Plasmoid Instability in Forming Current Sheets

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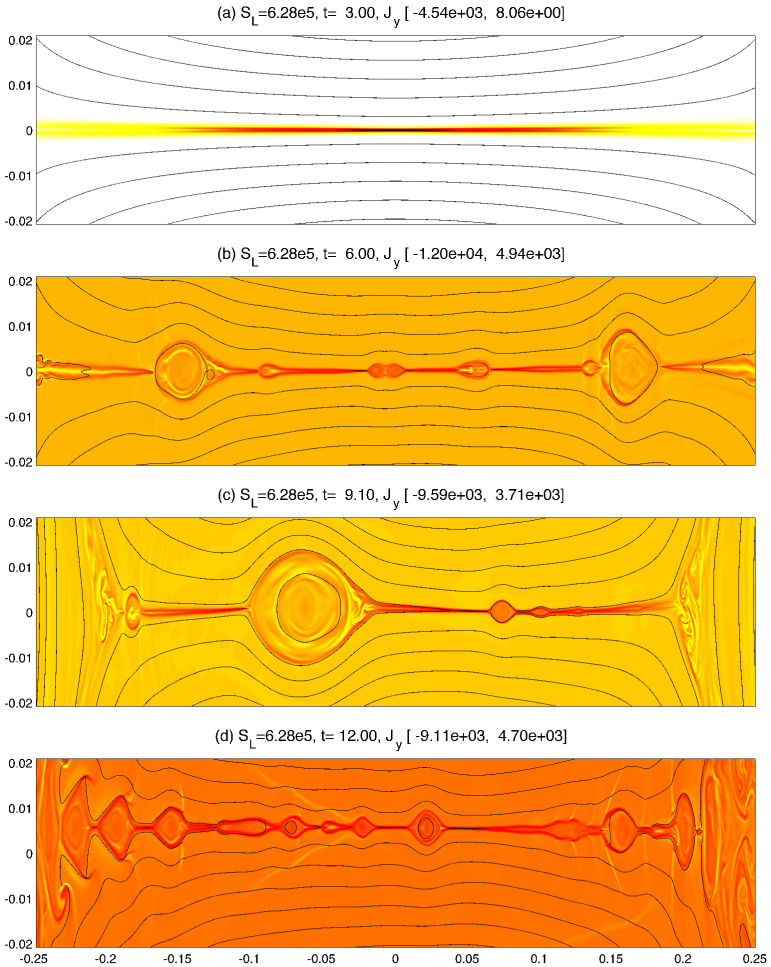
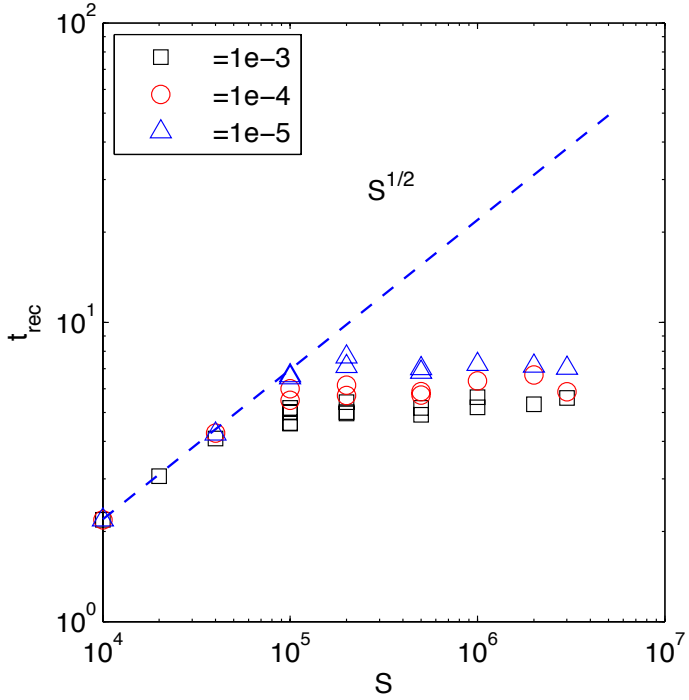
Outline

- Importance of the Plasmoid Formation
- Plasmoid Instability in Sweet-Parker Current Sheets
  - Scalings assuming Sweet-Parker
  - Problems with this approach
- A General Theory of the Plasmoid Instability
  - Principle of least time for plasmoids
  - Scaling laws
- Concluding Remarks

Sweet-Parker at large S is too slow to be true...

- Sweet-Parker scalings do not hold for $S = Lv_A/\eta > S_{critical}$ because of the *plasmoid instability*!





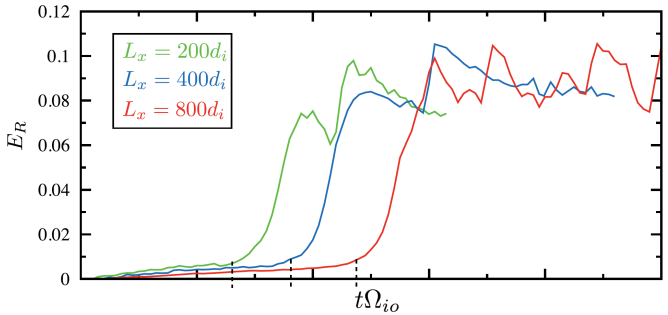
Bhattacharjee *et al.*, PoP 2009

Huang & Bhattacharjee, PoP 2010

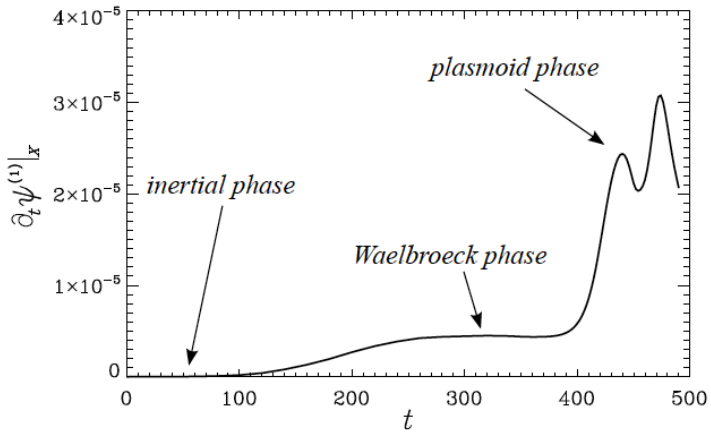
(also Uzdensky *et al.*, PRL 2010)

Breakdown of the Sweet-Parker model at large S

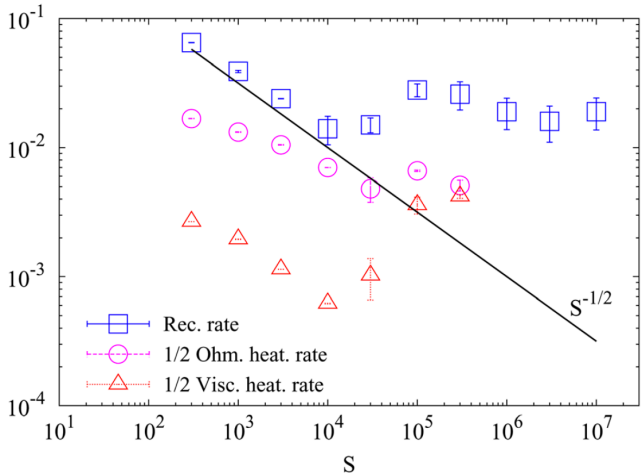
- Speed-up of the reconnection process due to plasmoid formation has been shown by many other research groups



