Recurrence in three dimensional magnetohydrodynamic plasma

Rupak Mukherjee[†*] Rajaraman Ganesh[*] Abhijit Sen[*]

[†]Princeton Plasma Physics Laboratory, Princeton, NJ, USA
[*]Institute for Plasma Research, HBNI, India

rmukherj@pppl.gov
rupakm@princeton.edu
rupakmukherjee01@gmail.com

3rd Asia-Pacific Conference on Plasma Physics

04th November, 2019

Direct Numerical Simulation (DNS) study of *Three dimensional* Single fluid MagnetoHydroDynamic equations have been carried out to explore

1. "Nonlinear Dispersion-free Alfven Waves (NDAW)" - via large amplitude oscillations of kinetic and magnetic energy
2. "Recurrence Phenomena" - a periodic reconstruction of initial flow of fluid and magnetic field variables mediated via NDAW.

1. NDAW exists in both 2D & 3D - But Recurrence ?
   [Key Parameter: Initial condition]
2. Rayleigh Quotient determines criteria of Recurrence.
   [Key Parameter: Initial Condition]

Single Fluid MHD equations evaluated by G-MHD3D

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[\rho \vec{u} \otimes \vec{u} + \left(P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \otimes \vec{B} \right] = \mu \nabla^2 \vec{u} + \vec{f}$$

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[\left(E + P + \frac{B^2}{2} \right) \vec{u} - \vec{u} \cdot \left(\vec{B} \otimes \vec{B} \right) - \eta \vec{B} \times \left(\vec{\nabla} \times \vec{B} \right) \right]$$

$$= \mu \left(\vec{\nabla} \cdot \vec{u} \right)^2; \qquad P = C_s^2 \rho$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u} \right) = \eta \nabla^2 \vec{B}$$

$$u_0 = \frac{L}{t_0},\ V_A = \frac{B_0}{4\pi\sqrt{\rho_0}},\ M_s = \frac{u_0}{C_s},\ M_A = \frac{u_0}{V_A},\ Re = \frac{\rho_0 u_0 L}{\nu},\ R_m = \frac{L u_0}{\eta}.$$

Scope and Algorithm

1. The DNS code G-MHD3D governs the time evolution of *Three dimensional* single fluid MagnetoHydroDynamic equations.
2. Equations are evolved in pseudo-spectrally in a periodic domain to achieve higher accuracy and performance over finite difference schemes. [*FFTW & cuFFT library*]

GPU Optimization with NVIDIA in DGX systems

1. Optimised GPU code with OpenACC and CUDA parallelisation shows two order of magnitude speedup over OpenMP code.
2. For high resolution runs multi-GPU parallalisation with NV-Link & accFFT library has been achieved to test on NVIDIA - DGX station and PSG cluster.

RM *et. al.*, IEEE 25th International Conference on High Performance Computing Workshops (HiPCW), 2018

Nonlinear Dispersion-free Alfven Wave (NDAW)

- Within the premise of Single fluid MHD, energy can cascade through both kinetic and magnetic channels simultaneously.
- With weak resistivity, MHD model predicts -
  1). irreversible conversion of magnetic energy into fluid kinetic energy (i.e. reconnection).
  2). conversion of kinetic energy into mean large scale magnetic field (i.e. dynamo).
- Question: For a given fluid type and magnetic field strength, are there fluid flow profiles which do neither?
- Answer: Yes. For a wide range of initial flow speeds or Alfven Mach number it is shown that the **coherent nonlinear** oscillation persist.
- Is it a new result? - **NO**!

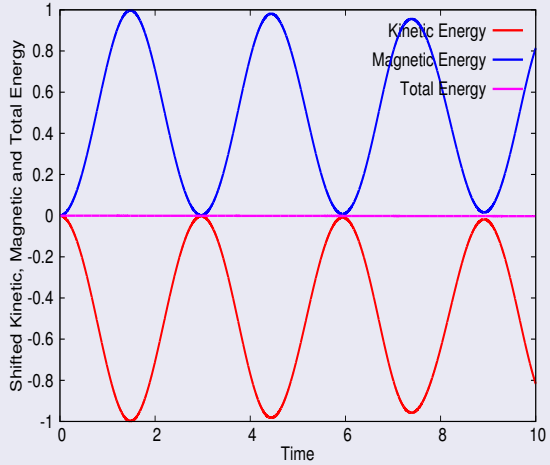
Earlier works with NDAW and the take-over

- The existence of such **dispersionless nonlinear** oscillations are known for some time [**H. Alfven**, Cosmical electrodynamics (Clarendon Press, Oxford, 1963)]
- More recently these results have been revisited by **Z Yoshida** [Communications in Nonlinear Science and Numerical Simulation **17**, 2223 (2012)] and in another work by **H M Abdelhamid & Z Yoshida** [Physics of Plasmas **23**, 022105 (2016)] from a point of view of Hamiltonian-Casimir formulation of ideal MHD and its extended models.
- *H M Abdelhamid & Z Yoshida had also suggested the necessity of special initial wave forms for sustaining such oscillations*- We started with these initial conditions!

NDAW in two spatial dimensions at Alfven resonance ($C_s = V_A$).

