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Energetic particle effects on FRC stability

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Fast ion effects in prolate FRC with $S^* \gg 1$ and $E \gg 1$

Initial studies on fast ions: use beams to stabilize the n=1 tilt mode (internal)
- large S*, MHD-like FRC (confinement)
- large E, for reduced n=1 tilt growth rate
- $V_b \gg V_A$, $n_b \ll n_e$ but $J_b \sim J_{pl}$

- Generalized energy principle [Lovelace'75, Finn and Sudan'93] calculated bulk plasma-beam interaction term: $I_b \sim 1/[n^2\Omega^2 - \omega_Z{}^2]$,

  Ω is average toroidal rotation frequency of beam ions,

  ω_Z is axial betatron frequency (assuming $\Omega \gg \omega$, rigid-tilt mode structure, and $f_b = f_b(\varepsilon - \Omega\, p_\varphi)$)

  - The beam ion contribution is stabilizing $I_b > 0$ provided: $n\Omega > \omega_Z$

- 3D simulations [D. Barnes, R. Milroy'91] found the stabilization of the tilt instability for prolate FRC with n_b~1-2% and $V_0 \sim 10 V_A$; destabilization of the *n*=4 kink mode has been found for cold beams [Nishimura`99].

Problems with beam stabilization of tilt in MHD

- Need cold super-Alfvenic beams to stabilize tilt in MHD regime when $\gamma_{tilt} \sim \gamma_{MHD} \sim V_A/Z_s = O(1)$
- Cold ($E_b \gg T_b$) beams can de-stabilize other modes in MHD:
  - in addition to n=1 tilt, there are 3 types of global modes unstable for each n,
  - if stability condition $n\Omega - \omega > \omega_Z$ is satisfied for one mode (n=1), other modes will be de-stabilized by the beam,
  - a more general resonant condition: $n\Omega - \omega = l\omega_Z$ (assuming $E \gg 1$ and $n\Omega \sim \omega_Z \ll \omega_R$)
  - modes with different symmetry relative to midplane (ie axial vs radial modes) have different resonances (l = odd/even) → different stability conditions.
- Experimental (prolate) FRCs have $S^* \lesssim 10$
  - no need to worry about high-n modes (thermal ion FLR stabilization)
  - but have $\omega_Z \ll n\Omega \sim \omega_R$, so need to account for radial beam ion motion $n\Omega - \omega = l\omega_Z + m\omega_R$

Generalized energy principle ($S^* \gg 1$): $\omega_Z, \Omega \ll \omega_R$

In general, $I = 1/2 \int \delta \mathbf{J}_b \cdot \mathbf{B}_0 \times \boldsymbol{\xi}^* \, d^3\mathbf{x}$ – self-ajoint provided $\omega \ll \Omega$ or $\mathrm{Im}(\omega) \ll \mathrm{Re}(\omega)$.

$$I = \frac{1}{2i\omega^*} \iint (\mathbf{v} \cdot \delta\mathbf{E}^*) \left[-\frac{\partial f_0}{\partial \varepsilon} - \frac{n}{\omega} \frac{\partial f_0}{\partial p_\phi} \right] Int(t) \, d^3\mathbf{v} \, d^3\mathbf{x} - \frac{1}{2|\omega|^2} \int R \, |\delta E_\phi|^2 \int v_\phi \frac{\partial f_0}{\partial p_\phi} d^3\mathbf{v} \, d^3\mathbf{x}$$

where $Int(t) = \int^t (\mathbf{v} \cdot \delta\mathbf{E}) \, dt'$. First term is responsible for resonant interactions, and the resonance condition includes secondary resonances: $n\Omega - \omega = l\omega_Z + m\omega_R$.

For $f_0 \sim \exp(-(\varepsilon - \Omega p_\varphi)/T)$, averaging over radial motion, and assuming modes with odd/even symmetry relative to midplane, it can be shown that [Belova, IAEA 2002]:

$$I_{\mathrm{odd}} = \frac{A_1}{\Omega_n^2 - \omega_Z^{\,2}} + \frac{A_3}{\Omega_n^2 - 9\omega_Z^{\,2}} + \ldots$$

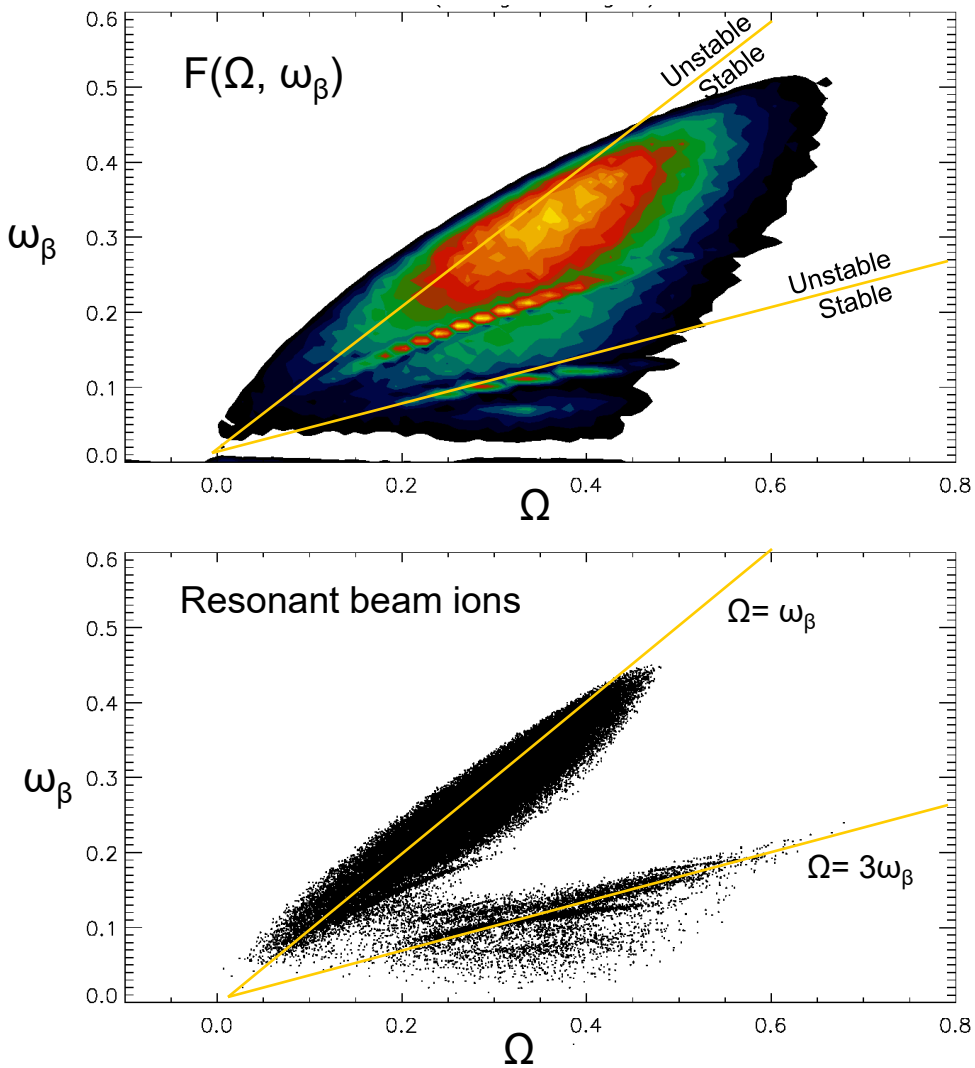
for odd modes
(antisymmetric relative to the midplane, ie with tilt-like polarization)

$$I_{\mathrm{even}} = \frac{A_2}{\Omega_n^2 - 4\omega_Z^{\,2}} + \frac{A_4}{\Omega_n^2 - 16\omega_Z^{\,2}} + \ldots$$

where $\Omega_{\mathrm{n}} = n\Omega - \omega$

for even modes
(symmetric, with radial polarization, radial shift)

Linear simulation results: n=1 tilt mode

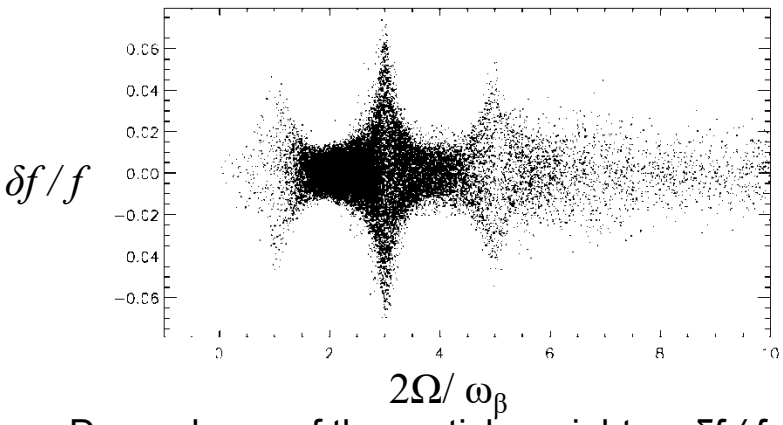


F(Ω, ω_β) – equilibrium distribution function in (Ω, ω_β) phase-space.

