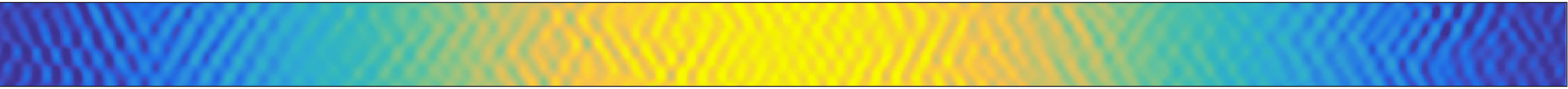
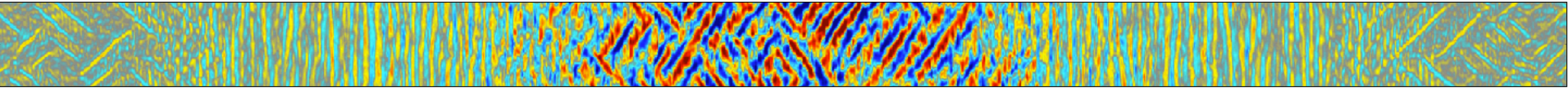
Self-defeating Alfvén waves



and

Self-sustaining Sound



in Collisionless, High-β Plasma

(or, how to reduce stress and not surf in a storm)

Matthew Kunz

PRINCETON UNIVERSITY

PPPL R&R Seminar 11/3/2020

primarily with

Jono Squire

Eliot Quataert

Alex Schekochihin

with support from NASA, DOE, XSEDE/NSF, and Sloan Foundation

I usually provide some astrophysical motivation,
but this is really just basic plasma physics…

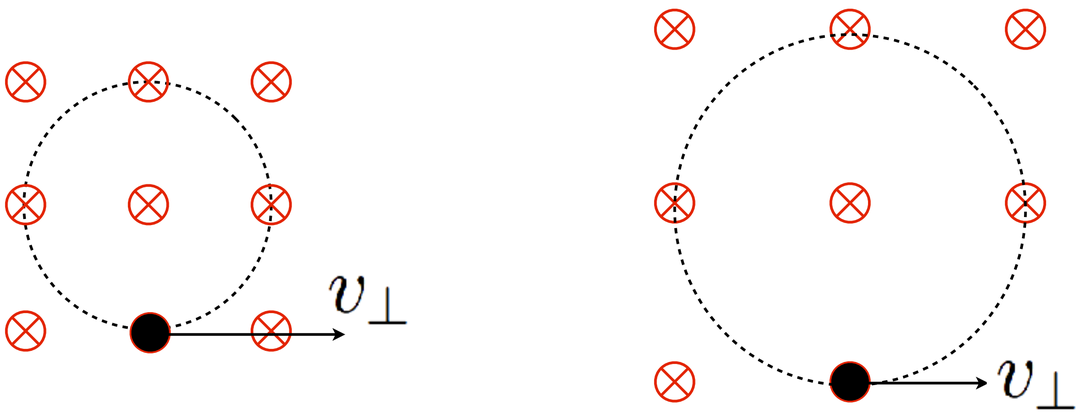
so let's jump right in.

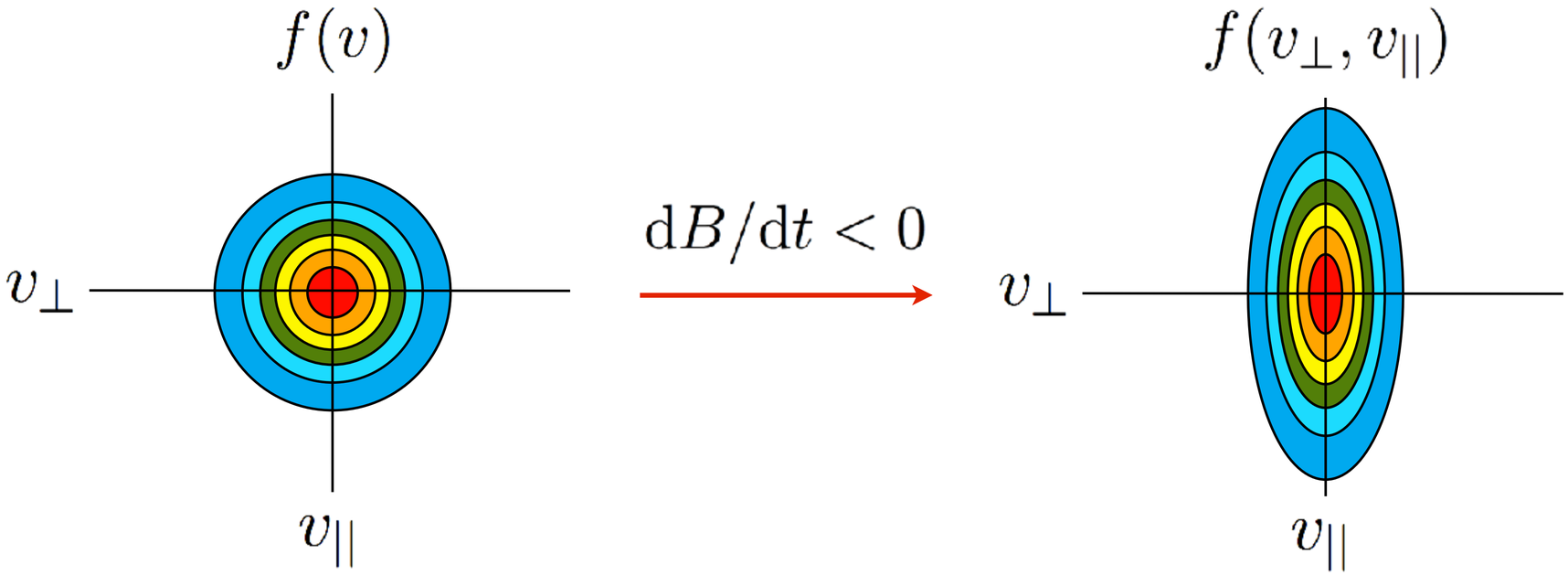
Consider a magnetized, weakly collisional plasma…

$$\lambda_{\mathrm{mfp}} \gtrsim L \gg \rho_i$$

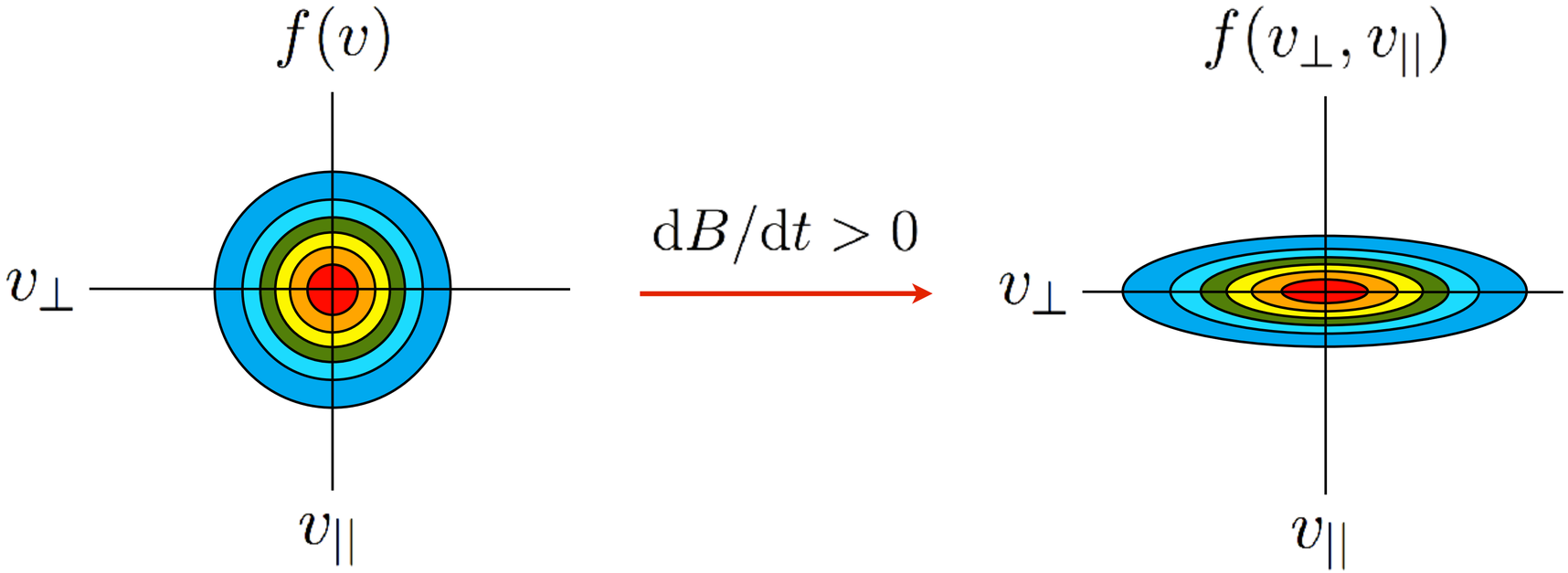
which, in the simplest case, just means…

suppose $\mu \equiv mv_\perp^2/2B$ is approximately conserved during some large-scale evolution





$$f(v_{||}, v_\perp) \propto \exp\left(-\frac{v_{||}^2}{v_{\mathrm{th}||}^2}\right) \exp\left(-\frac{v_\perp^2}{v_{\mathrm{th}\perp}^2}\right)$$



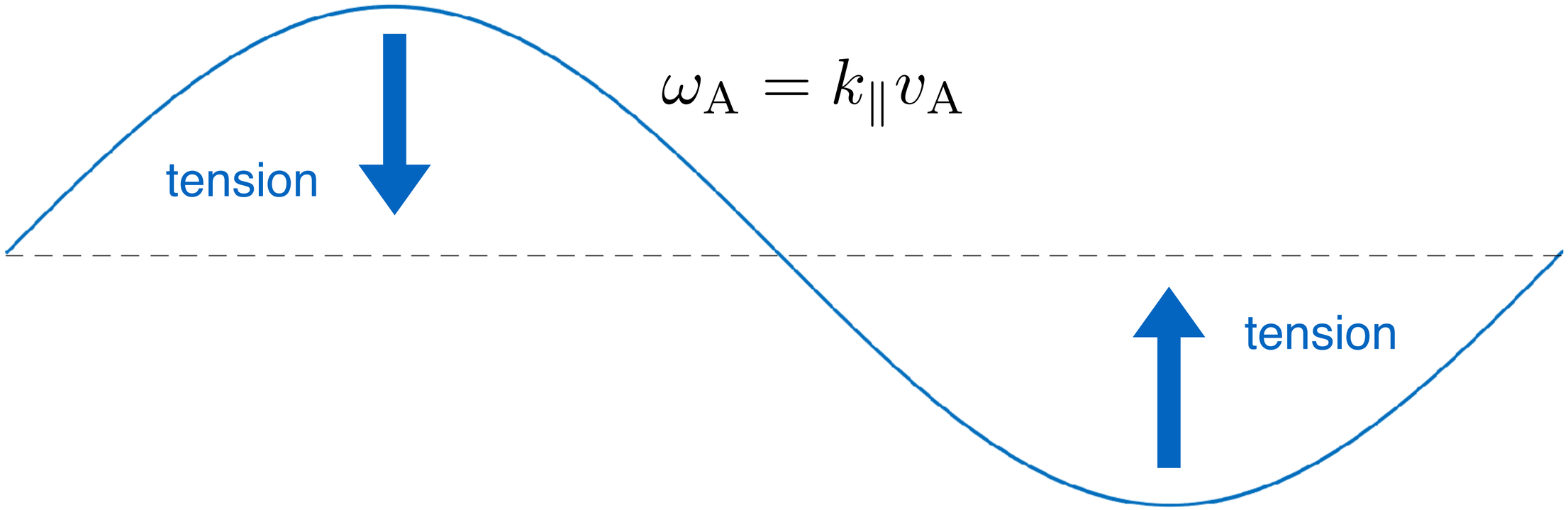
"pressure anisotropy"

$$T_{||} \neq T_\perp$$

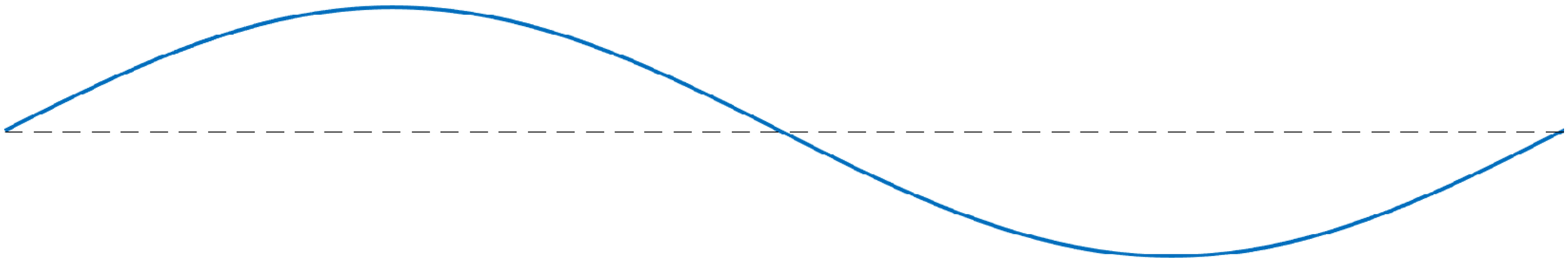
Q. How does fluctuation-driven pressure anisotropy feed back on the fluctuations themselves?