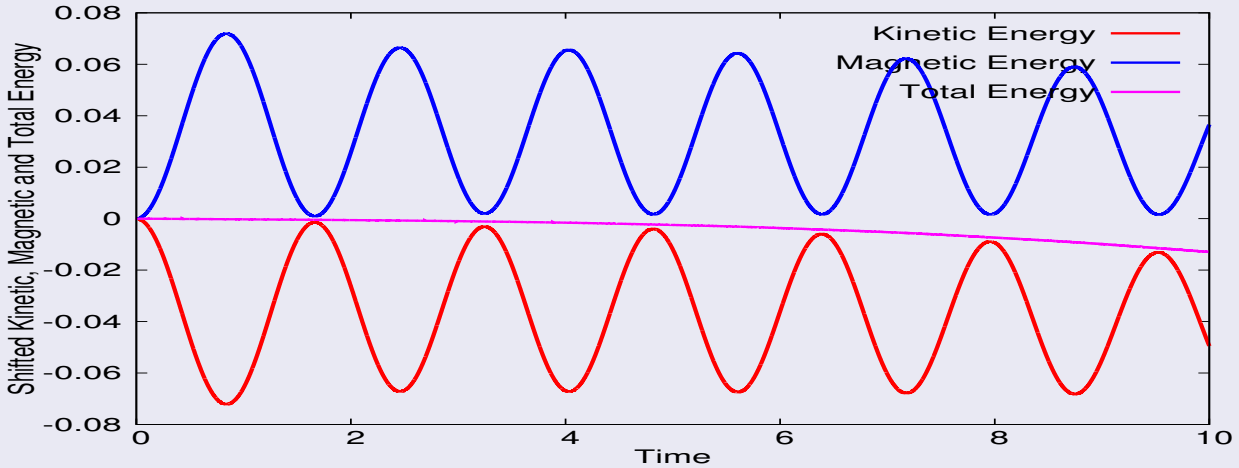
Orszag-Tang Flow

$$u_x = -A\sin(k_0 y)$$
$$u_y = +A\sin(k_0 x)$$



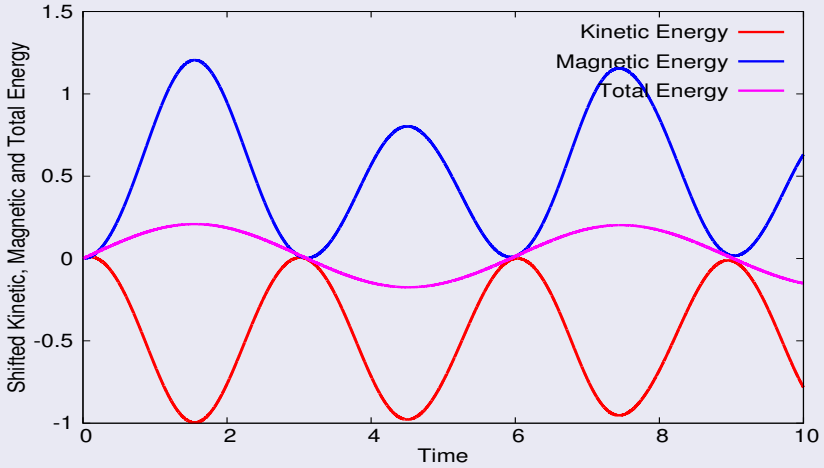
Cat's Eye Flow

$$u_x = +\sin(k_0 x)\cos(k_0 y) - A\cos(k_0 x)\sin(k_0 y)$$
$$u_y = -\cos(k_0 x)\sin(k_0 y) + A\sin(k_0 x)\cos(k_0 y)$$



2D Orszag-Tang Flow with External Forcing

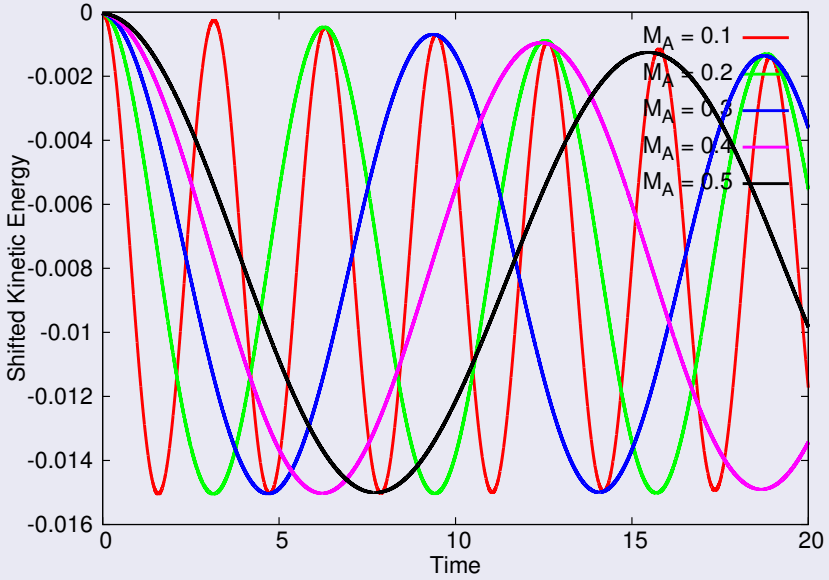
$$\vec{f} = \alpha \begin{bmatrix} -A\sin(k_f y) \\ +A\sin(k_f x) \end{bmatrix}, \quad \alpha = 0.1$$



3D ABC Flow with different M_A

$$u_x = U_0[A\sin(k_0 z) + C\cos(k_0 y)]$$
$$u_y = U_0[B\sin(k_0 x) + A\cos(k_0 z)]$$
$$u_z = U_0[C\sin(k_0 y) + B\cos(k_0 x)]$$

Frequency of energy exchange scales linearly with M_A.



- With external forcing similar to initial flow the plasma acts as a forced-relaxed system both in two and three dimensions.

