Parasitic Momentum Flux in the Tokamak Core

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Careful geometric analysis shows that energy transfer from the electrostatic potential to ion parallel flows breaks symmetry in the fully nonlinear toroidal momentum transport equation, causing countercurrent rotation peaking without applied torque.

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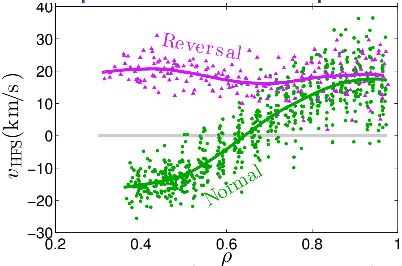
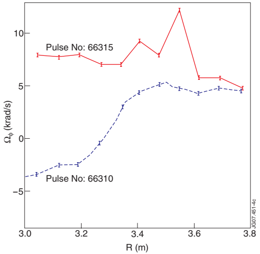
Outline

- Background
  - Experiment:
    - Intrinsic rotation and rotation reversals
  - Theory:
    - Intrinsic rotation: Vanishing momentum flux
    - Symmetry and symmetry-breaking mechanisms
- Rotation model
  - Intuitive cartoon of simple, axisymmetric example
  - Model equations and conservation properties
  - Symmetry breaking
    - Fluxtube coordinates and v_E^x
    - Free-energy flow in phase space⇒momentum flux
  - Predictions
    - Core rotation peaking: scaling, behavior
    - Conditions for peaked profiles

Tokamak plasmas rotate spontaneously without applied torque.



TCV Ohmic shots ($I_p \approx$155, 195kA)
Stoltzfus-Dueck et al PoP '15



JET ICRH shots ($I_p \approx$1.5, 2.6MA)
Eriksson et al PPCF '09

Important for stability against resistive wall modes at low torque (ITER).

Typical intrinsic rotation profiles have three regions:

- Edge: Co-rotating due to ion orbit shifts
- Mid-radius "gradient region": Countercurrent peaking or ~flat
  - Gradient exhibits sudden 'reversals' at critical parameter values.
- Sawtoothing region inside $q = 1$: Flat or weak cocurrent peaking

In axisymmetric geometry, neoclassical momentum transport is negligible.

Intrinsic rotation profiles result from vanishing momentum flux.

Axisymmetric steady state with no torque⇒zero momentum outflux:

$$0 = \Pi = -\underbrace{\nu \nabla L}_{\sim O(\rho_*^2 v_\varphi / v_{ti})} + \underbrace{v_{\text{pinch}} L}_{\sim O(\rho_*^3)} + \underbrace{\Pi^{\text{res}}}_{\sim O(\rho_*^3)} \implies \nabla L = (v_{\text{pinch}} L + \Pi^{\text{res}})/\nu$$

Toroidal momentum gradient ∇L is set by balancing

- Viscous flux ($-\nu \nabla L$) (saturation) against both
- Momentum pinch ($v_{\text{pinch}} L$) due to
  - 'Turbulent equipartition' due to ∇B (Hahm et al PoP '07)
  - Coriolis force (Peeters et al PoP '09)
- Residual stress (Π^{res}, independent of L)
  - Only explanation for peaked profiles that cross $L = 0$
  - $O(\rho_*^3)$ for up-down-symmetric geometry, so causes $v_\varphi \sim \rho_*^1 v_{ti}$.

In this talk, I identify and explore one contribution to Π^{res}.
I make no attempt to show that other contributions are small.

Symmetry restricts contributions to residual stress.

