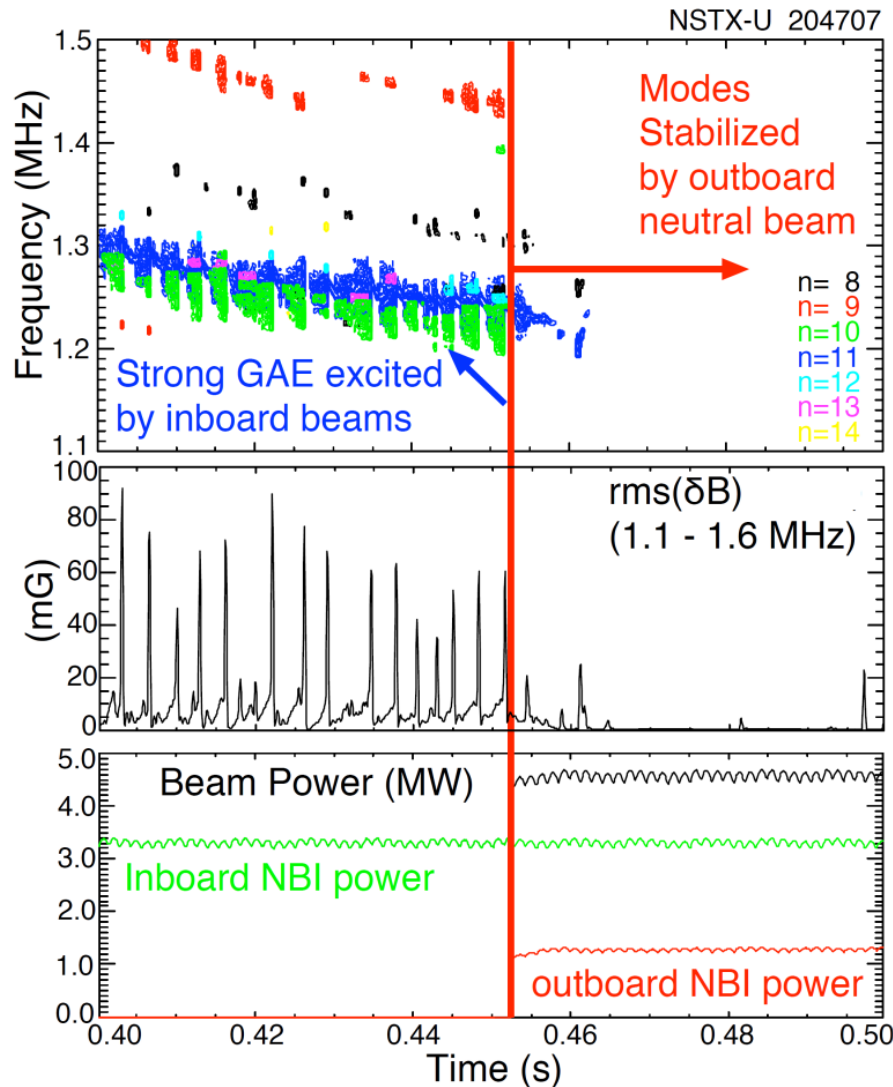# Numerical simulations of Global Alfven Eigenmode (GAE) stabilization in NSTX-U

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NSTX-U off-axis neutral beam suppresses Global Alfven Eigenmodes (GAEs) [E. Fredrickson, PRL 2017]

NSTX-U 204707

Modes Stabilized by outboard neutral beam

Strong GAE excited by inboard beams

n= 8
n= 9
n=10
n=11
n=12
n=13
n=14

rms(δB) (1.1 - 1.6 MHz)

Beam Power (MW)

Inboard NBI power

outboard NBI power

- Counter-propagating GAEs are frequently observed in the sub-cyclotron frequency range of $0.1f_{ci}$ up to $0.5f_{ci}$, in NSTX and NSTX-U.

- Driven by cyclotron resonance with beam ions.

- New neutral beam sources -> ability to control the fast ion distribution.

- Off-axis neutral beams inject fast ions onto trajectories largely parallel to the magnetic field, with pitch $0.8 < V_{||}/V < 1$.

- Reliable suppression of the counter-propagating GAE when an additional 1.3MW is injected using the outboard beam.

New neutral beam sources on NSTX-U allowed demonstration of GAEs stabilization

- NSTX-U had six beam sources, the original three from NSTX with tangency radii inboard of the magnetic axis ($R_{mag} \approx$ 1.1 m) at $R_{tan} \approx$ 0.7, 0.6, and 0.5 m, (labeled 1a, 1b, 1c), and three new outboard sources with R_{tan} = 1.3, 1.2, and 1.1 m (labeled 2a, 2b, 2c).

- Off-axis neutral beams inject fast ions onto trajectories largely parallel to the magnetic field, with pitch $0.8 < V_{||}/V < 1$.

