Recurrence in three dimensional magnetohydrodynamic plasma

Rupak Mukherjee†∗  Rajaraman Ganesh∗  Abhijit Sen∗

†Princeton Plasma Physics Laboratory, Princeton, NJ, USA
∗Institute for Plasma Research, HBNI, India

rmukherj@pppl.gov
rupakm@princeton.edu
rupakmukherjee01@gmail.com

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Direct Numerical Simulation (DNS) study of *Three dimensional* Single fluid MagnetoHydroDynamic equations have been carried out to explore

1. “Nonlinear Dispersion-free Alfven Waves (NDAW)” - via large amplitude oscillations of kinetic and magnetic energy
2. “Recurrence Phenomena” - a periodic reconstruction of initial flow of fluid and magnetic field variables mediated via NDAW.

1. NDAW exists in both 2D & 3D - But Recurrence ?
   [Key Parameter: Initial condition]
2. Rayleigh Quotient determines criteria of Recurrence.
   [Key Parameter: Initial Condition]
Motivation

G-MHD3D: DNS GPU code

Nonlinear Alfvén wave

Recurrence

Summary

Extra

Governing Equations in three dimensions

**Single Fluid MHD equations evaluated by G-MHD3D**

\[
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0
\]

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[ \rho \vec{u} \otimes \vec{u} + \left( P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \otimes \vec{B} \right] = \mu \nabla^2 \vec{u} + \vec{f}
\]

\[
\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[ \left( E + P + \frac{B^2}{2} \right) \vec{u} - \vec{u} \cdot \left( \vec{B} \otimes \vec{B} \right) - \eta \vec{B} \times \left( \vec{\nabla} \times \vec{B} \right) \right] = \mu \left( \vec{\nabla} \cdot \vec{u} \right)^2 \quad ; \quad P = C_s^2 \rho
\]

\[
\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left( \vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u} \right) = \eta \nabla^2 \vec{B}
\]

\[
u_0 = \frac{L}{t_0}, \quad V_A = \frac{B_0}{4\pi \sqrt{\rho_0}}, \quad M_s = \frac{u_0}{C_s}, \quad M_A = \frac{u_0}{V_A}, \quad Re = \frac{\rho_0 u_0 L}{\nu}, \quad Rm = \frac{Lu_0}{\eta}.
\]
Scope and Algorithm

1. The DNS code G-MHD3D governs the time evolution of *Three dimensional* single fluid MagnetoHydroDynamic equations.

2. Equations are evolved in *pseudo-spectrally* in a periodic domain to achieve higher *accuracy and performance* over finite difference schemes. [*FFTW & cuFFT library*]

GPU Optimization with NVIDIA in DGX systems

1. Optimised GPU code with OpenACC and CUDA parallelisation shows two order of magnitude speedup over OpenMP code.

2. For high resolution runs multi-GPU parallelisation with NV-Link & accFFT library has been achieved to test on NVIDIA - DGX station and PSG cluster.

*RM et. al.*, IEEE 25th International Conference on High Performance Computing Workshops (HiPCW), 2018
Nonlinear Dispersion-free Alfven Wave (NDAW)

- Within the premise of Single fluid MHD, energy can cascade through both kinetic and magnetic channels simultaneously.

- With weak resistivity, MHD model predicts -
  1). irreversible conversion of magnetic energy into fluid kinetic energy (i.e. reconnection).
  2). conversion of kinetic energy into mean large scale magnetic field (i.e. dynamo).

- **Question:** For a given fluid type and magnetic field strength, are there fluid flow profiles which do neither?

- **Answer:** Yes. For a wide range of initial flow speeds or Alfven Mach number it is shown that the coherent nonlinear oscillation persist.

- **Is it a new result?** - **NO!**
Earlier works with NDAW and the take-over

- The existence of such **dispersionless nonlinear** oscillations are known for some time [H. Alfven, *Cosmical electrodynamics* (Clarendon Press, Oxford, 1963)]

- More recently these results have been revisited by Z Yoshida [Communications in Nonlinear Science and Numerical Simulation 17, 2223 (2012)] and in another work by H M Abdelhamid & Z Yoshida [Physics of Plasmas 23, 022105 (2016)] from a point of view of Hamiltonian-Casimir formulation of ideal MHD and its extended models.

