NSTX-U

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U.S. Department of Energy Office of Science
Fusion Energy Sciences

Theory for control of sub-cyclotron Alfvén instabilities and implications for anomalous electron energy transport

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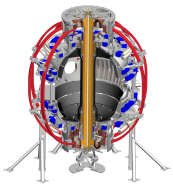
61st APS-DPP Meeting
Fort Lauderdale, FL
October 24, 2019

PPPL



Motivation and Main Results

- **Motivation**: sub-cyclotron Alfvén eigenmodes (AEs) have been experimentally linked to anomalous electron temperature flattening in NSTX
  - No theory quantitatively reproduces the observations
- **Goal**: predict instability conditions for realistic neutral beam (NBI) distributions using analytic theory and numerical simulations
- **Main result**: theory describes high frequency AE excitation and demonstrates how to stabilize modes with additional NBI source
  - Explains NSTX-U suppression of AEs with new beam source
  - Provides tool to control and study the associated electron energy transport

Outline

- Introduction: Alfvén Eigenmodes Connected to Anomalous Transport
- Hybrid Simulations Reveal Complicated Stability Boundaries
- Reduced Analytic Theory Explains Simulations and Observations
- Injecting Multiple Neutral Beams Can Control Alfvén Eigenmodes

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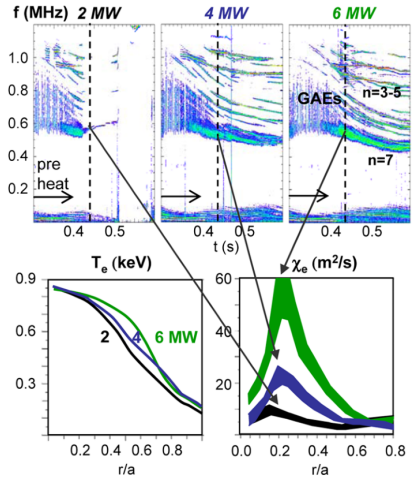
Anomalous T_e Flattening in NSTX Correlates with CAE/GAEs

- Beam-driven compressional (CAE) and global (GAE) Alfvén eigenmodes have been excited in NSTX(-U), MAST, DIII-D, and may be present in ITER
- Temperature profiles can not be explained by turbulence in gyrokinetic simulations
- Methods to control CAEs/GAEs are essential to studying and predicting the electron energy transport that they induce[1]

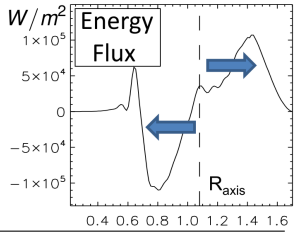


[1] D. Stutman *et al.* Phys. Rev. Lett. **102**, 115002 (2009)

How Can CAEs/GAEs Affect Temperature Profiles?

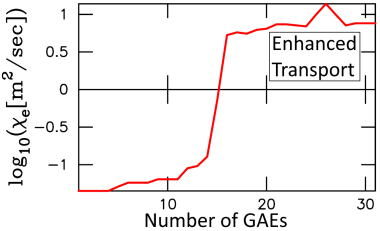
Energy Channeling

- AE in core can mode convert to KAW near edge, damping on electrons
- Modifies effective beam energy deposition profile[2,3]



Orbit Stochastization

- Sufficiently many unstable AEs can stochasticize electron orbits
- Enhances diffusion, transporting energy away from the core[4]



[2]Y.I. Kolesnichenko *et al.* Phys. Rev. Lett. **104**, 075001 (2010)
[3]E.V. Belova *et al.* Phys. Rev. Lett. **115**, 015001 (2015)
[4]N.N. Gorelenkov *et al.* Nucl. Fusion **50**, 084012 (2010)

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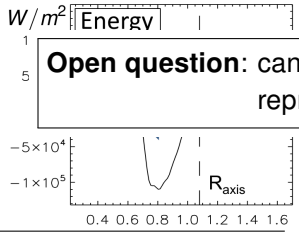

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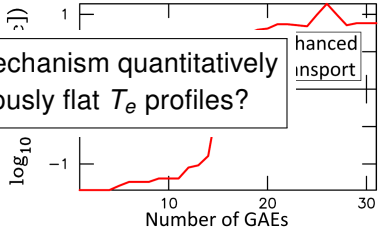

Open question: can either transport mechanism quantitatively reproduce the anomalously flat T_e profiles?