- New neutral beam sources -> ability to control the fast ion distribution.
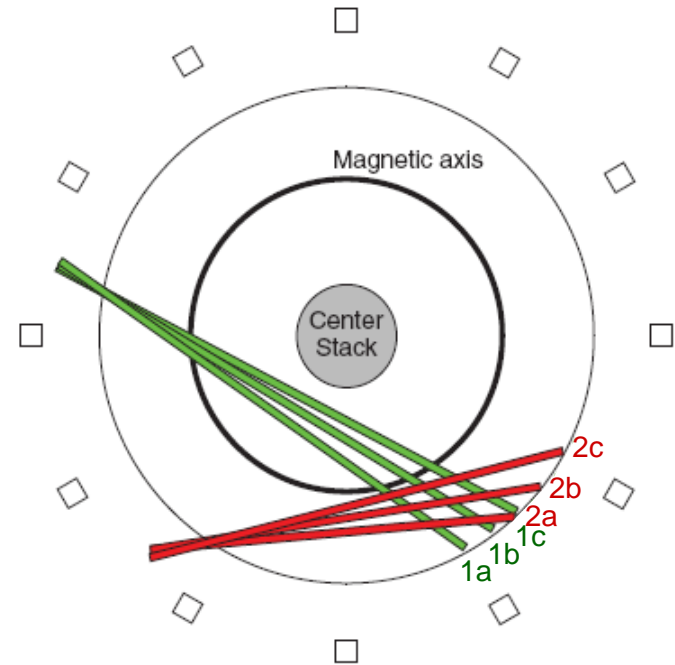
Sketch of neutral beam geometry. Original NSTX beams in green, labeled 1a, 1b, 1c; new beams for NSTX-U shown in red labeled 2a, 2b and 2c.

# GAE stabilization has been well documented for many NSTX-U shots
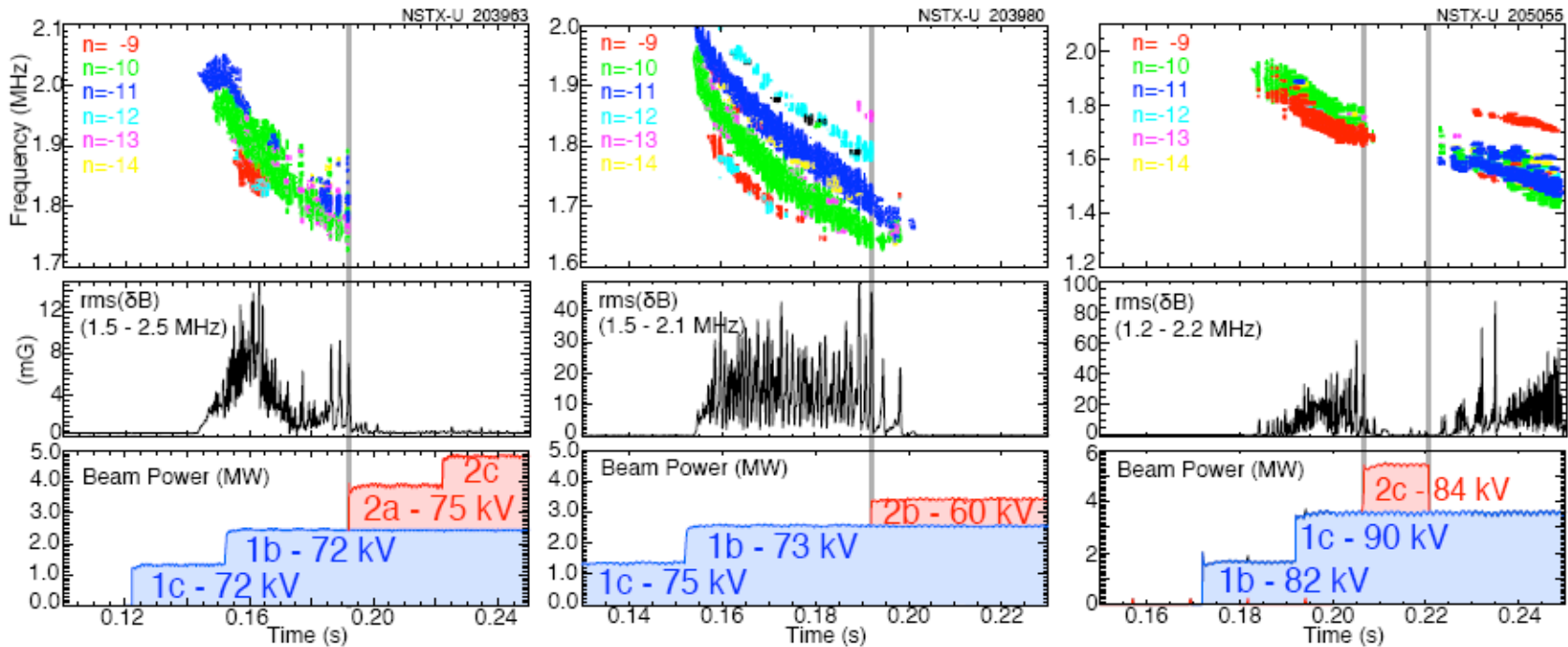
Three examples of GAE being suppressed by the injection of one of the three off-axis beam sources. Figure 1(c) also shows that the GAE can reappear when the 2c power is turned off.

The measured GAE suppression time ~ few ms is much smaller than slowing-down time (~50ms), suggesting that it takes relatively few high-pitch fast ions to suppress the GAE.

Correlation between strong GAE/CAE activity and flattening of the electron temperature profile has been observed in NSTX [Stutman, PRL 2009]

- Intense GAE/CAE activity (0.5-1.1MHz).