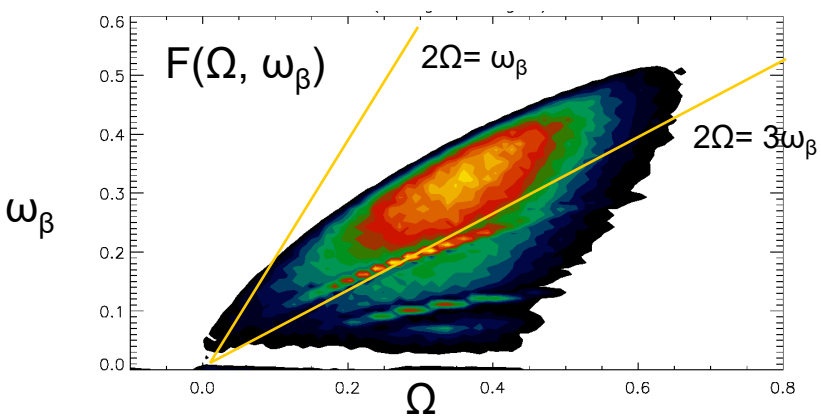
Ω is beam ion toroidal rotation frequency, ω_β is axial betatron (bounce) frequency, normalized by the ion cyclotron frequency.

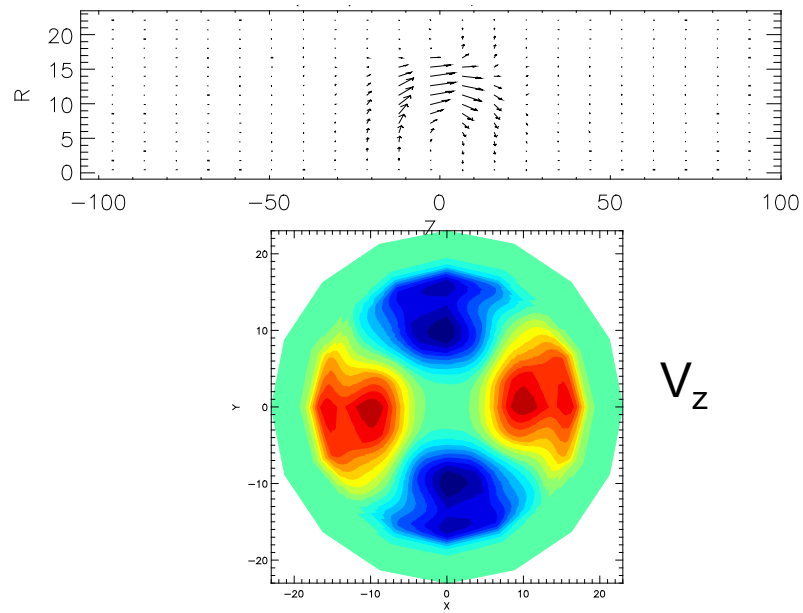
Most beam ions satisfy condition: $\Omega > \omega_\beta$, ie beam ion effect is stabilizing. For the n=1 tilt mode, the growth rate is reduced to $\gamma = 1.27 V_a/Z_s$ compared to MHD growth rate $\gamma = 1.47 V_a/Z_s$.

Linear simulation results: n=2 mode



Dependence of the particle weight $w=\delta f / f$ on the frequency ratio $2\Omega/\omega_\beta$ for n=2 odd mode. The resonant behavior is seen at $2\Omega/\omega_\beta=3$. The beam effect is <u>destabilizing</u>.

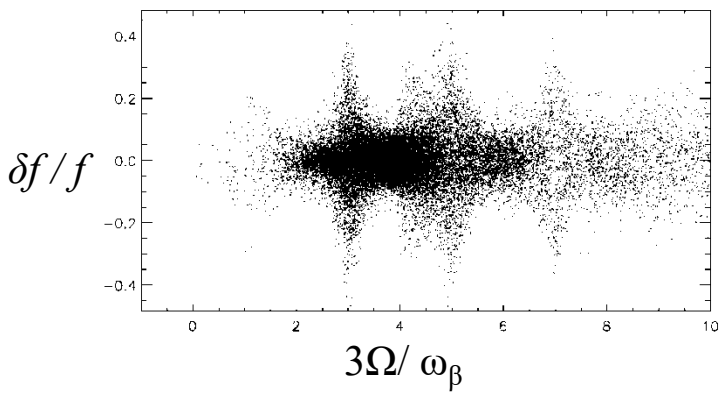




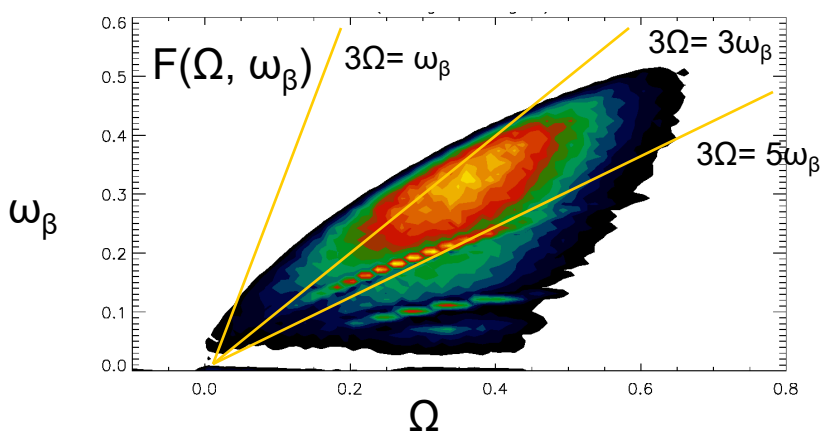
Linear mode structure from hybrid simulations for n=2 odd mode; vector plot of poloidal velocity, and contour plot of perturbed axial velocity in the midplane.

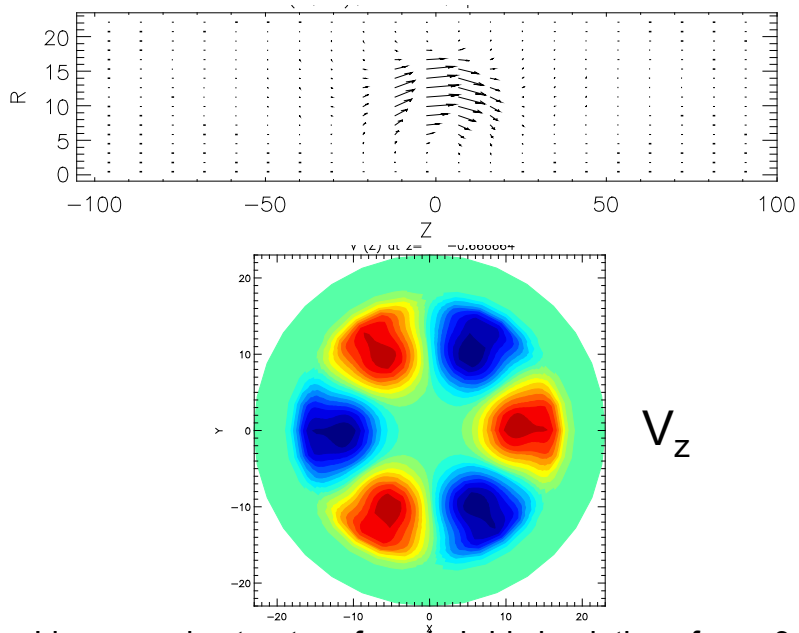
Most beam ions satisfy condition: $2\Omega<3\omega_\beta$, ie beam ion effect is destabilizing.
For the n=2 odd mode, the growth rate is increased to $\gamma=2.V_a/Z_s$ compared to MHD growth rate $\gamma=1.5V_a/Z_s$.

Linear simulation results: n=3 mode



Dependence of the particle weight $w=\delta f / f$ on the frequency ratio $3\Omega/\omega_\beta$ for n=3 odd mode.





Linear mode structure from hybrid simulations for n=3 odd mode; vector plot of poloidal velocity, and contour plot of perturbed axial velocity in the midplane.

Most beam ions satisfy condition: $3\Omega>3\omega_\beta$, but $3\Omega<5\omega_\beta$.
For the n=3 odd mode, the growth rate is increased to $\gamma=2.4V_a/Z_s$ compared to MHD growth rate $\gamma=1.55V_a/Z_s$.