[2]Y.I. Kolesnichenko *et al.* Phys. Rev. Lett. **104**, 075001 (2010)
[3]E.V. Belova *et al.* Phys. Rev. Lett. **115**, 015001 (2015)
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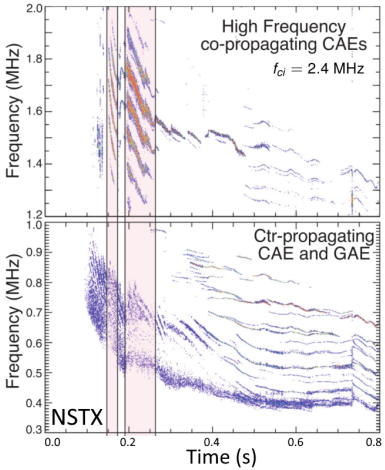
Sub-cyclotron Alfvén Eigenmodes in NSTX(-U)

- **Compressional Alfvén eigenmode** (CAE): ideal magnetosonic mode: $\omega \approx k v_A$
- **Global Alfvén eigenmode** (GAE): discrete shear Alfvén eigenmode existing below minimum of Alfvén continuum: $\omega \leq \left[k_{\|}(r) v_A(r)\right]_{\min}$
- CAEs/GAEs interact with fast ions through Doppler-shifted cyclotron resonance $\omega - \left\langle k_{\|} v_{\|} \right\rangle - \left\langle k_{\perp} v_{\mathrm{Dr}} \right\rangle = \ell \left\langle \omega_{ci} \right\rangle$
- Observed to be co- $(\ell = 0)$ and cntr- $(\ell = 1)$ propagating with $|n| = 3 - 15$, $\omega/\omega_{ci} \approx 0.1 - 1.2$



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Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)
- Initial value code in 3D toroidal geometry
- Thermal plasma: single fluid resistive MHD model
- Beam ions: full orbit kinetic particles with δF scheme
  - Captures cyclotron resonance which drives the modes
- Equilibrium includes fast ion effects self-consistently[5]
- Thermal plasma and beam ions coupled through current in momentum equation

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + \underbrace{(\boldsymbol{J}-\boldsymbol{J_b}) \times \boldsymbol{B} - e n_b (\boldsymbol{E} - \eta \delta \boldsymbol{J})}_{\text{current coupling}} + \mu \Delta \boldsymbol{V}$$

[5] E.V. Belova *et al.* Phys. Plasmas **10**, 3240 (2003)

Fast Ion Distribution Model

- Equilibrium distribution $F_0 = F_1(v)F_2(\lambda)F_3(p_\phi)$
  - Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx v_\perp^2/v^2$

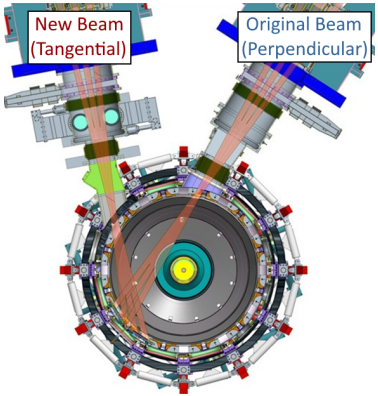
$$F_1(v) = \frac{1}{v^3 + v_c^3}$$

$$F_2(\lambda) = \exp\left(-(\lambda - \lambda_0)^2/\Delta\lambda^2\right)$$

$$F_3(p_\phi) = \left(\frac{p_\phi - p_{\min}}{m_i R_0 v - q_i \psi_0 - p_{\min}}\right)^\alpha$$

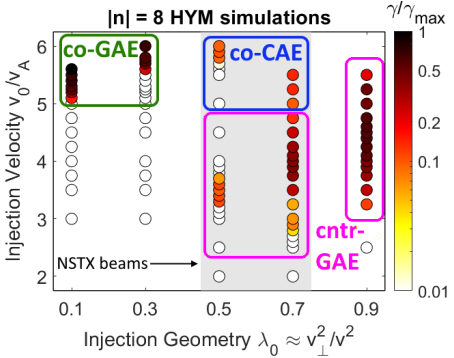
for $v < v_0$ and $p_\phi > p_{\min}$

- NSTX: $v_0/v_A \lesssim 5$, $\lambda_0 = 0.5 - 0.7$ for original beam
- NSTX-U: $v_0/v_A \lesssim 2$, $\lambda_0 = 0$ for new beam
- Distribution parameters matched to TRANSP



Hybrid Simulations Predict Rich Mixture of CAEs/GAEs

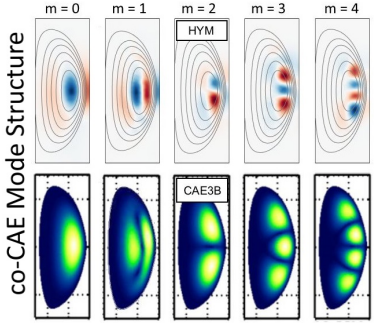
- Parameter scan of injection geometry (λ_0) and velocity (v_0/v_A) reveals complicated stability boundaries for different mode types
- GAEs excited at lower beam energy ($v_0/v_A \gtrsim 2.5$) than CAEs ($v_0/v_A \gtrsim 4$), typically with larger growth rates
- co-GAEs excited with very tangential beams
  - Anomalous cyclotron resonance ($\ell = -1$)
  - May exist in future NSTX-U experiments

