Fluctuation Dynamo in Collisionless and Weakly Collisional Magnetized Plasmas

Denis St-Onge
Matthew Kunz

in collaboration with

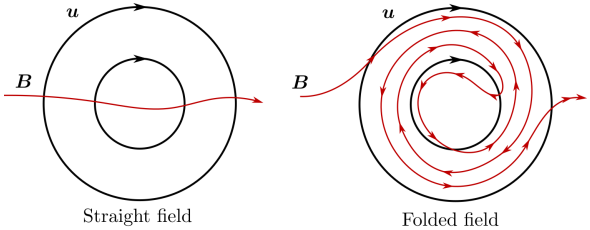
Alexander Schekochihin
Jonathan Squire

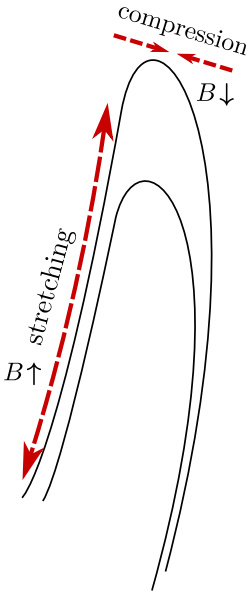
APS-DPP 2019
Fort Lauderdale Florida

October 25, 2019

PRINCETON
PLASMA PHYSICS
LABORATORY

Fluctuation dynamo and folded magnetic fields





Random velocity shears stretch and twist a seed magnetic field:

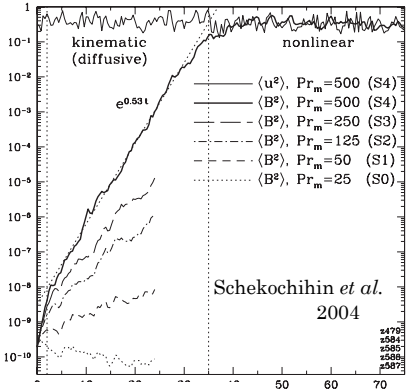
$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla}\boldsymbol{u} - \boldsymbol{\nabla}\cdot\boldsymbol{u},$$

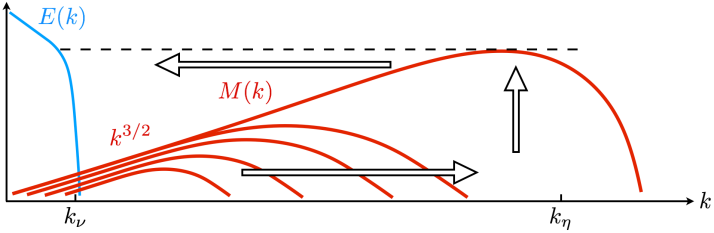
arranging the magnetic fields into long, thin folds.
B anti-correlated with field-line curvature $\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}\hat{\boldsymbol{b}}$
From Schekochihin *et al.*, Astrophy. J. **612**, 276 (2004).

The $\mathrm{Pm} \gg 1$ MHD fluctuation dynamo

Four phases:

1. Diffusion-free
2. Kinematic
   - Kazantsev $k^{3/2}$ spectrum.
3. Nonlinear
   - $\rho \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} \sim \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} / 4\pi$
   - smallest-scale stretching suppressed
   - Secular growth of $\langle B^2 \rangle$
4. Saturation
   - minimization of $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla} \boldsymbol{u}$
   - $v_{\mathrm{A}} \sim u_{\mathrm{rms}}$ (not scale-by-scale!)





Theoretical ingredients of the plasma dynamo

ICM only requires $B \sim 10^{-18}$ G to be magnetized (i.e. $\rho_\mathrm{i} \sim \lambda_\mathrm{mfp}$). Conservation of magnetic moment $\mu \doteq w_\perp^2/B \longrightarrow \mathrm{d}_t(p_\perp/nB) = 0$.