Generalized energy principle ($S^* \lesssim 10$): $\omega_Z << \Omega \sim \omega_R$

Radial resonances are more important for low S* configurations (because $V_b >> v_{th}$ implies $\Omega \sim \omega_R \sim \omega_{ci}$), and for modes with even polarization (radial modes).

$$I = \frac{R_0^2 \Omega^2}{2\,|\,\omega\,|^2\ T} \int n_b \frac{|\,\Omega_n\,|^2}{\Omega_n^2 - \omega_R^2} \left| \frac{\partial \delta E_\phi}{\partial R} \right|^2 \delta R^2 d^3\mathbf{x} \sim \frac{1}{\Omega_n^2 - \omega_R^2}$$

where resonant condition: $n\Omega - \omega = \omega_R$.

Numerical results using HYM code

- One-fluid MHD for thermal plasma; full-f PIC for beam ions
  - beam injection with finite temperature, $V_b > V_{th}$
  - scaling with S*
  - using monoenergetic beam with $F_b = \delta(\varepsilon - \varepsilon_0)\delta(p_\varphi - p_{\varphi 0})$
- Kinetic description for both thermal and beam ions
  - slowing down beam distribution with large T_b
- Oblate (E~1) FRC + beam ions

HYM – Parallel Hybrid/MHD Code

HYM used to investigate FRC formation and stability properties; beam-driven sub-cyclotron Alfven eigenmodes in NSTX(-U) and DIII-D

- 3-D nonlinear.
- Three different physical models:
  - Resistive MHD & Hall-MHD.
  - Hybrid (fluid electrons, particle ions).
  - MHD/particle (one fluid thermal plasma, + energetic particle ions)
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)[1].
- Self-consistent equilibria, including beam ion effects.

Grids

Field grid: arbitrary orthogonal coordinate system (xyz, cylindrical, dipole, or nonuniform).

Particle grid: xyz, uniform to avoid problems at R=0. Could be made same as field grid (incomplete).

Boundary conditions

Fields: periodic, symmetry, $\partial / \partial n=0$, etc. easy to change – specified by set of integer numbers.

Particles: periodic, reflection, or loss.

[1]Simulations are performed at NERSC.

Hybrid scheme (fluid electrons, particle ions)

Electron-fluid and fields equations

$$\mathbf{E} = -\mathbf{V}_e \times \mathbf{B} - \nabla p_e / n + \eta \mathbf{J}$$

$$\mathbf{V}_e = -(\mathbf{J} - \mathbf{J}_i) / e n_e$$

$$n_e = n_i$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\partial p_e^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V}_e p_e^{1/\gamma})$$

Kinetic ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

$$w = \delta F / F \quad \text{- particle weight}$$

$$\frac{dw}{dt} = -(1-w)\frac{d(\ln F_0)}{dt}$$

$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

$$\delta n_i(\mathbf{x}) = \sum_m w_m S(\mathbf{x}_m - \mathbf{x})$$

Where $\varepsilon = m_i v^2/2 + e\varphi$, $p_\varphi = m_i R v_\varphi - e\psi$ is the azimuthal angular momentum , ψ is poloidal flux, and φ is the electrostatic potential.
Equilibrium distribution function is assumed to be known, so the zero-order ion density n_{i0} and current J_{i0} are calculated analytically.

Self-consistent MHD + fast ions coupling scheme

Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_i) \times \mathbf{B} - n_i (\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$

$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

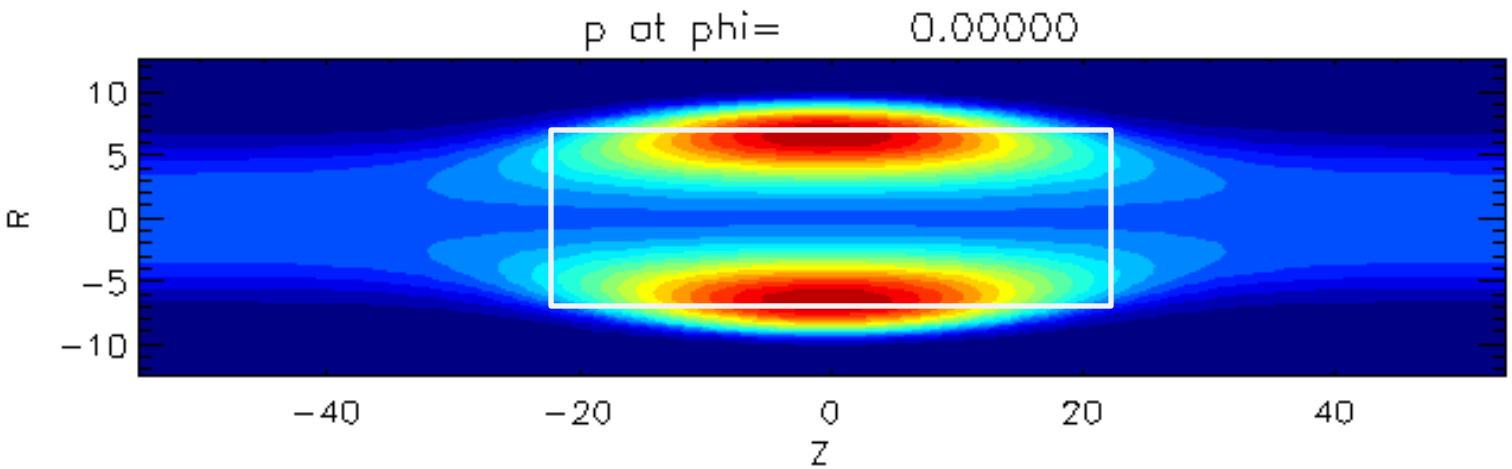
$w = \delta F / F$ - particle weight

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

ρ, **V** and p are bulk plasma density, velocity and pressure, n_i and $\boldsymbol{j}$ are fast ion density and current, $n_i << n$ – is assumed.

Full-f simulations with beam injection

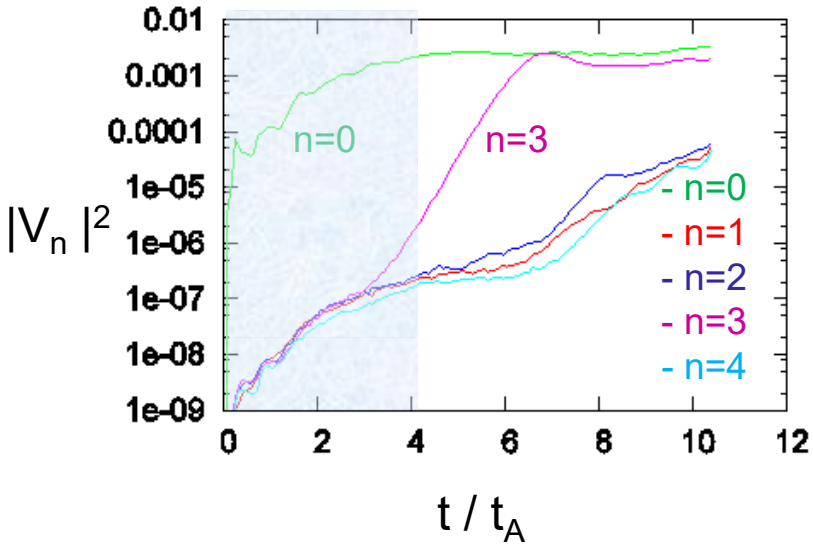


Numerical model: one-fluid MHD for thermal plasma; full-f PIC for beam ions.

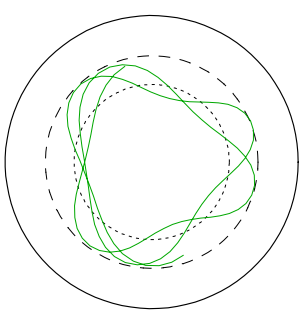
Initial FRC configuration: E= 3-4, S*= 9.5, x_s = 0.7.

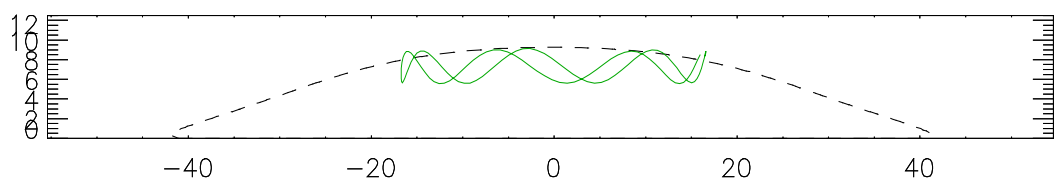
Beam parameters: V_0 =3.5-5 V_A, v_{th} = 0.5-2 V_A , n_b = 1-2% n_0 .