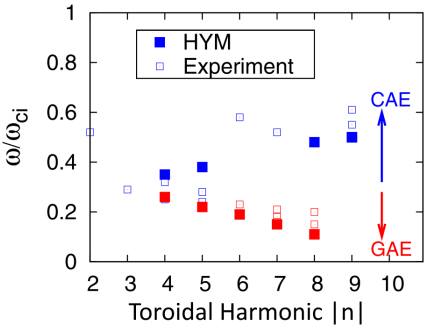


- Colored dot: growth rate of most unstable mode in each simulation
- White dot: no unstable modes

Realistic Simulations Motivate Development of Theory

- HYM simulations accurately model CAEs/GAEs in NSTX(-U) experiments and recover eigenmodes from spectral codes
- Theory is needed to interpret simulation results and improve understanding to develop predictive capability





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Theory Applied to Understand Experiments and Simulations

- Linear growth rate derived for realistic NBI distribution
  - Uncovers new instability regime – necessary to explain GAE excitation in NSTX-U
- Goal: simple expressions for fast ion drive depending on
  1. fast ion distribution parameters ($\lambda_0, v_0/v_A$)
  2. mode parameters ($\omega/\omega_{ci}, k_\parallel/k_\perp$)
- Approach: restrict to 2D velocity space to avoid assumptions about equilibrium profiles, mode structure, particle orbits, *etc.*
- Provides upper bound on total growth rate, since neglecting bulk damping sources

Simple Conditions Derived for Net Drive

- Fast ion drive depends on gradients of the distribution

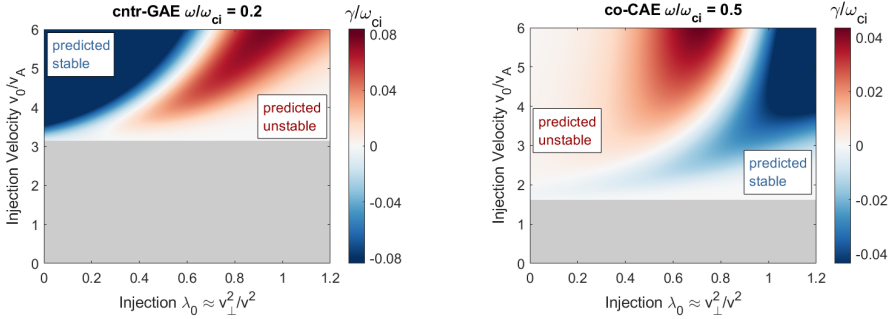
$$\gamma \propto \int \left[\left(\frac{\ell \omega_{ci}}{\omega} - \lambda \right) \frac{\partial}{\partial \lambda} + \frac{v}{2} \frac{\partial}{\partial v} \right] f_b(\lambda, v) > 0 \text{ for instability}$$

- cntr-propagating CAE/GAEs ($\ell = +1$) driven by $\partial f_b / \partial \lambda > 0$
  - $v_0 < v_{\parallel,\mathrm{res}} / (1 - \lambda_0 \langle \bar{\omega}_{ci} \rangle)^{3/4}$ necessary for instability[6]
- co-propagating CAEs ($\ell = 0$) driven by $\partial f_b / \partial \lambda < 0$
  - $v_0 > v_{\parallel,\mathrm{res}} / \left(1 - \frac{\langle \bar{\omega}_{ci} \rangle}{2} \left[\lambda_0 + \sqrt{\lambda_0^2 + 8\Delta\lambda^2/3} \right] \right)^{5/8}$ necessary for instability
- Due to resonance condition, $v_{\parallel,\mathrm{res}}(\omega/\omega_{ci}, k_\parallel/k_\perp) = (\omega - \ell \langle \omega_{ci} \rangle) / k_\parallel$

[6] J.B. Lestz *et al.* Phys. Plasmas (submitted)

Analytic Bounds Explain Simulation Results

- Numerically integrate full analytic expression for growth rate to predict instability
  - red: net fast ion drive, blue: net fast ion damping
  - gray: insufficient beam velocity for resonant interaction



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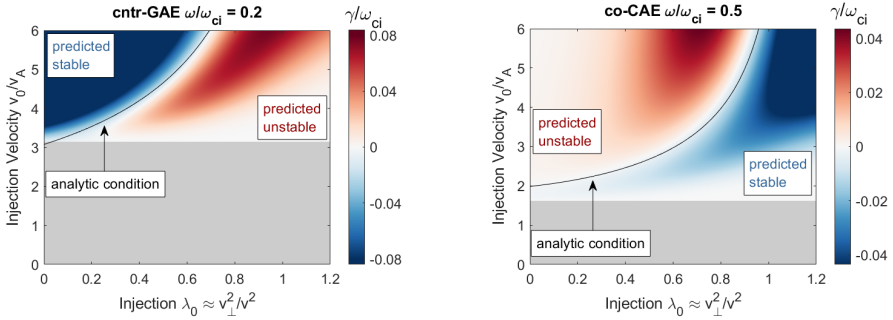