- **H M Abdelhamid & Z Yoshida had also suggested the necessity of special initial wave forms for sustaining such oscillations**—We started with these initial conditions!
NDAW in two spatial dimensions at Alfvén resonance ($C_s = V_A$).

**Orszag-Tang Flow**

\[
\begin{align*}
    u_x &= -A \sin(k_0y) \\
    u_y &= +A \sin(k_0x)
\end{align*}
\]

**Cat’s Eye Flow**

\[
\begin{align*}
    u_x &= + \sin(k_0x) \cos(k_0y) - A \cos(k_0x) \sin(k_0y) \\
    u_y &= - \cos(k_0x) \sin(k_0y) + A \sin(k_0x) \cos(k_0y)
\end{align*}
\]
2D Orszag-Tang Flow with External Forcing

\[ \vec{f} = \alpha \left[ -A \sin(k_f y) + A \sin(k_f x) \right], \quad \alpha = 0.1 \]

3D ABC Flow with different $M_A$

\[ u_x = U_0 [A \sin(k_0 z) + C \cos(k_0 y)] \]
\[ u_y = U_0 [B \sin(k_0 x) + A \cos(k_0 z)] \]
\[ u_z = U_0 [C \sin(k_0 y) + B \cos(k_0 x)] \]

Frequency of energy exchange scales linearly with $M_A$.

With external forcing similar to initial flow the plasma acts as a forced-relaxed system both in two and three dimensions.
NDAW in 3D MHD at Alfvén Resonance

Time evolution of kinetic & magnetic energy for TG, ABC flows.

Taylor-Green (TG) flow:
\[ u_x(0) = A \ U_0 [\cos(kx) \sin(ky) \cos(kz)] \]
\[ u_y(0) = -A \ U_0 [\sin(kx) \cos(ky) \cos(kz)] \]
\[ u_z(0) = 0 \]

Arnold-Beltrami-Childress (ABC) flow:
\[ u_x(0) = U_0[A \sin(k_f z) + C \cos(k_f y)] \]
\[ u_y(0) = U_0[B \sin(k_f x) + A \cos(k_f z)] \]
\[ u_z(0) = U_0[C \sin(k_f y) + B \cos(k_f x)] \]

<table>
<thead>
<tr>
<th>( N )</th>
<th>( L )</th>
<th>( dt )</th>
<th>( \rho_0 )</th>
<th>( U_0 )</th>
<th>( Re )</th>
<th>( Rm )</th>
<th>( M_s )</th>
<th>( M_A )</th>
<th>( A = B = C )</th>
<th>( k_0 )</th>
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<tbody>
<tr>
<td>( 64^3 )</td>
<td>( 2\pi^3 )</td>
<td>( 10^{-5} )</td>
<td>1</td>
<td>0.1</td>
<td>450</td>
<td>450</td>
<td>0.1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

RM, R Ganesh, A Sen, Phys Plasmas, 26, 042121 (2019)
Recurrence - TG (√) flow

**Kinetic Isosurfaces**

- (a) t = 0
- (b) t = 6.2
- (c) t = 12.5
- (d) t = 18.7
- (e) t = 25
- (f) t = 31.2
- (g) t = 37.5
- (h) t = 43.7
- (i) t = 49.9
- (j) t = 56.2
- (k) t = 62.4
- (l) t = 68.7
- (m) t = 74.9
- (n) t = 81.1
- (o) t = 87.4
- (p) t = 93.6
- (q) t = 99.9
- (r) t = 106.1
- (s) t = 112.3
- (t) t = 118.6
- (u) t = 124.8
- (v) t = 131.4
- (w) t = 137.3
- (x) t = 143.5
- (y) t = 149.2

**Magnetic Isosurfaces**

- (a) t = 0
- (b) t = 6.2
- (c) t = 12.5
- (d) t = 18.7
- (e) t = 25
- (f) t = 31.2
- (g) t = 37.5
- (h) t = 43.7
- (i) t = 49.9
- (j) t = 56.2
- (k) t = 62.4
- (l) t = 68.7
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- (n) t = 81.1
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- (y) t = 149.2
Kinetic Isosurfaces

Magnetic Isosurfaces

Rupak Mukherjee, PPPL, USA

Fundamental Plasma Physics, Vortex Dynamics, F-O11, Willow, 14:00-16:00

AAPPS-DPP 2019, Crowne Plaza, Hefei, China
Rayleigh quotient \( Q(t) \) measures the number of effective ‘active degrees of freedom’.

\[ Q(t) = \frac{\int_V \left( (\nabla \times \vec{u})^2 + \frac{1}{2} (\nabla \times \vec{B})^2 \right) dV}{\int_V \left( |\vec{u}|^2 + \frac{1}{2} |\vec{B}|^2 \right) dV} = \frac{k^2 |c_k|^2}{|c_k|^2} \]

where, \( \vec{u} \) & \( \vec{B} \) are expanded in a Fourier series.