3D stability simulations: n=3 is beam-destabilized



- Beam ions are loaded with $V_0 = 3.5 v_A$ and $v_{th} = 0.5 v_A$ at $t = 0 - 4 t_A$.
- n=3 is most unstable mode with $\gamma = 4\ V_A/Z_s$.
- More unstable than n=3 in MHD: $\gamma_{mhd} = 1.7\ V_A/Z_s$.
- n=3 is a beam-driven mode.

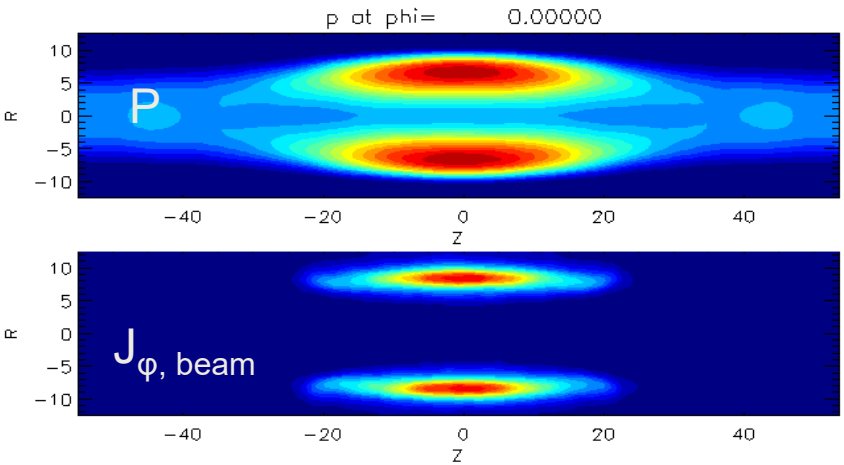




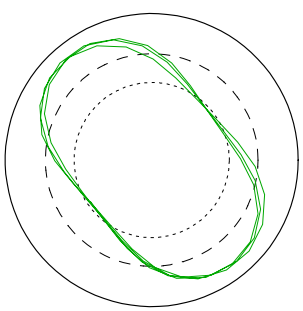
Ratio of radial orbit frequency ω_R to toroidal frequency Ω is $\omega_R / \Omega \sim 2.5$-3, so that resonant condition is approximately satisfied: $\mathbf{n\Omega \approx \omega_R}$ for the n=3 mode.

3D stability simulations (larger V_{beam})

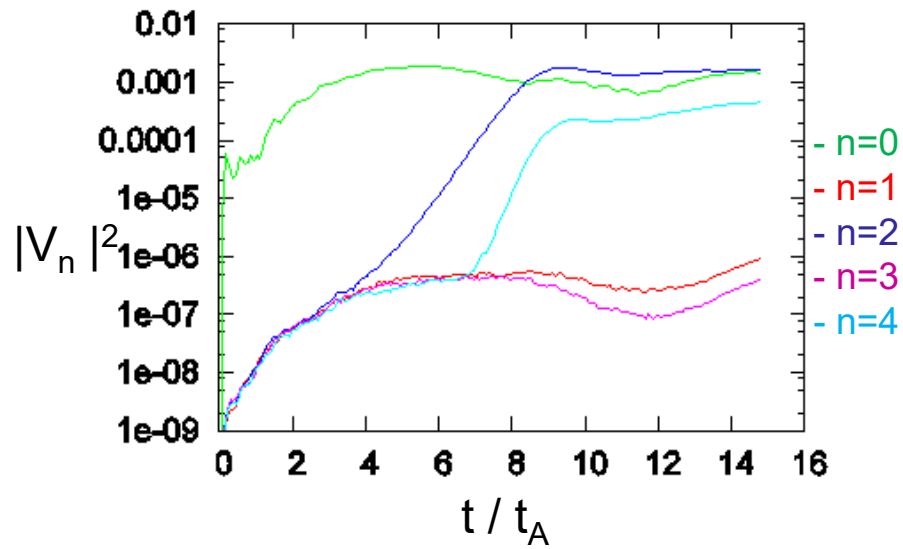
Beam parameters: V_0 =5.6 V_A, v_{th} = 1.5 V_A , n_b = 1% n_0 .



After relaxation, FRC elongates. The beam ions have wider axial profiles for larger v_{th}.



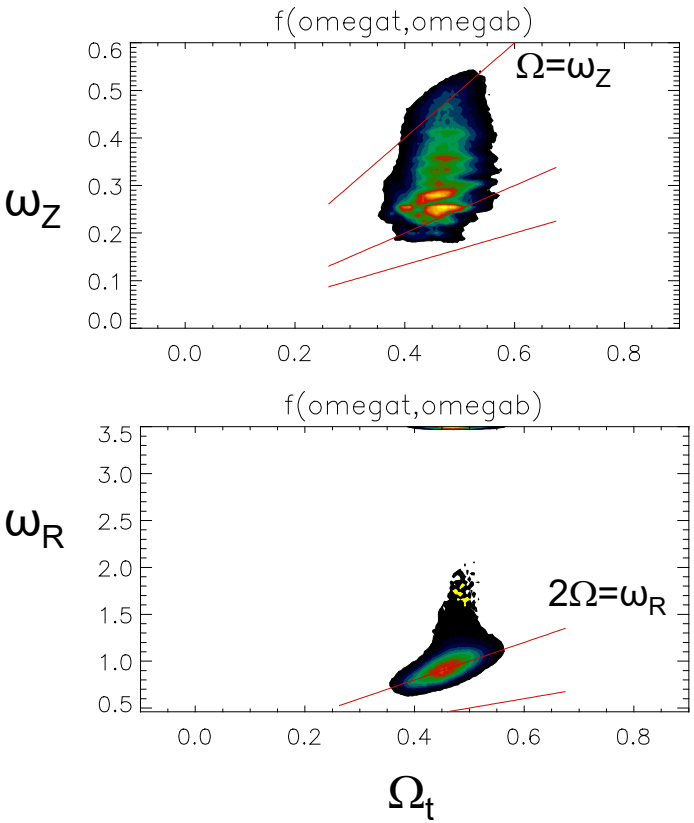
$\omega_R / \Omega \sim 2$, so that resonant condition is satisfied:
$\mathbf{n\Omega \approx \omega_R}$ for the n=2 mode.



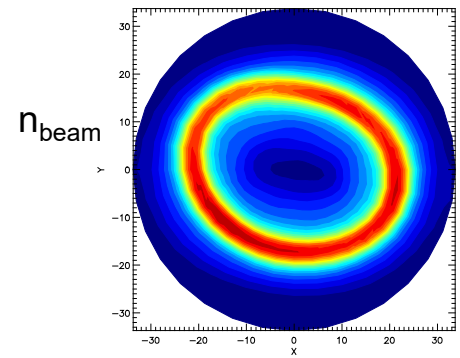
For these beam parameters, the n=1 tilt mode and n=3 modes are completely stabilized, but the n=2 rotational mode is still unstable with $\gamma = \gamma_{mhd}$(n=2) – beam has little effect on n=2 mode.

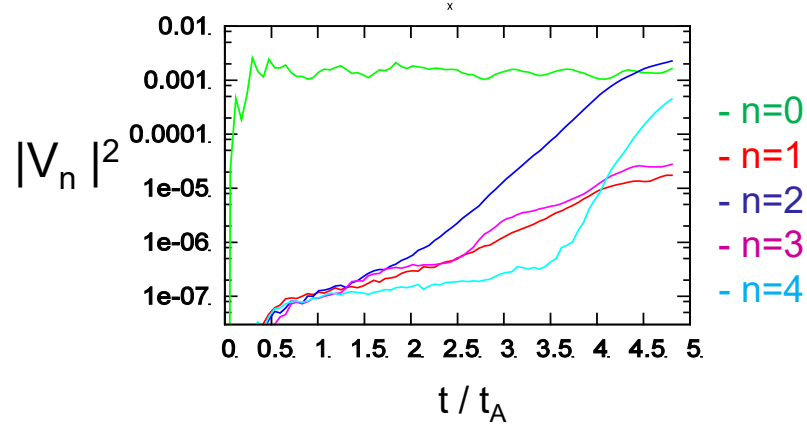
Dependence of betatron frequencies on S* and Ω_{tor}