In the simplest radially local fluxtube limit with

- up-down symmetric magnetic geometry,
- no background rotation or rotation shear, and
- no background $\boldsymbol{E} \times \boldsymbol{B}$ shear,

the delta-f gyrokinetic equations satisfy a symmetry $[y \propto (\zeta - q\theta), s \propto \theta]$:

If $f(x, y, s, v_\parallel, \mu, t)$, $\phi(x, y, s, t)$ is a solution
so is $-f(-x, y, -s, -v_\parallel, \mu, t)$, $-\phi(-x, y, -s, \varphi, t)$

with opposite sign of the dominant toroidal momentum flux.
(Peeters and Angioni PoP '05, Parra et al PoP '11)

This implies: toroidal momentum flux should vanish for terms that flip sign, but does *not* imply that invariant terms *must* drive momentum flux.

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What drives symmetry-breaking and momentum flux, in the absence of rotation and of rotation shear?

Symmetry-breaking mechanisms to drive residual stress include:

- Background $\boldsymbol{E} \times \boldsymbol{B}$ shear (Dominguez and Staebler Phys. Fluids B '93)
- Up-down asymmetric magnetic geometry (Camenen et al PRL '09)
- Quasilinear: assume phase between $\tilde{v}_r$ and $\tilde{v}_\parallel$ from a linear eigenmode
  - Drift waves (Coppi NF '02)
  - With intensity gradient (Gürcan PoP '10)
- Radially global effects via gyrokinetic simulation
  - GTS: magnetic & $\boldsymbol{E} \times \boldsymbol{B}$ shear, intensity gradients, neoclassical effects (Wang et al PRL '09, '11)
  - XGC1: avalanche momentum & heat transport (Ku et al NF '12)
- Corrections to fluxtube gyrokinetics (Parra and Barnes PPCF '15)
  - Neoclassical perturbation to turb mom transport
  - Turbulence inhomogeneity & finite orbit widths

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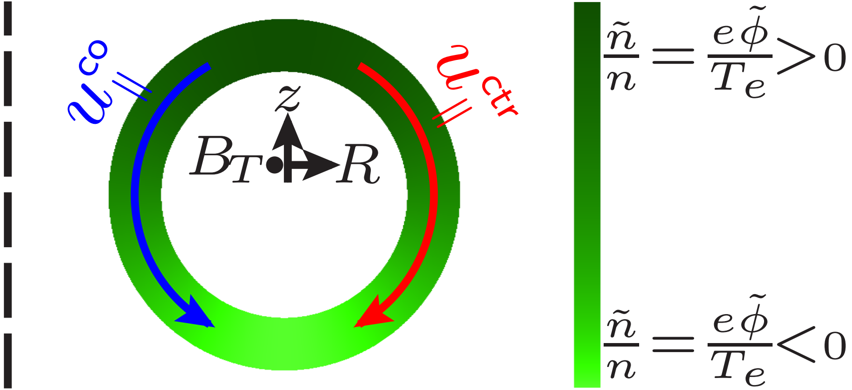
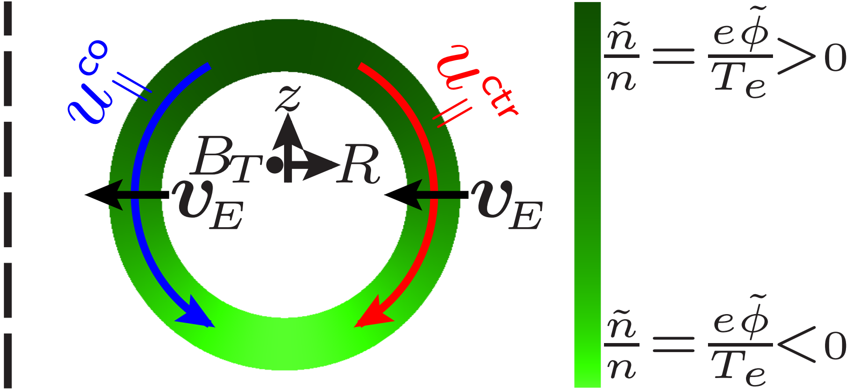
Free-energy flow in phase space + higher-order part of $\boldsymbol{E} \times \boldsymbol{B}$ drift $\Rightarrow$ residual stress

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.