Thus

1. As B increases, $p_\perp$ increases $\longrightarrow p_\perp \neq p_\parallel$ (*Bad for dynamo! – Helander et al. 2016*)
2. Estimate size of $\Delta p \doteq p_\perp - p_\parallel$ in weakly collisional plasmas using CGL equations and collisions:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \approx \underbrace{3\frac{\mathrm{d}\ln B}{\mathrm{d}t}}_{\text{adiabatic production}} - \underbrace{\nu_\mathrm{i}\frac{p_\perp - p_\parallel}{p}}_{\text{collisional relaxation}}$$

Recall:

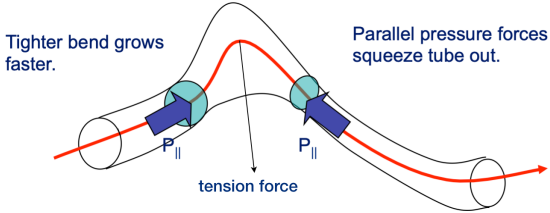
$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla}\boldsymbol{u} - \boldsymbol{\nabla}\cdot\boldsymbol{u}.$$

Results in $\sim 1\%$ deviations from local thermodynamic equilibrium.

Mirror and firehose instabilities

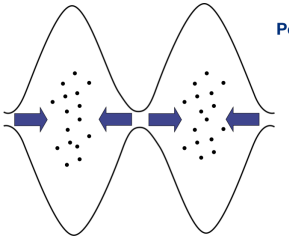
These instabilities arise in high-β ($\doteq 8\pi p/B^2$) plasmas. Defining $\Delta \doteq p_\perp/p_\parallel - 1$,

Firehose ($\Delta < -2/\beta$):



Rosenbluth 1956
Parker 1958

Mirror ($\Delta > 1/\beta$):



Perpendicular pressure forces blow out field lines.

$$-\hat{\boldsymbol{b}}\,(p_\perp - p_\parallel)\nabla_\parallel \delta B_\parallel$$

Rudakov and Sagdeev 1961
Southwood & Kivelson 1993

Saturation at $\nu_{\rm eff} \sim |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla u}|\beta$ (Kunz+ 2014; Melville+ 2016).

Three regimes for plasma dynamo

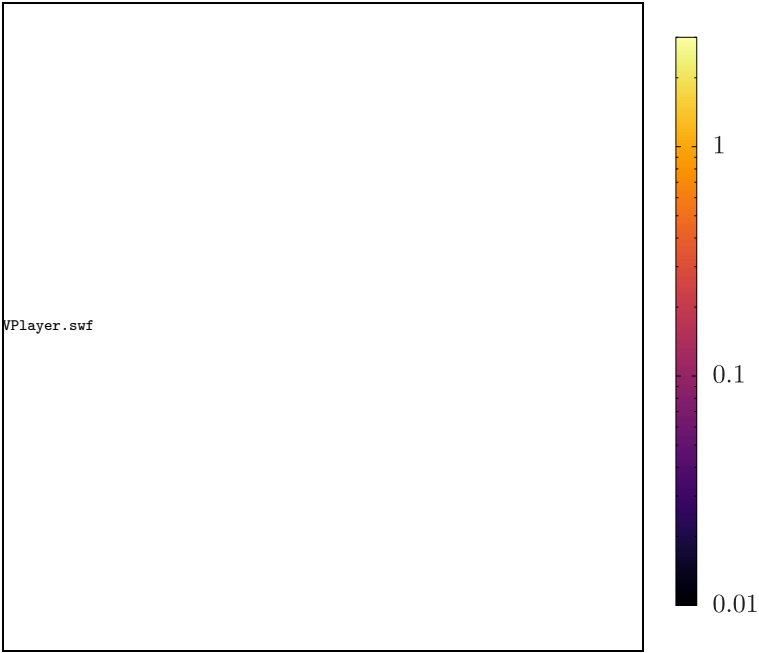
This physics suggests three dynamo regimes:

1. Unmagnetized regime ($\Omega_\mathrm{i} \ll \nu_\mathrm{i}$, see Rincon *et al.* 2016)
2. magnetized 'kinetic' regime ($\Omega_\mathrm{i} \ll |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla u}|\beta$)
3. magnetized 'fluid' regime ($\Omega_\mathrm{i} \gg |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla u}|\beta$)