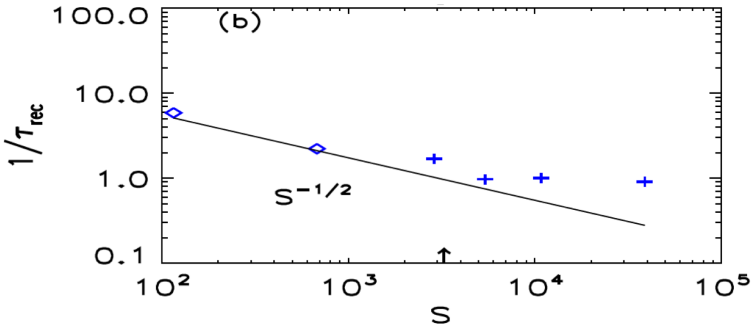
Daughton *et al.*, PRL 2009



Comisso *et al.*, PoP 2015



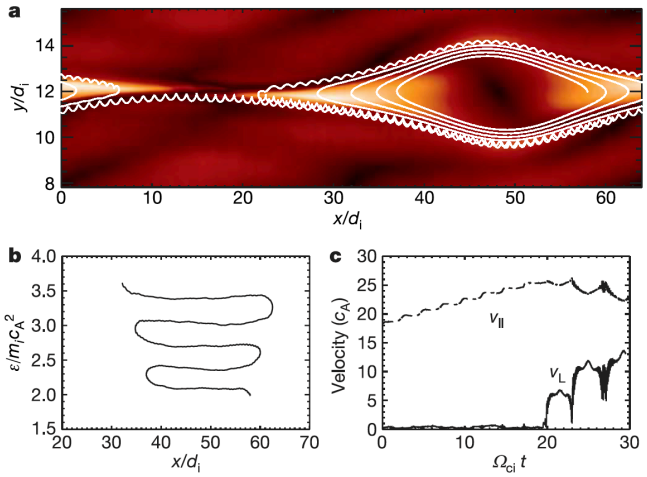
Loureiro *et al.*, PoP 2012



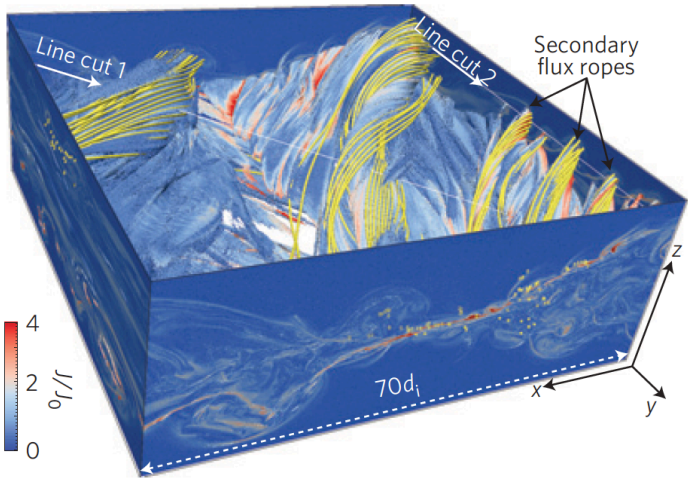
Ebrahimi & Raman, PRL 2015

Other important effects of the plasmoid formation

- The formation of plasmoids has other crucial implications:
  - particle acceleration
  - self-generated turbulent reconnection
  - .....



Drake *et al.*, Nature 2006



Daughton *et al.*, Nature 2011

- Sironi & Spitkovsky 2014
- Guo *et al.* 2014/15/16
- Werner *et al.* 2016, ...

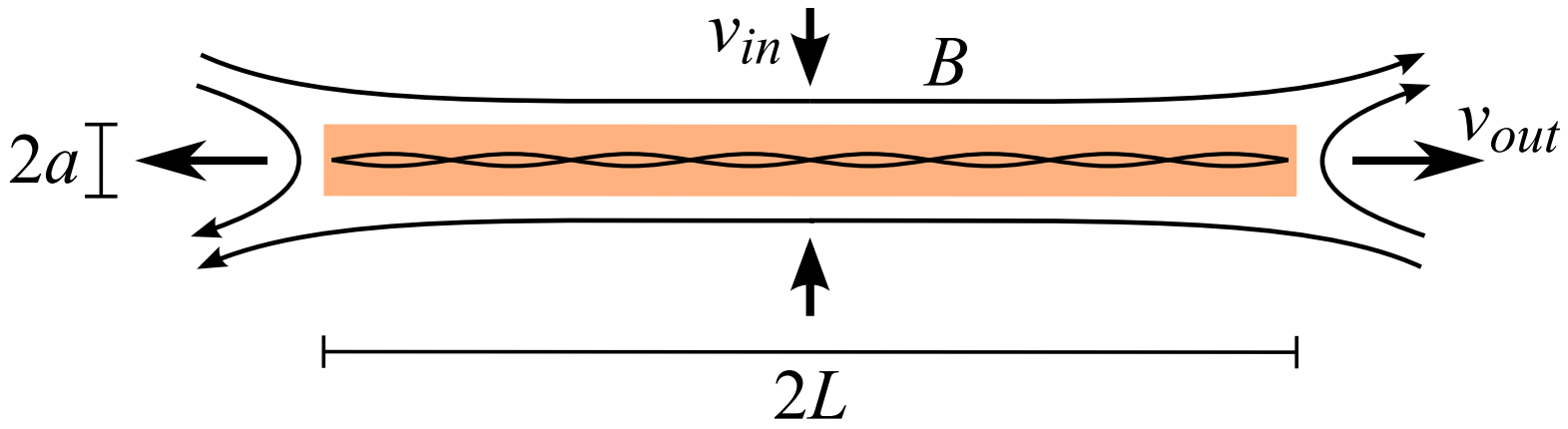
- Oishi *et al.* 2015
- Huang & Bhattacharjee 2016
- .....

An important issue to address

- We have seen that the formation of plasmoids plays a crucial role in magnetic reconnection

BUT

- What is the dynamical picture behind the onset and development of the plasmoid instability?



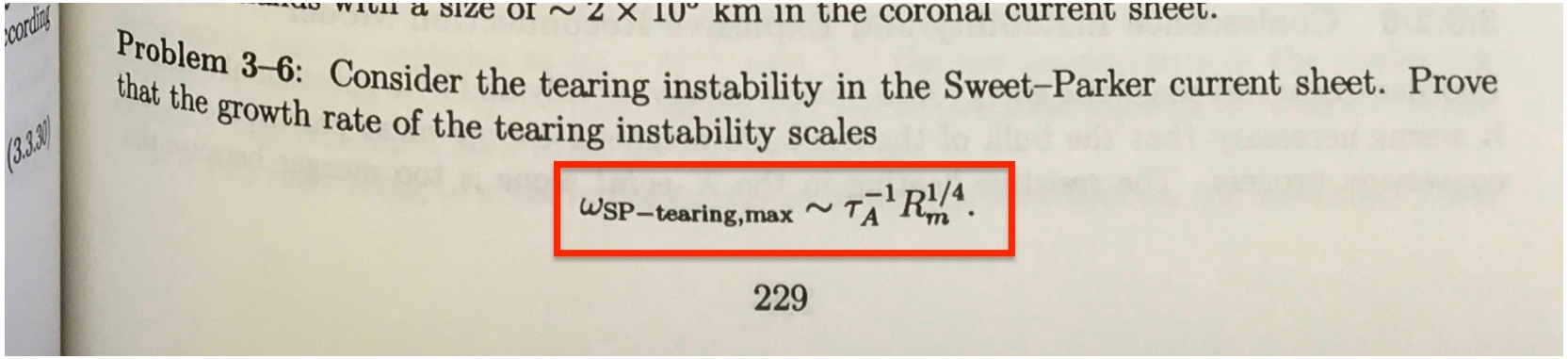
- We will see why the previous knowledge of the plasmoid instability was unsatisfactory

AND

- We will see what are the properties of this instability.

Assuming a Sweet-Parker aspect ratio...