I. Example: axisymmetric, $m = 1$, low-frequency density fluctuations.
$E_\parallel = -b_p(\partial_\theta \phi)/r$ accelerates ions out of density hump.
$E_\parallel v_\parallel = -b_p v_\parallel(\partial_\theta \phi)/r$ transfers energy to ion parallel flows.

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.

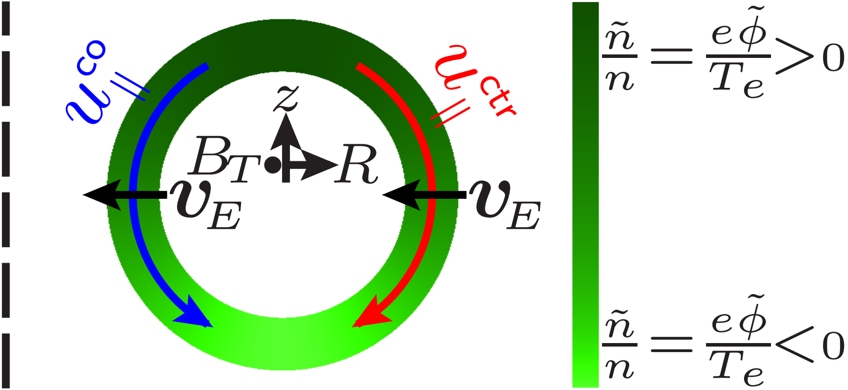


II.Weak radial $\boldsymbol{E} \times \boldsymbol{B}$ drift $v_E^x \sim -(cb_T/Br)\partial_\theta \phi$ advects ions.

Net exhaust of cocurrent momentum: $\Pi_{\varphi i}^{(2)} \sim -m_i R b_T^2 c(\partial_\theta \phi) f_i v_\parallel / B_p$

Momentum flux∝energy transfer because $E_\parallel / b_p = -\partial_\theta \phi = E_\perp / b_T$.

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.



III. Slow poloidal potential variation in $\partial_\theta \phi \sim k_\parallel \phi / b_p$:

- neglected by fluxtube orderings, but
- breaks symmetry because $\hat{b}$ neither parallel nor perp to $\hat{\varphi}$.

Model: energy- and momentum-conserving gyrokinetics

Full-F gyrokinetic equation conserves energy and toroidal angular momentum. After exact cancellations, toroidal angular momentum evolves as

$$\partial_t \underbrace{\left\langle F_s m_s v_\| b_\varphi \right\rangle}_{\| \text{ tor mom}} - \partial_t \underbrace{\left\langle \boldsymbol{P} \cdot \nabla A_\varphi \right\rangle / c}_{\boldsymbol{E}\times\boldsymbol{B} \text{ tor mom}} = -\partial_V \underbrace{\left\langle F_s m_s v_\| b_\varphi \dot{\boldsymbol{R}} \cdot \nabla V \right\rangle}_{\text{flux of } \| \text{ tor mom and } \ldots} - \underbrace{\left\langle F_s \partial_\varphi H \right\rangle}_{\text{of } \boldsymbol{E}\times\boldsymbol{B} \text{ tor mom}} .$$

Simplify to delta-f system using small-amplitude and fluxtube approximations:

$$\partial_t f_s + \frac{c}{B_0}\{J_0\phi, h_s\} + v_\| \nabla_\| h_s - \frac{\mu \nabla_\| B}{m_s} \partial_{v_\|} h_s - \frac{m_s v_\|^2 + \mu B}{2Ze} \mathscr{K}(h_s) = F'_{sM} \frac{c}{B_0} \partial_y J_0 \phi,$$

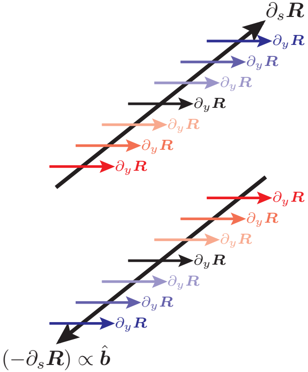
$$h_s \doteq f_s + (F_{sM}/T_{s0}) Ze J_0 \phi, \qquad \sum_s \int \mathrm{d}W \, Ze J_0 f_s = \sum_s n_{s0} Z^2 e^2 \frac{1-\Gamma_{0s}}{T_{s0}} \phi,$$