$S^*=20$, $\Omega_0=0.8$, $n_b/n_0=0.013$, $J_b/J_0=0.7$



$\omega_Z : \Omega : \omega_R = 0.7:1:2$

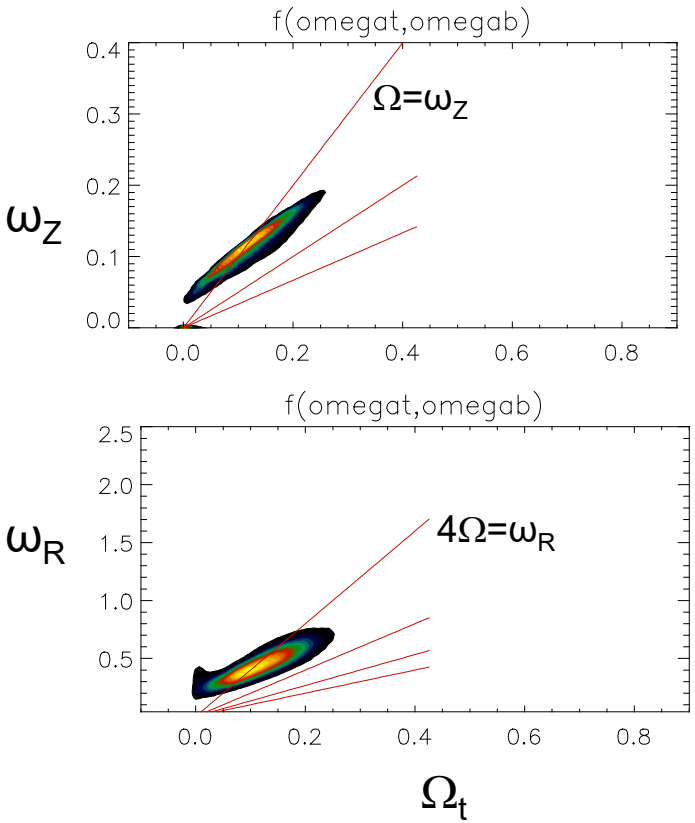


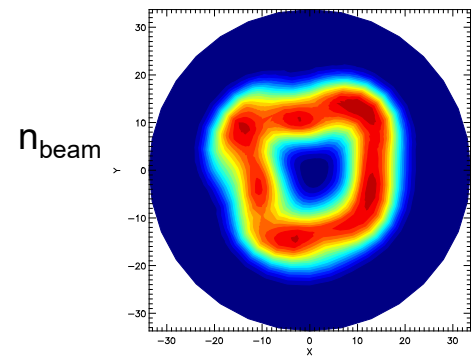


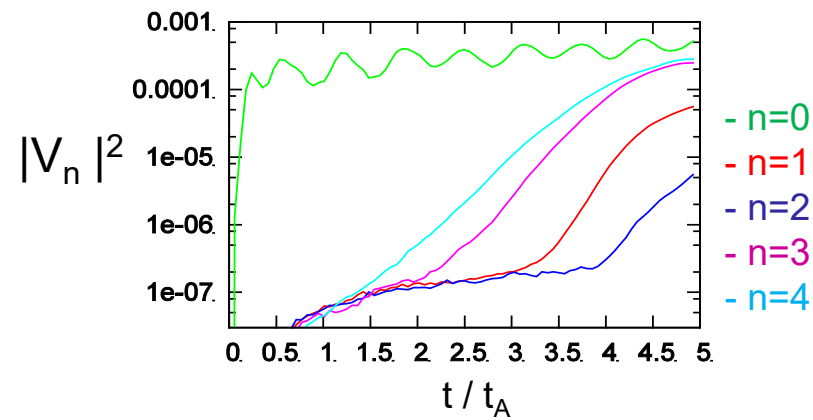
Ratio of radial orbit frequency ω_R to toroidal frequency Ω is $\omega_R/\Omega \sim 2$, and the n=2 mode is the most unstable mode.

Dependence of betatron frequencies on S* and Ω_{tor}

$S^*=20$, $\Omega_0=0.2$, $n_b/n_0=0.05$, $J_b/J_0=0.7$



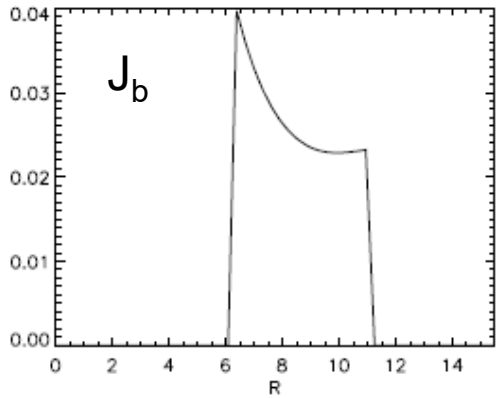




$\omega_Z : \Omega : \omega_R = 1:1:4$

Ratio of radial orbit frequency ω_R to toroidal frequency Ω is $\omega_R/\Omega \sim 4$, so that resonant condition is satisfied: $\mathbf{n\Omega \approx \omega_R}$ for the n=4 mode.

Simulations with beam $F_b=\delta(\varepsilon-\varepsilon_0)\delta(p_\varphi-p_{\varphi 0})$



FRC parameters: S*=9, E~3;
beam parameters: $V_0 = 5V_A$, $v^* = 4.5\ V_A$, $n_{beam}<1\%\ n_0$.

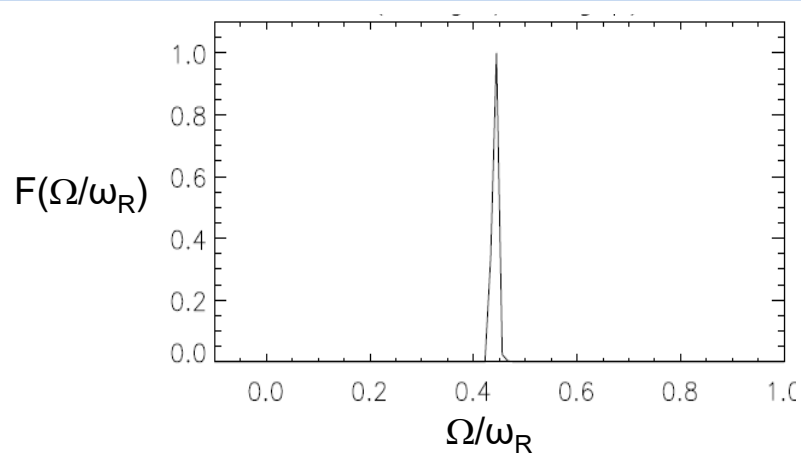
Equilibrium is calculated and beam ions are loaded with delta-function distribution function: $F_b=\delta(\varepsilon-\varepsilon_0)\delta(p_\varphi-p_{\varphi 0})$, where $\varepsilon_0= v_0^2/2$ and $p_{\varphi 0}= R_0 v^* - \psi_0$, $v^*<v_0$.

$n_b = n_{b0}(R_0/R)$, $\quad |\psi + p_{\varphi 0}|<Rv_0$,
$J_b = n_{b0}(R_0/R^2)(\psi + p_{\varphi 0})$,

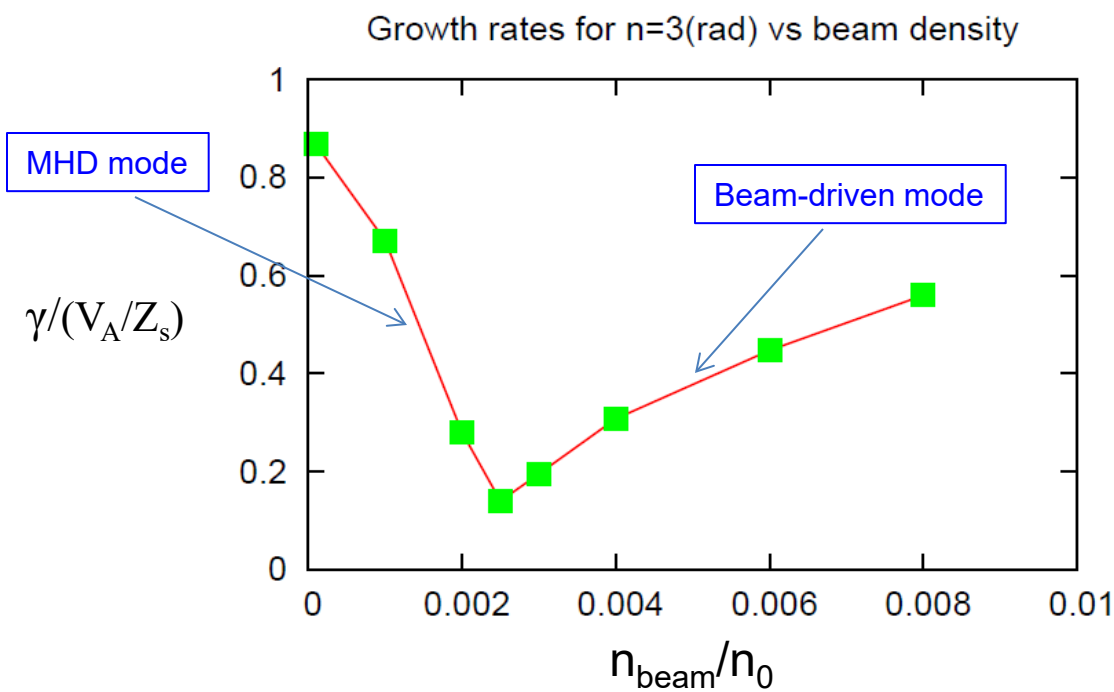
Beam parameters were chosen to satisfy resonant condition: $n\Omega \approx \omega_R + \omega_z$ for n=2 and n=3 modes.