- Tajima & Shibata, *Plasma Astrophysics* (1997)



Problem 3–6: Consider the tearing instability in the Sweet–Parker current sheet. Prove that the growth rate of the tearing instability scales

$$\boxed{\omega_{\mathrm{SP-tearing,max}} \sim \tau_A^{-1} R_m^{1/4}.}$$

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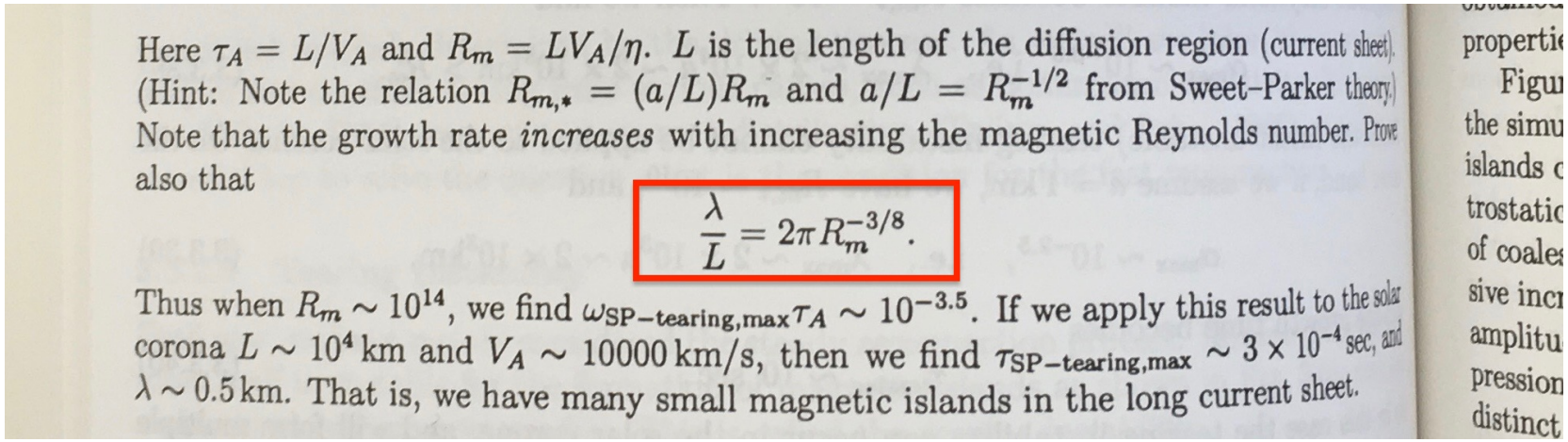


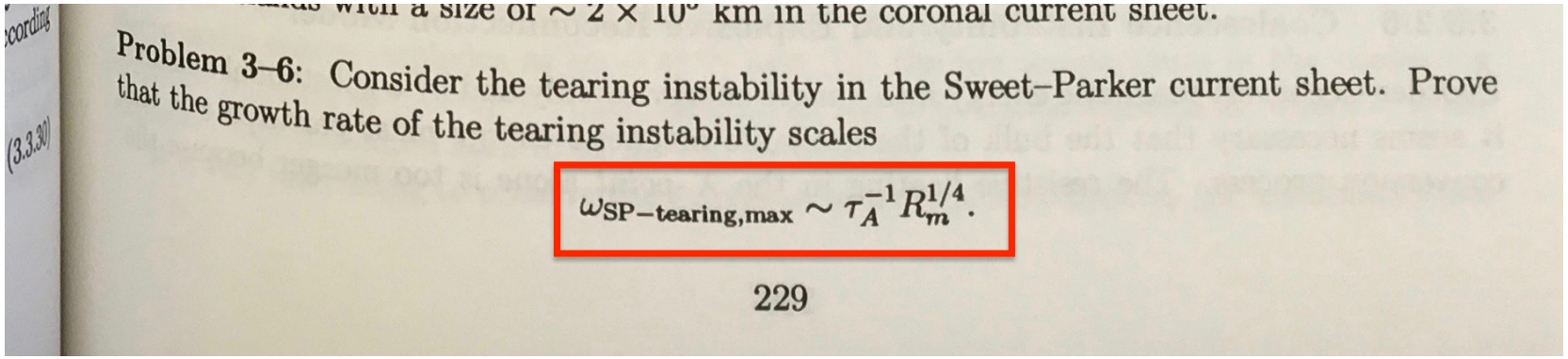
Here $\tau_A = L/V_A$ and $R_m = LV_A/\eta$. L is the length of the diffusion region (current sheet). (Hint: Note the relation $R_{m,*} = (a/L)R_m$ and $a/L = R_m^{-1/2}$ from Sweet–Parker theory.) Note that the growth rate *increases* with increasing the magnetic Reynolds number. Prove also that

$$\boxed{\frac{\lambda}{L} = 2\pi R_m^{-3/8}.}$$

Thus when $R_m \sim 10^{14}$, we find $\omega_{\mathrm{SP-tearing,max}}\tau_A \sim 10^{-3.5}$. If we apply this result to the solar corona $L \sim 10^4$ km and $V_A \sim 10000$ km/s, then we find $\tau_{\mathrm{SP-tearing,max}} \sim 3 \times 10^{-4}$ sec, and $\lambda \sim 0.5$ km. That is, we have many small magnetic islands in the long current sheet.

(Note that the same result has been re-obtained 10 years later by Loureiro *et al.*, PoP 2007)

But there is a problem with this result...



with a size of $\sim 2 \times 10^5$ km in the coronal current sheet.

Problem 3–6: Consider the tearing instability in the Sweet–Parker current sheet. Prove that the growth rate of the tearing instability scales

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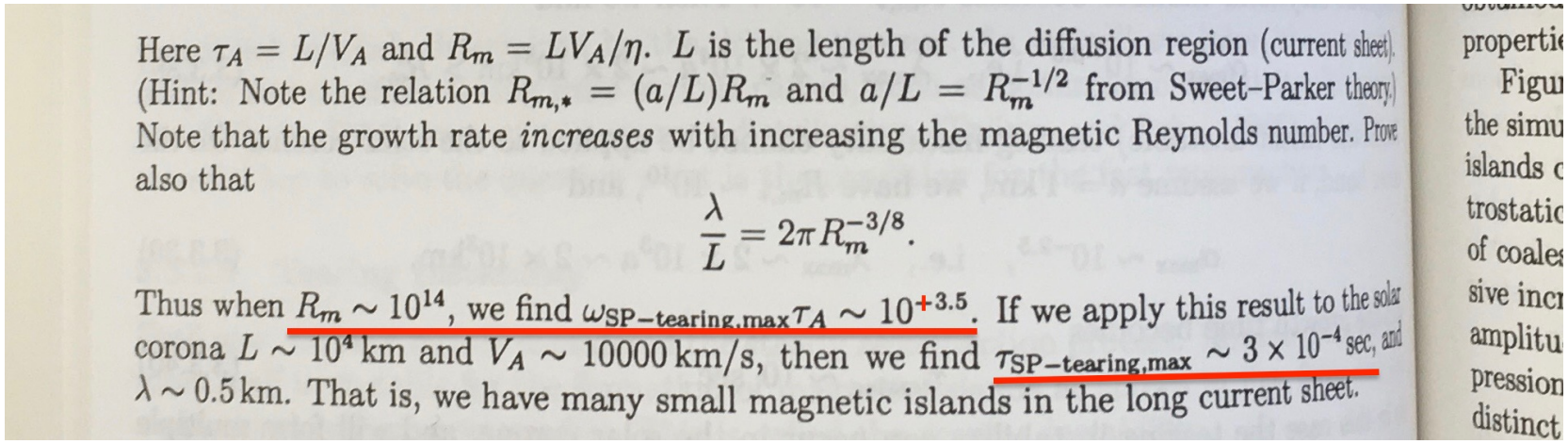