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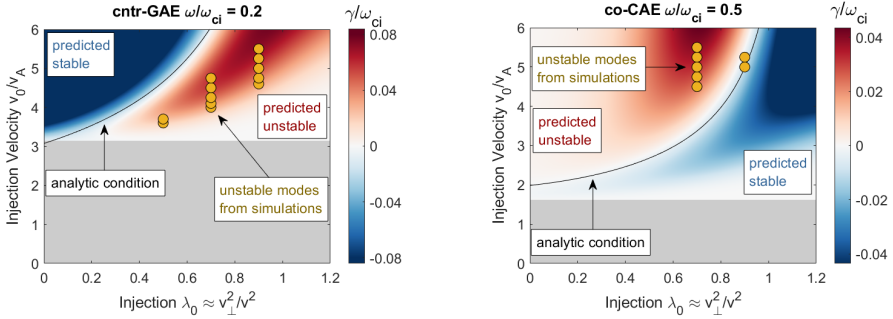
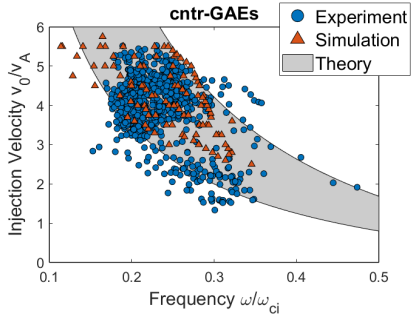
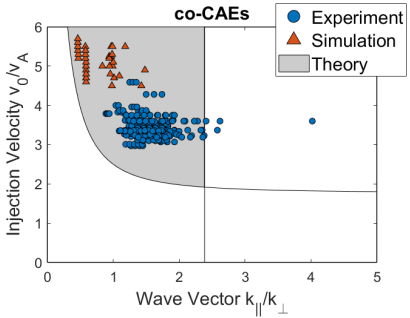
- black curve: approximate analytic conditions reproduce numerical calculation
- gold: unstable modes from HYM simulations agree with theory



Agreement Between Experiment, Theory, and Simulation

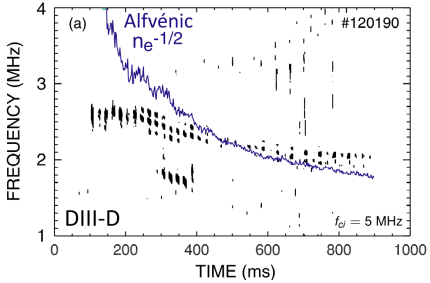
- For fixed NBI parameters, instability conditions constrain the spectrum of modes
- Cross-comparison with NSTX database of cntr-GAEs and co-CAEs demonstrates greater than 80% agreement with theory
- blue: NSTX observations, red: HYM simulations
  gray: unstable region predicted by theory





Theory Explains Observations on DIII-D

- DIII-D low field experiments ($v_0/v_A \approx 1.5$) observed cntr-modes with $\omega/\omega_{ci} \approx 0.6$
  - Tentatively identified as CAEs, with unexplained $k_\perp \rho_{\perp b} < 0.8$
  - Density scaling also sub-Alfvénic, conflicting with dispersion relation[7]



[7] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

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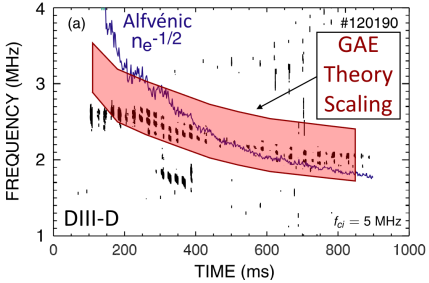
- Theory predicts narrow range of unstable GAEs: $\frac{1}{1+v_0/v_A} < \frac{\omega}{\omega_{ci}} < \frac{1}{1+v_0/v_A(1-\lambda_0)^{3/4}}$
  - Compatible with any value of $k_\perp \rho_{\perp b}$
  - Weaker density scaling agrees with data
- Theory predicts higher frequencies for larger λ_0
- Unstable CAEs require higher frequencies



- High frequency observations in DIII-D may be more consistent with GAEs

[7] W.W. Heidbrink *et al.* Nucl. Fusion **46**, 324 (2006)

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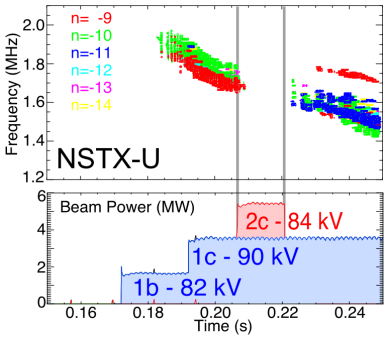
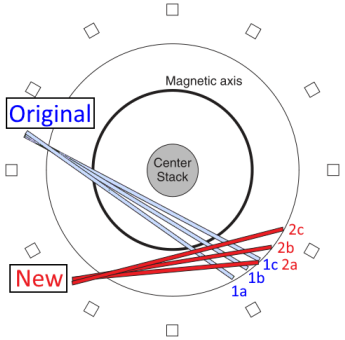
cntr-GAE Suppression Discovered on NSTX-U



