRANSP.
Thank you