- Flattening of T_e profile with
  - increased beam power;
  - beam energy scanned between 60 and 90 keV [Stutman, PRL 2009].

- Was attributed to
  - enhanced electron transport due to orbit stochasticity in the presence of multiple GAEs [Gorelenkov, NF 2010].
  - energy channeling due to CAE coupling to KAW [Belova, PRL 2015].

- Anomalously low T_e potentially can have significant implications for future fusion devices, especially low aspect ratio tokamaks.
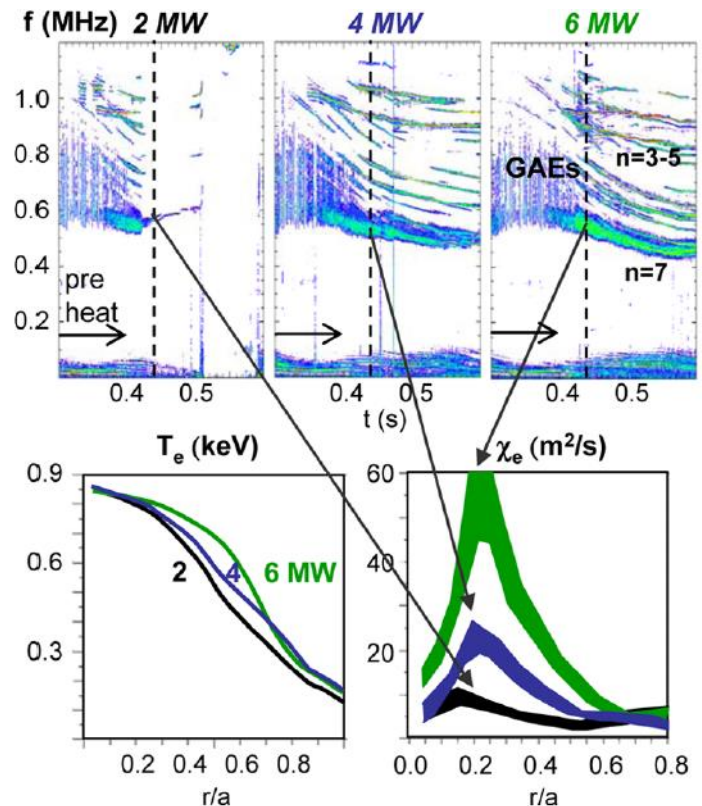
Correlation between GAE activity, T_e flattening, and central electron heat diffusivity χ_e in NSTX H modes with 2, 4, and 6MW neutral beam.

NSTX-U

HYM – HYbrid and MHD code

Applications

- NSTX
  - Sub-cyclotron frequency Alfven eigenmodes (GAE and CAE)
- ICC Theory and Modeling
  - Hybrid simulations of spheromak merging
  - FRC: Effects of beam ions on stability
    - Rotation control
    - n=2 rotational and n=1 wobble modes

Code description

- 3-D nonlinear.
- Physical models:
  - Resistive MHD & Hall-MHD
  - Hybrid (fluid electrons, particle ions)
  - **MHD/particle (one-fluid thermal plasma, + energetic particle ions)**
    - Drift-kinetic particle electrons
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)

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Self-consistent MHD + fast ions coupling scheme

Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b) \times \mathbf{B} - n_b (\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$

$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

$$w = \delta F / F \quad \text{- particle weight}$$

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

ρ, $\mathbf{V}$ and p are thermal plasma density, velocity and pressure, n_b and j_b are beam ion density and current, and $n_b \ll n_e$ – is assumed.

Self-consistent anisotropic equilibrium including the NBI ions

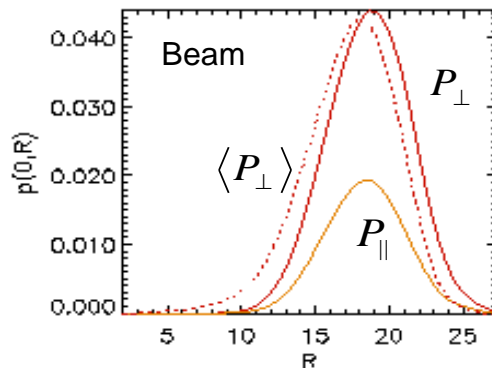Grad-Shafranov equation for two-component plasma:

MHD plasma (thermal) and fast ions [Belova et al, Phys. Plasmas 2003].

$$\frac{\partial^2 \psi}{\partial z^2} + R \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) = -R^2 p' - HH' - \underbrace{GH' + RJ_{b\phi}}$$

Beam effects

$$\mathbf{B} = \nabla\phi \times \nabla\psi + h\nabla\phi$$

$$h(R,z) = H(\psi) + G(R,z)$$

$$\mathbf{J}_{bp} = \nabla G \times \nabla\phi \; , \; G - \text{poloidal stream function}$$

Modifications of equilibrium due to beam ions:

- more peaked current profile,
- anisotropic pressure,
- increase in Shafranov shift

might have indirect effect on stability.