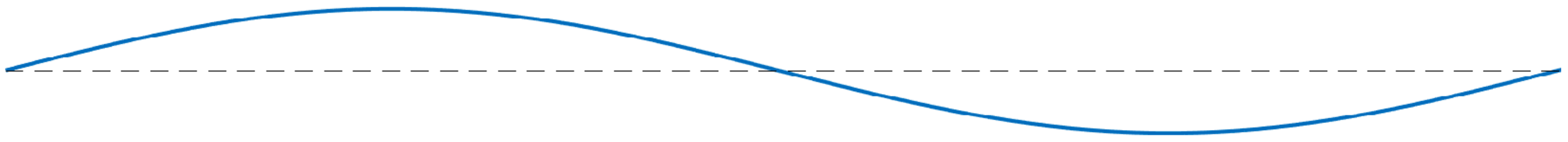
I will illustrate the answer to this question via two simple waves:
shear-Alfvén wave and **ion-acoustic wave**

Following Squire *et al.* (2016 ApJL),
consider a shear-Alfvén wave in an initially Maxwellian, collisionless plasma:



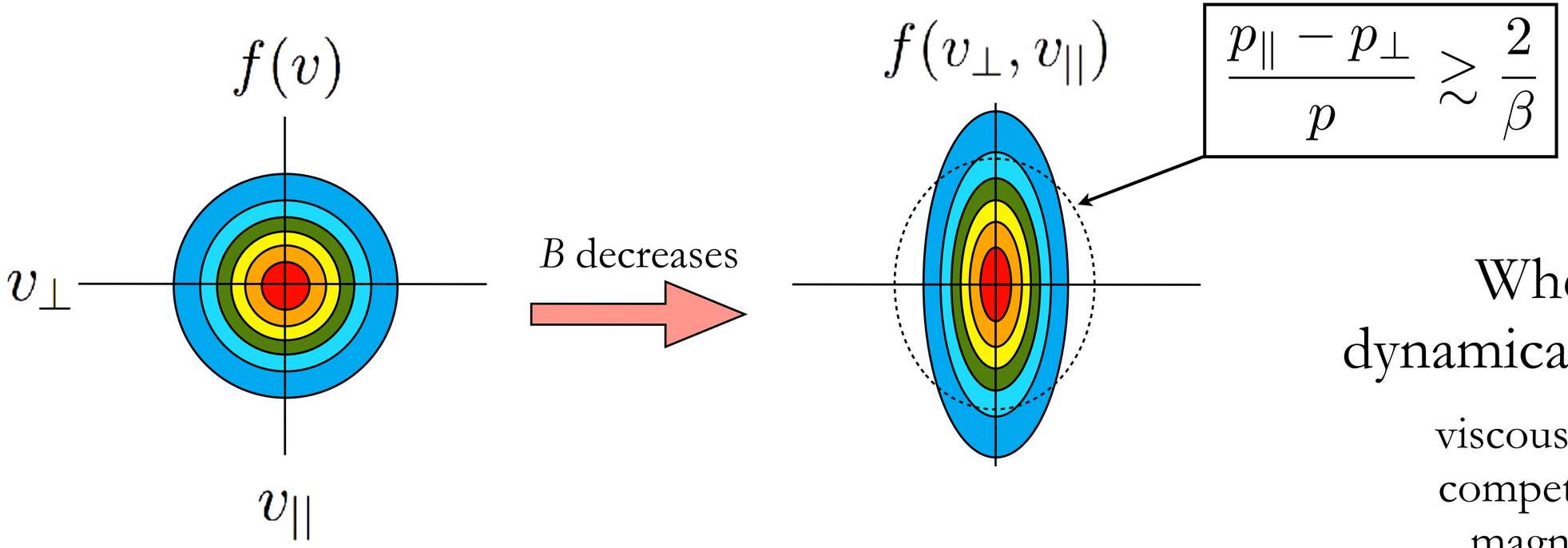
$$\omega_{\mathrm{A}} = k_{\parallel} v_{\mathrm{A}}$$





Question:

How much pressure anisotropy was adiabatically driven by this decrease in field strength?



When is this dynamically significant?

viscous stress is then competitive with the magnetic tension

if collisionless: $$\frac{P_\perp}{P_{||}} - 1 \simeq -\left|\frac{\delta B_\perp}{B_0}\right|^2 \quad \Longrightarrow \quad \left|\frac{\delta B_\perp}{B_0}\right| \gtrsim \frac{1}{\sqrt{\beta}}$$

if weakly collisional: $$\frac{P_\perp}{P_{||}} - 1 \simeq -\frac{\omega_{\rm A}}{\nu_{\rm c}}\left|\frac{\delta B_\perp}{B_0}\right|^2 \quad \Longrightarrow \quad \left|\frac{\delta B_\perp}{B_0}\right| \gtrsim \frac{\sqrt{\omega_{\rm A}/\nu_{\rm c}}}{\sqrt{\beta}}$$

What happens at this wave-amplitude threshold?

1. Wave is "interrupted" and can't oscillate/propagate.

$$\nabla \cdot \left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \left(\frac{B^2}{4\pi} + P_\perp - P_\parallel \right) \right]$$

magnetic tension

nullified if this is $-B^2/4\pi$

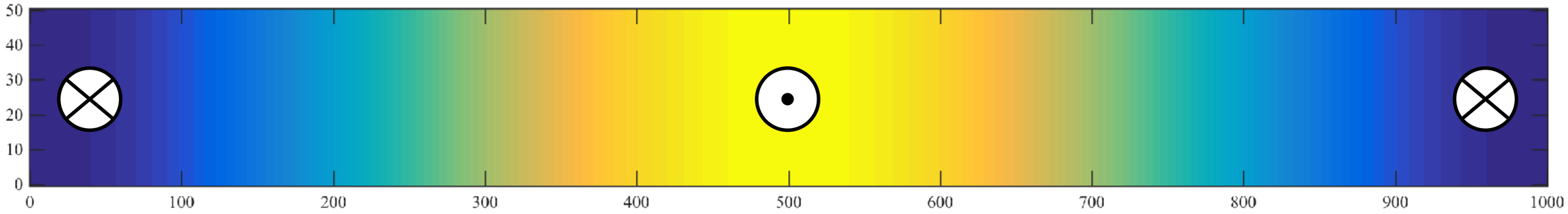
Alfvén wave nonlinearly removes its own restoring force.

2. Plasma is unstable to a sea of ion-Larmor-scale fluctuations (e.g., firehose), which trap and scatter particles and viscously decay the wave.

Note that this can happen even when $\delta B_\perp / B_0 \ll 1$ as long as $\beta \gg 1$!

linearly polarized, standing Alfvén wave

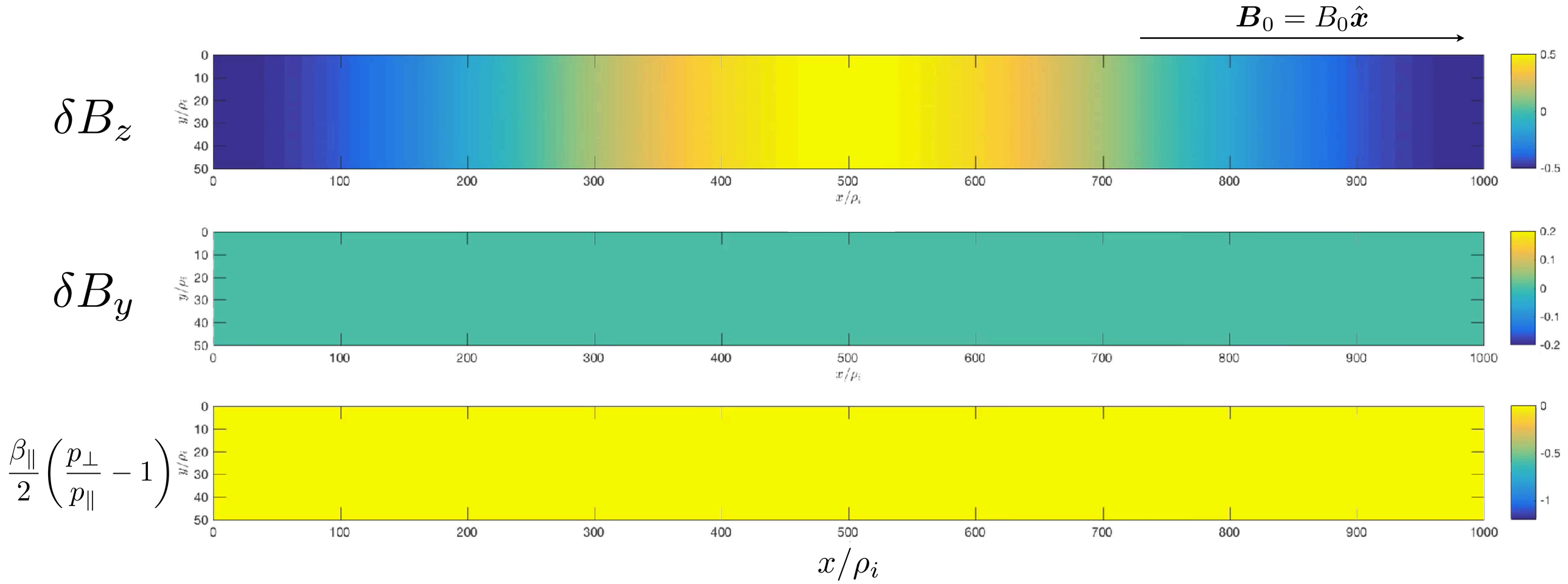
MHD:



$$\boldsymbol{B}_0 = B_0 \hat{\boldsymbol{x}}$$

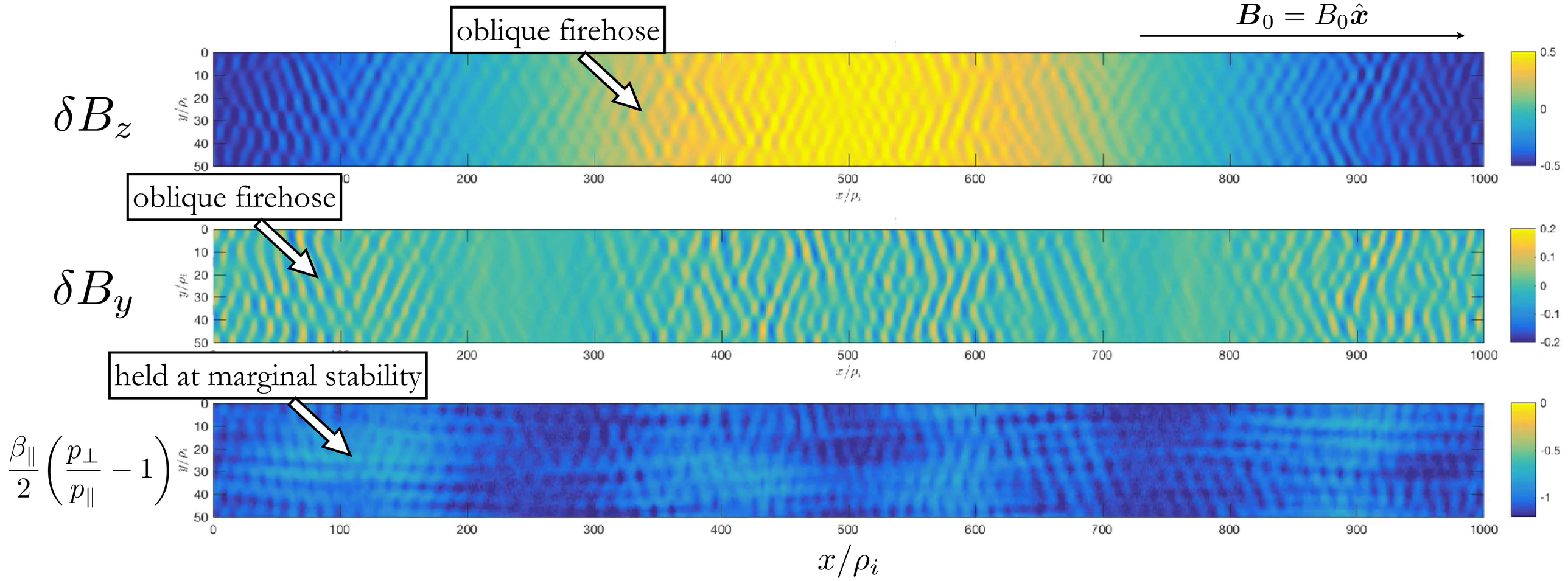
color is δB_z, which is out of plane

long-wavelength, linearly polarized, standing Alfvén wave



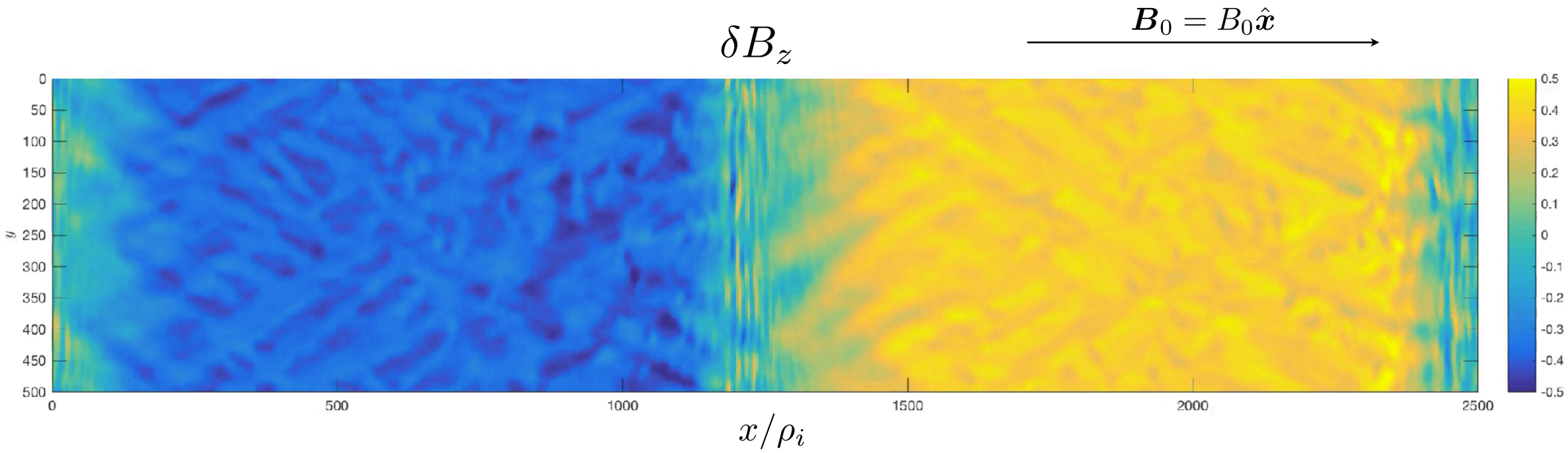
using hybrid-kinetic particle-in-cell code **Pegasus**; Squire, Kunz, Quataert & Schekochihin (2017), PRL

long-wavelength, linearly polarized, standing Alfvén wave



using hybrid-kinetic particle-in-cell code **Pegasus**; Squire, Kunz, Quataert & Schekochihin (2017), PRL

long-wavelength, linearly polarized, *traveling* Alfvén wave



interrupts, slows way down, then viscously decays,
leaving a trail of scattering firehoses in its wake

using hybrid-kinetic particle-in-cell code **Pegasus**; Squire, Kunz, Quataert & Schekochihin (2017), PRL

linearly polarized Alfvén waves cannot be sustained with amplitudes $\delta B_\perp / B_0 \gtrsim \beta^{-1/2}$.

(Nonlinear pressure anisotropy nullifies magnetic tension, drives firehose/mirror that cause viscous decay through effective collision frequency, $\nu \sim \beta\, \omega_{\rm A} (\delta B_\perp / B_0)^2$.)

"interruption" of shear-Alfvén waves at high β

(Squire *et al.*, 2016; Squire, Kunz, Quataert & Schekochihin, 2017 PRL)

can play a similar game with
compressive fluctuations…

Consider a small-amplitude sound wave…

In a magnetized, weakly collisional plasma: $\omega^2 = k^2 a^2 - \mathrm{i}\omega k^2 \mu$

But for viscous losses (and steepening), sound waves propagate just fine

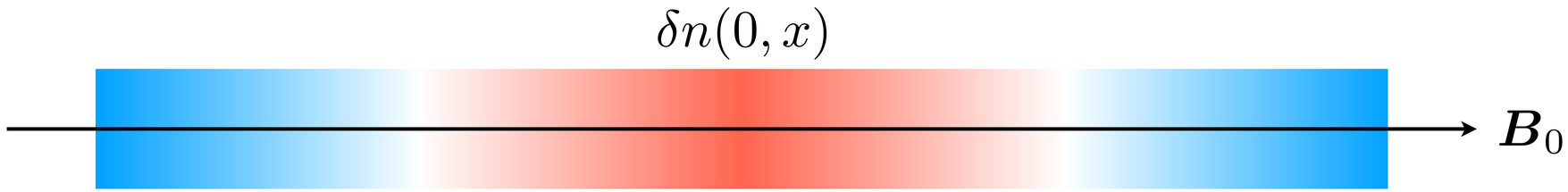
In a magnetized, collisionless plasma: $\dfrac{\omega}{|k_{\|}|\, v_{\mathrm{th}i}} Z\left(\dfrac{\omega}{|k_{\|}|\, v_{\mathrm{th}i}}\right) = -\left(1 + \dfrac{T_i}{T_e}\right)$

solving this… $\dfrac{\gamma}{|\omega|} \sim -1$ if $T_i \sim T_e$

Alex Schekochihin: *"[in a collisionless hot plasma] no one will hear you scream"*

well, not necessarily…

punchline: ***no one will hear you whisper,***
but everyone will hear you scream!



using linear Vlasov with isothermal electrons:

$$\delta f_i(t,k_\parallel,v) = f_{\mathrm{M},i}(v)\,\mathrm{e}^{-\mathrm{i}k_\parallel v_\parallel t}\left[\frac{\delta n(0,k_\parallel)}{n_0} - \mathrm{i}k_\parallel v_\parallel \int_0^t \mathrm{d}t'\,\mathrm{e}^{\mathrm{i}k_\parallel v_\parallel t'}\frac{T_e}{T_i}\frac{\delta n(t',k_\parallel)}{n_0}\right]$$

can calculate evolution of pressure anisotropy:

anisotropic phase mixing of initial signal

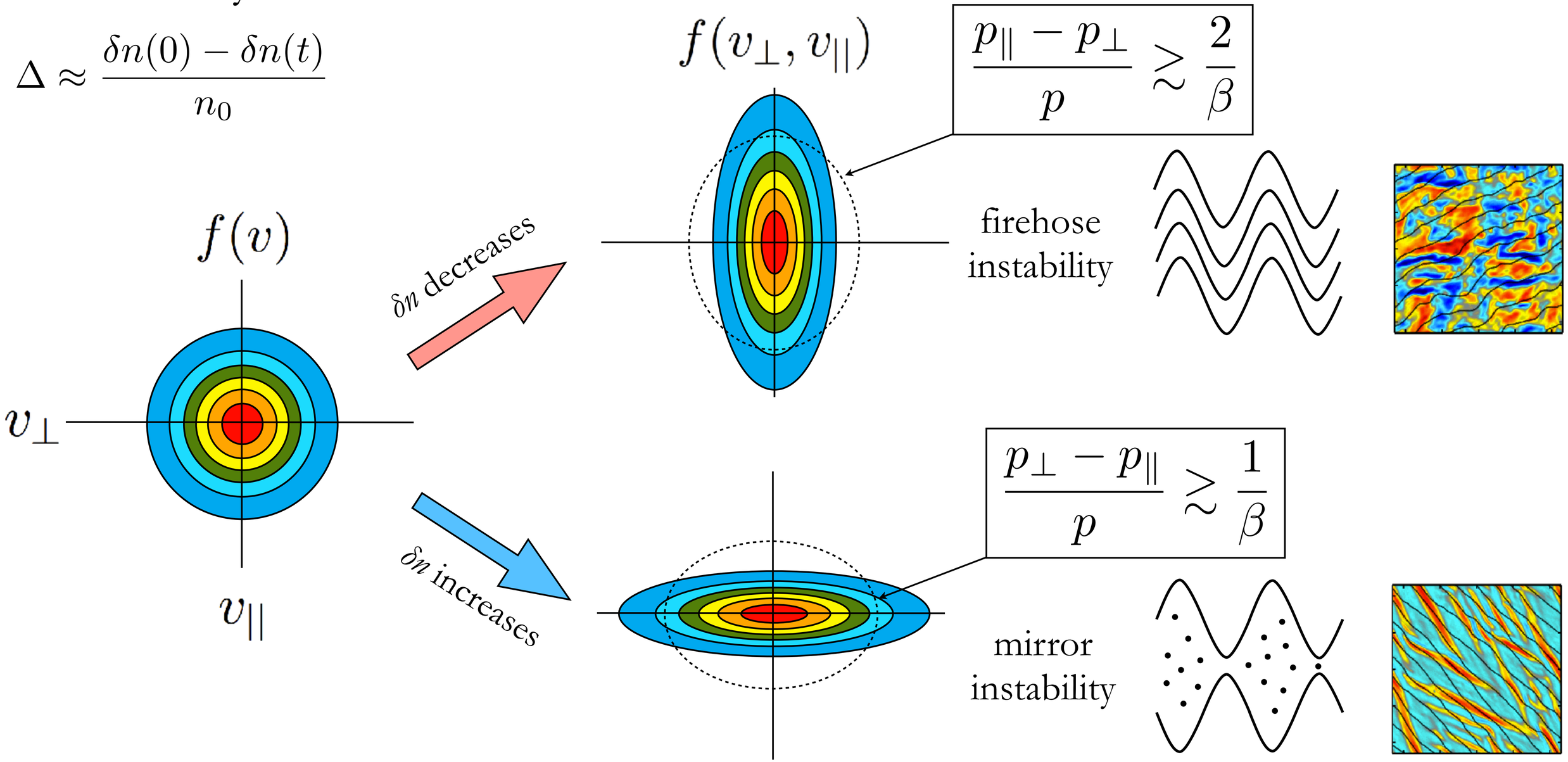
$$\left(\frac{\delta p_{\perp i} - \delta p_{\parallel i}}{p_0}\right)(t,k_\parallel) \equiv \Delta_i(t,k_\parallel) = \boxed{2\left(\frac{k_\parallel v_{\mathrm{th}i} t}{2}\right)^2 \mathrm{e}^{-(k_\parallel v_{\mathrm{th}i} t/2)^2}\left(1+\frac{T_e}{T_i}\right)\frac{\delta n(0,k_\parallel)}{n_0}}$$

$$\boxed{+\,2\int_0^t \mathrm{d}t'\left[\frac{k_\parallel v_{\mathrm{th}i}(t-t')}{2}\right]^2 \mathrm{e}^{-[k_\parallel v_{\mathrm{th}i}(t-t')/2]^2}\frac{T_e}{T_i}\frac{\partial}{\partial t'}\frac{\delta n(t',k_\parallel)}{n_0}}$$

eigenmode response (+ anisotropic phase mixing)