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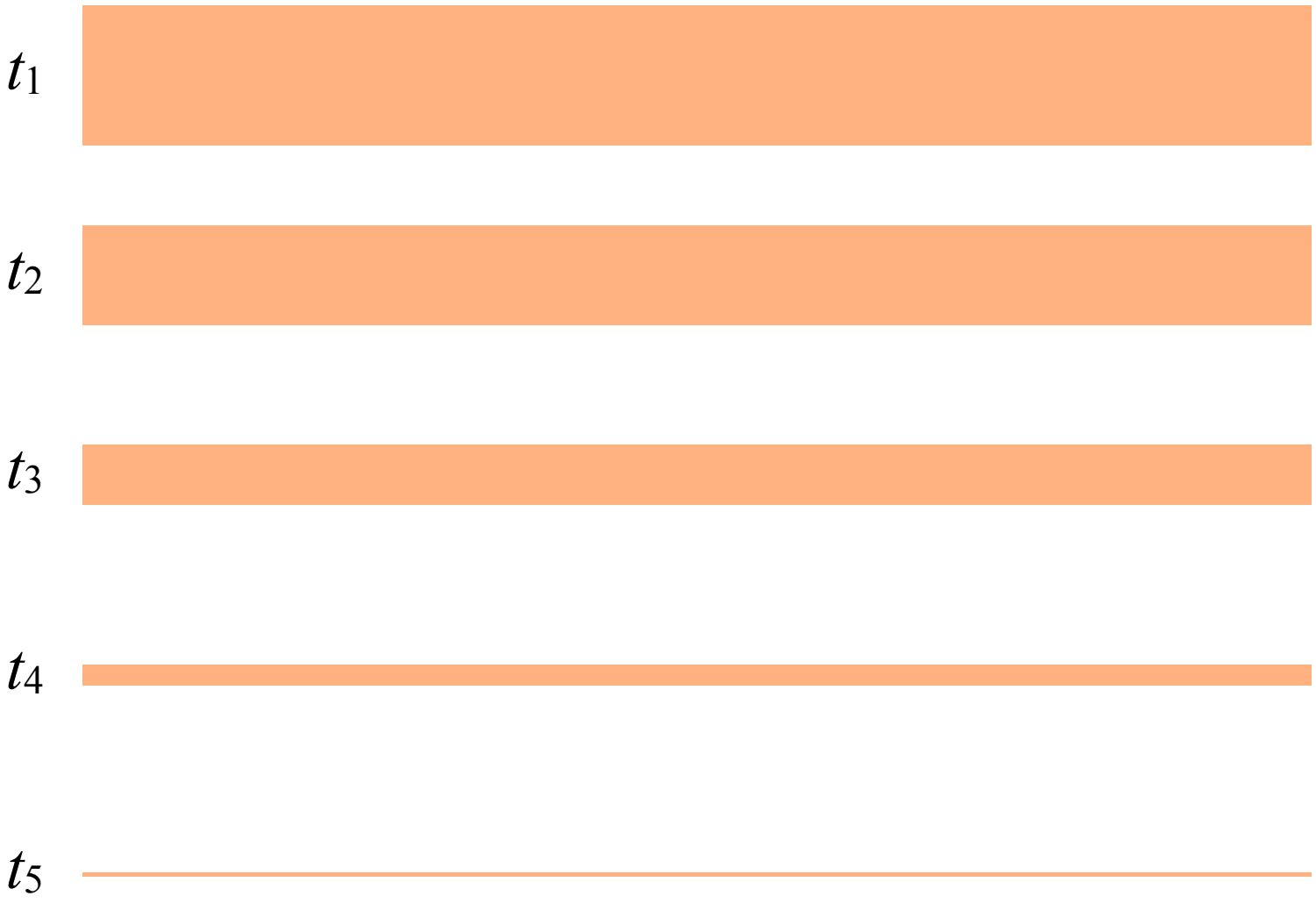
The instability growth rate is too fast for large S-values!

Sweet-Parker sheets *cannot form* in large S plasmas!

Since in reality current sheets form over time...

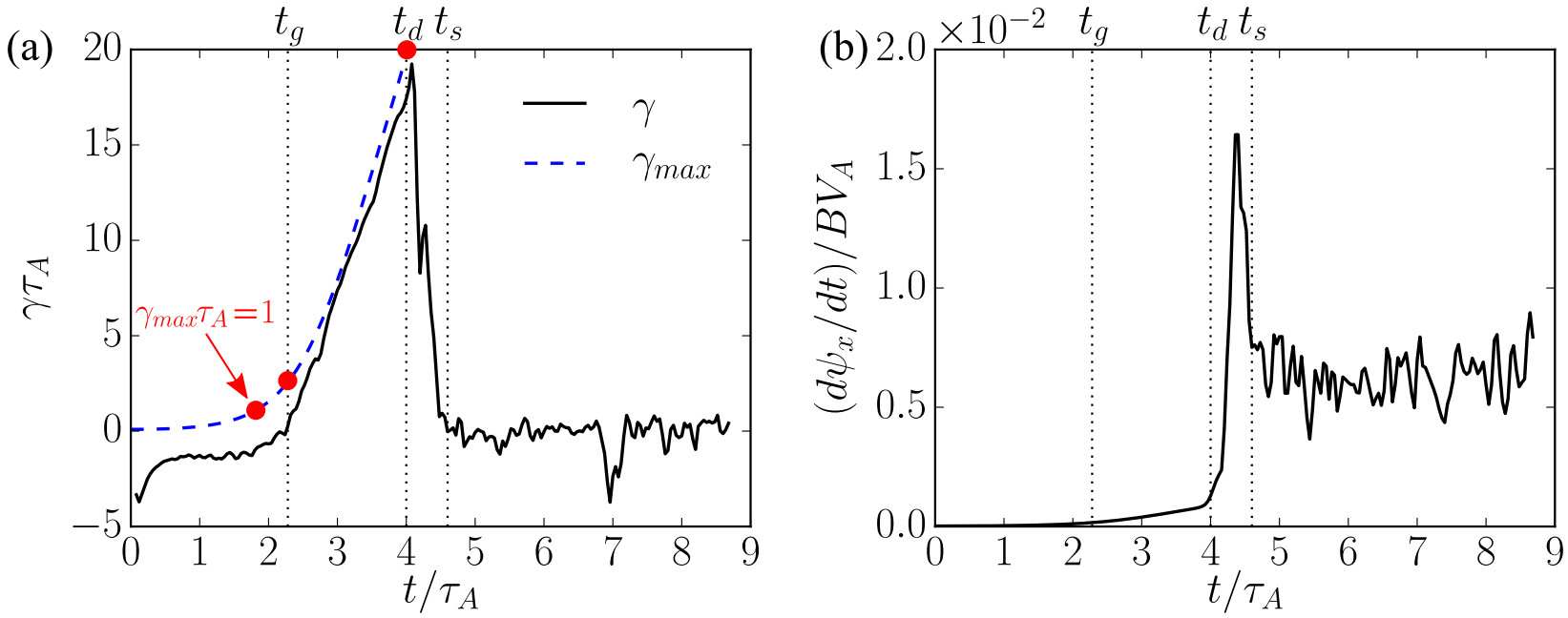
We need to consider a *time-evolving* current sheet



[1] Different recent theory: Uzdensky and Loureiro (2016)

A plot to keep well in mind

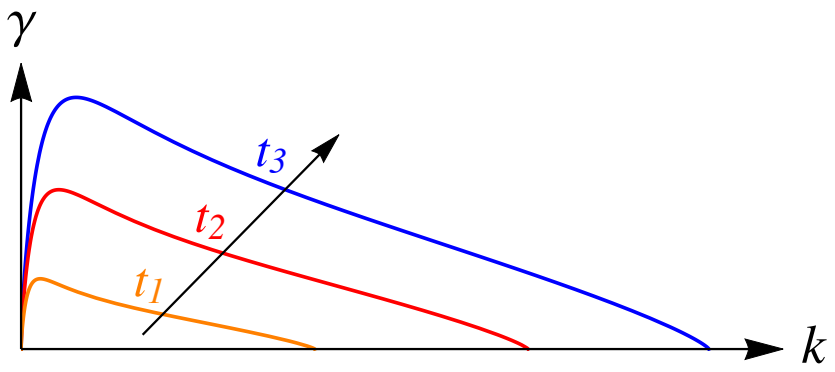
- The plasmoid instability remains quiescent for a certain time, and the fluctuation amplitude starts to grow only when $\gamma_{\max}\tau_A > O(1)$.
- Fast reconnection occurs at $t > t_d$.