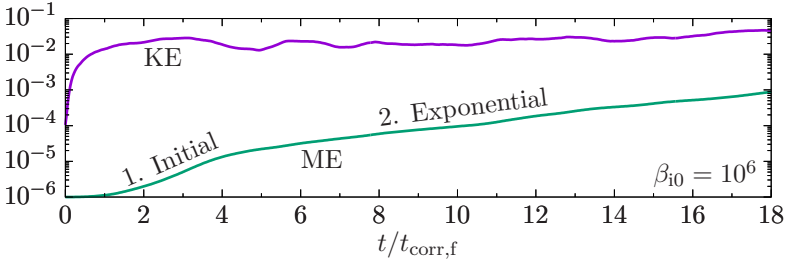
I now present results from:

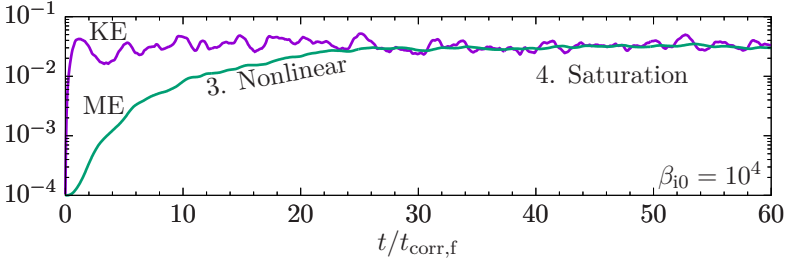
1. Hybrid-kinetic simulations (St-Onge & Kunz 2017)
   - How does the dynamo operate in a collisionless plasma?
   - *Ab initio* measurement of ν_eff motivates...
2. Braginskii-MHD simulations (St-Onge+, in prep.)
   - *Given* a prescribed viscosity, how does the plasma self-organize itself to amplify the magnetic field?
3. Analytic Modeling
   - Predicting the dynamo in certain asymptotic regimes.

$|\boldsymbol{B}|/B_{\mathrm{rms}}$



The Punchline[1]...





[1] St-Onge & Kunz, ApJ Lett. 2018, **863** (2), L25

Mirror and firehose instabilities

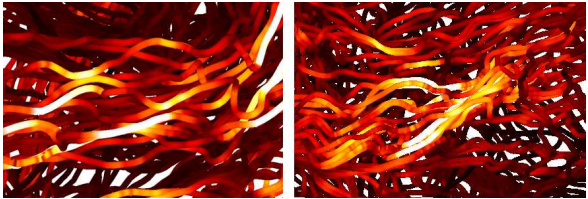
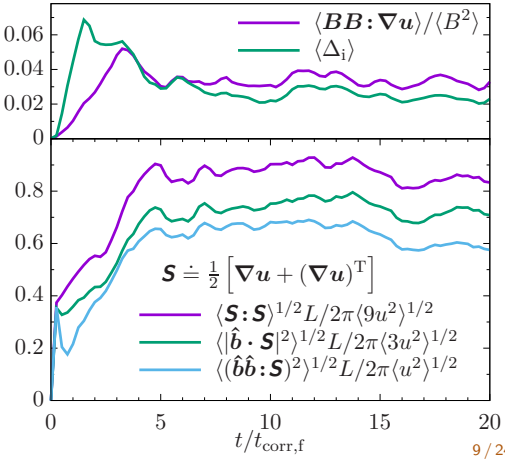


Figure: Visual evidence of mirror instabilities.