Schematically…

$$\Delta \approx \frac{\delta n(0) - \delta n(t)}{n_0}$$



graphically…

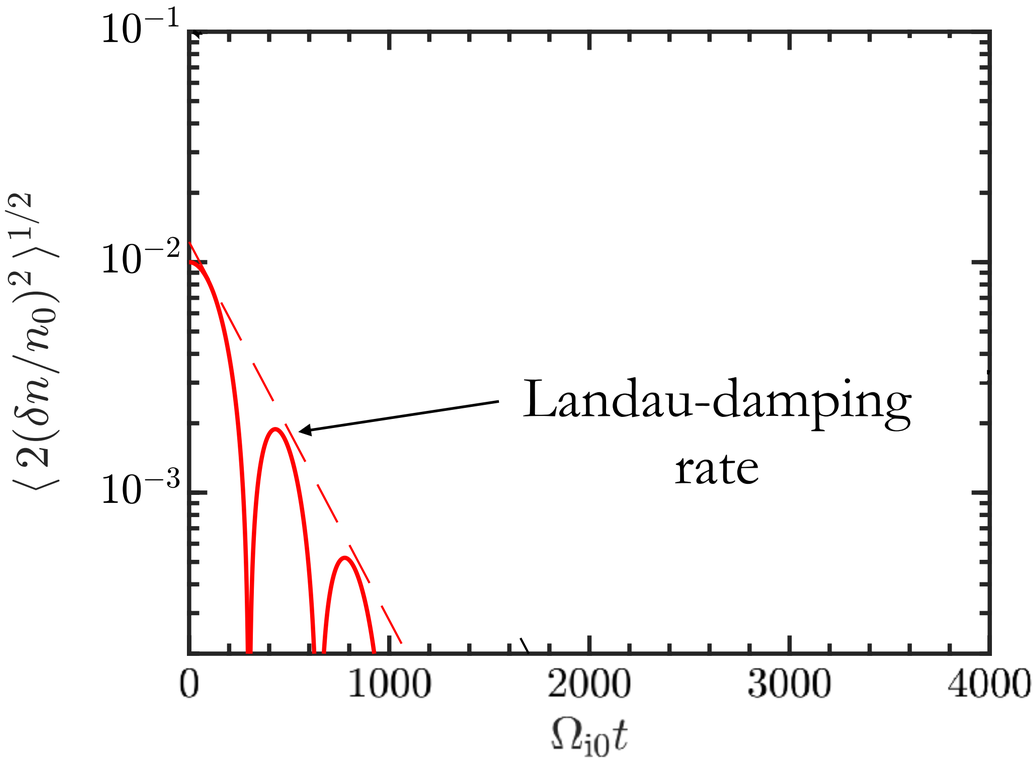
resonant surfer
demonstrating
Landau damping

not so much

ion-acoustic wave

(should just oscillate and Landau damp)

rms density fluctuation vs time



$$f(0, v, x) = f_{\mathrm{M}}(v) \left[1 + \alpha \cos(k_{\parallel} x)\right]$$

pressure anisotropy

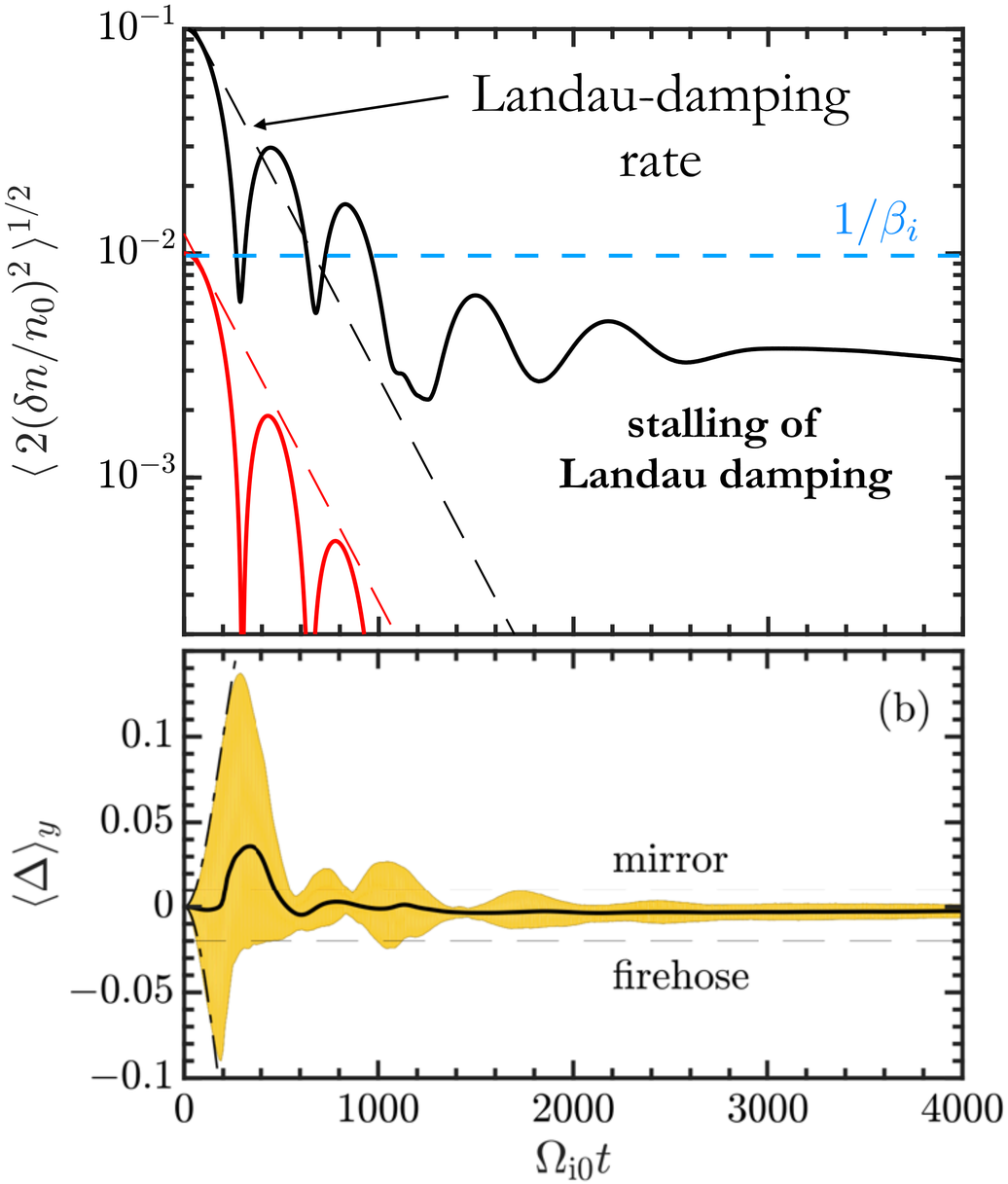
$\beta_i = 100$ Pegasus simulation

ion-acoustic wave

(should just oscillate and Landau damp)

rms density fluctuation vs time

pressure anisotropy



$$f(0,v,x) = f_{\mathrm{M}}(v)\left[1+\alpha\cos(k_{\|}x)\right]$$

density fluctuation stops Landau damping, then slowly decays on (long) viscous timescale

$$\Delta \approx \frac{\delta n(0)-\delta n(t)}{n_0}$$

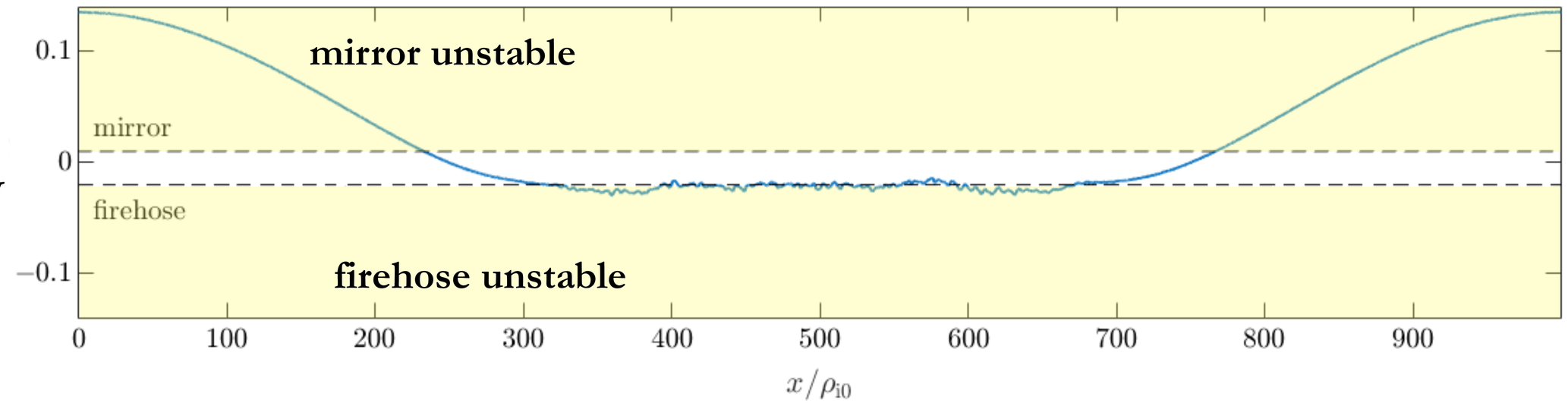
compare with $\frac{1}{\beta}$ and $-\frac{2}{\beta}$

$\beta_i = 100$ Pegasus simulation

Why does Landau damping stall?

vs distance along guide field

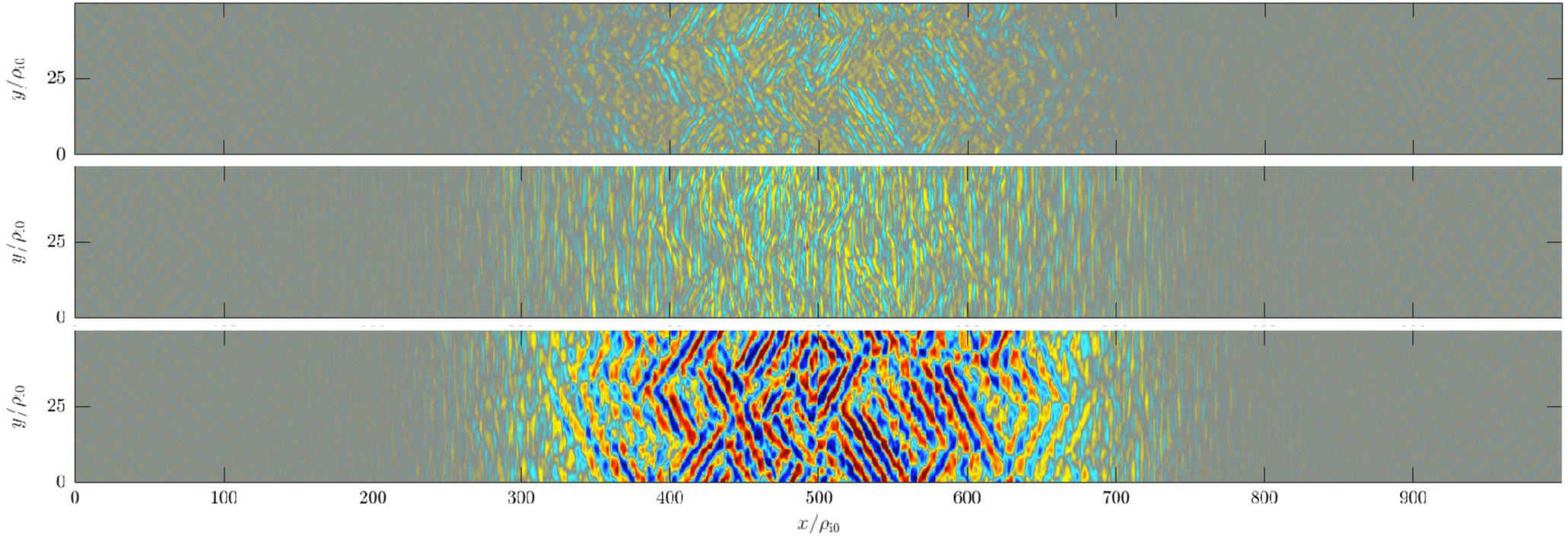
pressure anisotropy



δB_x

δB_y

δB_z



Why does Landau damping stall?

