Fluctuation Dynamo in Collisionless and Weakly Collisional Magnetized Plasmas

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Fluctuation dynamo and folded magnetic fields

Random velocity shears stretch and twist a seed magnetic field:

\[ \frac{d \ln B}{dt} = \hat{b} \cdot \nabla u - \nabla \cdot u, \]

arranging the magnetic fields into long, thin folds.

\( B \) anti-correlated with field-line curvature \( \hat{b} \cdot \nabla \hat{b} \)

The $\mathcal{P}_m \gg 1$ MHD fluctuation dynamo

Four phases:
1. Diffusion-free
2. Kinematic  
   ▶ Kazantsev $k^{3/2}$ spectrum.
3. Nonlinear  
   ▶ $\rho u \cdot \nabla u \sim B \cdot \nabla B / 4\pi$
   ▶ smallest-scale stretching suppressed
   ▶ Secular growth of $\langle B^2 \rangle$
4. Saturation  
   ▶ minimization of $\hat{b} \cdot \nabla u$
   ▶ $v_A \sim u_{\text{rms}}$ (not scale-by-scale!)
Theoretical ingredients of the plasma dynamo

ICM only requires $B \sim 10^{-18}$ G to be magnetized (i.e. $\rho_i \sim \lambda_{\text{mfp}}$). Conservation of magnetic moment $\mu = \frac{w^2}{B} \rightarrow \frac{d}{dt}(p_\perp/nB) = 0$.

Thus

1. As $B$ increases, $p_\perp$ increases $\rightarrow p_\perp \neq p_\parallel$ (*Bad for dynamo! – Helander et al. 2016*)

2. Estimate size of $\Delta p = p_\perp - p_\parallel$ in weakly collisional plasmas using CGL equations and collisions:

$$\frac{d}{dt} \frac{p_\perp - p_\parallel}{p} \approx 3 \frac{d \ln B}{dt} - \nu_i \frac{p_\perp - p_\parallel}{p}$$

Recall:

$$\frac{d \ln B}{dt} = \hat{b} \cdot \nabla u - \nabla \cdot u.$$ 

Results in $\sim 1\%$ deviations from local thermodynamic equilibrium.
Mirror and firehose instabilities

These instabilities arise in high-\(\beta\) (\(\simeq 8\pi p/B^2\)) plasmas. Defining \(\Delta = p_\perp/p_\parallel - 1\),

**Firehose** \((\Delta < -2/\beta)\):

**Mirror** \((\Delta > 1/\beta)\):

Saturation at \(\nu_{\text{eff}} \sim |\mathbf{\hat{b}} \cdot \nabla u|\beta\) (Kunz+ 2014; Melville+ 2016).
Three regimes for plasma dynamo

This physics suggests three dynamo regimes:

1. Unmagnetized regime \((\Omega_i \ll \nu_i, \text{ see Rincon et al. 2016})\)
2. Magnetized ‘kinetic’ regime \((\Omega_i \ll |\hat{b}\cdot \nabla u| \beta)\)
3. Magnetized ‘fluid’ regime \((\Omega_i \gg |\hat{b}\cdot \nabla u| \beta)\)

I now present results from:

1. Hybrid-kinetic simulations (St-Onge & Kunz 2017)
   - How does the dynamo operate in a collisionless plasma?
   - *Ab initio* measurement of \(\nu_{\text{eff}}\) motivates...

2. Braginskii-MHD simulations (St-Onge+, in prep.)
   - *Given* a prescribed viscosity, how does the plasma self-organize itself to amplify the magnetic field?

3. Analytic Modeling
   - Predicting the dynamo in certain asymptotic regimes.
\frac{|\mathbf{B}|}{B_{\text{rms}}}$
The Punchline

1. Initial

2. Exponential

$\beta_{i0} = 10^6$

3. Nonlinear

4. Saturation

$\beta_{i0} = 10^4$

Mirror and firehose instabilities

Rate of strain is **anisotropic** with respect to the magnetic field as if it were a weakly collisional, magnetized plasma (Braginskii 1965):

\[
S \equiv \frac{1}{2} \left[ \nabla u + (\nabla u)^T \right]
\]

- **Kinetic instabilities generate magnetic energy above** \(\rho_i\).
- **Evidence of firehose visually and in curvature PDFs.**
- **Plasma becomes Braginskii-like**
  \[
  (3\hat{b}\hat{b} : \nabla u \sim \nu_{\text{eff}} \Delta_i)
  \]

**Figure:** Visual evidence of mirror instabilities.
Regulation of the pressure anisotropy is *imperfect*:

\[
\Delta_i = \frac{p_i^\perp}{p_i^\parallel} - 1
\]

This suggests ‘hard-wall’ limiters may not be the ideal closure for kinetic microphysics.
Braginskii-MHD simulations