- Kinetic instabilities generate magnetic energy above ρ_{i}.
- Evidence of firehose visually and in curvature PDFs.
- Plasma becomes Braginskii-like ($3\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla u} \sim \nu_{\mathrm{eff}}\Delta_{\mathrm{i}}$)

Rate of strain is **anisotropic** with respect to the magnetic field as if it were a weakly collisional, magnetized plasma (Braginskii 1965):



Regulation of the pressure anisotropy is *imperfect*:

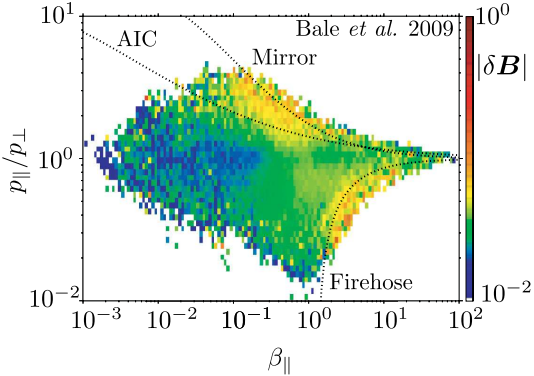


Figure: Solar wind measurements – *strong regulation*

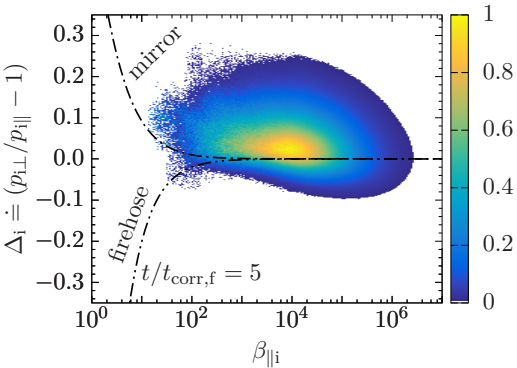


Figure: Plasma dynamo simulation – *weak regulation*

This suggests 'hard-wall' limiters may not be the ideal closure for kinetic microphysics.

Braginskii-MHD simulations[2]

- Dilute magnetized plasma ($\Omega_\mathrm{i} \gg \nu_\mathrm{i} \gg \omega$)
- Incompressible Braginskii MHD equations

$$\mathrm{d}_t \boldsymbol{u} = \boldsymbol{B}\cdot\boldsymbol{\nabla}\boldsymbol{B} - \boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\Delta p) + \mu\nabla^2\boldsymbol{u} + \widetilde{\boldsymbol{f}},$$
$$\mathrm{d}_t \boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{\nabla}\boldsymbol{u} + \eta\nabla^2\boldsymbol{B}.$$

- Nonhelical, incompressible, time-correlated forcing
- Pressure anisotropy $\Delta p = 3\mu_\mathrm{B}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla}\boldsymbol{u}$, both:
  - unlimited (parameter study on μ_B)
  - hard-wall limited:

$$\Delta p = \begin{cases} \min\left(B^2/2,\, 3\mu_\mathrm{B}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla}\boldsymbol{u}\right), & \Delta p > 0 \\ \max\left(-B^2, 3\mu_\mathrm{B}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla}\boldsymbol{u}\right), & \Delta p < 0 \end{cases}$$

[2] To be submitted to JPP

VPlayer.swf

Figure: $|\boldsymbol{B}|/B_\mathrm{rms}$ of the unlimited Braginskii-MHD dynamo.

Hard-walled Braginskii looks like $\mathrm{Pm} \gtrsim 1$ MHD

(in box-averaged evolution)