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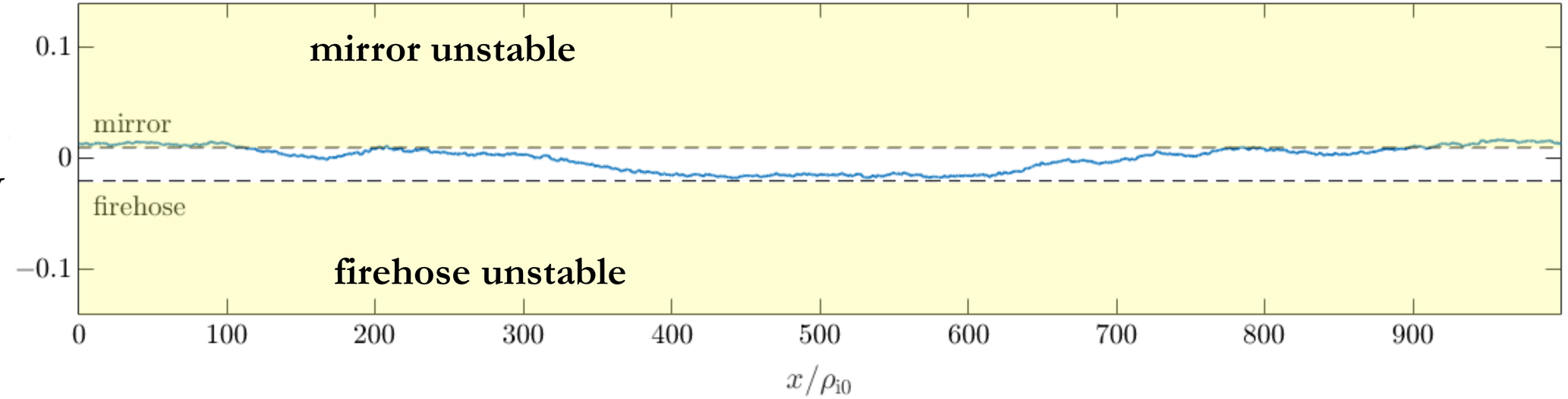
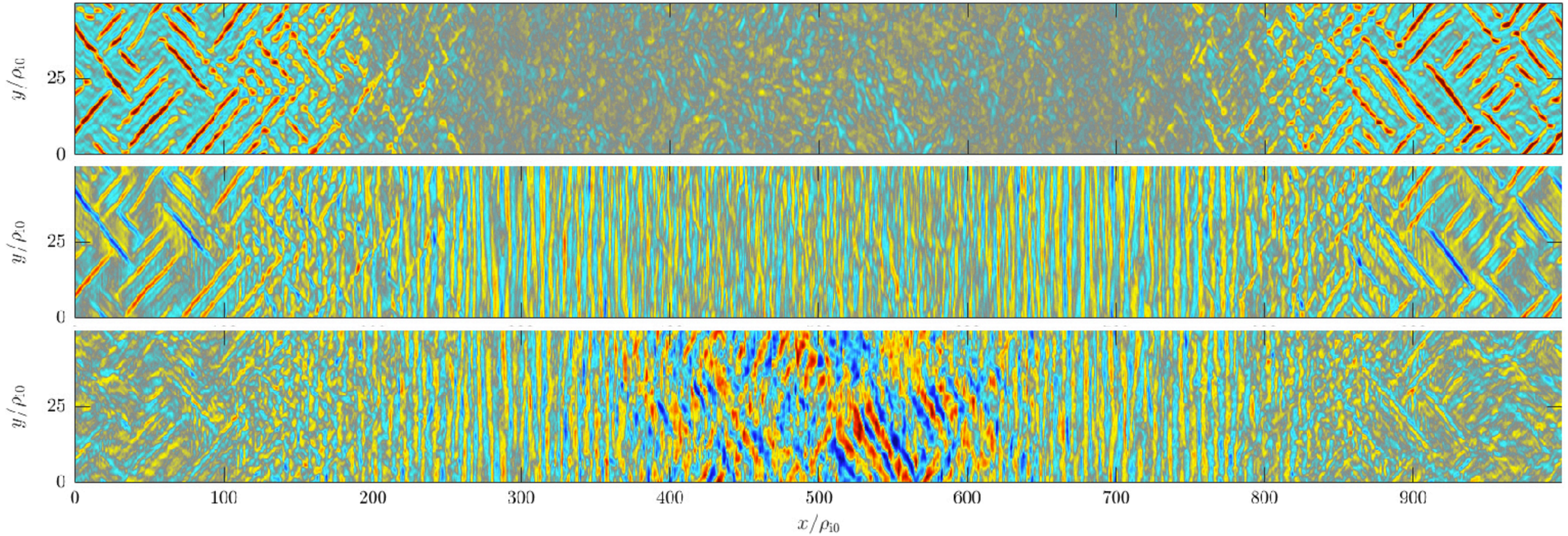