NDAW in 3D MHD at Alfven Resonance

Time evolution of kinetic & magnetic energy for TG, ABC flows.

Taylor-Green (TG) flow:

$$u_x(0) = A\ U_0\ [\cos(kx)\sin(ky)\cos(kz)]$$

$$u_y(0) = -A\ U_0\ [\sin(kx)\cos(ky)\cos(kz)]$$

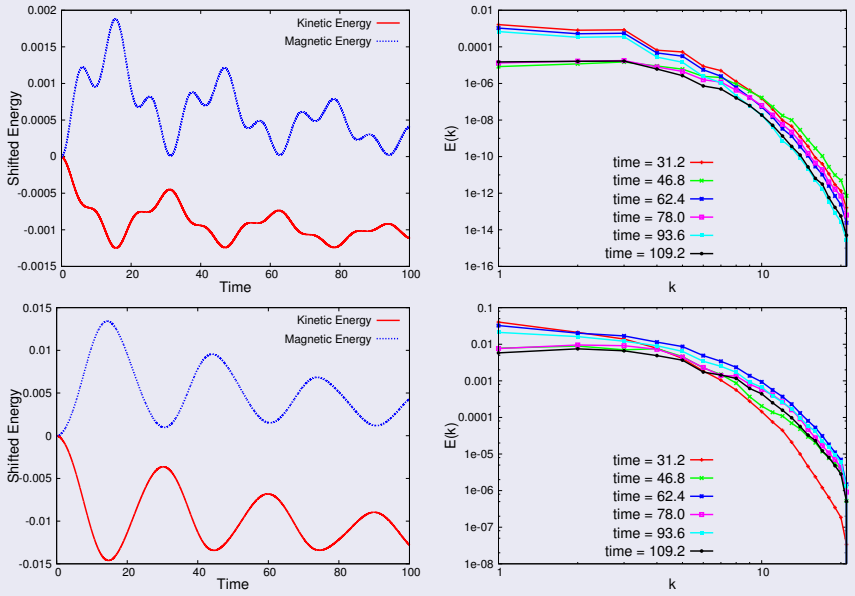
$$u_z(0) = 0$$

Arnold-Beltrami-Childress (ABC) flow:

$$u_x(0) = U_0[A\sin(k_f z) + C\cos(k_f y)]$$

$$u_y(0) = U_0[B\sin(k_f x) + A\cos(k_f z)]$$

$$u_z(0) = U_0[C\sin(k_f y) + B\cos(k_f x)]$$

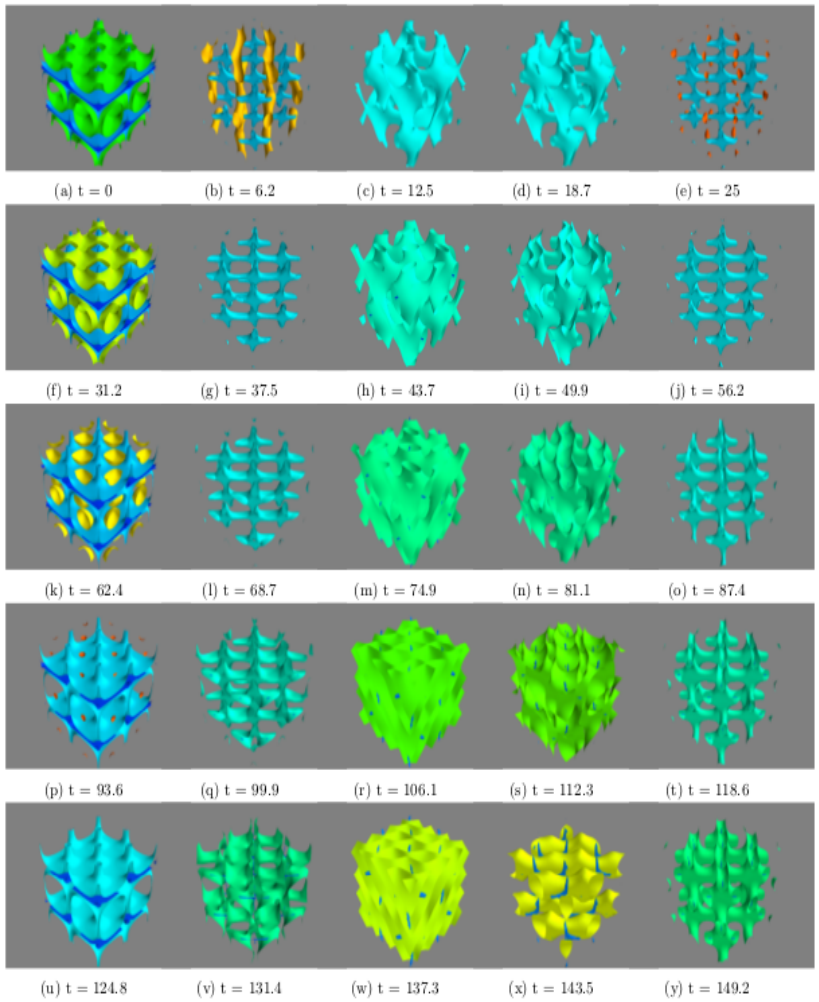


N	L	dt	ρ_0	U_0	Re	Rm	M_s	M_A	A = B = C	k_0
64^3	$2\pi^3$	10^{-5}	1	0.1	450	450	0.1	1	1	1