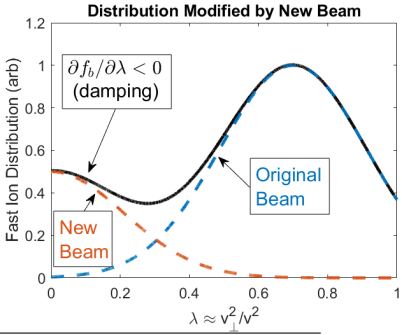


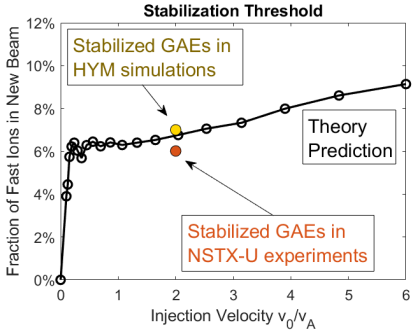
- NSTX-U found robust suppression of cntr-GAEs with addition of new off-axis/tangential beams[8]

[8] E.D. Fredrickson *et al.* Phys. Rev. Lett. **118**, 265001 (2017)

Analytic Theory Explains cntr-GAE Stabilization on NSTX-U

- Tangential injection flips sign of $\partial f_b/\partial\lambda \rightarrow$ damping
- Stabilization: damping from new beam balances drive from original beam
  - 7% of fast ions in new beam predicted for cntr-GAE stabilization (similar for cntr-CAEs)
    - Very close to experiment and HYM simulations[9]

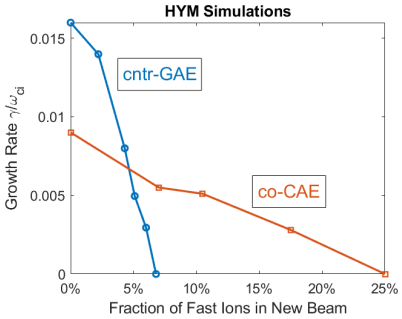




[9]E.V. Belova *et al.* Phys. Plasmas **26**, 092507 (2019)

co-CAE Stabilization More Subtle than for cntr-GAEs

- Theory predicts that very perpendicular injection will decrease drive for co-CAEs
  - Difficult to verify in simulations since such conditions destabilize cntr-GAEs
- Surprisingly, simulations show that tangential injection also stabilizes co-CAEs
  - Requires $\sim$ 25% fast ions in new beam vs 7% for cntr-GAEs
- Large radiative damping ($\gamma_{damp}/\gamma_{drive} \sim 60\%$) may be sensitive to changes in the equilibrium or mode structure.
  - Theory is less applicable near marginal stability and for non-cyclotron resonance
- **Open question**: what is the dominant mechanism for co-CAE stabilization by tangential injection?



Stabilization Techniques for Studying Electron Transport

1. Add a new beam in a different geometry (increase damping from $\partial f_b/\partial\lambda$)
   - To suppress **cntr**-GAEs/CAEs, add a more tangential beam
     - explains NSTX-U GAE suppression observations
   - To suppress **co**-CAEs, add a very tangential or perpendicular beam
     - driven by $\partial f_b/\partial\lambda < 0$, opposite condition for cntr-CAEs/GAEs
     - near marginal stability, large radiative damping is sensitive to beam distribution
   - To suppress **either**, counter-inject a new beam
     - Accesses new resonance for same mode, with opposite contribution to drive
2. Add a new beam at a different voltage without changing geometry
   - Adding a beam at a lower voltage should suppress **co**-CAEs
3. Add resonant particles which are stabilizing ($\lambda > \lambda_0$ for cntr-GAEs)
   - Can be achieved by lengthening the tail of the distribution – RF heating?

Summary and Outlook

- CAEs/GAEs were investigated in NSTX(-U) with hybrid simulations and theory
- A reduced analytic theory of sub-cyclotron Alfvén eigenmode instability has been developed for realistic NBI distributions
  - **Perpendicular injection**: drives cntr-propagating CAEs/GAEs and damps co-modes
  - **Tangential injection**: damps cntr-CAEs/GAEs and can drive or damp co-modes
  - Explains experimental observations and simulations of CAE/GAE excitation and stabilization in multiple devices (NSTX, NSTX-U, DIII-D)
- **Impact**: theory for control of CAEs/GAEs will enable investigation of their role in electron energy transport and help identify transport mechanisms
- **Future Directions**: (1) incorporate radiative damping to apply theory closer to marginal stability and (2) develop reduced analytic theory to interpret observations of higher frequency ion cyclotron emission (ICE).