If \( Q(t) \) is bounded \( \Rightarrow \) Recurrence can happen.

For TG flow, \( Q(t) \) is bounded.

For ABC flow \( Q(t) \) increases without bound.

RM, R Ganesh, A Sen, Phys Plasmas 26, 022101 (2019) [Editor’s Pick]
Summary & Discussions

- Ideally very low probability of trapping in phase space in high dimensional systems (e.g. 3D MHD systems).
- Recurrence is observed for flows involving few number of active degrees of freedom. [Birkhoff, Dynamical Systems, 1927, Chapter 7]
- Recurrence can be helpful to make short time forecasts once typical initial profiles are experimentally obtained.
- Energy exchange between fluid and magnetic variables opens up new cascading paths for energy amongst different scales.
- Nonlinear Dispersionless Alfven Waves generated via certain specific flow and field profiles cause Recurrence.
Acknowledgement

- A Thyagaraja, Culham Labs, UK
  (for several Physics discussions related to MHD)
- Nagavijayalakshmi Vydyathan, NVIDIA
  (for teaching GPU computing)
- PPPL & AAPPS-DPP
  (for financial support)

Thank You
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Thank You
Recurrence - TG (√) & ABC (✗) flow
Motion of Tracer particles during Recurrence with TG flow
GPU Performance of G-MHD3D (With NVIDIA)
GPU Performance of G-MHD3D with Passive Tracers (With NVIDIA)

G-MHD3D & 3D Poisson solver
Performance – Hackathon - 2018

Performance of Multi-GPU 3D Poisson solver using accFFT library.
GMHD3D has been openACC paralalised using cuFFT library.

<table>
<thead>
<tr>
<th>3D Poisson Solver: Number of GPUs (Resolution)</th>
<th>Total Time (ms)</th>
<th>Local FFT (ms)</th>
<th>Communication (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (512 X 512 X 512)</td>
<td>19.9</td>
<td>19.8</td>
<td>0</td>
</tr>
<tr>
<td>2 (512 X 512 X 512)</td>
<td>25.4</td>
<td>9.84</td>
<td>14.0</td>
</tr>
<tr>
<td>4 (512 X 512 X 512)</td>
<td>14.6</td>
<td>5.01</td>
<td>8.59</td>
</tr>
<tr>
<td>4 (1024X1024X1024)</td>
<td>153</td>
<td>82.0</td>
<td>61.0</td>
</tr>
</tbody>
</table>

GMHD3D: Grid Resolution
Run time of GMHD3D (in second) (10 iterations)

64 X 64 X 64: 1.54
128 X 128 X 128: 14.6

<table>
<thead>
<tr>
<th>GMHD3D: Grid Resolution</th>
<th>Run time of GMHD3D (in seconds) (10 iterations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 X 64 X 64</td>
<td>0.464</td>
</tr>
<tr>
<td>128 X 128 X 128</td>
<td>0.861</td>
</tr>
<tr>
<td>256 X 256 X 256</td>
<td>4.009</td>
</tr>
<tr>
<td>512 X 512 X 512</td>
<td>646.6</td>
</tr>
</tbody>
</table>
Poisson using MPI & OpenACC

- **Resolution**: 1024 X 512 X 256 – Run on DGX

<table>
<thead>
<tr>
<th>Number of GPU</th>
<th>Total Time (ms)</th>
<th>Local FFT (ms)</th>
<th>Communication (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.6</td>
<td>44.6</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>38.6</td>
<td>22.7</td>
<td>14.3</td>
</tr>
<tr>
<td>4</td>
<td>21.6</td>
<td>11.6</td>
<td>8.74</td>
</tr>
</tbody>
</table>

- Cuda aware MPI accFFT libraries are used.
- Pipelined FFTs for overlap of communication.
- Implement the changes in the full code – GMHD3D.