Fast ions – delta-f scheme: $F_0 = F_0(\varepsilon, \mu, p_\varphi)$

Equilibrium distribution function $F_0 = F_1(v)\, F_2(\lambda)\, F_3(p_\varphi, v)$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \quad \text{for} \quad v < v_0$$

$$F_2(\lambda) = \exp(-(\lambda - \lambda_0)^2 / \Delta\lambda^2)$$

$$F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}, \quad \text{for} \quad p_\phi > p_0$$

where $v_0 = 2\text{-}5v_A$, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, $\lambda_0 = 0.5\text{-}0.7$,

and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn'81]:

$$\mu = \frac{(\mathbf{v}_\perp - \mathbf{v_d})^2}{2B} - \frac{\mu_0 v_\parallel}{2B}[\hat{b}\cdot\nabla\times\hat{b} - 2(\hat{a}\cdot\nabla\hat{b})\cdot\hat{c}]$$

$\mathbf{v}_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = \mathbf{v}_\perp/v_\perp$, $\hat{a} = \hat{b}\times\hat{c}$.

Parameters are chosen to match TRANSP beam profiles.

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Simulations have been performed to study the excitation and stabilization of GAEs in the NSTX-U
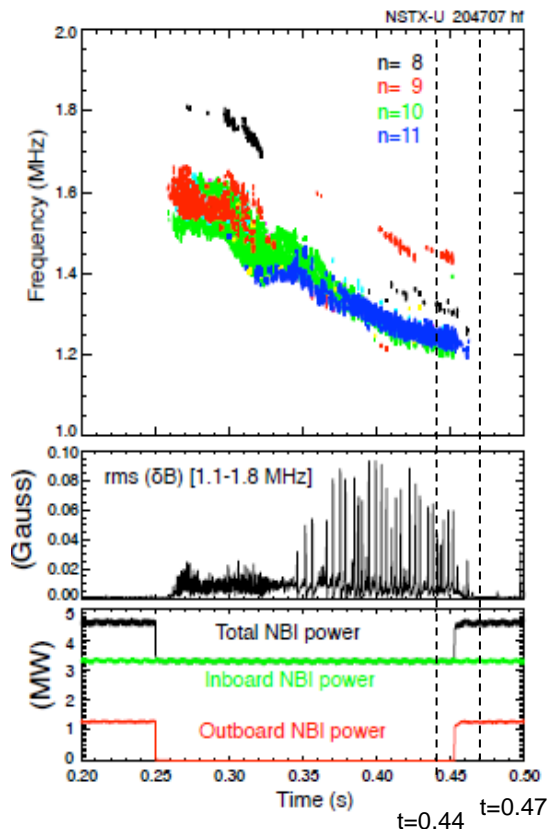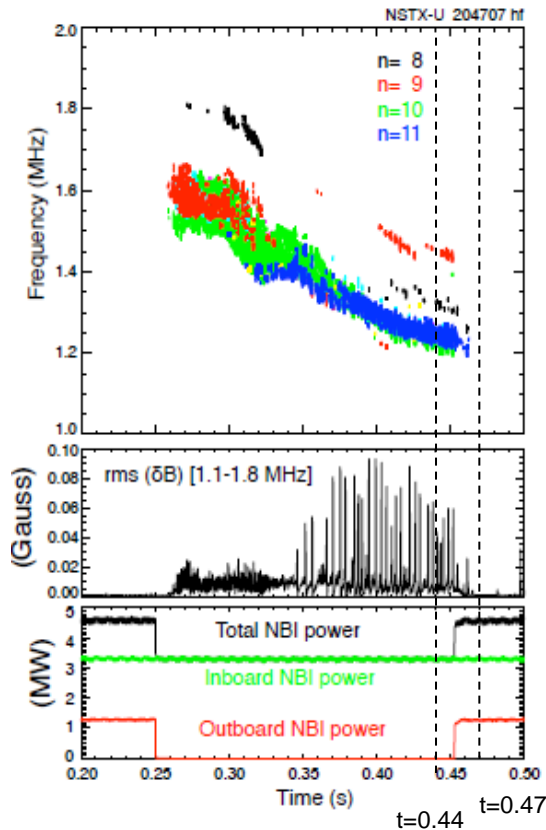

(a) Spectrogram on magnetic fluctuations (n=8-11 counter-GAEs).
(b) Rms magnetic fluctuations;
(c) Injected beam power.

• Simulations using the HYM code have been performed for NSTX-U shot #204707 right before (t=0.44s) and shortly after (t=0.47s) the additional off-axis beam injection.

• Plasma and beam profiles have been chosen to match TRANSP profiles for t=0.44s and t=0.47s.