Backup Slides

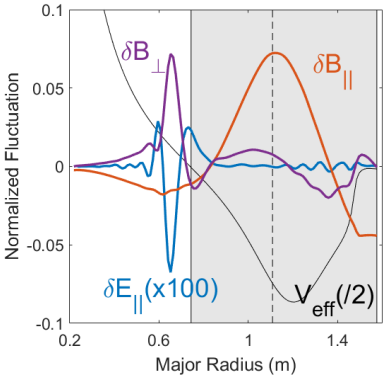
Compressional Alfvén Eigenmodes (CAE)

- Ideal magnetosonic mode in toroidal geometry
  - Compressional polarization
  - In uniform, low β limit, $\omega = kv_A$
- Localized by 2D wave equation

$$\left[\nabla_\perp^2 - V_{\text{eff}}(r,\theta)\right]\delta B_\parallel = 0$$

$$V_{\text{eff}}(r,\theta) = k_\parallel^2 - \frac{\omega^2}{v_A^2} \approx \left(\frac{n}{R}\right)^2 - \left(\frac{\omega}{v_A}\right)^2$$

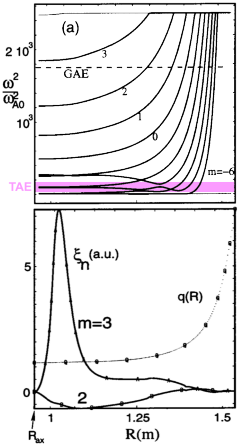
- $V_{\text{eff}} = 0$ coincides with Alfvén resonance, where CAE couples to kinetic Alfvén wave



$n = 4$ CAE calculated by HYM. $\delta B_\parallel$ corresponds to the CAE. Coherent $\delta B_\perp$, $\delta E_\parallel$ structures show the KAW.

Global Alfvén Eigenmodes (GAE)

- Discrete shear Alfvén eigenmode solutions may exist below minimum of Alfvén continuum
  - Approximate dispersion $\omega \leq \left[k_{\|}(r)v_A(r)\right]_{min}$
  - Weakly damped due to separation from continuum
- Dominant shear polarization: $\delta B_{\perp} \gg \delta B_{\|}$
  - In NSTX conditions, also have large compressional component $\delta B_{\|} \approx \delta B_{\perp}$ near edge
- CAEs/GAEs routinely observed in NSTX with $|n| = 3 - 12$ and $\omega/\omega_{ci} \approx 0.1 - 1.2$
  - ICE with $\omega > \omega_{ci}$ also present



GAE calculated by NOVA

HYM Physics Model

Fluid thermal plasma

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla P + (\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - en_b(\boldsymbol{E} - \eta\delta\boldsymbol{J}) + \mu\Delta\boldsymbol{V}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta\delta\boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{V})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

Kinetic fast ions

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{q_i}{m_i}(\boldsymbol{E} - \eta\delta\boldsymbol{J} + \boldsymbol{v} \times \boldsymbol{B})$$

δF Scheme

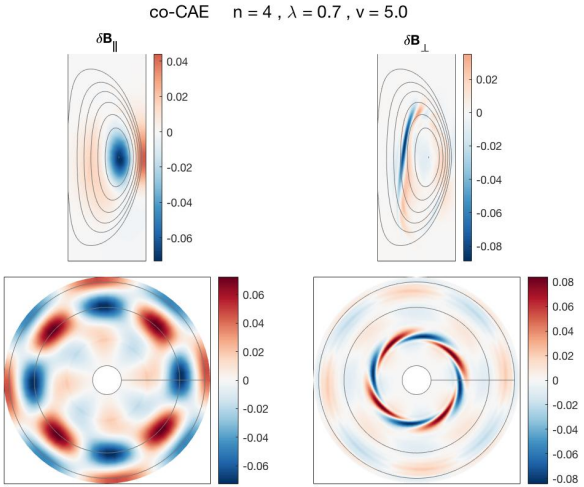
$$F = F_0(\mathcal{E}, \mu, p_\phi) + \delta F(t)$$

$$w \equiv \delta F / F$$

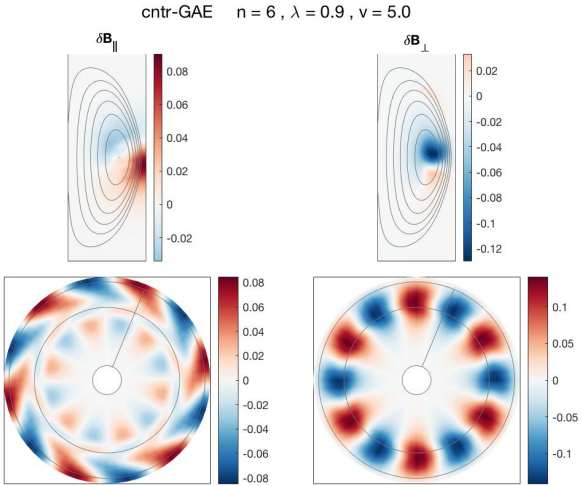
$$\frac{dw}{dt} = -(1 - w)\frac{d \ln F_0}{dt}$$