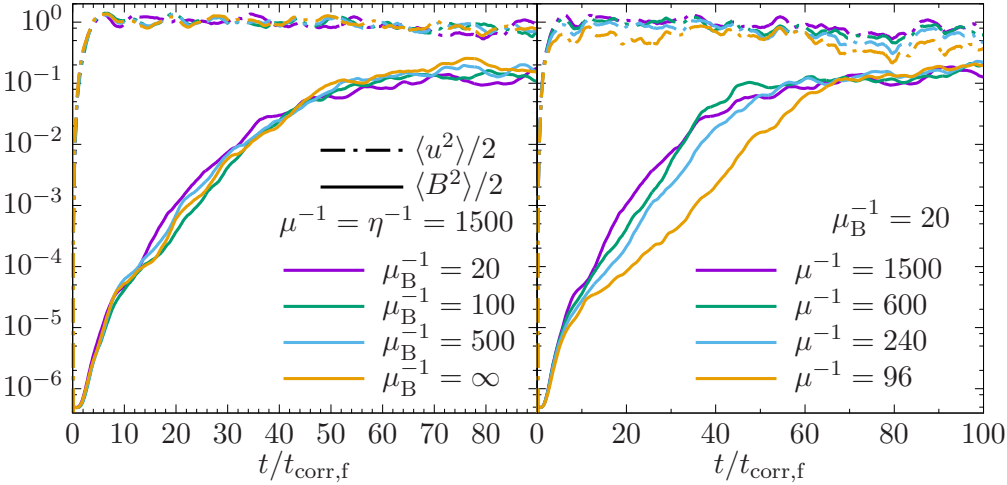


Figure: Evolution of magnetic energy

Hard-walled Braginskii looks like $\mathrm{Pm} \gtrsim 1$ MHD

(in spectra)

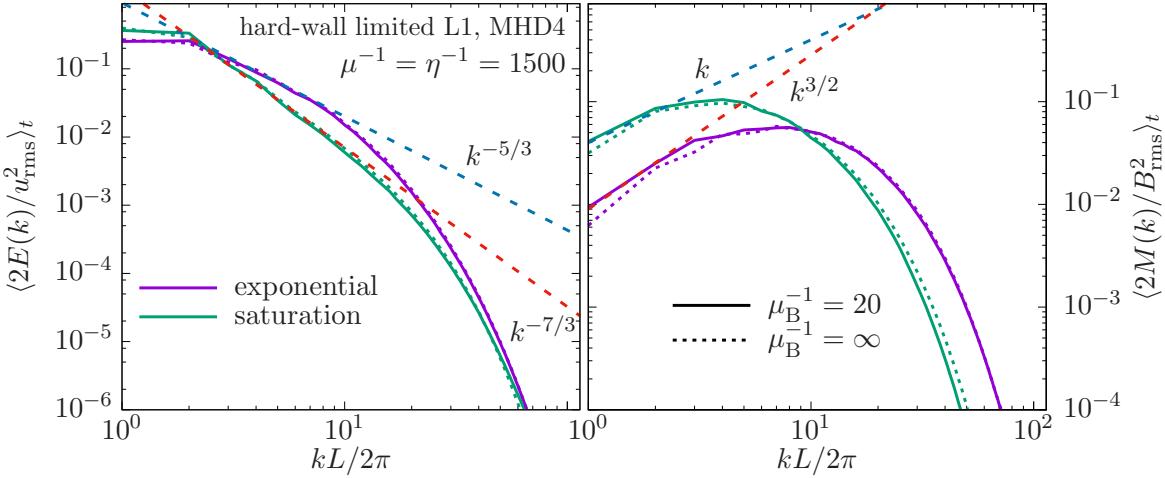


Figure: Kinetic and magnetic energy spectra

Unlimited Braginskii dynamo mimics saturated MHD

- Unlimited regime relevant to early stages of plasma dynamo
- Mimics saturated MHD in:
  - statistics of $\boldsymbol{\nabla u}$ and alignment with respect to $\hat{\boldsymbol{b}}$
  - magnetic spectrum
  - fold geometry (including PDF of $\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} \hat{\boldsymbol{b}}$)
  - spectral anisotropy of turbulent velocity

Why is this? Compare

$$\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} = \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} B^2)$$

to

$$\boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \Delta p) \propto \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\, \mathrm{d}_t \ln B).$$

Pressure anisotropy plays the role of magnetic-field strength in tension force.