• The beam ion distribution function matches TRANSP data, with pitch distribution in the form $F_b \sim \exp[-(\lambda-\lambda_0(\varepsilon))^2/\Delta\lambda(\varepsilon)^2]$.
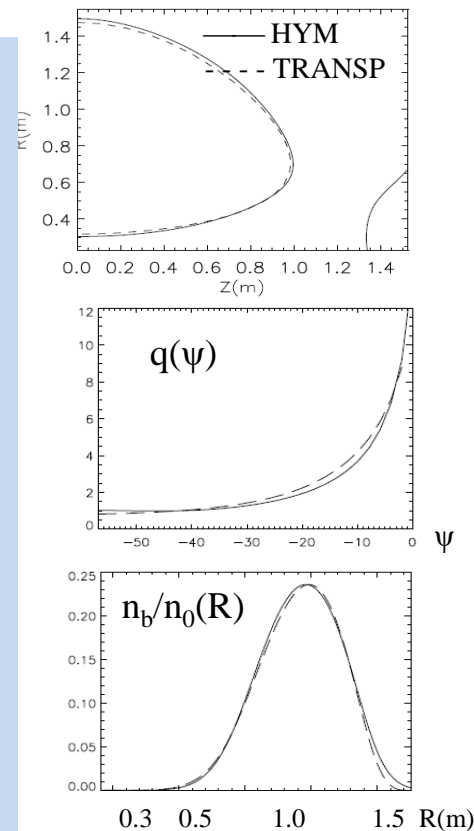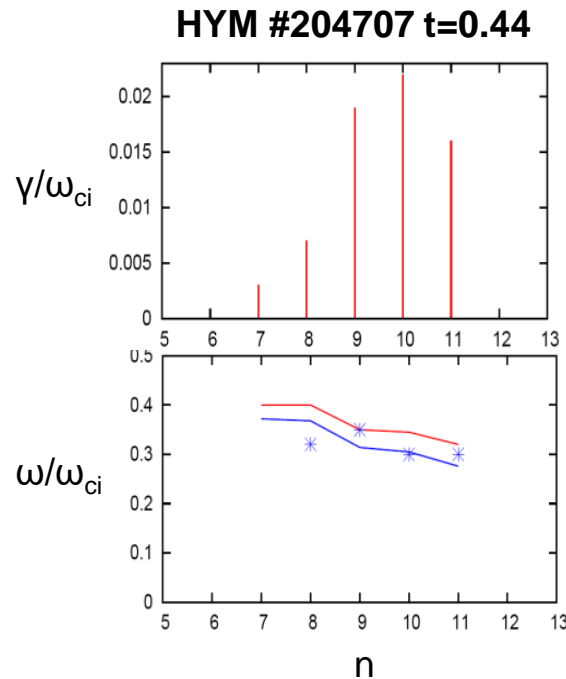

Plasma shape, q- and n_b profiles for NSTX-U shot 204707 t=0.44 from TRANSP and HYM GS solver + FREE_FIX.

HYM reproduces experimentally observed unstable GAEs

(a) Spectrogram on magnetic
fluctuations (n=8-11 counter GAEs).
(b) Rms magnetic fluctuations;
(c) Injected beam power.

HYM #204707 t=0.44

γ/ω_{ci}

ω/ω_{ci}

n

(a) Growth rates and (b)
frequencies of unstable counter-
GAEs from HYM simulations for
t=0.44s. Blue line is Doppler-shift
corrected frequencies, points –
experimental values.

• Simulations reproduce
most unstable toroidal
mode numbers and GAEs
frequencies.

• HYM overestimates
growth rates compared
experimental analysis by
2-3 times.

• Experiment:
n=-10 γ/ω_{ci}=0.84%
n=-11 γ/ω_{ci}=0.6%

• Growth rates are
sensitive to distribution
function parameters –
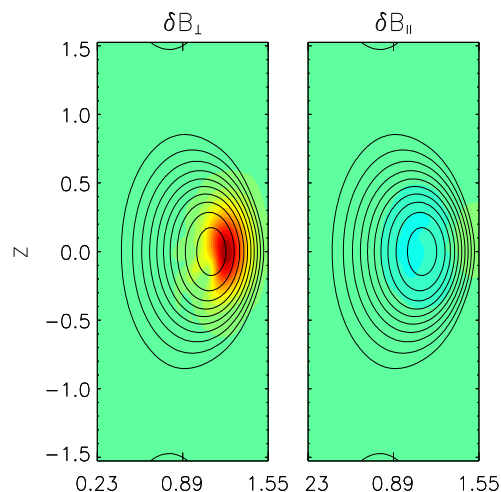resonance particles are in
'tail' of distribution.