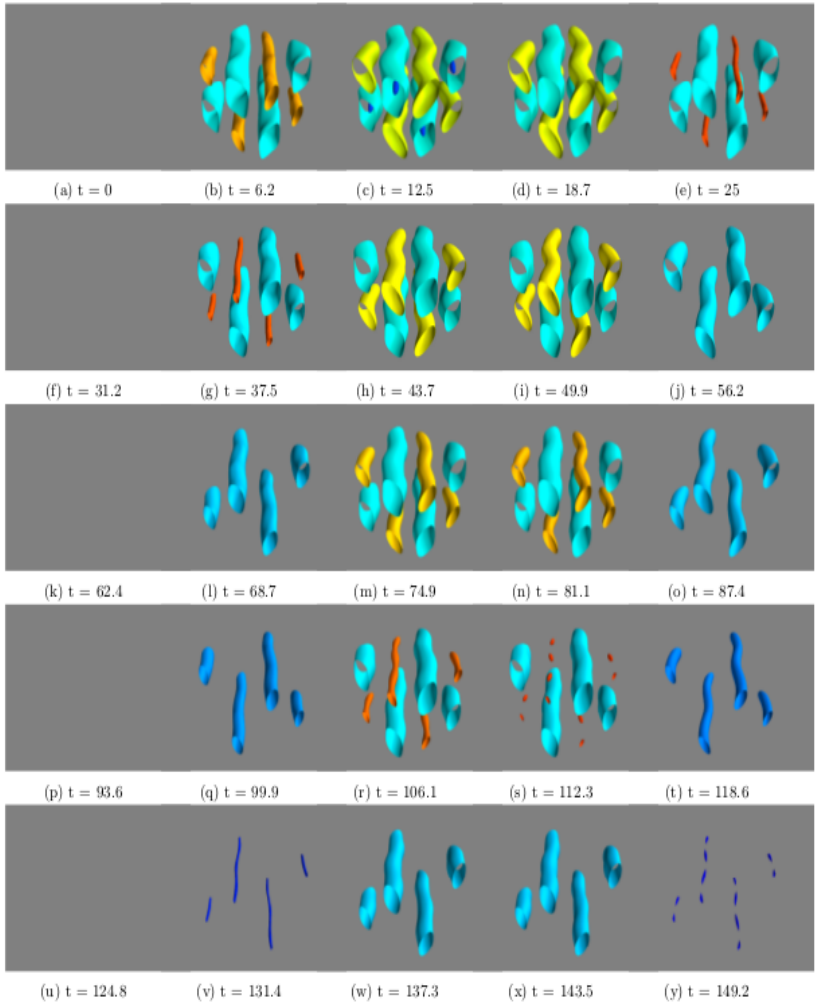
RM, R Ganesh, A Sen, Phys Plasmas, **26**, 042121 (2019)