- ρ, $\boldsymbol{V}$, P are plasma mass density, velocity, and pressure
- n_b, $\boldsymbol{J_b}$ are beam ion density and current

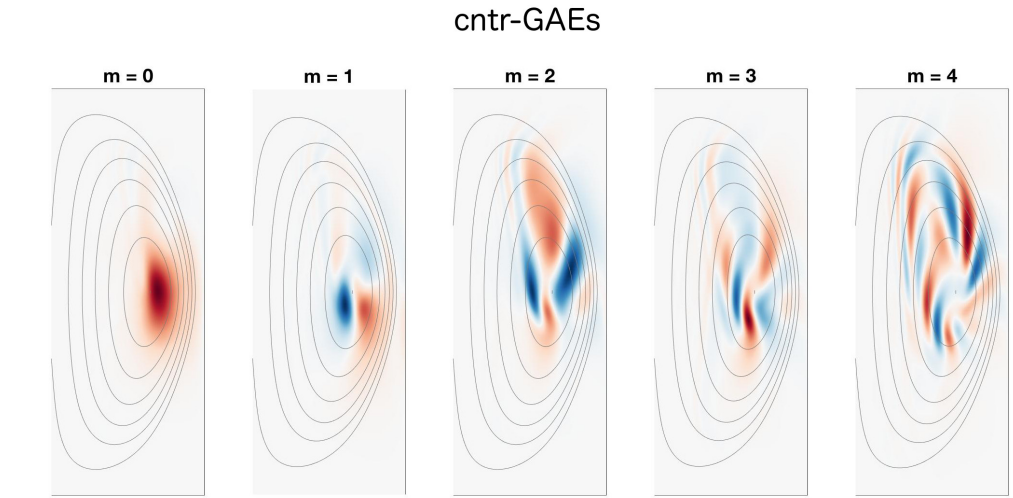
CAE Mode Structure from HYM



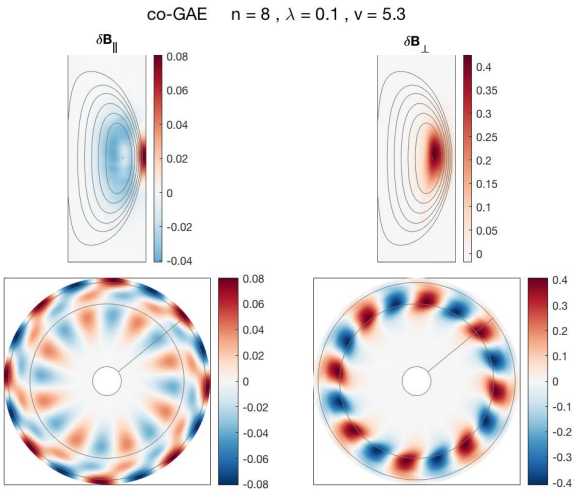
cntr-GAE Mode Structure from HYM



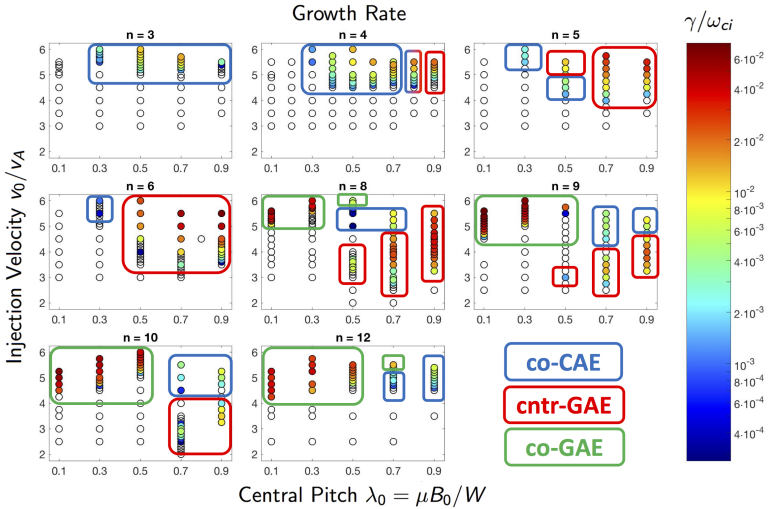
cntr-GAE Mode Structure from HYM



co-GAE Mode Structure from HYM



Linear Simulation Stability Results



Growth Rate for Beam Distribution

- For a beam-like distribution, with $x \equiv \mathcal{E}_\perp/\mathcal{E}$, $u \equiv \mathcal{E}/\mathcal{E}_0 = \mathcal{E}_\parallel^{res}/\mathcal{E}_0(1-x)$