Huang *et al.*, ApJ (2017)

General Theory [Comisso *et al.*, PoP 2016]

Tearing modes become unstable at different times and exhibit different instantaneous growth rates $\gamma(k,t)$

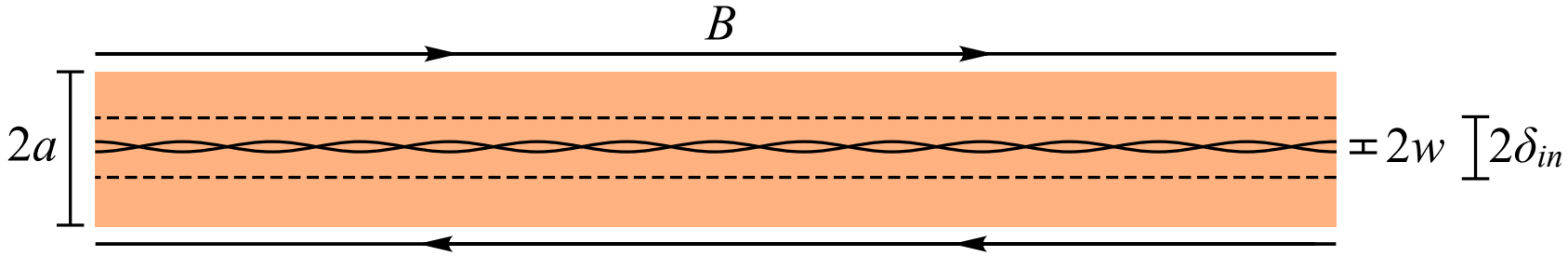


- Their amplitude changes in time according to

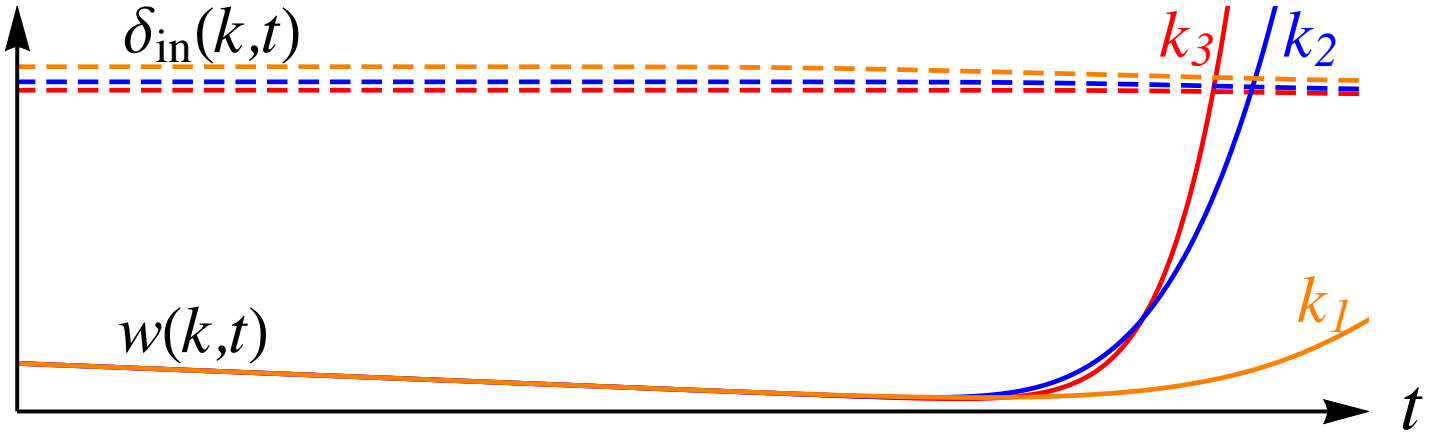
$$\psi(k,t) = \psi_0(k) \exp\left(\int_{t_0}^{t} \gamma(k,t')dt' \right).$$

- Their evolution becomes nonlinear when

$$w(k,t) = 2\sqrt{\frac{\psi a}{B}} \quad > \quad \delta_{\rm in}(k,t) = \left[\eta\gamma a^2/(kv_A)^2\right]^{1/4}.$$



General Theory [Comisso *et al.*, PoP 2016]



- Principle of Least Time for the Plasmoid Instability, i.e., *the mode of the plasmoid instability that emerges from the linear phase is the one that traverses it in the least time.*
- To implement this formulation, we introduce the function

$$F(k,t) := \delta_{\rm in}(k,t) - w(k,t)\,.$$

- Then, the least time principle is formulated as

$$F(k,t)|_{k_*,t_*} = 0\,, \qquad dt_*/dk_* = 0\,.$$

General Theory [Comisso *et al.*, PoP 2016]

- From Eqs. (1)-(1) we obtain the *least time plasmoid Eqs:*

$$\left\{\left(\gamma\bar{t}-\frac{1}{2}\right)\frac{\partial\gamma}{\partial k}+\frac{\gamma}{k}+2\frac{\gamma}{w_0}\frac{\partial w_0}{\partial k}\right\}\Bigg|_{k_*,t_*}=0$$

$$\left\{\ln\left(\frac{\delta_{\rm in}}{w_0}\frac{g^{1/2}}{f^{1/2}}\right)-\frac{1}{2}\int_{t_0}^{t}\gamma(t')dt'\right\}\Bigg|_{k_*,t_*}=0$$

with:

$$w_0=2\sqrt{\frac{\psi_0 a_0}{B_0}}\,,\quad \bar{t}=\frac{\partial}{\partial\gamma}\int_{t_0}^{t}\gamma(t')dt'\,,\quad f=\frac{a(t)}{a_0}\,,\quad g=\frac{B(t)}{B_0}\,.$$

- From these Eqs. it is possible to arrive at:

γ_*	(final growth rate)	L_*/a_*	(final aspect ratio)
k_*	(final wavenumber)	t_*	(elapsed time from t_0)
$\delta_{\rm in*}$	(final inner layer)	τ_p	(time from $\hat{\gamma}(\hat{k}_*,\hat{t}_{\rm on})>1/\tau$)

General Theory [Comisso *et al.*, PoP 2016]

Until now the equations are very general.

Let's try to be more specific...

- We are interested in the case where:

$$L \approx \text{const.}, \quad B_0 \approx \text{const.}, \quad a(t) = a_0 f(t).$$

- For the moment, we consider an exponentially thinning current sheet (**this will be generalized later**) of the form

$$\hat{a}(\hat{t})^2 = (\hat{a}_0^2 - \hat{a}_\infty^2)e^{-2\hat{t}/\tau} + \hat{a}_\infty^2 \,, \qquad \text{with } \hat{a}_\infty = S^{-1/2}$$

- Here and in the following:
  - lengths normalized by the current sheet half-length L
  - time normalized by the Alfvén time $\tau_A = L/v_A$
  - magnetic field normalized by the upstream field B_0

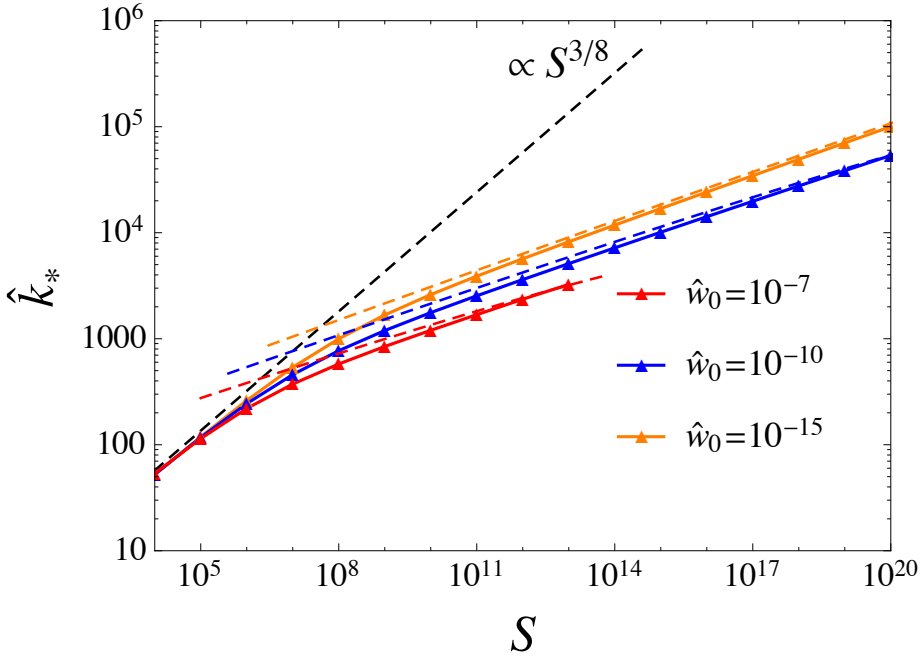
General Theory [Comisso *et al.*, PoP 2016]

- With some algebra we can see that there is a *Transitional S*

$$S_T = \frac{1}{\tau^4}\left[\ln\left(\frac{\tau^9}{C}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^4$$

above which the plasmoid instability change behavior!