- Solution (f_i, f_e, ϕ) used to evaluate fluxes on RHS of momentum eqn.
- Conserves a free energy, essential for momentum result.
- In fluxtube limit, satisfies symmetry $(x, y, s, v_\|, \mu) \to (-x, y, -s, -v_\|, \mu)$.

Field-aligned coordinates enable the symmetry argument.

Most gyrokinetic models use a variant of the following coordinates:

- Flux label (radial coordinate) x
  - Axisymmetric & satisfies $\hat{b} \cdot \nabla x = 0$
- "Binormal" coordinate $y \propto (\zeta - q\theta)$
  - Satisfies $\hat{b} \cdot \nabla y = 0$
- Poloidal angle (parallel coordinate) $s \propto \vartheta$
  - Axisymmetric



These choices have the key properties that:

- $\hat{b} \cdot \nabla = (\hat{b} \cdot \nabla s)\partial_s$ (from definitions of x, y)
  - Only slow variation in $s \propto \vartheta$
- $\partial_y \propto \hat{\varphi} \cdot \nabla$ (from definitions of x, s)
  - Retains simple periodicity in y
  - ∂_y vanishes for all equilibrium quantities

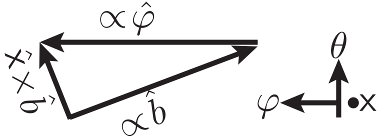
These properties underlie the Peeters/Parra symmetry argument, which is seen to use a somewhat "skew" transform, giving rise to symmetry breaking.

The radial $\boldsymbol{E} \times \boldsymbol{B}$ drift with true $\nabla_\perp \phi$ breaks the symmetry.

Define convenient directions

$$\hat{x} \doteq \frac{\nabla x}{|\nabla x|}, \quad \hat{p} \doteq \hat{\varphi} \times \hat{x}$$

and decompose $\hat{b} = b_T \hat{\varphi} + b_p \hat{p}$.



Use $\hat{x} \times \hat{b} = (\hat{\varphi} - b_T \hat{b})/b_p$ to evaluate

$$\boldsymbol{v}_E \cdot \hat{x} = \frac{c}{B} \hat{b} \times \nabla (J_0 \phi) \cdot \hat{x} = \frac{c}{b_p B} (\hat{\varphi} - b_T \hat{b}) \cdot \nabla (J_0 \phi),$$

Symmetry prevents first term $\propto \hat{\varphi} \cdot \nabla J_0 \phi \propto \partial_y J_0 \phi$ from driving residual stress.
Second term cancels the parallel gradient included in $\hat{\varphi} \cdot \nabla J_0 \phi \neq \hat{x} \times \hat{b} \cdot \nabla J_0 \phi$:

- Nominally smaller than the first term, by $k_\parallel / k_\perp b_p$, but
- Contributes a symmetry-breaking term to momentum flux $[f_s m_s v_\parallel b_\varphi v_E^x]$:*

$$\Pi_{\varphi i}^{(2)} = \frac{1}{V_{\mathrm{pl}}} \frac{-2\pi c}{B^\theta V'} \int \mathrm{d}\Lambda \, f_i m_i v_\parallel b_\varphi^2 \nabla_\parallel J_0 \phi$$

*T. Sung et al, Phys. Plasmas **20**, 042506 (2013).