$$\frac{\gamma}{\omega_{ci}} \propto -\sum_\ell \int_0^{1-\mathcal{E}_\parallel^{res}/\mathcal{E}_0} \underbrace{\frac{x}{(1-x)^2}\overbrace{\mathcal{J}_\ell^m\left(\frac{k_\perp v_{\parallel,\mathrm{res}}}{\omega_{ci}}\sqrt{\frac{x}{1-x}}\right)}^{\text{FLR terms}}\overbrace{\frac{e^{-(x-\lambda_0\langle\bar\omega_{ci}\rangle)^2/\Delta\lambda^2\langle\bar\omega_{ci}\rangle^2}}{1+(4u)^{3/2}}}^{\text{fast ion distribution}}}_{\text{non-negative}} \times$$

$$\underbrace{\left[\frac{2}{\Delta\lambda^2\langle\bar\omega_{ci}\rangle^2}\left(x-\lambda_0\langle\bar\omega_{ci}\rangle\right)\left(\frac{\ell}{\bar\omega}-x\right)\right.}_{\substack{\partial f_b/\partial\lambda \text{ drive/damping}\\ (\text{has sign of } x-\lambda_0\langle\bar\omega_{ci}\rangle)}} \underbrace{\left.+\frac{3}{2}\left(1-\frac{1}{1+(4u)^{3/2}}\right)\right]}_{\substack{\partial f_b/\partial\mathcal{E} \text{ damping}\\ (\text{negligible for } \ell\neq 0)}} dx$$

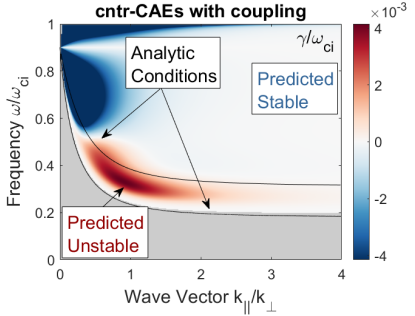
- If $1-\mathcal{E}_\parallel^{res}/\mathcal{E}_0 < \lambda_0\langle\bar\omega_{ci}\rangle$, then the integrand does not change sign.
  - **co**-propagating modes ($\ell = 0, -1$) will be strictly **damped**
  - **cntr**-modes ($\ell = +1$) will be strictly **driven** by fast ions

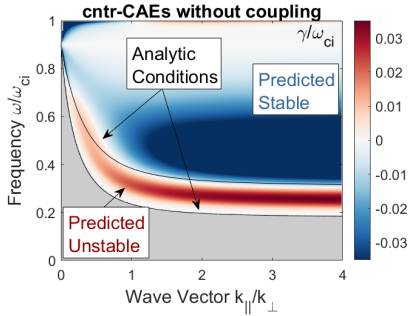
CAE/GAE Coupling Can Alter Most Unstable Modes

- Including two fluid effects, dispersions are coupled, modifying the polarization

$$\left[1 - \frac{k_{\|}^2 v_A^2}{\omega^2}\left(1 - \frac{\omega^2}{\omega_{ci}^2}\right)\right]\left[1 - \frac{k^2 v_A^2}{\omega^2}\left(1 - \frac{\omega^2}{\omega_{ci}^2}\right)\right] = \frac{\omega^2}{\omega_{ci}^2}$$

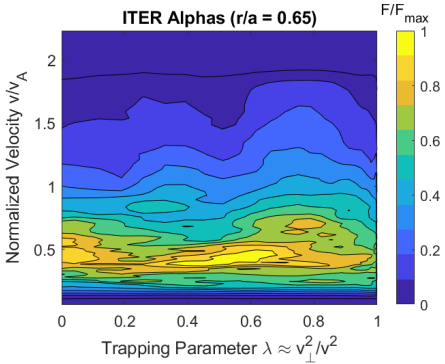
  - Changes growth rate, most unstable parts of spectrum
    - Most important for cntr-CAEs, also co-CAEs at smaller v_0/v_A
- $\ell = 0$ co-GAE can not exist without this coupling





CAEs/GAEs May Be Present in Burning Plasmas

- ITER will have super-Alfvénic NBI and alpha particles ($v_0/v_A = 1.5 - 2$)
- Anisotropy of alphas near the edge could destabilize cntr-GAEs/CAEs
  - Similar to NSTX(-U) beam parameters
- ITER NBI distribution has $\lambda_0 = 0.3 - 0.8$ depending on radius
  - Could be either destabilizing or stabilizing (NSTX-U multi-beam suppression)



- **Open question**: if the modes are excited, will the anomalous electron transport also be present in ITER or is it unique to spherical tokamaks?

Open Questions

1. Which transport mechanism is the dominant cause of anomalous flat T_e profiles?
2. Will CAEs/GAEs be unstable in ITER? Will they induce anomalous transport?
3. What is the dominant mechanism for co-CAE stabilization by tangential injection?
4. Can the analytic stability boundaries be generalized to $\omega \gg \omega_{ci}$ in order to interpret ion cyclotron emission (ICE)?