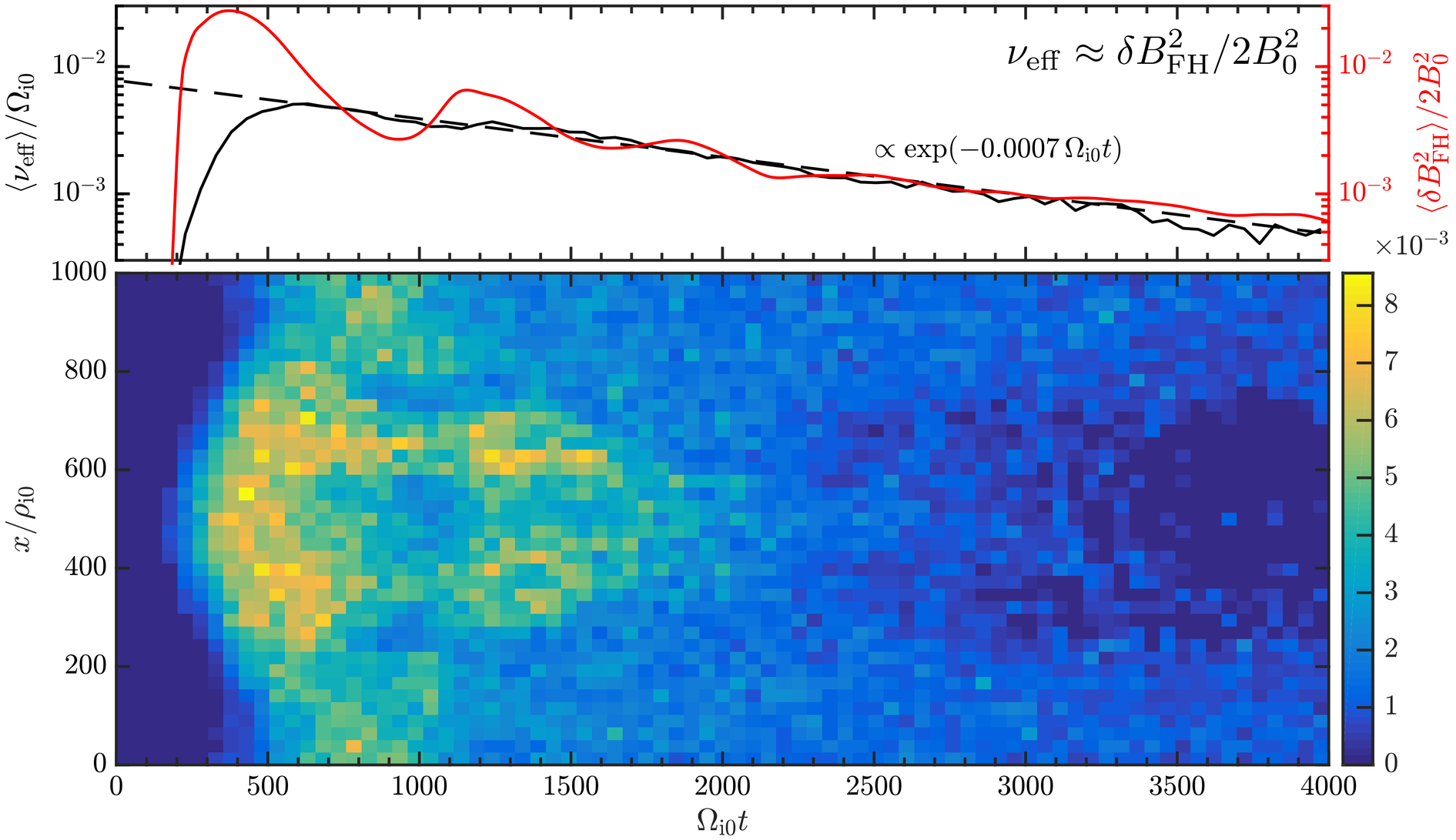
δB_x

δB_y

δB_z



mean of measured scattering frequency



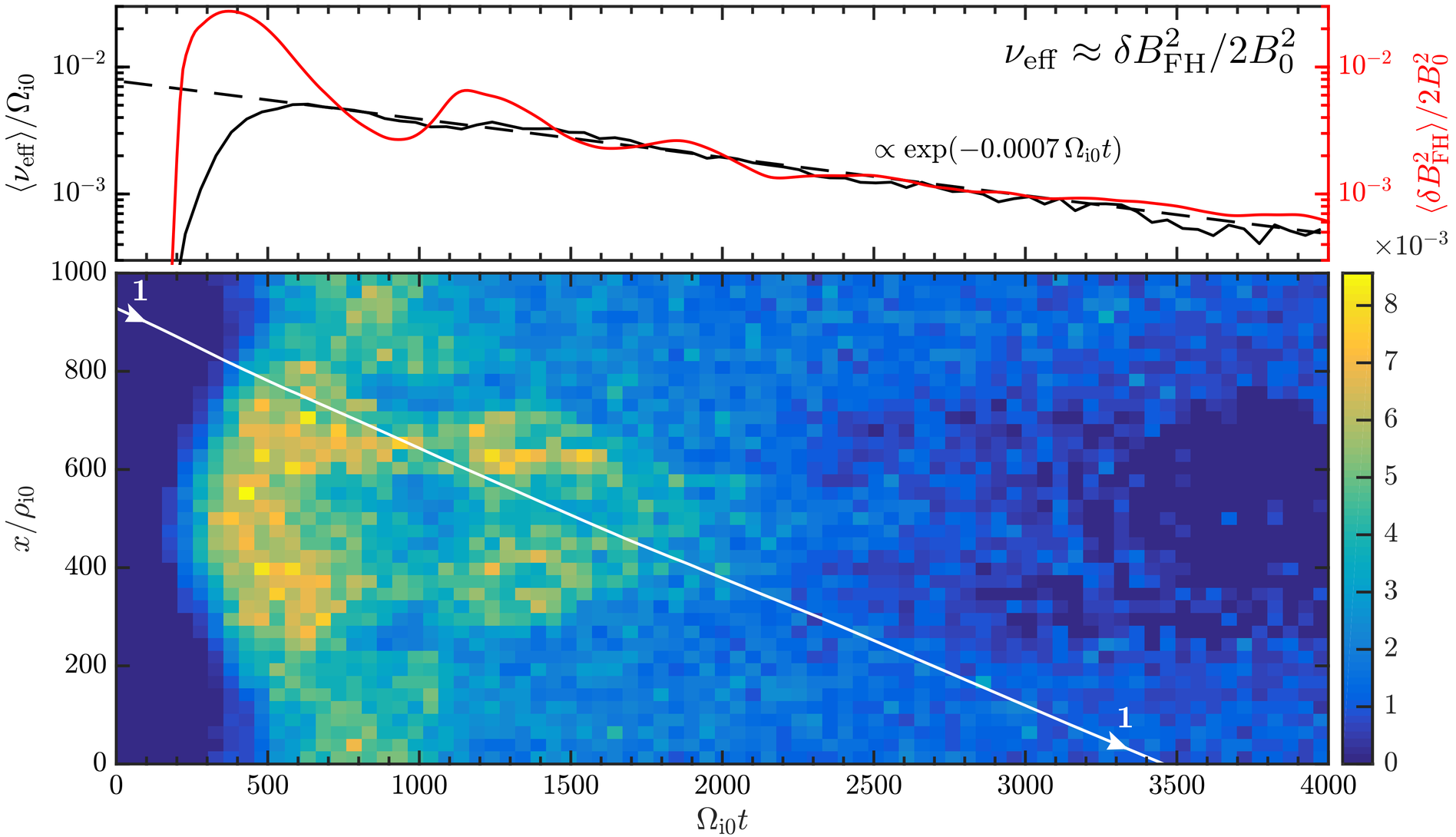
firehose magnetic energy

space-time plot of scattering frequency

color is effective collision frequency, $\nu_{\rm eff}/\Omega_{\rm i0}$

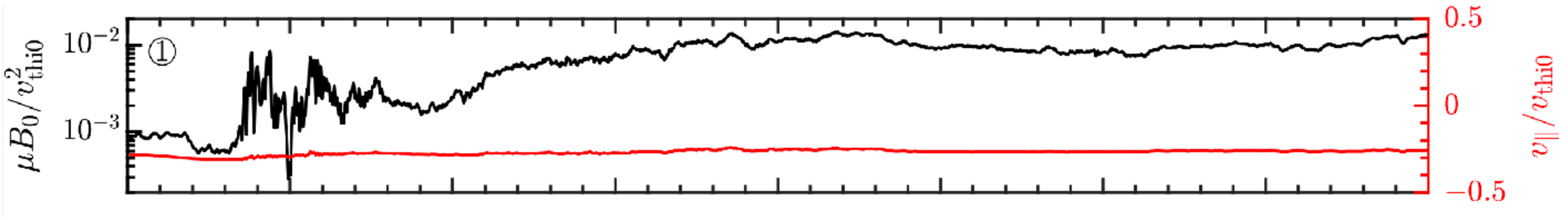
mean of measured scattering frequency

firehose magnetic energy



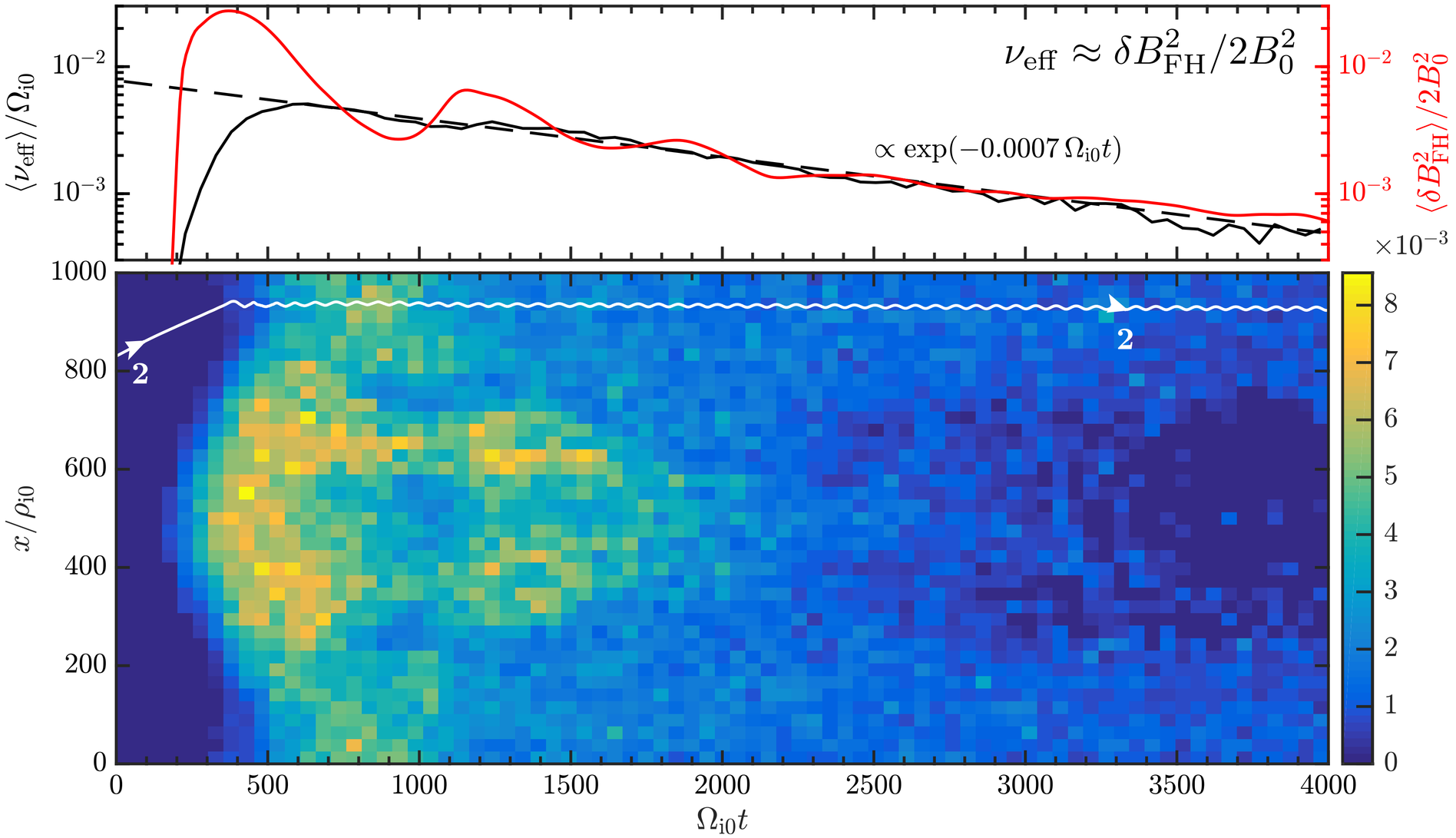
space-time plot of scattering frequency

magnetic moment



parallel velocity

mean of measured scattering frequency



firehose magnetic energy

space-time plot of scattering frequency

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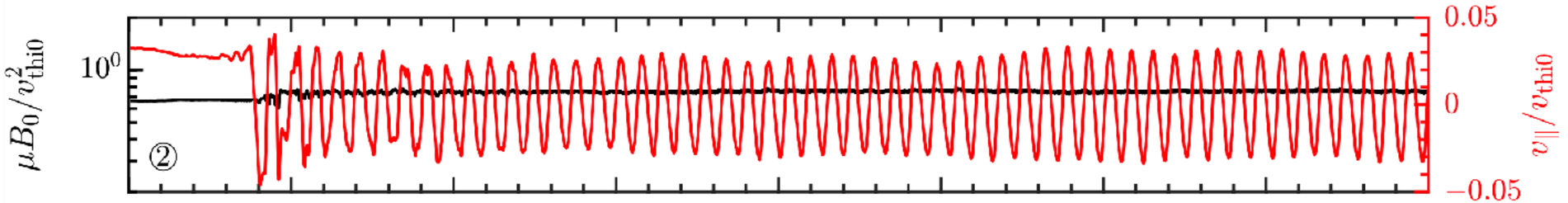

parallel velocity

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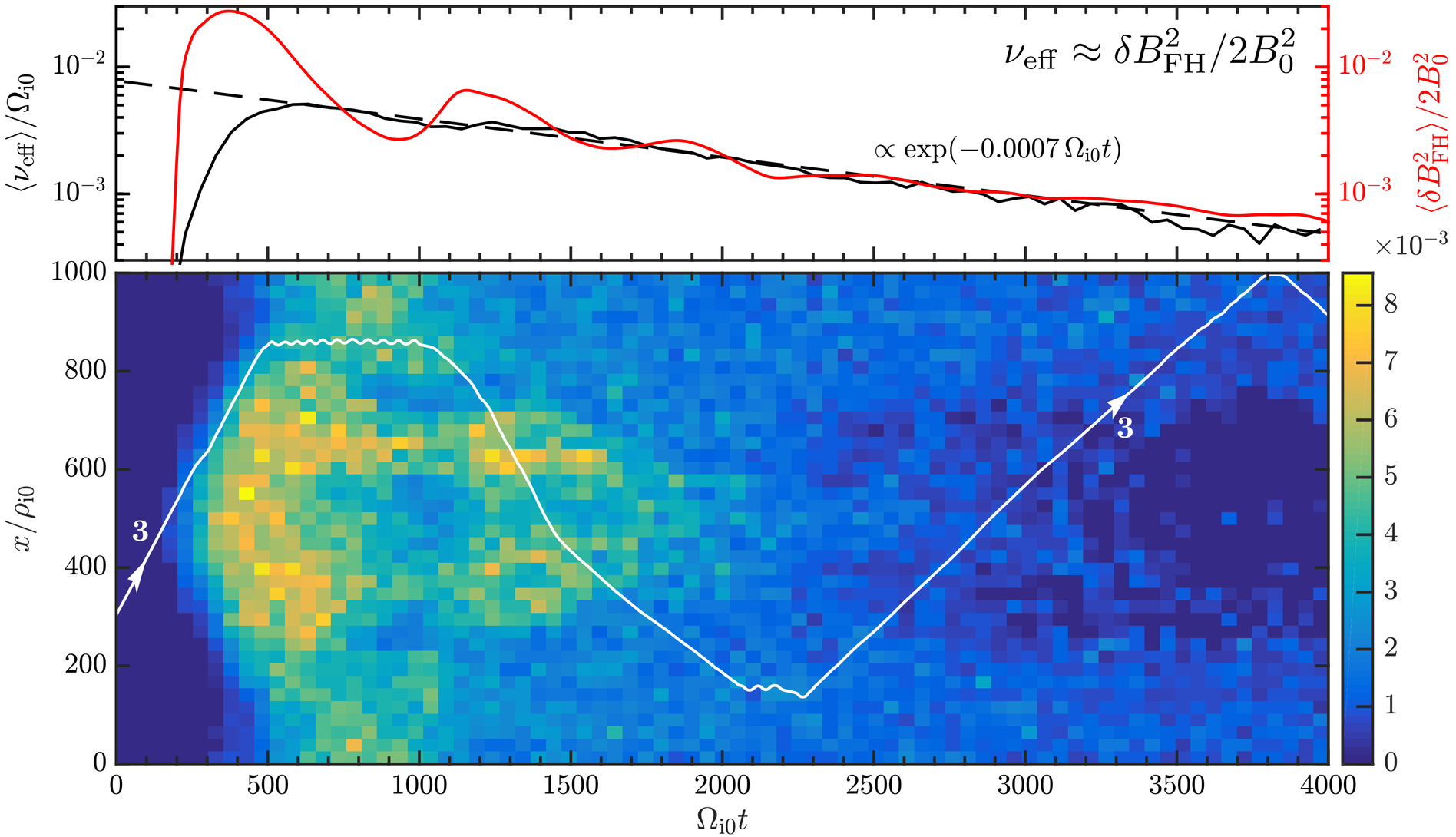
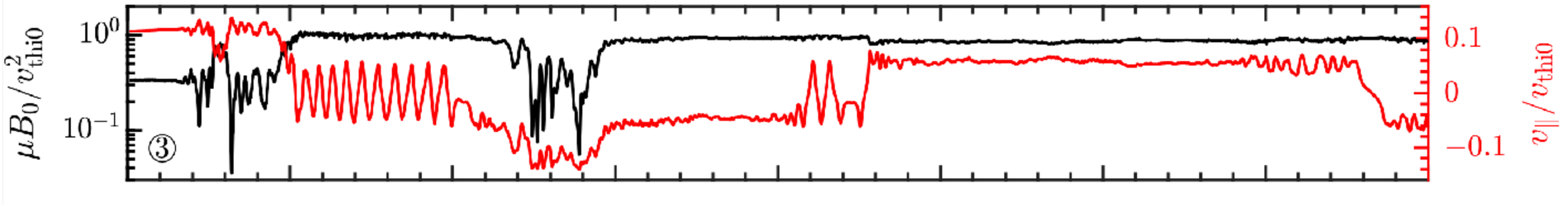
firehose magnetic energy