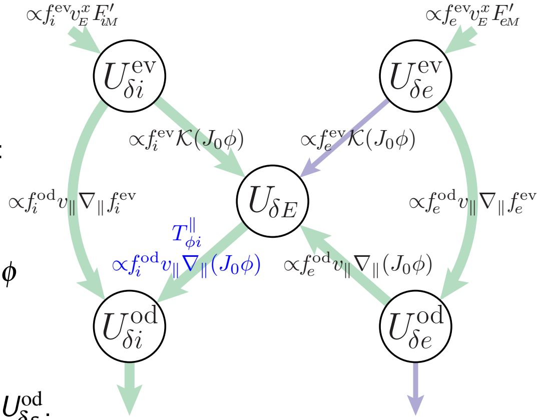
If ion parallel flows are excited, co-current momentum flows out.

$f_s^{\mathrm{ev}}(v_\parallel) \doteq [f_s(v_\parallel)+f_s(-v_\parallel)]/2$

$f_s^{\mathrm{od}}(v_\parallel) \doteq [f_s(v_\parallel)-f_s(-v_\parallel)]/2$

Free energy has 5 parts ($s = i,e$):

$U_{\delta s}^{\mathrm{ev}} \doteq \int \mathrm{d}\Lambda\, T_{s0}(f_s^{\mathrm{ev}})^2/2F_{sM}$
$U_{\delta s}^{\mathrm{od}} \doteq \int \mathrm{d}\Lambda\, T_{s0}(f_s^{\mathrm{od}})^2/2F_{sM}$
$U_{\delta E} \doteq \frac{1}{2}\int \mathrm{d}V \sum_s n_{s0}\frac{Z^2e^2}{T_{s0}}\phi(1-\Gamma_{0s})\phi$



- Heat flux down the gradient ($\propto f_s^{\mathrm{ev}} v_E^x F'_{sM}$)$\Rightarrow$energy to $U_{\delta s}^{\mathrm{ev}}$.
- Parallel dissipation$\Rightarrow$sink for $U_{\delta s}^{\mathrm{od}}$.
- Necessitates transfer $U_{\delta s}^{\mathrm{ev}} \rightarrow U_{\delta s}^{\mathrm{od}}$, at low frequencies mostly $U_{\delta s}^{\mathrm{ev}} \rightarrow U_{\delta i}^{\mathrm{od}}$.
- This forces $T_{\phi i}^{\parallel} \doteq -V_{\mathrm{pl}}^{-1}\int \mathrm{d}\Lambda Zev_\parallel f_i \nabla_\parallel J_0\phi = -V_{\mathrm{pl}}^{-1}\int \mathrm{d}\Lambda Zev_\parallel f_i^{\mathrm{od}} \nabla_\parallel J_0\phi > 0$
- Implies co-current outflux: $\Pi_\varphi^{(2)} \approx (2\pi c m_i R_0^2/ZeB^\theta V')T_{\phi i}^{\parallel} \sim (R_0/\Omega_{ci\theta})T_{\phi i}^{\parallel}$

When ion Landau damping is significant, one obtains counter-current rotation peaking with a simple scaling.

Let $f_L \doteq T^{\parallel}_{\phi i}/[V_{\rm pl}^{-1}\sum_s \int \mathrm{d}\Lambda\, f_s \frac{c}{B_0}(\partial_y J_0\phi)\, T_{s0} F'_{sM}/F_{sM}] \sim T^{\parallel}_{\phi i}/[\sum_s Q_s/L_{Ts}]$ be the fraction of free energy that passes through $T^{\parallel}_{\phi i}$, then

$$\Pi^{(2)}_{\varphi} \sim \frac{R_0}{\Omega_{ci\theta}} T^{\parallel}_{\phi i} \sim f_L \frac{R_0}{\Omega_{ci\theta}} \sum_s \frac{Q_s}{L_{Ts}} \sim f_L \frac{\rho_{i\theta}}{L_\perp}\left(\frac{R_0}{v_{ti}} Q\right).$$

Balance viscosity against this residual stress:

$$\chi_\varphi n_{i0} m_i R_0 \partial_r v_\varphi = \Pi^{(2)}_{\varphi} \sim f_L \frac{R_0}{\Omega_{ci\theta}} \sum_s \frac{Q_s}{L_{Ts}}.$$