An example: alignment of $(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^{\mathrm{T}})/2$ and $\hat{\boldsymbol{b}}$

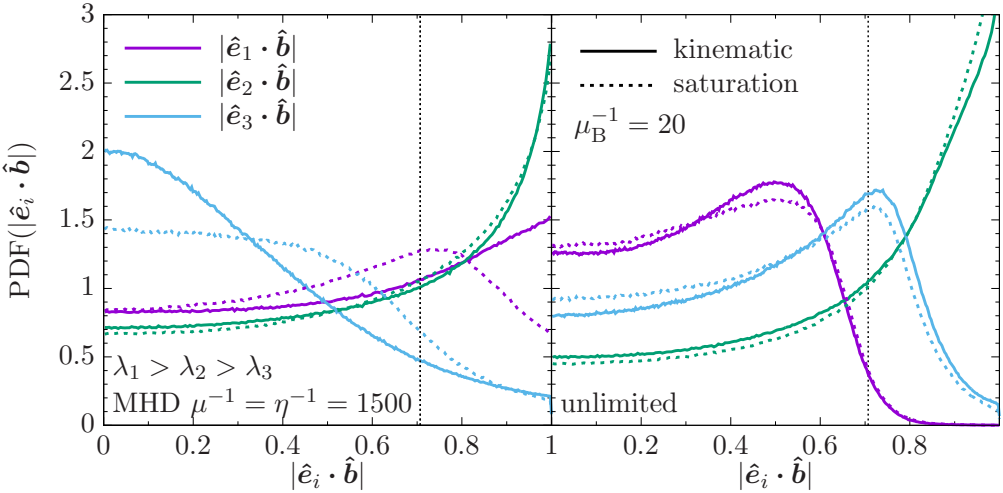


Figure: Alignment of $\hat{\boldsymbol{b}}$ and eigenvectors $\hat{\boldsymbol{e}}_i$ of $(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^{\mathrm{T}})/2$ with eigenvalues λ_i. Dotted vertical line denotes 45° alignment.

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

Consider a velocity field with prescribed statistics

$$\overline{u^i(t,\boldsymbol{x})} = 0, \qquad \overline{u^i(t,\boldsymbol{x})u^j(t',\boldsymbol{x}')} = \delta(t-t')\kappa^{ij}(\boldsymbol{x}-\boldsymbol{x}'),$$

which are anisotropic with respect to $\hat{\boldsymbol{b}}$:

$$\kappa^{ij}(\boldsymbol{k}) = \kappa^{(\mathrm{i})}(k,|\xi|)(\delta^{ij} - \hat{k}_i\hat{k}_j) + \kappa^{(\mathrm{a})}(k,|\xi|)(\hat{b}^i\hat{b}^j + \xi^2\hat{k}_i\hat{k}_j - \xi\hat{b}^i\hat{k}_j - \xi\hat{k}_i\hat{b}^j),$$

where $\xi \doteq \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{b}}$. We derive an equation for the joint PDF of B, $\boldsymbol{k}$ and $\hat{\boldsymbol{b}}$:

$$\mathcal{P}(B,\boldsymbol{k},\hat{\boldsymbol{b}}) = \delta(|\hat{\boldsymbol{b}}|^2 - 1)\delta(\hat{\boldsymbol{b}}\cdot\boldsymbol{k})(4\pi^2 k)^{-1}P(B,k).$$

This model was originally developed for the saturated state in MHD by Schekochihin (2004)

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

We then derive an equation for the magnetic energy spectrum $M(k) \doteq (1/2)\int_0^\infty \mathrm{d}B\, B^2 P(B,k)$:

$$\frac{\partial M}{\partial t} = \underbrace{\frac{\gamma_\perp}{8}\frac{\partial}{\partial k}\left[(1+2\sigma_\parallel)k^2\frac{\partial M}{\partial k} - (1+4\sigma_\perp+10\sigma_\parallel)kM\right]}_{\textbf{drift–diffusion in } k\textbf{-space}} + \underbrace{2(\sigma_\perp+\sigma_\parallel)\gamma_\perp M}_{\textbf{growth}} - \underbrace{2\eta k^2 M}_{\textbf{decay}},$$

where

$$\gamma_\perp = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\perp^2 \kappa_\perp(\boldsymbol{k}), \qquad \textbf{mixing}$$

$$\sigma_\perp = \frac{1}{\gamma_\perp}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\parallel^2 \kappa_\perp(\boldsymbol{k}), \qquad \textbf{shearing}$$

$$\sigma_\parallel = \frac{1}{\gamma_\perp}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\parallel^2 \kappa_\parallel(\boldsymbol{k}). \qquad \textbf{stretching}$$