Recurrence - TG (✓) flow

Kinetic Isosurfaces

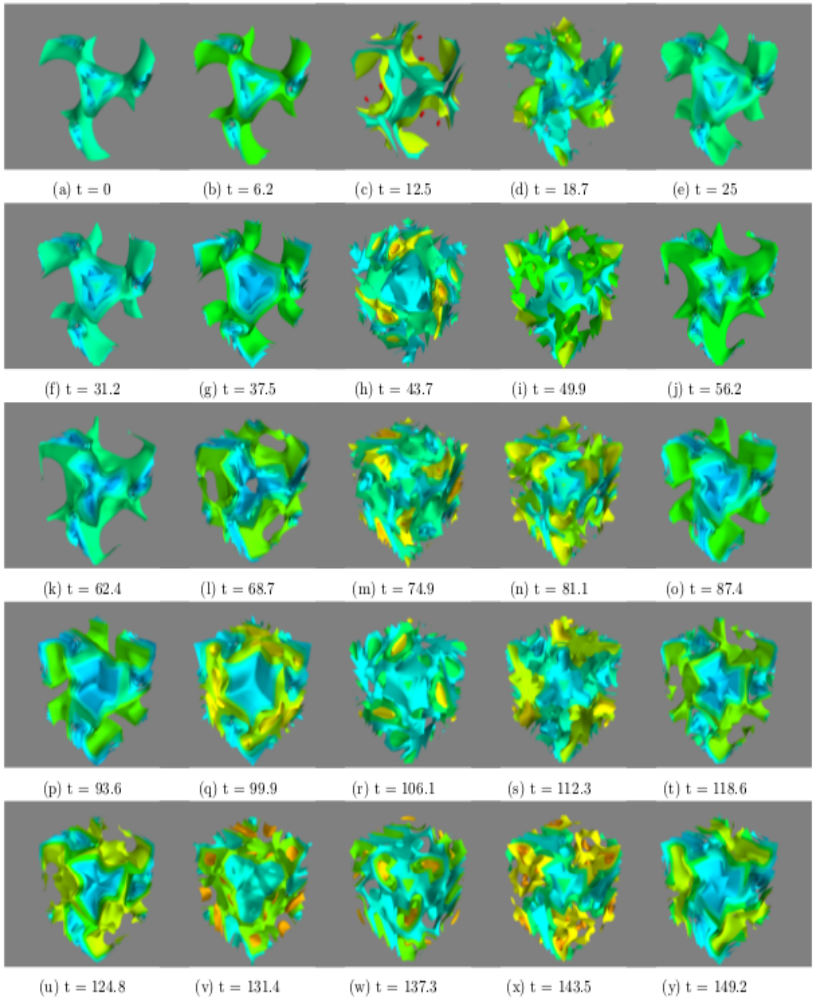


Magnetic Isosurfaces

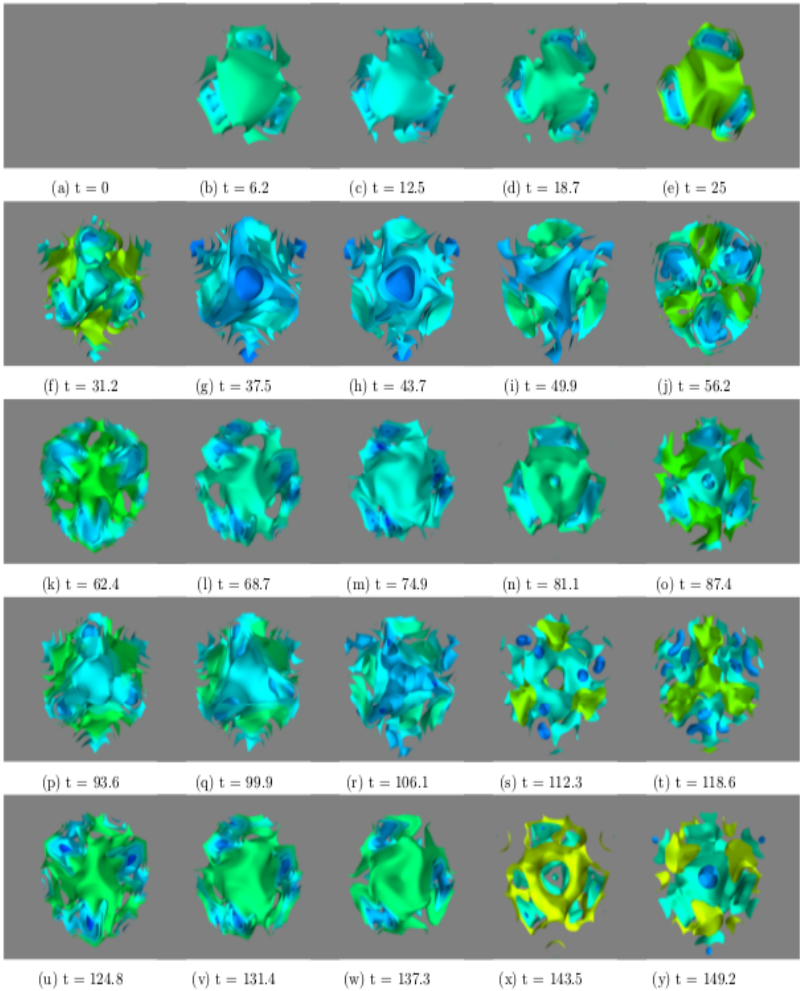


NO Recurrence - ABC (✗) flow

Kinetic Isosurfaces



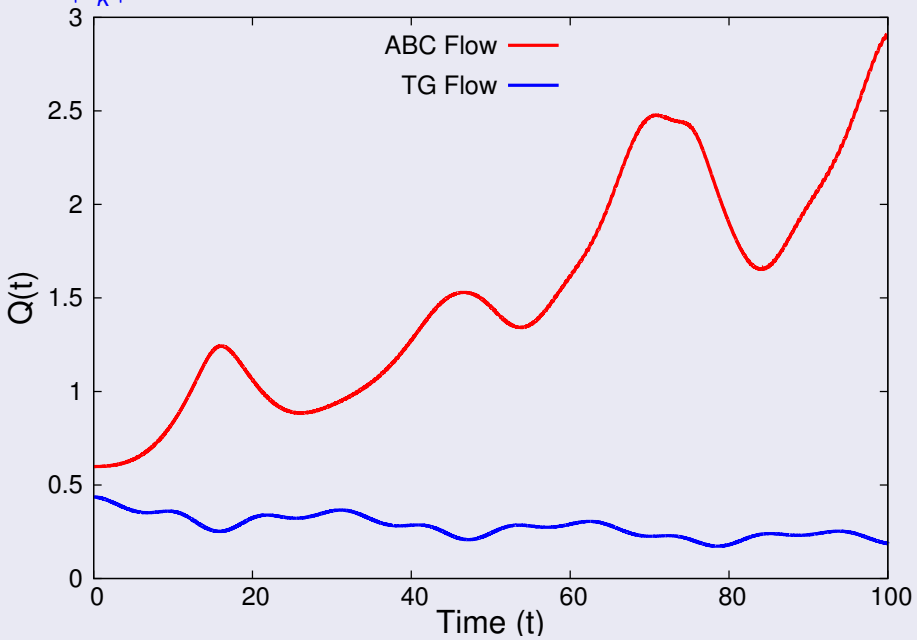
Magnetic Isosurfaces



WHY DOES IT RECUR?

- Rayleigh quotient $[Q(t)]$ measures the number of effective **'active degrees of freedom'**. [A. Thyagaraja, Physics of Fluids, **22**, 2093 (1979); AND A. Thyagaraja, Physics of Fluids, **24**, 1973 (1981)]
  - $Q(t) = \frac{\int_V \left[\left(\vec{\nabla}\times\vec{u}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\times\vec{B}\right)^2 \right] dV}{\int_V \left(|\vec{u}|^2 + \frac{1}{2}|\vec{B}|^2 \right) dV} = \frac{k^2 |c_k|^2}{|c_k|^2}$ where, $\vec{u}$ & $\vec{B}$ are expanded in a Fourier series.

- If $Q(t)$ is bounded $\Rightarrow$ Recurrence can happen.
- For TG flow, $Q(t)$ is bounded.
- For ABC flow $Q(t)$ increases without bound.