- The slope of $\hat{k}_*$ substantially *decreases* at large S !!!!!!!!!!!



For $S < S_T$:

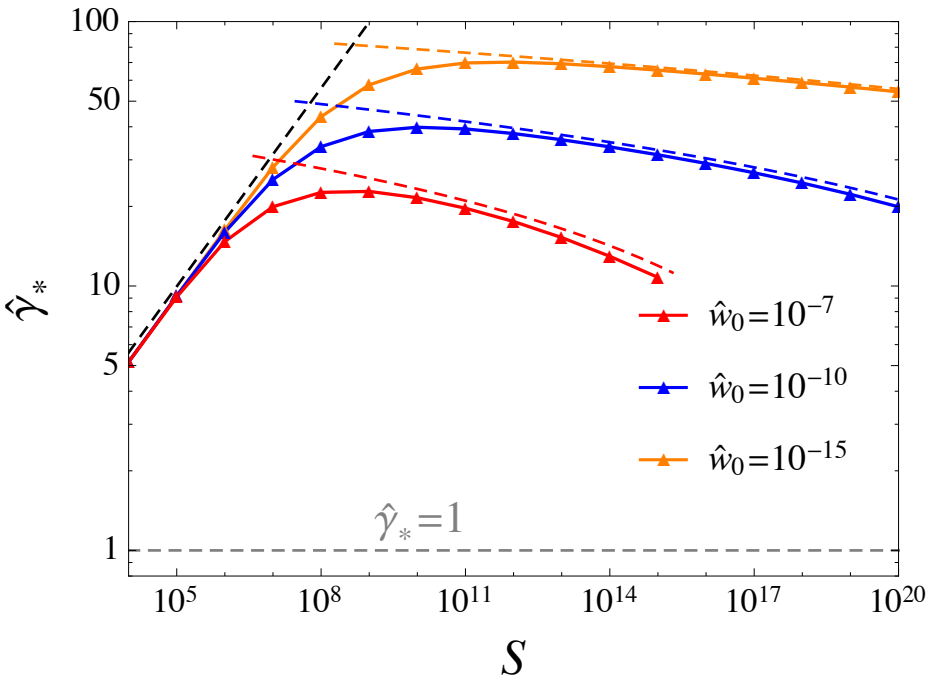
$$\hat{k}_* \sim S^{3/8}$$

For $S > S_T$:

$$\hat{k}_* \simeq c_k \frac{S^{1/6}}{\tau^{5/6}}\left[\ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{5/6}$$

General Theory [Comisso *et al.*, PoP 2016]

- The behavior of $\hat{\gamma}_*$ is *non-monotonic* in S !!!!!!!!!!!



For $S < S_T$:

$$\hat{\gamma}_* \sim S^{1/4}$$

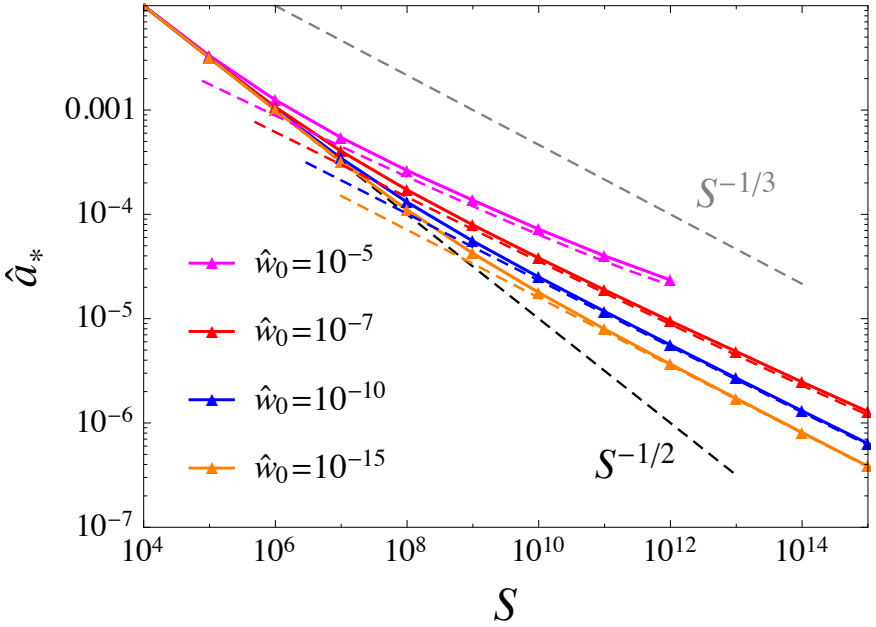
For $S > S_T$:

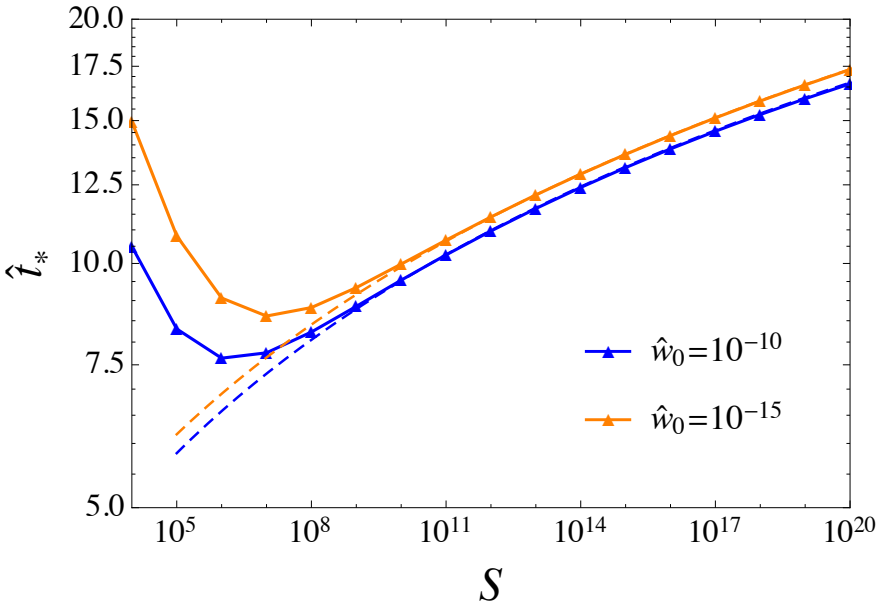
$$\hat{\gamma}_* \simeq \frac{c_\gamma}{\tau} \ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)$$

- The earlier scaling (black dashed line) is *not applicable* for large-S plasmas.
- $\hat{\gamma}_*$ can decrease with S because $\hat{\delta}_{\rm in}$ decreases with S $\Rightarrow$ less "space" to accelerate the perturbation growth

General Theory [Comisso *et al.*, PoP 2016]

- What about the disruption of the current sheet?





$$\hat{a}_* \simeq \frac{\tau^{2/3}}{S^{1/3}}\left[\ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{-2/3}, \quad \hat{t}_* \simeq \tau \ln\left\{\hat{a}_0 \frac{S^{1/3}}{\tau^{2/3}}\left[\ln\left(\frac{\tau}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{2/3}\right\}$$