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

Original : small-scale stretching motions suppressed by magnetic tension in saturation:

$$\int_{k_\mathrm{s}}^{k_\mu} \mathrm{d}k\, E(k) \sim \frac{1}{2}\langle B^2\rangle,$$

New idea: balance throughout dynamo evolution occurs between the hydrodynamic nonlinearity and the Braginskii viscous stress $\boldsymbol{\nabla}\cdot(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\Delta p)$:

$$\int_{k_0}^{k_\mu} \mathrm{d}k\, E(k) = 3\mu_\mathrm{B} c_1 \langle|\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla}\boldsymbol{u}|^2\rangle^{1/2} \sim 3\mu_\mathrm{B}'(\sigma_\perp + \sigma_\parallel)\gamma_\perp.$$

Resulting magnetic field growth rate:

$$\gamma = \frac{\gamma_\perp}{8}\left[\frac{14\varpi_0 - 1}{1 + (2/5)\varpi_0} - \frac{6}{5}\varpi_0\right],$$

where $\varpi_0 = W_0/\mu_\mathrm{B}'\gamma_\perp$ and $W_0 = \int_{k_0}^{k_\mu} \mathrm{d}k\, E(k)$.

Sufficiently large $\gamma_\perp$ (mixing) or μ_B (parallel viscosity) kills the dynamo!

A new "Prandtl" number

Unlimited Braginskii MHD has two important dimensionless numbers:

$$\underbrace{\frac{\mu_\parallel}{\eta}}_{\textbf{MHD Pm}}, \qquad \underbrace{\frac{\mu_\parallel}{\mu_\perp}}_{\textbf{NEW!}}.$$

Ratio of stretching and mixing in the dynamo matters, and is controlled by $\mu_\parallel/\mu_\perp$.

Predictions of the model for $\mathrm{Re}_\perp \gg 1$, $\mathrm{Re}_\parallel \sim 1$