RM, R Ganesh, A Sen, Phys Plasmas **26**, 022101 (2019) **[Editor's Pick]**

Summary & Discussions

- Ideally very low probability of trapping in phase space in high dimensional systems (e.g. 3D MHD systems).
- Recurrence is observed for flows involving few number of **active** degrees of freedom. [Birkhoff, Dynamical Systems, 1927, Chapter 7]
- Recurrence can be helpful to make short time forecasts once typical initial profiles are experimentally obtained.
- Energy exchange between fluid and magnetic variables opens up new cascading paths for energy amongst different scales.
- Nonlinear Dispersionless Alfven Waves generated via certain specific flow and field profiles cause Recurrence.

Acknowledgement

- A Thyagaraja, Culham Labs, UK
  (for several Physics discussions related to MHD)
- Nagavijayalakshmi Vydyanathan, NVIDIA
  (for teaching GPU computing)
- PPPL & AAPPS-DPP
  (for financial support)

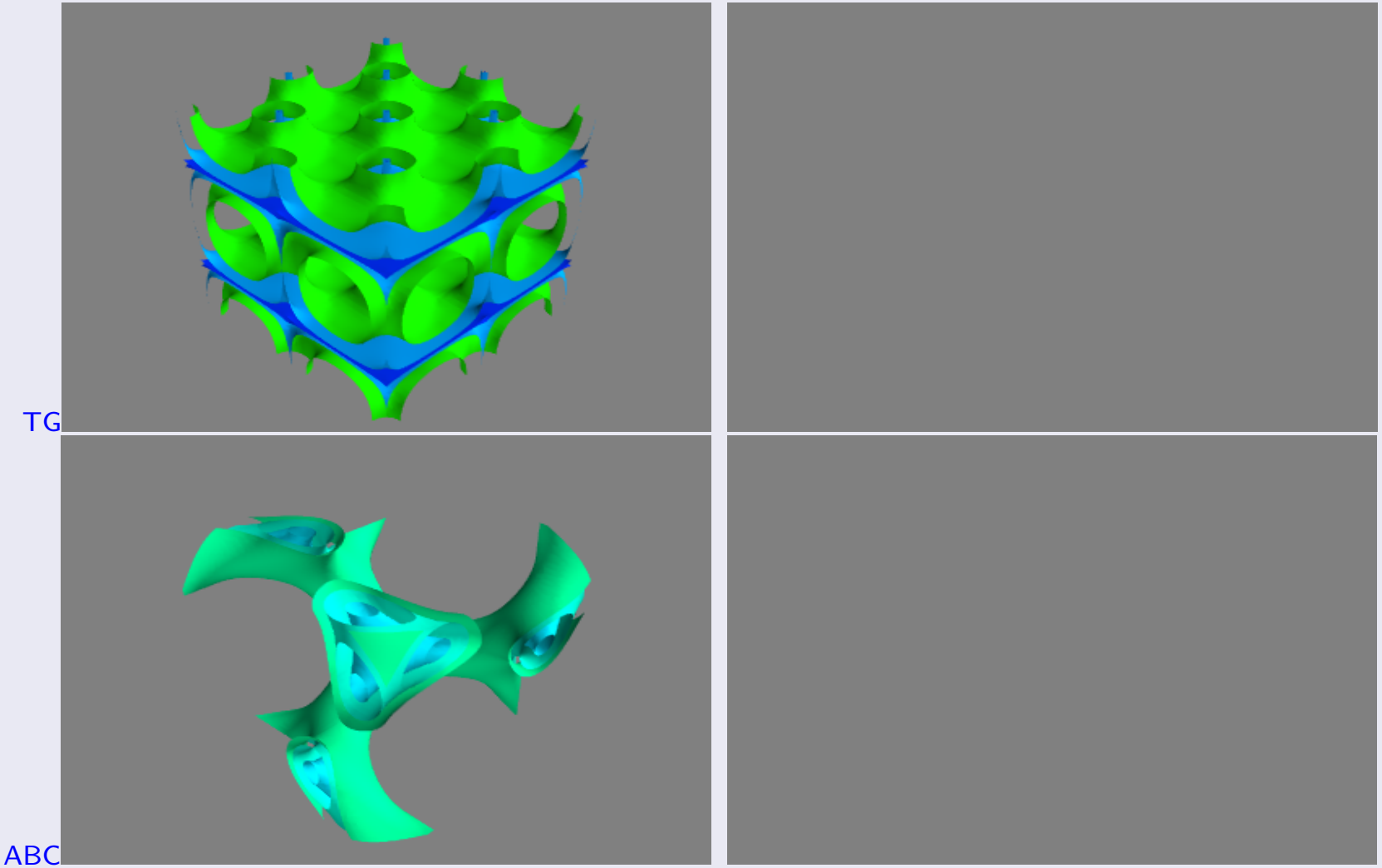
Thank You

Acknowledgement

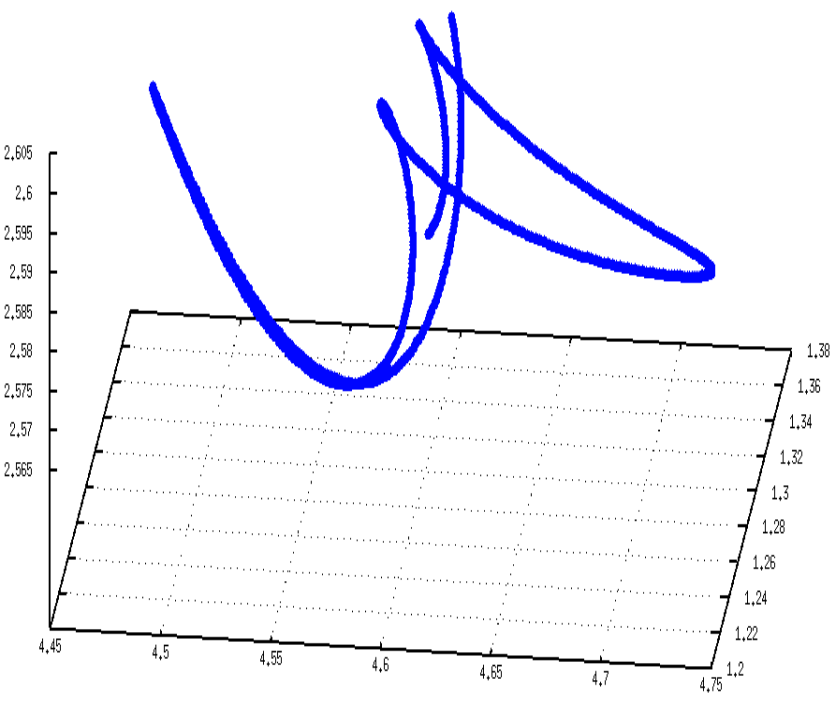
- A Thyagaraja, Culham Labs, UK
  (for several Physics discussions related to MHD)
- Nagavijayalakshmi Vydyanathan, NVIDIA
  (for teaching GPU computing)
- PPPL & AAPPS-DPP
  (for financial support)