NSTX-U simulations: n=-10 counter-GAE (t=0.44s)
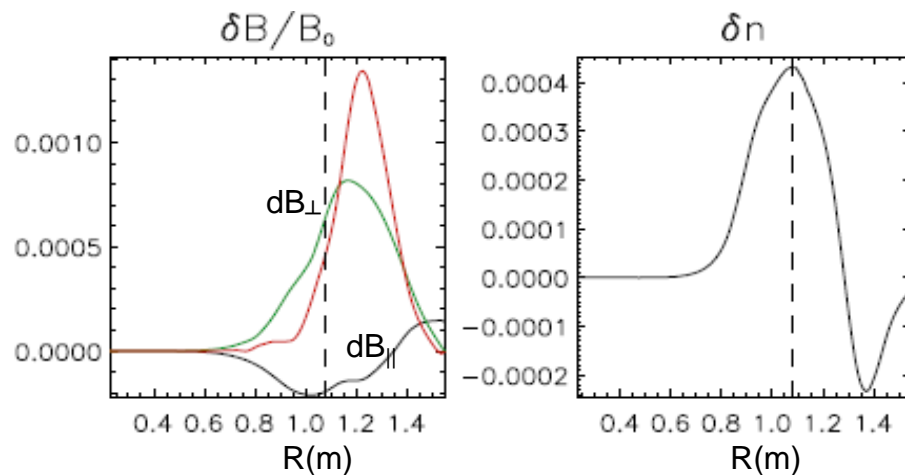
Radial profile of **δB** and δn for n=-10 counter-GAE.
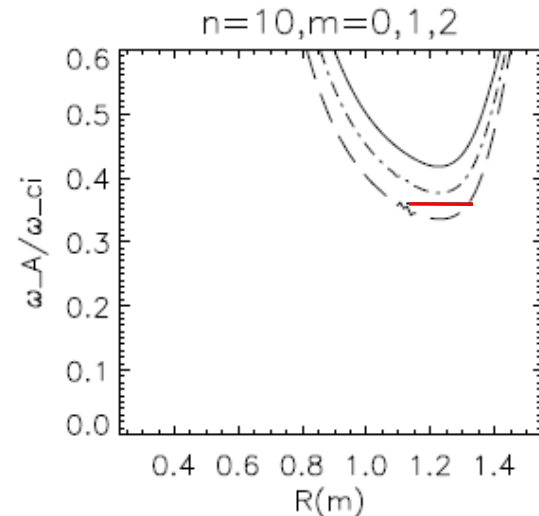
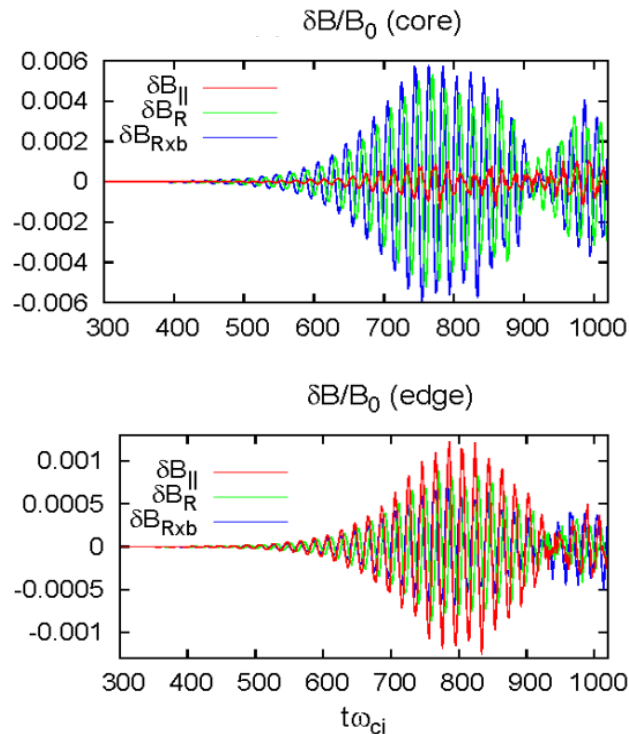Radial profile of ω_A/ω_{ci} for n=-10, m=0-2; location of GAE.

- Unstable modes are counter-rotating and have shear Alfven polarization.
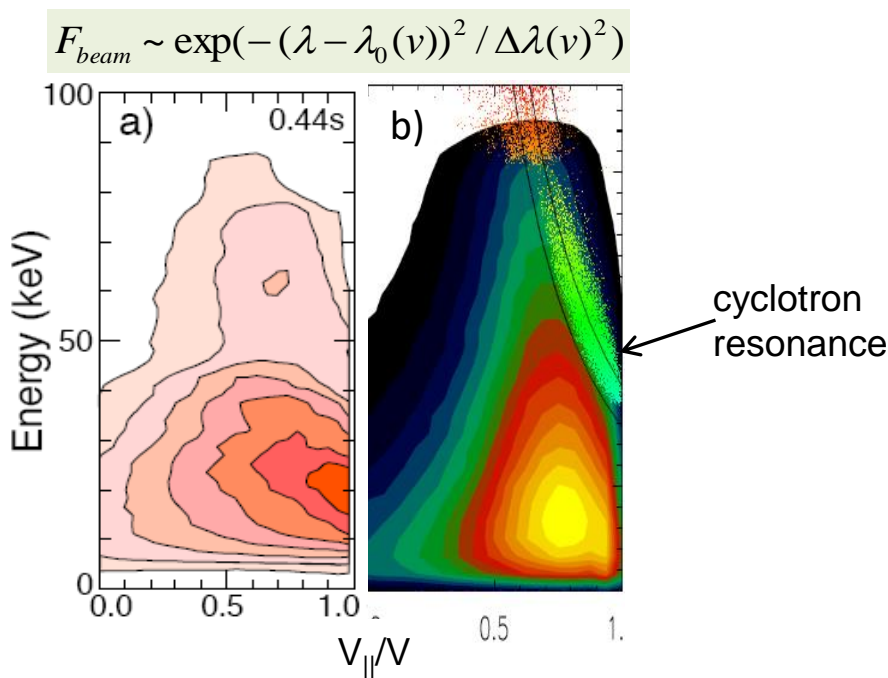- Located near min of ω_A.

Nonlinear simulations: n=-11 counter-GAE (t=0.44s)

$\delta B/B_0$ (core)

$\delta B/B_0$ (edge)

$t\omega_{ci}$

Time evolution of perturbed magnetic field components from nonlinear simulations for n=-11 GAE.

- Nonlinear simulations show peak saturation amplitudes of $\delta B/B_0 \sim 5 \times 10^{-3}$ at R~1.2m close to the minimum of the Alfvén continuum, and $\delta B/B_0 \sim 10^{-3}$ near the edge at the midplane.