Beam ions with $F_b=\delta(\varepsilon-\varepsilon_0)\delta(p_\varphi-p_{\varphi 0})$, have very narrow distribution in orbit frequencies.

For given beam parameters:
$\langle\Omega\rangle = 0.54\ \omega_{ci}$
$\langle\omega_R\rangle = 1.2\ \omega_{ci}$
$\langle\omega_z\rangle = 0.25\ \omega_{ci}$



Stabilization of MHD n=3 radial mode



Growth rate of n=3 radial mode vs beam ion density.

From 3D nonlinear MHD-hybrid (full-f) simulations of FRC with E~3. Modes other than n=3 radial mode are filtered out.

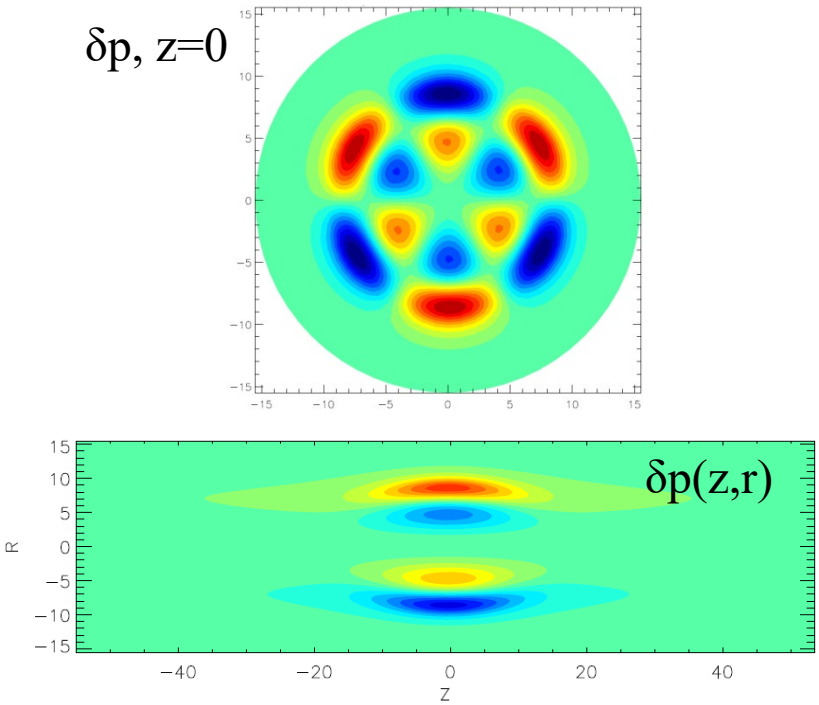
Beam ion parameters:
$V_0 = 5V_A$, $v^* = 4.5\ V_A$,
$n_{beam} < 1\%\ n_0$, $I_b/I_{tot} < 2.5\%$.

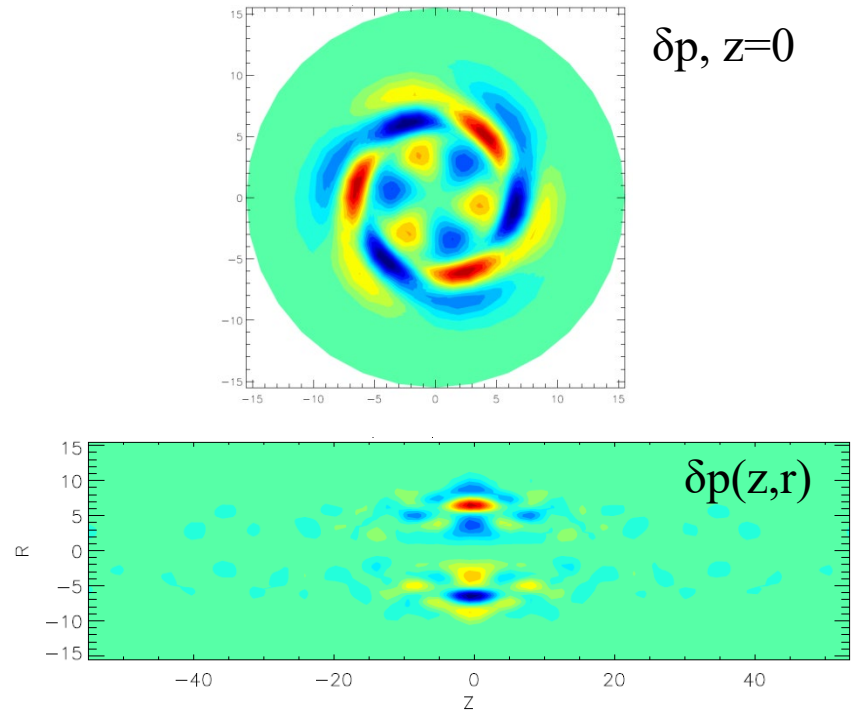
- Beam ions have strong stabilizing effect on n=3 radial MHD mode for very small values of n_{beam}/n_0.
- Larger values of n_{beam}/n_0 result in growth of different mode with large radial and axial mode numbers.
- Beam ion parameters satisfy stability condition: $n\Omega > \omega_R + \omega_Z$ for n=3.

n=3 radial MHD mode vs beam-driven mode

MHD mode

Beam-driven mode



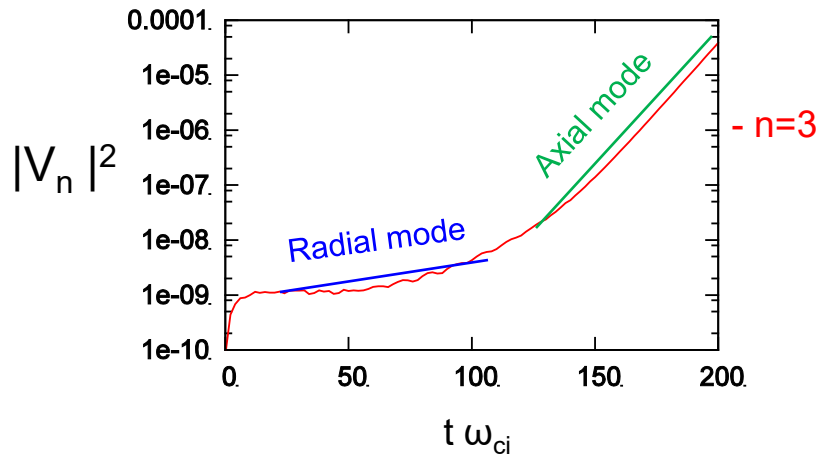


- MHD mode is stabilized by beam which satisfy stability condition: $n\Omega > \omega_R + \omega_Z$.
- Beam-driven mode is excited for larger values of n_{beam} due to high-order resonances: $n\Omega = l\omega_R + k\omega_Z$.

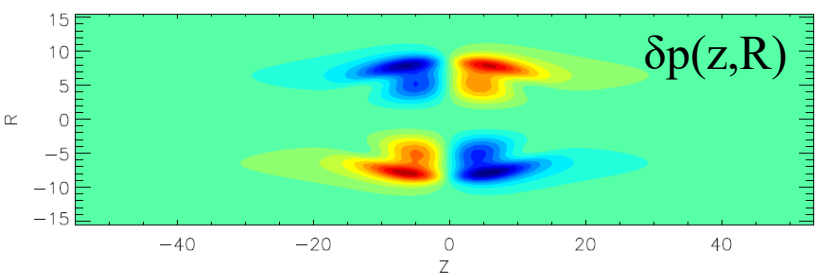
Beam effect on n=3 MHD mode (no symmetry filter)

- The n=3 radial mode growth rate is reduced by a factor of 3 compared to MHD.
- At later time, the n=3 axial mode grows with MHD-like growth rate.
- Stabilizing effect of the beam depends on the mode symmetry.
- Beam ion parameters:

$V_0 = 5V_A$, $v^* = 4.5\ V_A$, $n_{beam} = 0.004\%\ n_0$, $I_b/I_{tot} = 1.4\%$.