- Dilute magnetized plasma ($\Omega_i \gg \nu_i \gg \omega$)
- Incompressible Braginskii MHD equations
  \[
  \frac{d}{dt} u = B \cdot \nabla B - \nabla p + \nabla \cdot (\hat{b} \hat{b} \Delta p) + \mu \nabla^2 u + \tilde{f}, \\
  \frac{d}{dt} B = B \cdot \nabla u + \eta \nabla^2 B.
  \]

- Nonhelical, incompressible, time-correlated forcing
- Pressure anisotropy $\Delta p = 3\mu_B \hat{b} \hat{b} : \nabla u$, both:
  - unlimited (parameter study on $\mu_B$)
  - hard-wall limited:
    \[
    \Delta p = \begin{cases} 
    \min \left( \frac{B^2}{2}, 3\mu_B \hat{b} \hat{b} : \nabla u \right), & \Delta p > 0 \\
    \max \left( -B^2, 3\mu_B \hat{b} \hat{b} : \nabla u \right), & \Delta p < 0 
    \end{cases}
    \]

\(^2\)To be submitted to JPP
Hard-walled Braginskii looks like $\mathbb{P}_m \gtrsim 1$ MHD

(in box-averaged evolution)

Figure: Evolution of magnetic energy
Hard-walled Braginskii looks like $P_m \gtrsim 1$ MHD

(in spectra)

Figure: Kinetic and magnetic energy spectra
Unlimited Braginskii dynamo mimics saturated MHD

- Unlimited regime relevant to early stages of plasma dynamo
- Mimics saturated MHD in:
  - statistics of $\nabla u$ and alignment with respect to $\hat{b}$
  - magnetic spectrum
  - fold geometry (including PDF of $\hat{b} \cdot \nabla \hat{b}$)
  - spectral anisotropy of turbulent velocity

Why is this? Compare

$$B \cdot \nabla B = \nabla \cdot (\hat{b} \hat{b} B^2)$$

to

$$\nabla \cdot (\hat{b} \hat{b} \Delta p) \propto \nabla \cdot (\hat{b} \hat{b} \frac{d}{dt} \ln B).$$

Pressure anisotropy plays the role of magnetic-field strength in tension force.
An example: alignment of $(\nabla u + \nabla u^T)/2$ and $\hat{b}$

Figure: Alignment of $\hat{b}$ and eigenvectors $\hat{e}_i$ of $(\nabla u + \nabla u^T)/2$ with eigenvalues $\lambda_i$. Dotted vertical line denotes $45^\circ$ alignment.
A modified Kazantsev model for $\text{Re}_\parallel / \text{Re}_\perp \ll 1$

Consider a velocity field with prescribed statistics

$$u^i(t, x) = 0, \quad u^i(t, x)u^j(t', x') = \delta(t - t')\kappa^{ij}(x - x'),$$

which are anisotropic with respect to $\hat{b}$:

$$\kappa^{ij}(k) = \kappa^{(i)}(k, |\xi|)(\delta^{ij} - \hat{k}_i\hat{k}_j) + \kappa^{(a)}(k, |\xi|)(\hat{b}^i\hat{b}^j + \xi^2\hat{k}_i\hat{k}_j - \xi\hat{b}^i\hat{k}_j - \xi\hat{k}_i\hat{b}^j),$$

where $\xi \equiv \hat{k} \cdot \hat{b}$. We derive an equation for the joint PDF of $B$, $k$ and $\hat{b}$:

$$\mathcal{P}(B, k, \hat{b}) = \delta(|\hat{b}|^2 - 1)\delta(\hat{b} \cdot k)(4\pi^2k)^{-1}P(B, k).$$

This model was originally developed for the saturated state in MHD by Schekochihin (2004)
A modified Kazantsev model for $\Re_{\parallel}/\Re_{\perp} \ll 1$

We then derive an equation for the magnetic energy spectrum $M(k) \doteq (1/2) \int_0^\infty dB \, B^2 P(B, k)$:

$$\frac{\partial M}{\partial t} = \frac{\gamma_{\perp}}{8} \frac{\partial}{\partial k} \left[ (1 + 2\sigma_{\parallel}) k^2 \frac{\partial M}{\partial k} - (1 + 4\sigma_{\perp} + 10\sigma_{\parallel}) k M \right] + 2(\sigma_{\perp} + \sigma_{\parallel}) \gamma_{\perp} M - 2\eta k^2 M,$$

where

$$\gamma_{\perp} = \int \frac{d^3k}{(2\pi)^3} k_{\perp}^2 \kappa_{\perp}(k),$$

$$\sigma_{\perp} = \frac{1}{\gamma_{\perp}} \int \frac{d^3k}{(2\pi)^3} k_{\parallel}^2 \kappa_{\perp}(k),$$

$$\sigma_{\parallel} = \frac{1}{\gamma_{\perp}} \int \frac{d^3k}{(2\pi)^3} k_{\parallel}^2 \kappa_{\parallel}(k).$$
A modified Kazantsev model for $\text{Re}_\parallel/\text{Re}_\perp \ll 1$