- Unstable modes have shear Alfven polarization in the core, and mixed polarization at the edge.

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Improved F_{beam} fit allows more accurate description

$$F_{beam} \sim \exp(-(\lambda - \lambda_0(v))^2 / \Delta\lambda(v)^2)$$

cyclotron resonance

(a) TRANSP fast-ion distribution before the outboard beam injection t=0.44.
(b) HYM fast-ion distribution from n=-11 GAE simulations; dots show resonant particles.
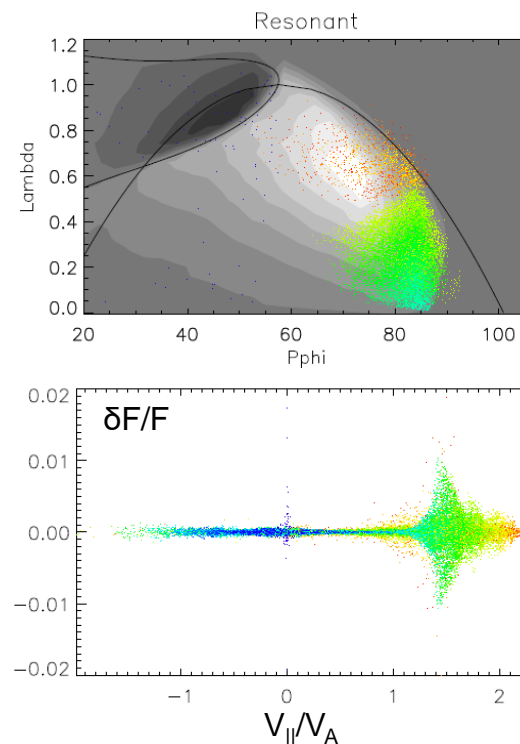
(a) Location of resonant particles in phase space: $\lambda = \mu B_0/\varepsilon$ vs p_φ.
(b) Particle weight $w \sim \delta F/F$ vs orbit-averaged parallel velocity. Particle color corresponds to different energies: from E=0 (purple) to E=90keV (red).

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HYM simulations reproduce experimental finding: off-axis neutral beam injection reliably and strongly suppresses unstable GAEs

• The beam ion distribution function with pitch distribution in the form $F_b \sim \exp[-(\lambda-\lambda_0(\varepsilon))^2/\Delta\lambda(\varepsilon)^2]$.

• Additional off-axis beam injection modeled by adding beam ions with distribution $F_{add} \sim \exp[-\lambda^2/\Delta\lambda_a(\varepsilon)^2]$, i.e. with $\lambda_0=0$, $\Delta\lambda_a<\Delta\lambda$ and about 1/3 of the total beam ion inventory or less.

• HYM shows complete linear stabilization of n= 7-12 counter-GAEs by additional off-axis beam injection.
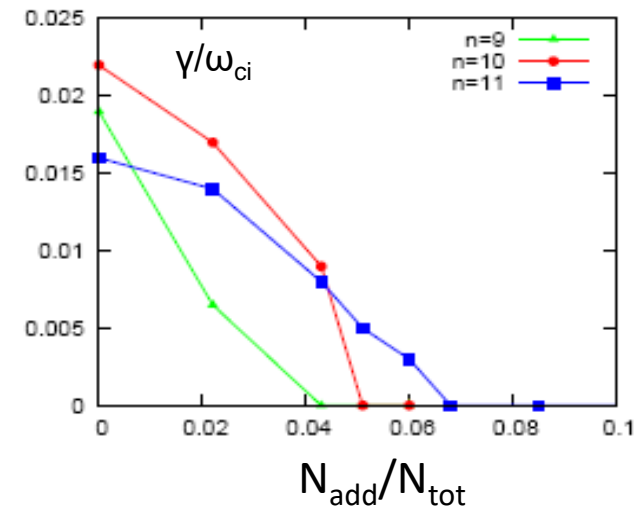
HYM #204707,   t=0.44, t=0.47

n= -11

$|dB|^2$

t=0.44s

t=0.47s

$t\omega_{ci}$
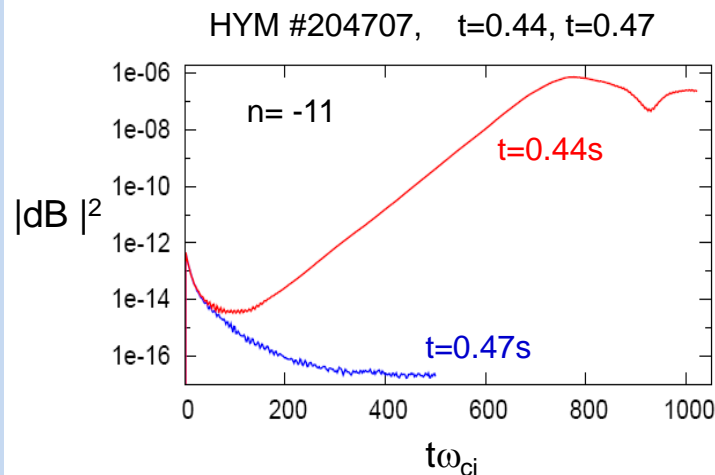
Time evolution of magnetic energy of n=-11 GAE from HYM simulations for t=0.44s (red), and t=0.47s (blue).