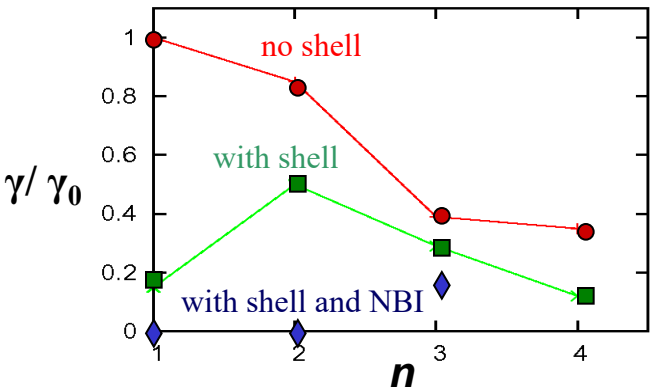
Time evolution of kinetic energy of n=3 mode, initial conditions correspond to radial mode.



Mode structure (pressure perturbation) of the n=3 axial mode.

FRC stability including close-fitting conducting shell and energetic beam ion effects: I. Linear results

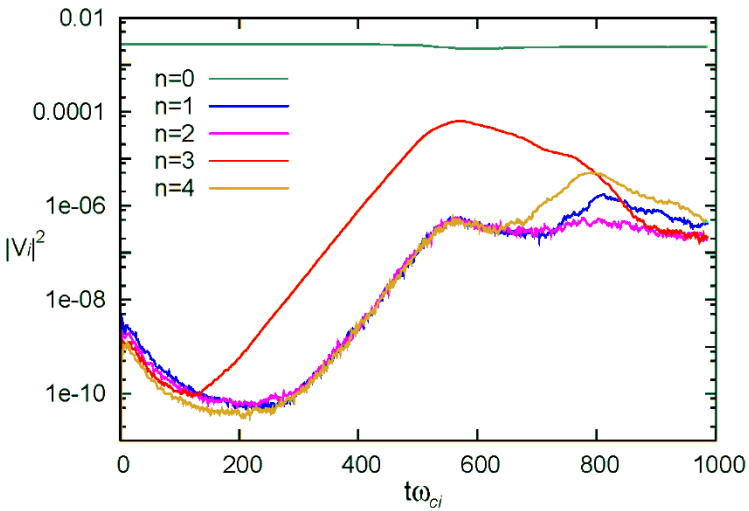
- Close-fitting conducting shell stabilizes all low-*n* radially-polarized (even) modes.
- Due to localization, the ion beams are effective in stabilizing the residual low-*n* instabilities, except for relatively cold beams which have a destabilizing effect on *n*≥3 modes.
- The NBI effects are stronger for lower-*n* modes (*n*=1 and *n*=2), and smaller V_0.
- The n=1 tilt mode and the n=2 mode are stabilized, and the growth rate of the n=3 mode is reduced for E~1, S*~18, n_b/n_i= 0.03, and V_0= $6V_A$.

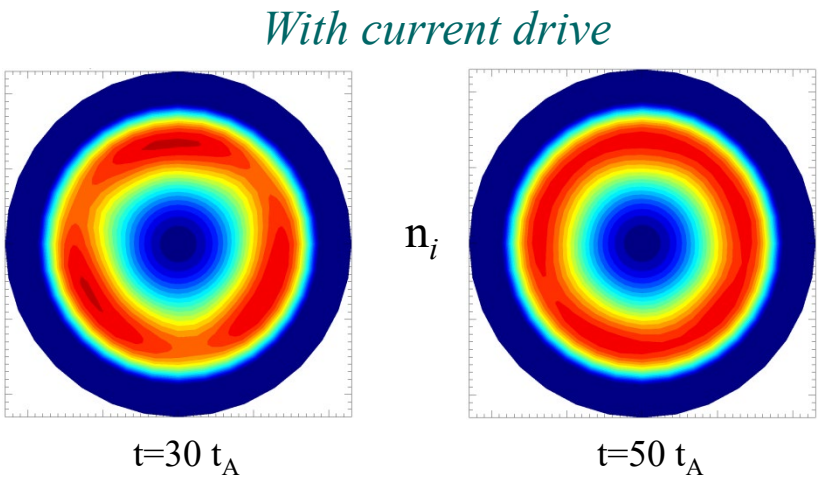


Normalized growth rates of the *n*=1-4 modes from 3D hybrid simulations including the effects of conducting shell and NBI stabilization.

FRC stability including close-fitting conducting shell and energetic beam ion effects: II. Nonlinear simulations

- Nonlinear 3D simulations show that the residual instabilities (n=3 mode) saturate at small amplitudes.
- FRC remains stable with respect to **all MHD modes**, as long as it is sustained.





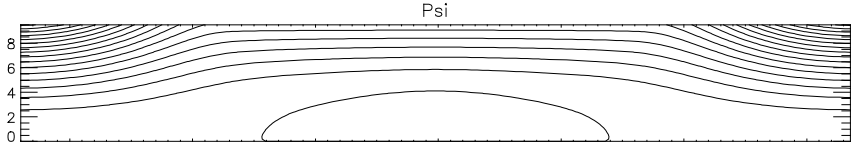
Nonlinear hybrid simulations of an FRC with E=1.1, including the effects of the beam ions and the close-fitting conducting shell. (a) Time evolution of n=0-4 modes kinetic energy; and (b) contour plots of plasma density in the toroidal cross sections.

Simulations have been performed in support of MRX-FRC experimental proposal.

HYM simulations for C2-like equilibrium

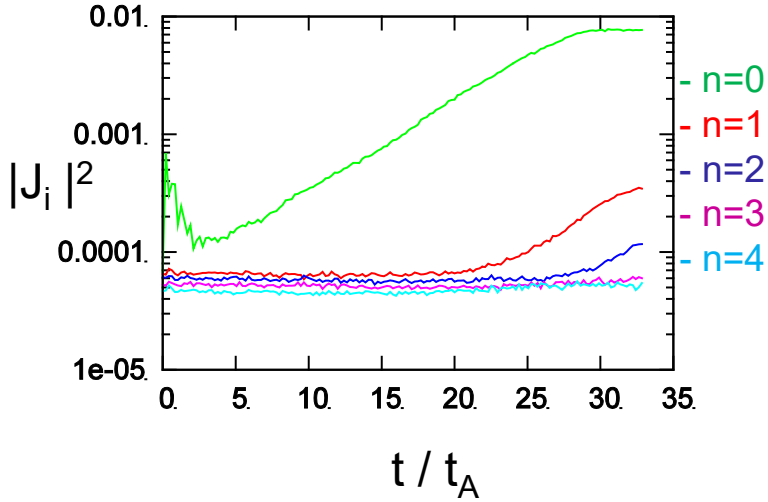
Plasma parameters: $n_e = 2 \cdot 10^{19}\ m^{-3}$, $\lambda_i = 7.4cm$, $R_s = 0.3m$, $\beta_s = 0.84$.

Normalized: $S^* = 4$, $E \sim 3$, mirror ratio ~3

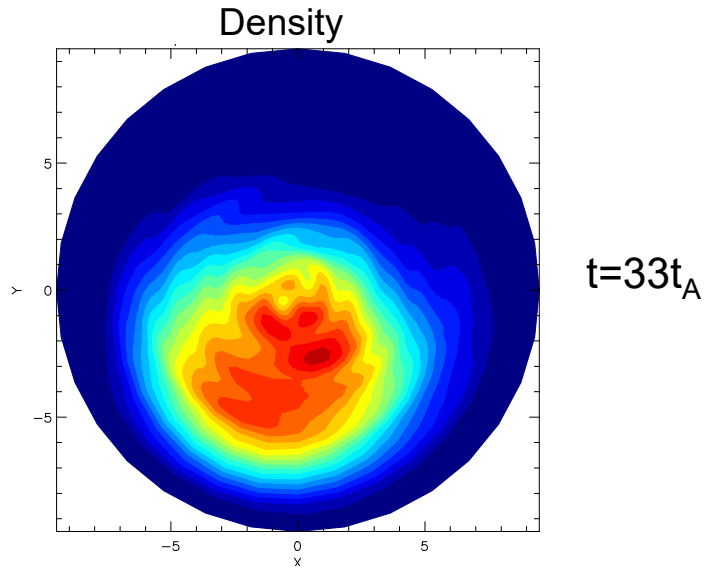


P. flux

Hybrid simulations (no beam)



Time evolution of different Fourier harmonics from full-f hybrid simulations using HYM code for S*=4. The n=1 wobble mode is unstable.



- The n=1 wobble mode is unstable (no saturation) in simulations with C2-like equilibrium with mirror ratio ~3, and S*=4.
- Periodic BCs are used for E, so no end-shorting.
- Similar simulations with mirror ratio =1 do not show the wobble mode, instead there is weak instability of the n=1 tilt mode, and n=2 rotational mode.