Original: small-scale stretching motions suppressed by magnetic tension in saturation:

$$\int_{k_s}^{k_\mu} dk \ E(k) \sim \frac{1}{2} \langle B^2 \rangle,$$

New idea: balance throughout dynamo evolution occurs between the hydrodynamic nonlinearity and the Braginskii viscous stress $\nabla \cdot (\hat{b}\hat{b} \Delta p)$:

$$\int_{k_0}^{k_\mu} dk \ E(k) = 3 \mu_B c_1 \langle |\hat{b}\hat{b} : \nabla \mathbf{u}|^2 \rangle^{1/2} \sim 3 \mu'_B (\sigma_\perp + \sigma_\parallel) \gamma_\perp.$$

Resulting magnetic field growth rate:

$$\gamma = \frac{\gamma_\perp}{8} \left[ \frac{14 \varpi_0 - 1}{1 + (2/5) \varpi_0} - \frac{6}{5} \varpi_0 \right],$$

where $\varpi_0 = W_0/\mu'_B \gamma_\perp$ and $W_0 = \int_{k_0}^{k_\mu} dk \ E(k)$.

Sufficiently large $\gamma_\perp$ (mixing) or $\mu_B$ (parallel viscosity) kills the dynamo!
A new “Prandtl” number

Unlimited Braginskii MHD has two important dimensionless numbers:

\[
\frac{\mu_\parallel}{\eta}, \quad \frac{\mu_\parallel}{\mu_\perp}
\]

MHD Pm NEW!

Ratio of stretching and mixing in the dynamo matters, and is controlled by \(\mu_{\parallel}/\mu_{\perp}\).
Predictions of the model for $\text{Re}_\perp \gg 1$, $\text{Re}_\parallel \sim 1$

$\langle u^2 \rangle/2$

$\langle B^2 \rangle/2$

$\eta^{-1} = 1500$

$\nu^{-1} = 1500$

$\nu^{-1} = 600$

$\nu^{-1} = 240$

$\nu^{-1} = 96$

$W_0 = 0.6$

$\nu'_B = 0.57$

$r_{2D} = 0.005$

$r_{2D} = 0.01$

$r_{2D} = 0.02$

$r_{2D} = 0.03$

$r_{2D} = 0.04$

Figure: Evolution of magnetic energy (left) and growth rates of the Kazantsev model (right).
Examine the opposite limit: $Re_{\parallel} \ll 1$ Stokes flow

In the Stokes flow regime, viscosity is so large that the velocity is determined by a balance between dissipation and driving alone:

$$-\mu \nabla^2 u = \tilde{f}$$

As $\nu \to \infty$, flow becomes $\delta$-correlated in time.
Unlimited Braginskii Dynamo for $\text{Re}_{\parallel} \ll 1$ Stokes flow.

Figure: Evolution of the magnetic energy for MHD and unlimited Braginskii-MHD in the Stokes flow regime for fixed $u_{\text{rms}}$. 

$\langle B^2 \rangle / 2 \propto \left( \frac{u_{\text{rms}}^2}{t/L} \right)^{1/2}$

$\mu^{-1} = \infty$  
$\mu^{-1} = 1500$  
$\mu_B^{-1} = 20$  
$\mu_B^{-1} = 4$  
$\mu_B^{-1} = 2$  
$\mu_B^{-1} = 0.5$  
$\mu^{-1} = 0.5$  
$\mu^{-1} = 4$  

$\eta_H^{-1} = 1.8 \times 10^7$
The take-away points

To summarize,

- **Dynamo exists in a collisionless magnetized plasma.** (See also Rincon et al. 2016 for unmagnetized regime)
- Larmor-scale instabilities play a crucial role.
- Many features appear MHD-like ($Pm \gtrsim 1$), despite collisionless plasma.
- Saturation at $u \sim v_A$.

For weakly collisional plasmas,

- Too anisotropic a viscosity is deleterious for the dynamo
  \textit{(controls ratio of mixing to stretching)}
- Perfect pressure-anisotropy regulation $\rightarrow Pm \sim 1$ MHD
- Weak pressure-anisotropy regulation $\rightarrow$ saturated MHD
Future research directions

- Exact determination of \( \nu_{\text{eff}} \) in the magnetized kinetic regime
- Other components of the Braginskii viscosity (i.e. gyro-viscosity)
- Kinetic electron effects
  - Dynamo relies on magnetized electrons (flux-freezing)!
  - Resistive scale (i.e. fold separation) set by electron physics
- Interplay between mean-field and fluctuation dynamos:
  - Historical anxiety about mean-field dynamo in the face of fluctuation dynamo
  - Fluctuation dynamo can lead to catastrophic \( \alpha \) quenching!
  - Could kinetic effects alleviate these concerns?
Questions?