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Complete stabilization of all unstable GAEs requires an additional beam power significantly lower than what was used in NSTX-U

• Off-axis beam injection has been modelled by adding beam ions with distribution $F_{add} \sim \exp[-\lambda^2/\Delta\lambda\,(\epsilon)^2]$, and varying density.

• Fraction of off-axis beam population of the total fast ion inventory has been varied from 4% to 17%.

• Unstable n=-11 GAE is stabilized when the fraction of the additional beam ions is larger than 7%.

• Stabilization threshold is lower for lower n modes.

N_{add}/N_{tot}

Growth rates of n=-11 (blue), -10 (red), -9 (green) GAEs vs fraction of outboard beam ion population.

Experimental values of fraction of outboard beam power vs total beam power in NSTX-U were ~24-30%.

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GAE stabilization is consistent with analytic predictions
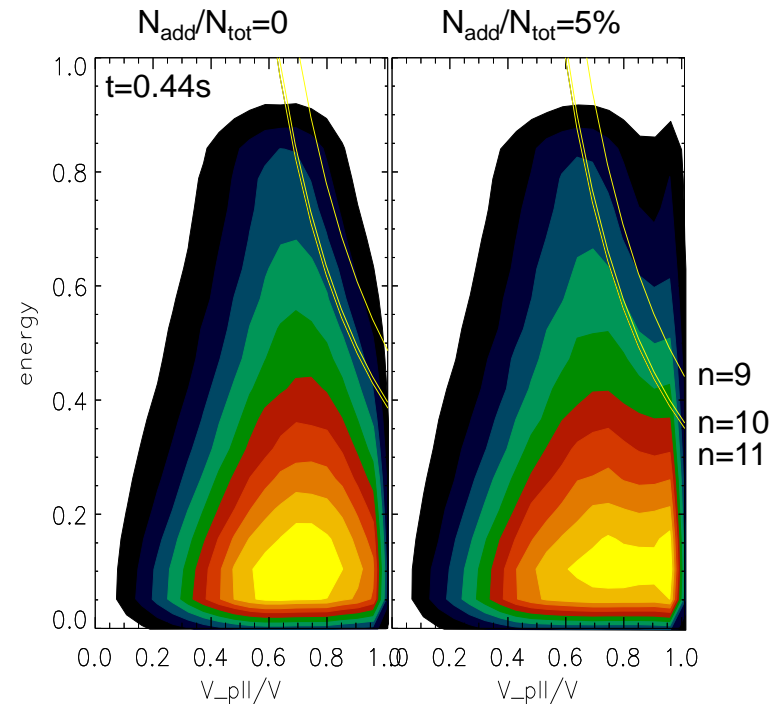
Analytical instability condition: $2 < k_\perp \rho_b < 4$ [Gorelenkov,2003] – large pitch particles are stabilizing.
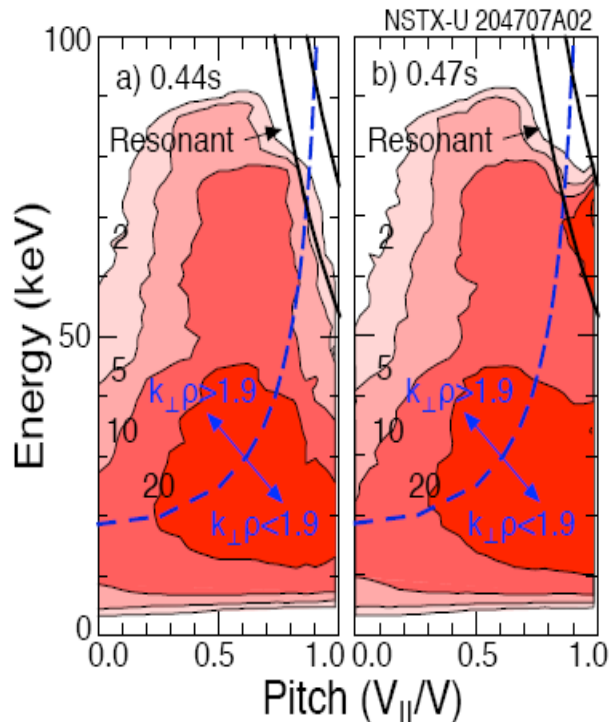
TRANSP fast-ion distribution before and after the outboard beam injection. Fast ions with pitch $v_\parallel/v \sim 1$ are responsible for GAE suppression [Fredrickson,2017].

Fast-ion distribution from HYM simulations before the outboard beam injection and for case with $N_{add}/N_{tot}=5\%$ (n=9,10 are stable). Cyclotron resonance lines are shown for n=9,10,11 GAEs.

Summary and Future Work

- HYM simulations show range of toroidal mode numbers, and frequencies of unstable GAEs that match the experimentally observed GAEs in NSTX-U.

- Growth rates and saturation amplitudes of GAEs are overestimated in simulations.

- Simulations reproduce experimental finding, namely it is shown that off-axis neutral beam injection reliably and strongly suppresses all unstable GAEs.

- A very effective mechanism for stabilizing GAEs - threshold for stabilization for additional beam is less than 7% of total beam power.

Future work:

- Bulk plasma rotation and Hall term can have effect on GAE stability and mode structure.

- Comparison with experimental results including mode structure, saturation amplitudes and etc for several shots.

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