space-time plot of scattering frequency

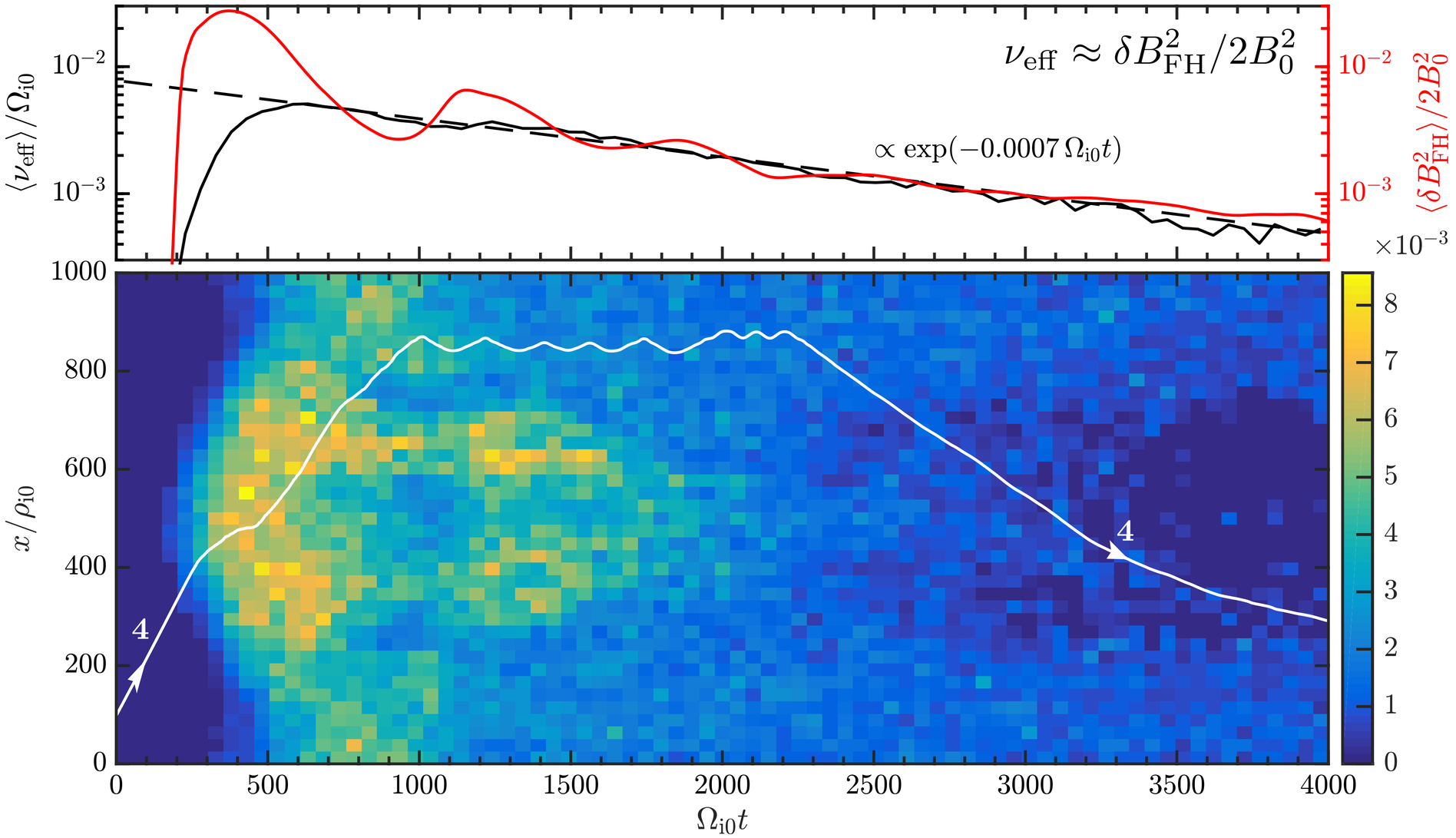
magnetic moment



parallel velocity

mean of measured scattering frequency

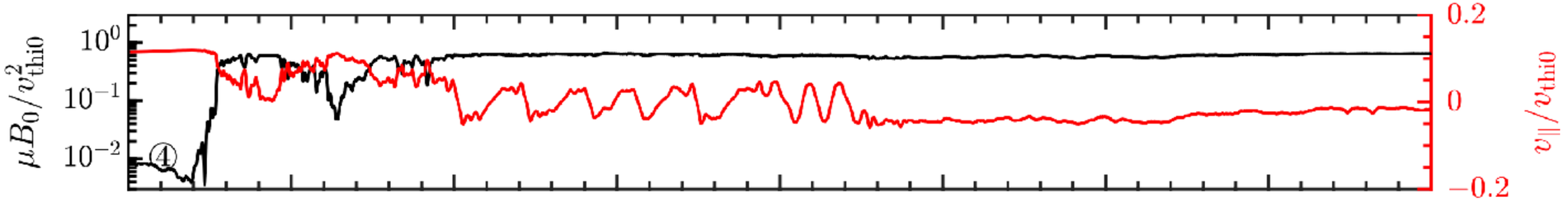
firehose magnetic energy



space-time plot of scattering frequency

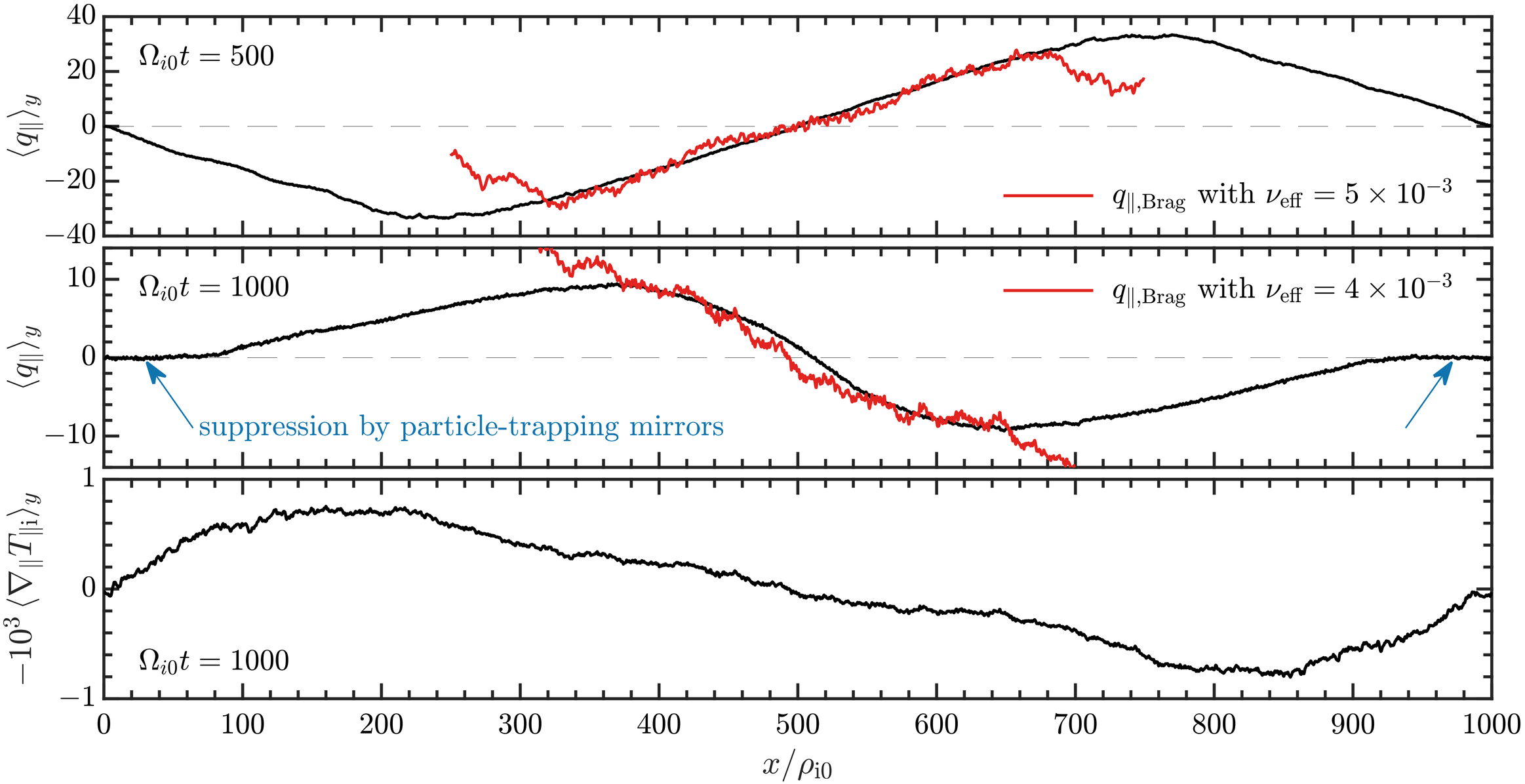
magnetic moment

parallel velocity



parallel heat flux: well described by Braginskii in firehose regions

suppressed in mirror regions (factor of ~200 from Landau-fluid model)



We did an extremely thorough investigation of dependence on scale separation, L/ρ_i

fit analytically predicted scalings, but most importantly:

$$\boxed{\frac{\nu_{\mathrm{eff}}}{|k_{\parallel}|v_{\mathrm{th}i}} \approx 2\beta\frac{\delta n}{n_0}}$$

makes plasma behave as though it were weakly collisional

What if IAWs are continuously driven?