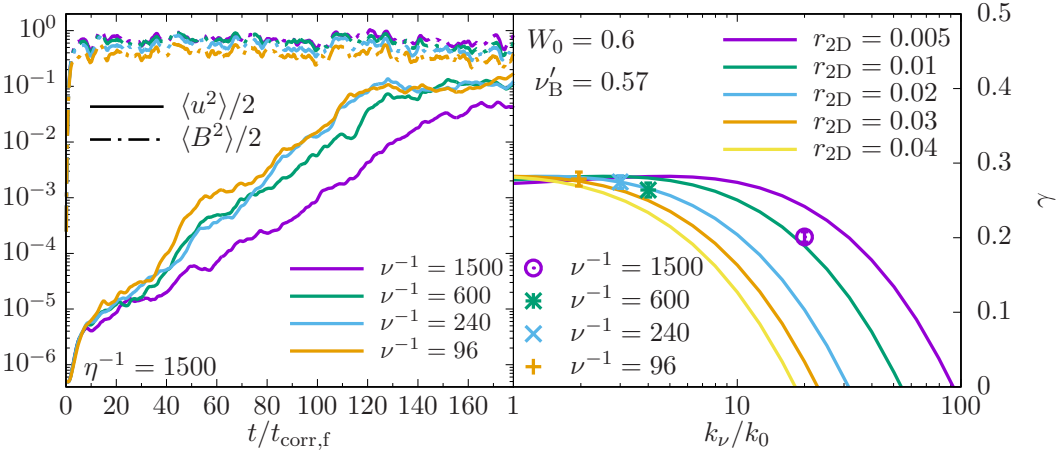


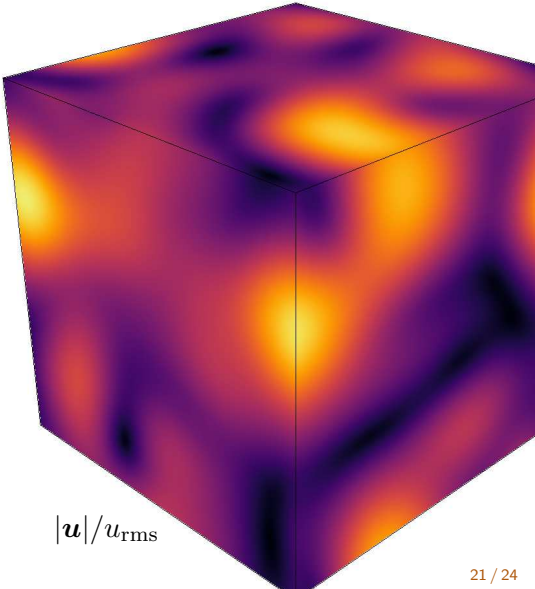
Figure: Evolution of magnetic energy (left) and growth rates of the Kazantsev model (right).

Examine the opposite limit: $\mathrm{Re}_{\|} \ll 1$ Stokes flow

In the Stokes flow regime, viscosity is so large that the velocity is determined by a balance between dissipation and driving alone:

$$-\mu \nabla^2 \boldsymbol{u} = \widetilde{\boldsymbol{f}}$$

As $\nu \to \infty$, flow becomes δ-correlated in time.



Unlimited Braginskii Dynamo for $\mathrm{Re}_{\parallel} \ll 1$ Stokes flow.

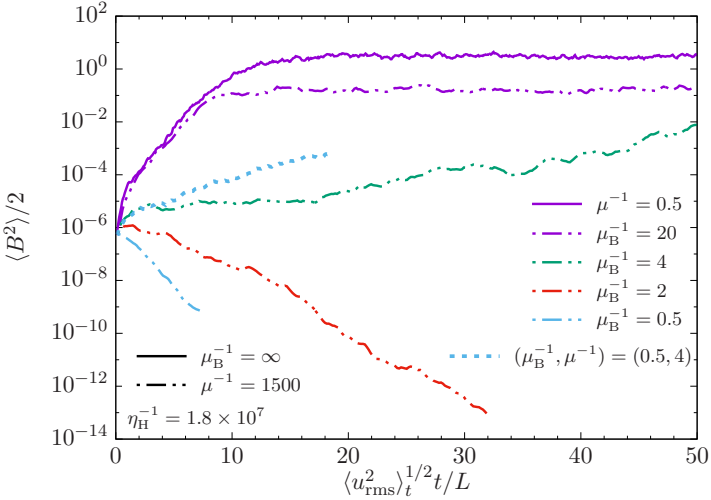


Figure: Evolution of the magnetic energy for MHD and unlimited Braginskii-MHD in the Stokes flow regime for fixed u_{rms}.

The take-away points

To summarize,

- **Dynamo exists in a collisionless *magnetized* plasma.** (See also Rincon *et al.* 2016 for unmagnetized regime)
- Larmor-scale instabilities play a crucial role.
- Many features appear MHD-like (Pm $\gtrsim 1$), despite collisionless plasma.
- Saturation at $u \sim v_{\mathrm{A}}$.

For weakly collisional plasmas,

- Too anisotropic a viscosity is deleterious for the dynamo
  (controls ratio of mixing to stretching)
- Perfect pressure-anisotropy regulation $\longrightarrow \mathrm{Pm} \sim 1$ MHD
- Weak pressure-anisotropy regulation $\longrightarrow$ saturated MHD

Future research directions

- Exact determination of ν_{eff} in the magnetized kinetic regime
- Other components of the Braginskii viscosity (i.e. gyro-viscosity)
- Kinetic electron effects
  - Dynamo relies on magnetized electrons (flux-freezing)!
  - Resistive scale (i.e. fold separation) set by electron physics
- Interplay between mean-field and fluctuation dynamos:
  - Historical anxiety about mean-field dynamo in the face of fluctuation dynamo
  - Fluctuation dynamo can lead to catastrophic α quenching!
  - Could kinetic effects alleviate these concerns?

Questions?

Questions?[3]

[3] questions?