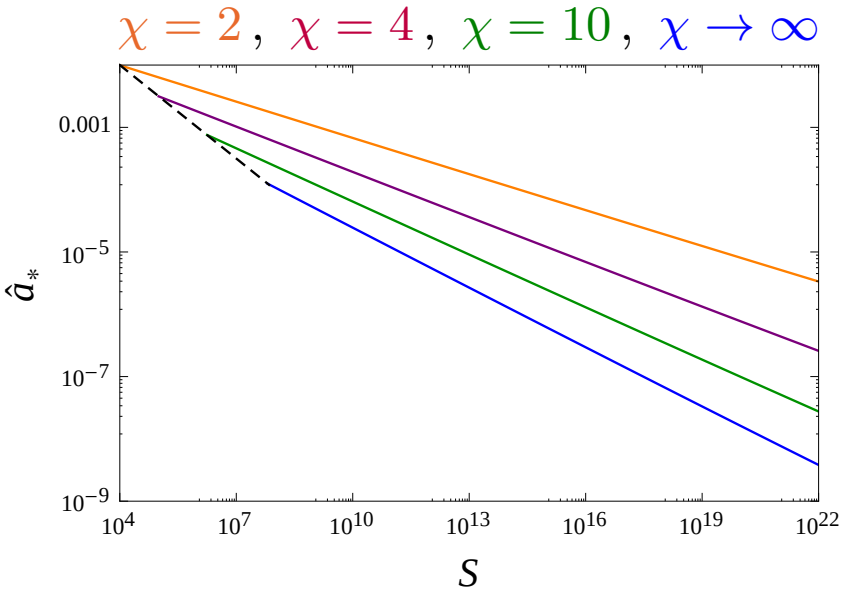
- Current sheets disrupt before the Sweet-Parker state can be achieved (as expected!)

General Theory [Comisso *et al.*, PoP 2016]

- To generalize the previous scaling laws, we consider the generalized current thinning function

$$\hat{a}(\hat{t})^2 = (\hat{a}_0^2 - \hat{a}_\infty^2)\left(\frac{\tau}{\tau + 2\hat{t}/\chi}\right)^\chi + \hat{a}_\infty^2 \,, \qquad \text{with } \hat{a}_\infty = S^{-1/2}$$

- The final aspect-ratio ($\frac{1}{\hat{a}_*}$) depends on the thinning process!



$[\tau = 1,\ \hat{w}_0 = 10^{-12},\ \hat{a}_0 = 1/2\pi]$

$$\hat{a}_* \sim \hat{a}_0^{\frac{2\zeta}{\chi}} \frac{\tau^\zeta}{S^{\frac{\zeta}{2}}} \left[\ln\left(\frac{\tau^{\frac{3\zeta}{2}}}{S^{\frac{6+3\zeta}{4}}} \frac{\hat{a}_0^{\frac{3\zeta}{\chi}}}{2^6 \varepsilon^3}\right)\right]^{-\zeta}$$

where $\zeta := \frac{2\chi}{4+3\chi}$

General Theory [Comisso *et al.*, PoP 2016]

- And finally... also the scaling laws of the plasmoid instability at large S depend on the thinning process!

$$\hat{\gamma}_* \sim S^{(3\zeta-2)/4} \left(\frac{\ln(\theta_R)}{\hat{a}_0^{2/\chi} \tau} \right)^{3\zeta/2} ,$$

$$\hat{k}_* \sim S^{(5\zeta-2)/8} \left(\frac{\ln(\theta_R)}{\hat{a}_0^{2/\chi} \tau} \right)^{5\zeta/4} ,$$

$$\hat{\delta}_{\mathrm{in}*} \sim S^{-(3\zeta+2)/8} \left(\frac{\hat{a}_0^{2/\chi} \tau}{\ln(\theta_R)} \right)^{3\zeta/4} ,$$

where

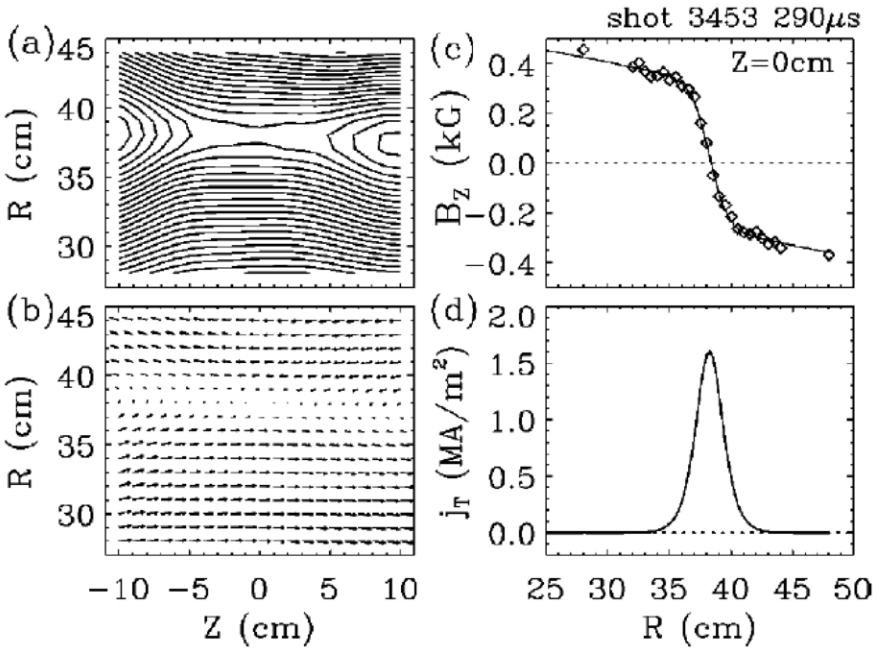
$$\theta_R := \frac{\tau^{3\zeta/2}}{S^{3(2+\zeta)/4}} \frac{\hat{a}_0^{3\zeta/\chi}}{2^6 \varepsilon^3} .$$

Connections with experiments and observations

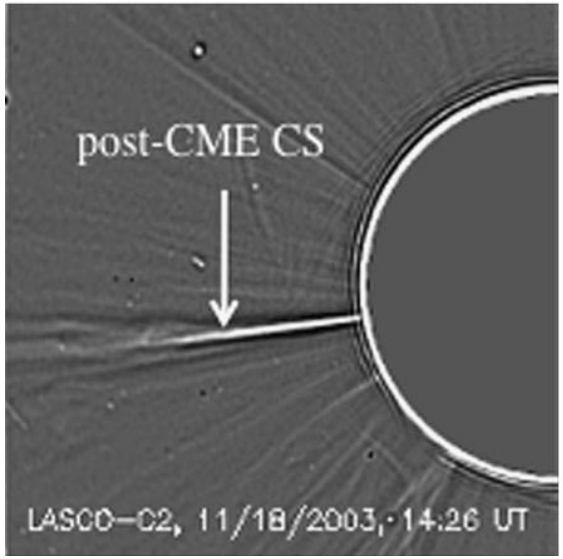
- Now we have a theory that can potentially predict the onset of fast magnetic reconnection.

  Opportunities to check the theory in the real world...

- The "easiest" quantities to check should be $\hat{a}_*$ and $\hat{t}_*$.



Ji *et al.*, PoP (1999)



Lin *et al.*, SSRv (2015)