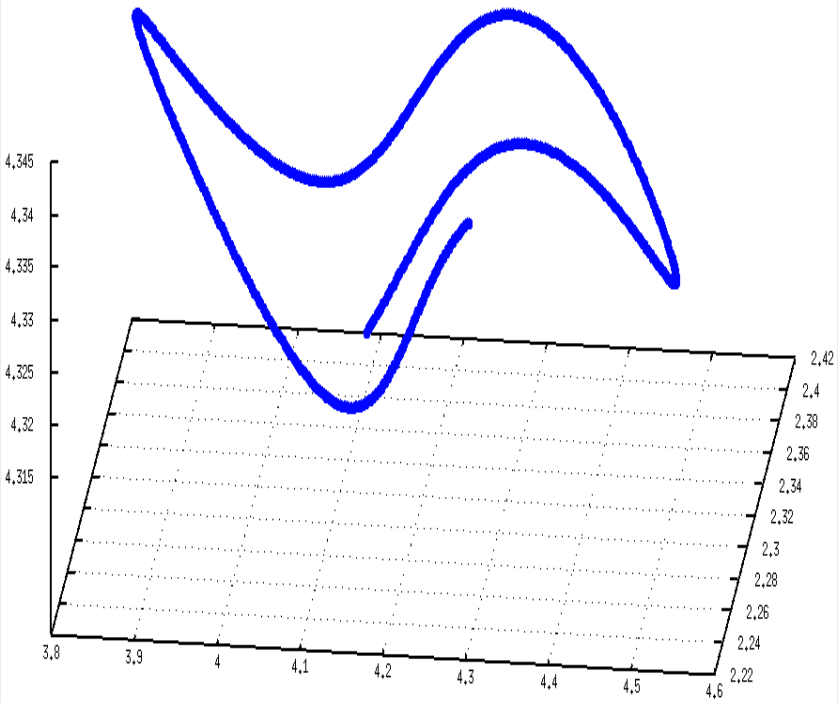
Thank You

Recurrence - TG (✓) & ABC (✗) flow

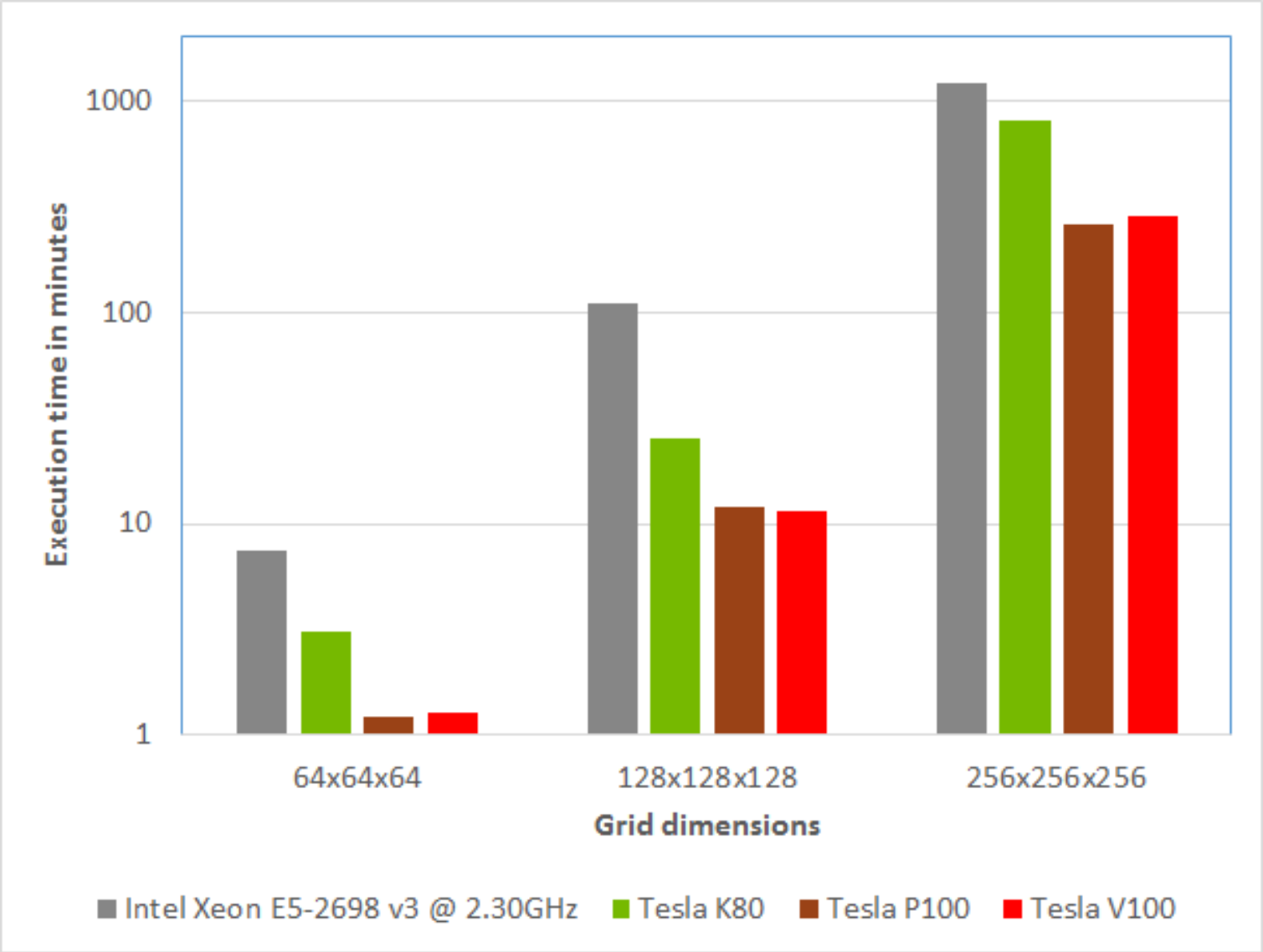


Motion of Tracer particles during Recurrence with TG flow

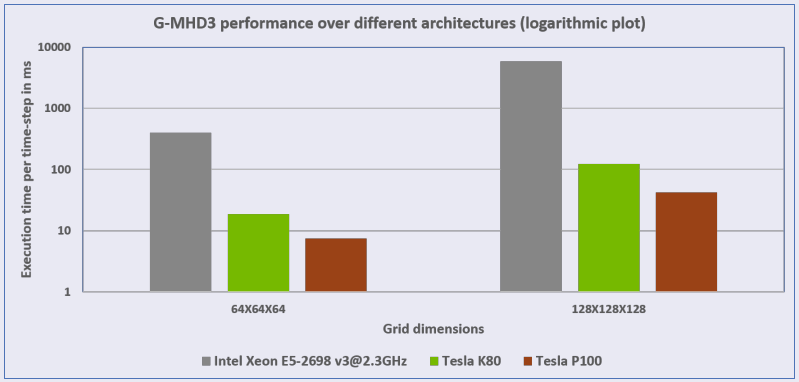


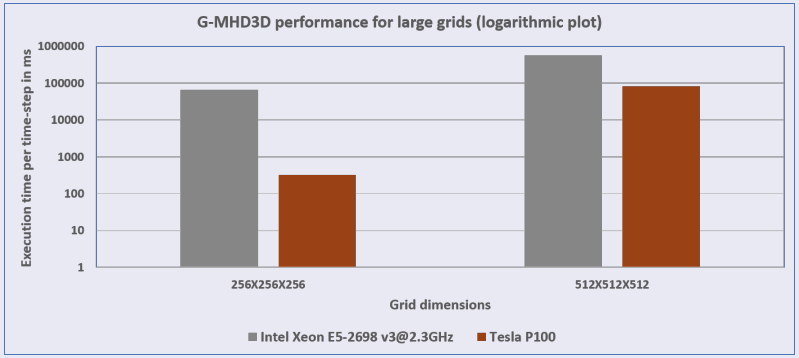


GPU Performance of G-MHD3D (With NVIDIA)



GPU Performance of G-MHD3D with Passive Tracers (With NVIDIA)





G-MHD3D & 3D Poisson solver Performance – Hackathon - 2018

3D Poisson Solver: Number of GPUs (Resolution)	Total Time (ms)	Local FFT (ms)	Communication (ms)
1 (512 X 512 X 512)	19.9	19.8	0
2 (512 X 512 X 512)	25.4	9.84	14.0
4 (512 X 512 X 512)	14.6	5.01	8.59
4 (1024X1024X1024)	153	82.0	61.0

GMHD3D: Grid Resolution Out-of-place FFT	Run time of GMHD3D (in second) (10 iterations)
64 X 64 X 64	1.54
128 X 128 X 128	14.6

Performance of Multi-GPU 3D Poisson solver using accFFT library.

GMHD3D has been openACC parallalised using cuFFT library.

GMHD3D: Grid Resolution In-place FFT	Run time of GMHD3D (in seconds) (10 iterations)
64 X 64 X 64	0.464
128 X 128 X 128	0.861
256 X 256 X 256	4.009
512 X 512 X 512	646.6

Poisson using MPI & OpenACC

- Resolution: 1024 X 512 X 256 – Run on DGX

Number of GPU	Total Time (ms)	Local FFT (ms)	Communication (ms)
1	44.6	44.6	0
2	38.6	22.7	14.3
4	21.6	11.6	8.74

- Cuda aware MPI accFFT libraries are used.
- Pipelined FFTs for overlap of communication.
- Implement the changes in the full code – GMHD3D.