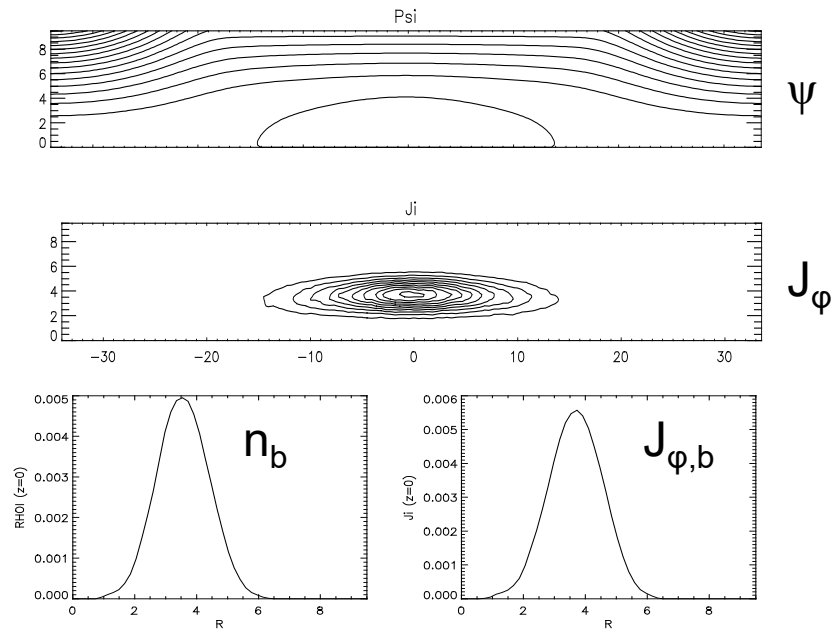
Injection of fast ions with slowing-down distribution

Beam ions were loaded into simulation region between t=0 and $t=5t_A$.

Particles were loaded with slowing-down distribution: $f_b = f(v, p_\varphi)$, where

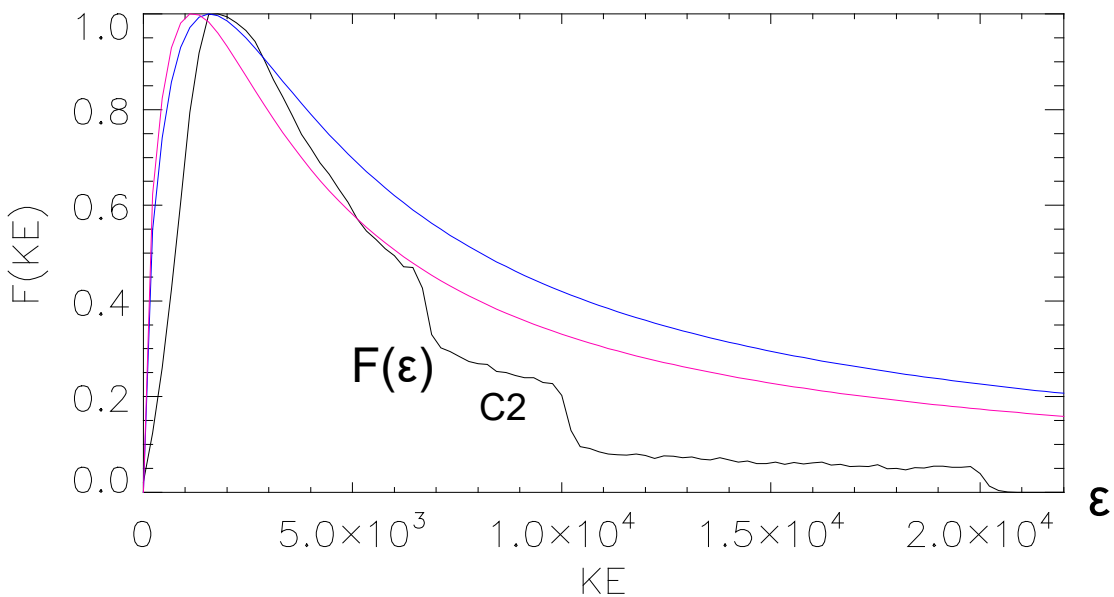
$$f(v, p_\varphi) \sim 1/(v^3 + v^{*3}) \cdot [(p_\varphi - p_m)/(Rv - \psi_0 - p_m)]^4$$

$p_\varphi = Rv_\varphi - \psi$ is toroidal angular momentum, $p_m = -0.1\psi_0$, and ψ is poloidal flux.



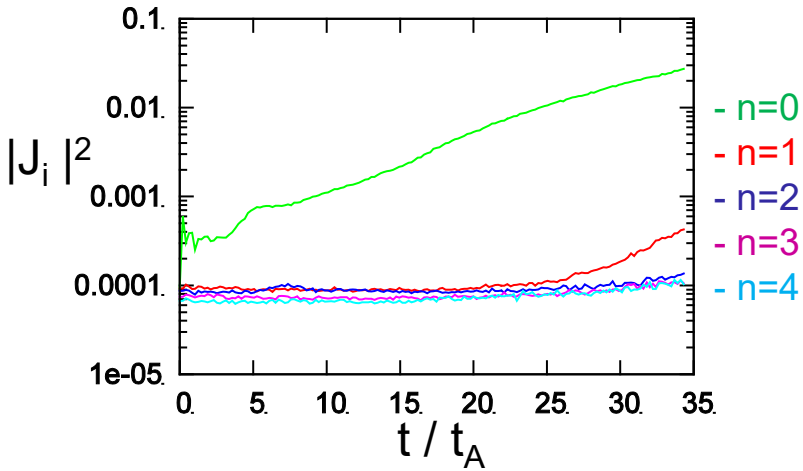
(a-b) Contour plots of poloidal flux and beam ion toridal current; (c-d) radial profiles of beam ion density and current corresponding to slowing-down distribution function $f_b = f(v, p_\varphi)$.

Injection of fast ions with slowing-down distribution



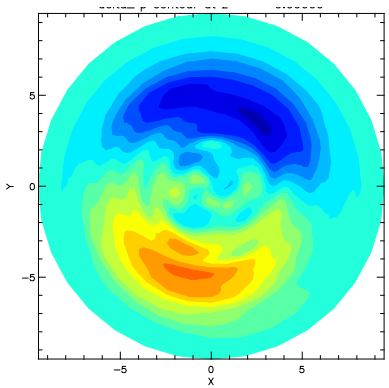
C2 beam parameters: ε~10keV, v_0~ 2.2 V_A, $n_b/n_e \leq 0.3$.

Simulations for C2 equilibrium including fast ions with slowing-down distribution

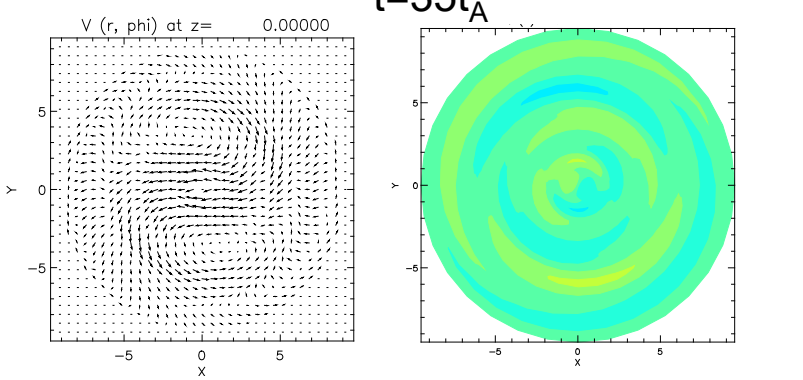


Time evolution n=0-4 mode energies from simulations with beam injection.

Perturbed density $t=35t_A$



$t=35t_A$



Perturbed velocity at midplane

Polarization of unstable n=1 mode corresponds to radial shift mode, as can be seen from δn and δV plots. Perturbed axial velocity is small compared to δV_φ and δV_R. Beam injection does not cause beam-driven modes, and reduces the growth rate of n=1 wobble mode.

Conclusions

- For large beam ion toroidal rotation frequencies (small S*) radial resonances are more important than axial betatron resonances.

- Simulations for the n=2 and n=3 radial modes validate stability condition

- Full-f simulations for a delta-function distribution of the beam ions and the MHD description of the thermal plasma demonstrate why cold beams ($T_b \ll E_b$) are always de-stabilizing.

- MHD growth rates are too large for beam ion stabilization for reasonable beam parameters – need kinetic description of thermal plasma.

- A stable regime can be found for oblate FRCs with a close-fitting if conducting shell and energetic beam ions are used stabilization.

- In lower S* FRCs, beams with large thermal spread could be used for stabilization. Beam injection does not cause beam-driven modes, and reduces the growth rate of n=1 wobble mode in C2-like equilibrium.