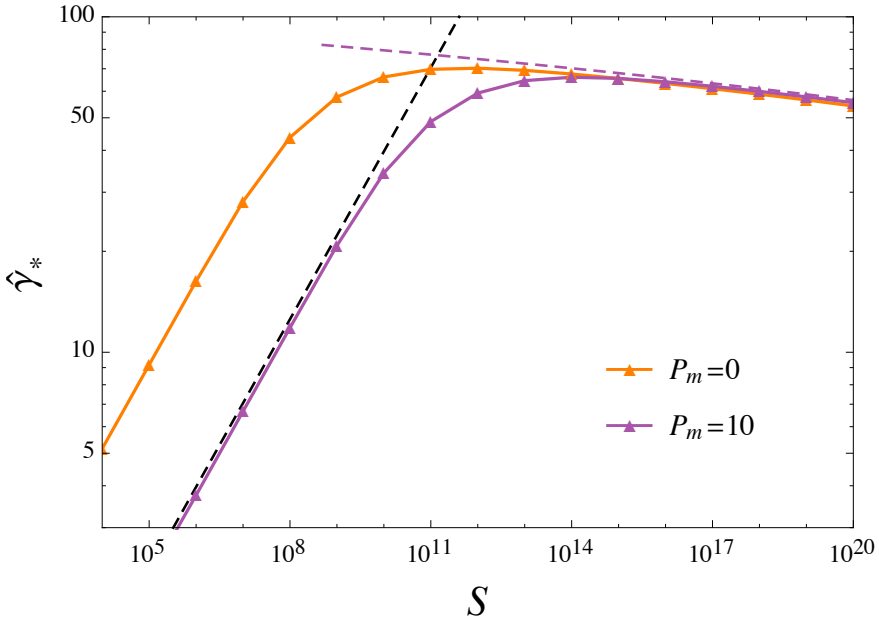
Concluding Remarks

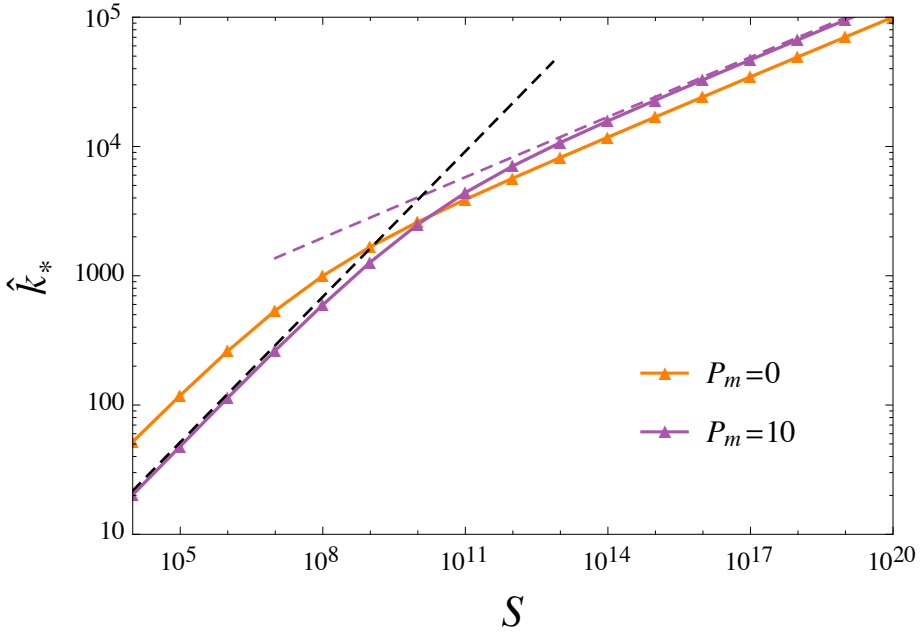
- *The scaling laws of the plasmoid instability are not simple power laws*, and depend on:
  - The Lundquist number (S)
  - The noise of the system (ψ_0)
  - The characteristic rate of current sheet evolution ($1/\tau$)
  - The thinning process (χ)
- In astrophysical systems, reconnecting current sheets break up before they can reach the Sweet-Parker aspect ratio.
  - The scaling laws of the plasmoid instability obtained assuming a Sweet-Parker current sheet are *inapplicable* to the vast majority of the astrophysical systems.
- How these scaling laws for the plasmoid instability affect the turbulent energy cascade? (stay tuned)

Supplement: plasma viscosity [arXiv:1707.01862]

Also plasma viscosity could be important in several systems.

- Plasma viscosity allows to extend the validity of the Sweet-Parker based scalings to larger S-values ($S_T \uparrow$).

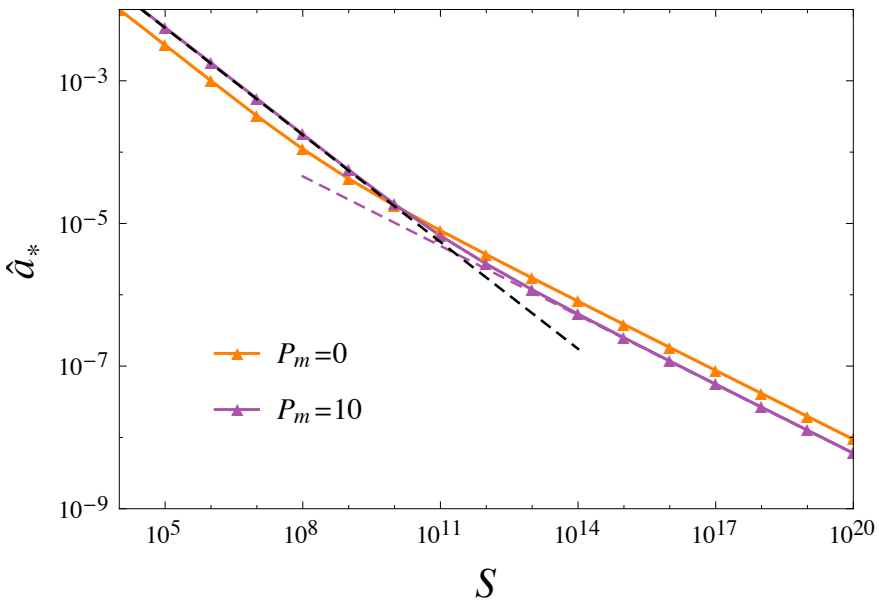


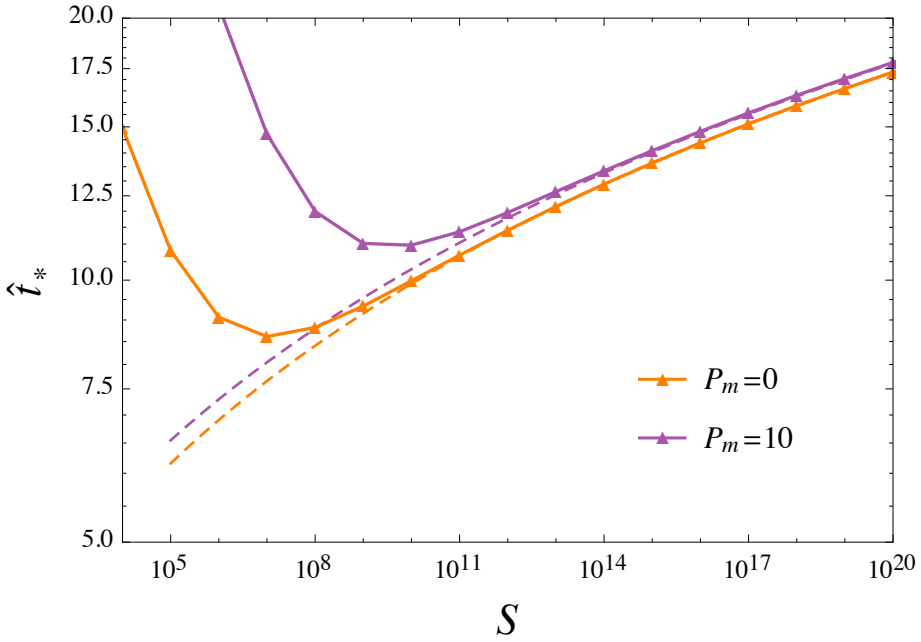


$$\hat{\gamma}_* \simeq \frac{\lambda_\gamma}{\tau} \ln\left(\frac{\tau P_m^{1/2}}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right), \qquad \hat{k}_* \simeq \lambda_k \frac{S^{1/6} P_m^{1/3}}{\tau^{5/6}} \left[\ln\left(\frac{\tau P_m^{1/2}}{S^2}\frac{\hat{a}_0^3}{\hat{w}_0^6}\right)\right]^{5/6}$$

Supplement: plasma viscosity [arXiv:1707.01862]

- At large S, plasma viscosity allows to reach *larger aspect ratios* of the reconnecting current sheets.





$$\hat{a}_* \simeq \frac{\tau^{2/3}}{S^{1/3} P_m^{1/6}} \left[\ln \left(\frac{\tau P_m^{1/2}}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{-2/3}, \quad \hat{t}_* \simeq \tau \ln \left\{ \frac{S^{1/3} P_m^{1/6}}{\tau^{2/3}/\hat{a}_0} \left[\ln \left(\frac{\tau P_m^{1/2}}{S^2} \frac{\hat{a}_0^3}{\hat{w}_0^6} \right) \right]^{2/3} \right\}$$