Define Prandtl number $\mathrm{Pr} \doteq \chi_\varphi/\chi_i$, then solve for peaking:

$$\partial_r v_\varphi \sim \frac{f_L}{\mathrm{Pr}} \frac{v_{ti}}{\Omega_{ci\theta} L_{Ti}} \left[\sum_s \frac{Q_s/L_{Ts}}{Q_i/L_{Ti}}\right] \frac{v_{ti}}{L_{Ti}}.$$

Assume flat current $2\pi r B_p \sim (4\pi/c) I_p (r^2/a^2)$ to get dimensional peaking

$$a\partial_r v_\varphi \sim 5 \frac{f_L}{\mathrm{Pr}} \frac{a^3}{L_{Ti}^2 r} \frac{T_{i0}(\mathrm{keV})}{Z I_p(\mathrm{MA})} \left[\sum_s \frac{Q_s/L_{Ts}}{Q_i/L_{Ti}}\right] \mathrm{km/s}.$$

comparable with peaking measured on DIII-D, C-mod, TCV, and KSTAR.

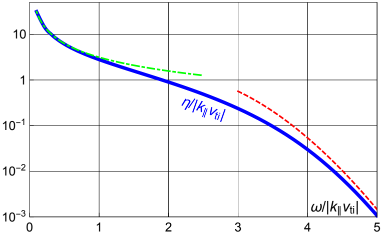
Landau's calculation shows when $T^{\parallel}_{\phi i}$ may be large or small.

Landau* solved dissipationless equations isomorphic to a simple limit of our delta-f system, with energy equations

$$\partial_t U_{\delta i} = -\int \mathrm{d}\Lambda\, f_i Zev_{\parallel}\nabla_{\parallel}\phi = V_{\mathrm{pl}} T^{\parallel}_{\phi i},$$

$$\partial_t U_{\delta E} = \int \mathrm{d}\Lambda\, f_i Zev_{\parallel}\nabla_{\parallel}\phi = -V_{\mathrm{pl}} T^{\parallel}_{\phi i},$$

finding $T^{\parallel}_{\phi i} > 0$ for all solutions, so $U_{\delta E} \to 0$ but $U_{\delta i} \neq 0$ for $t \to \infty$.



He further evaluated the Landau damping rate $\eta = V_{\mathrm{pl}} T^{\parallel}_{\phi i}/U_{\delta E}$, finding:

- strong damping for low frequencies $\omega \lesssim k_{\parallel} v_{ti}$, and
- exponentially small damping for $\omega \gg k_{\parallel} v_{ti}$.

Rotation drive by $T^{\parallel}_{\phi i}$ is therefore sensitive to turbulent frequencies, and turns off for $\omega \gg k_{\parallel} v_{ti}$.

*L. Landau, J. Phys. (U.S.S.R.) **10**, 25 (1946).

Summary

A geometrically higher-order portion of the $\boldsymbol{E} \times \boldsymbol{B}$ drift causes a nondiffusive momentum flux that:

- results from symmetry-breaking by excitation of ion parallel flows
  - does not require $\langle v_\varphi \rangle$ or $\nabla \langle v_\varphi \rangle \Rightarrow$residual stress
  - a fully nonlinear mechanism, not quasilinear
- causes counter-current rotation peaking in the core
- may drive experimentally relevant rotation peaking around

$$a\partial_r v_\varphi \sim 5 \frac{f_L}{\mathrm{Pr}} \frac{a^3}{L_{Ti}^2 r} \frac{T_{i0}(\mathrm{keV})}{Z I_p(\mathrm{MA})} \Big[\sum_s \frac{Q_s / L_{Ts}}{Q_i / L_{Ti}} \Big] \mathrm{km/s}.$$

- acts only when turbulence is at low enough frequencies to excite ion parallel flows
  - allows for both hollow and flat rotation profiles