Consider the following (linear) plasma-kinetic Langevin problem:
(see Kanekar, Schekochihin, Dorland, Loureiro, 2015, JPP)

stochastically force parallel momentum ($k = k_{\|}$):

$$\left(\frac{\partial}{\partial t} + \mathrm{i}kv_{\|}\right)\delta f_i(k) + \frac{e\varphi(k)}{T_i}\mathrm{i}kv_{\|} f_{\mathrm{M},i} = \frac{2v_{\|}a(t)}{v_{\mathrm{th}i0}^2} f_{\mathrm{M},i} \qquad \text{with} \qquad \overline{a(t)a(t')} = \varepsilon(k)v_{\mathrm{th}i0}^2\delta(t-t')$$

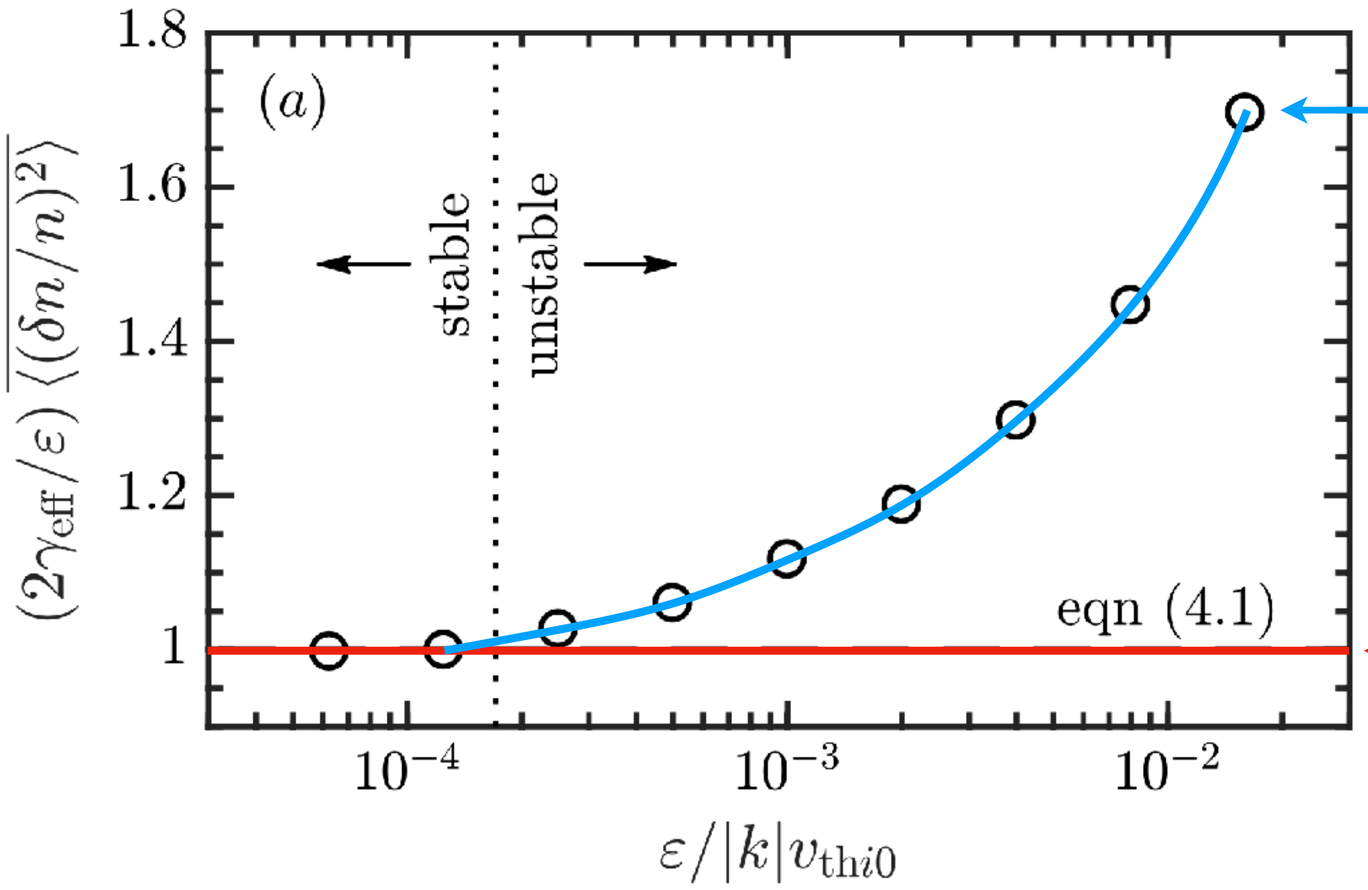
can show that steady-state fluctuations satisfy

$$\overline{\left|\frac{\delta n(k)}{n_0}\right|^2} = \frac{2}{\pi}\frac{\varepsilon(k)}{|k|v_{\mathrm{th}i}}\left(\frac{T_i}{T_e}\right)^2 \int_{-\infty}^{+\infty} \mathrm{d}\zeta \left|\frac{1+\zeta Z(\zeta)}{D(\zeta)}\right|^2 \doteq \frac{\varepsilon(k)}{2\gamma_{\mathrm{eff}}}\frac{T_i}{T_e}$$

and can derive steady-state pressure anisotropy

Solved this problem with a series of Pegasus++ simulations

rms density fluctuation in steady state



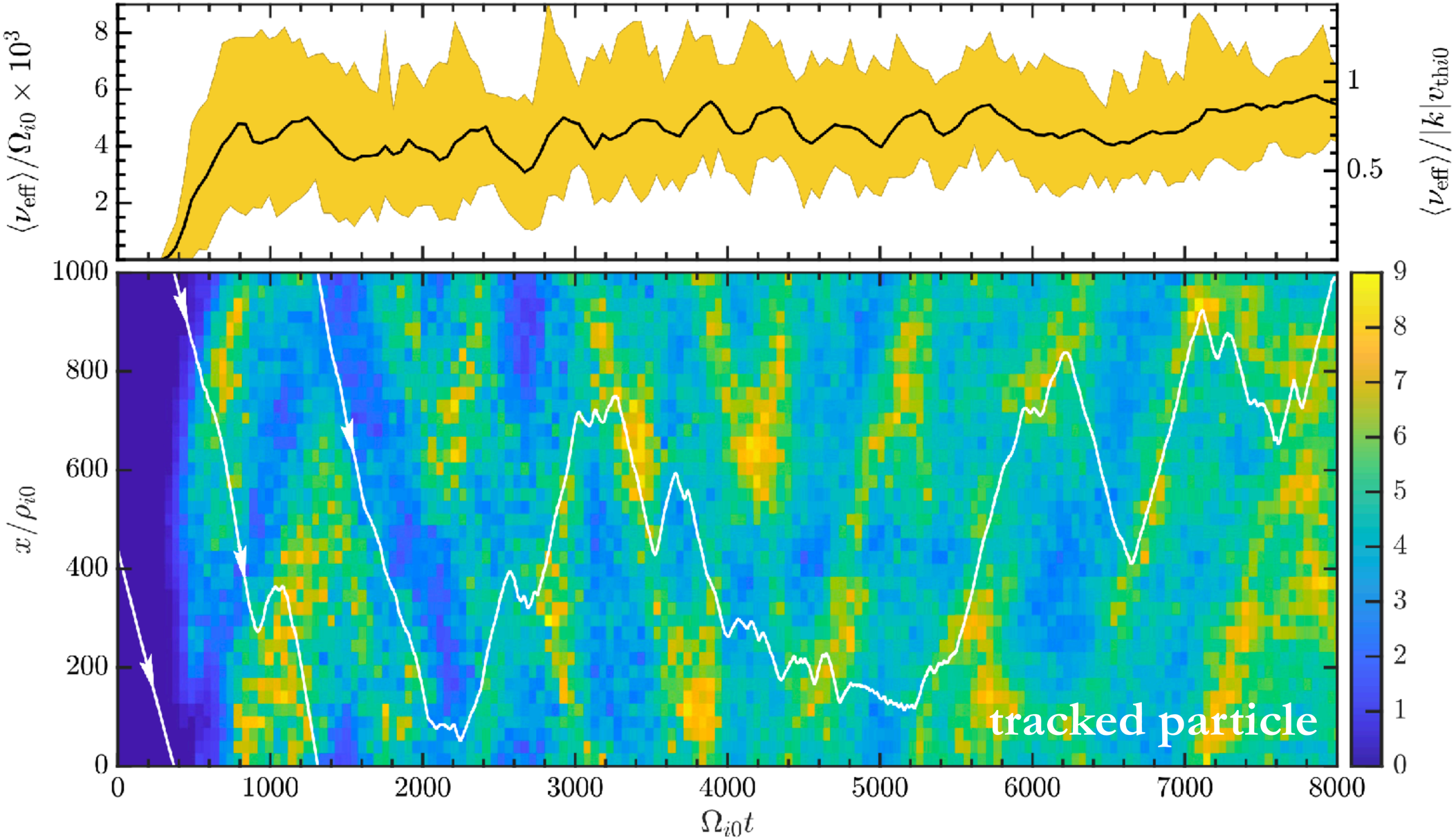
enhanced fluctuation level due to reduced damping when above $\delta n/n \gtrsim 2/\beta$ threshold

if collisionless

energy injection

mean of measured scattering frequency vs time

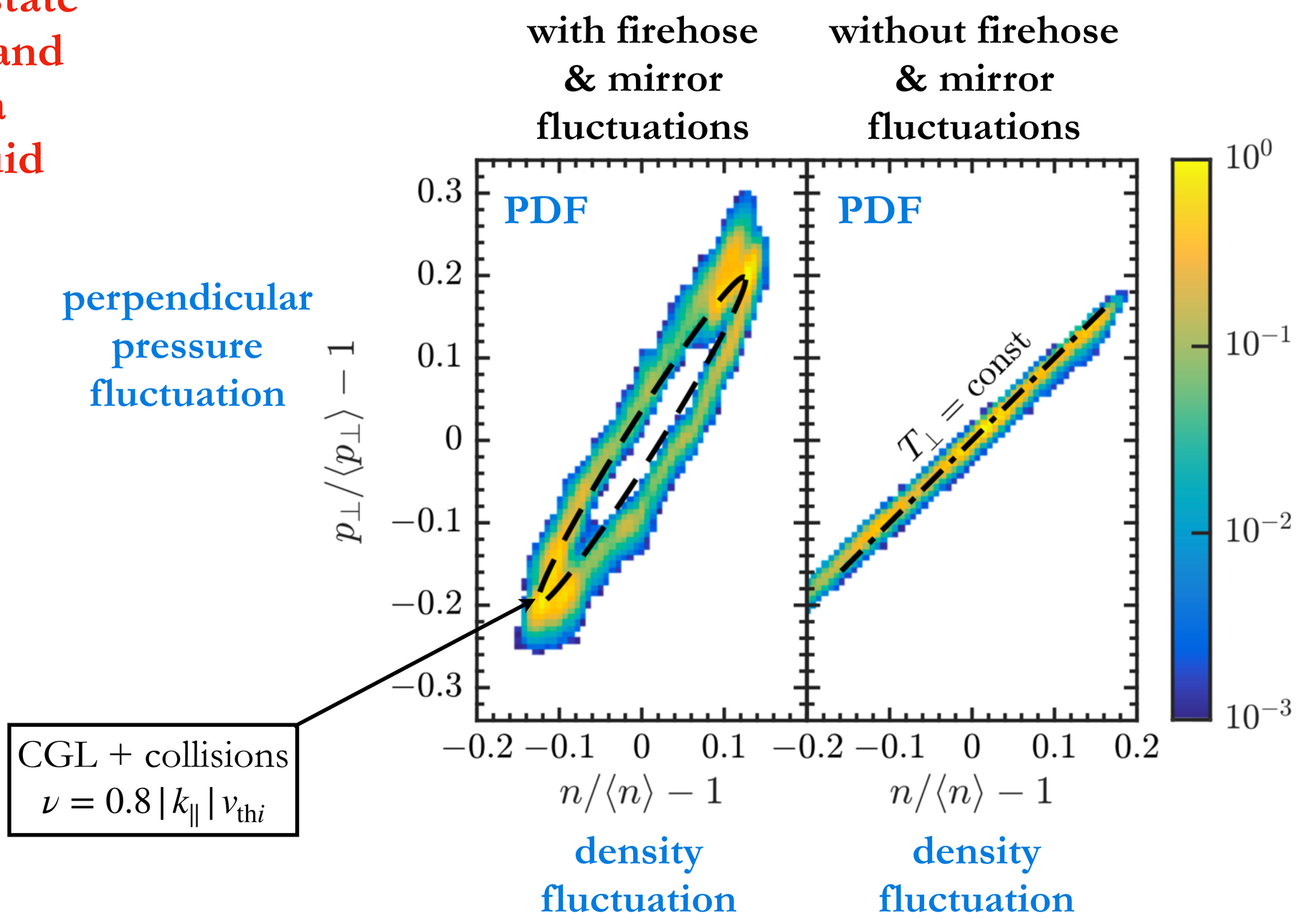
space-time plot of scattering frequency



implies weakly collisional fluid

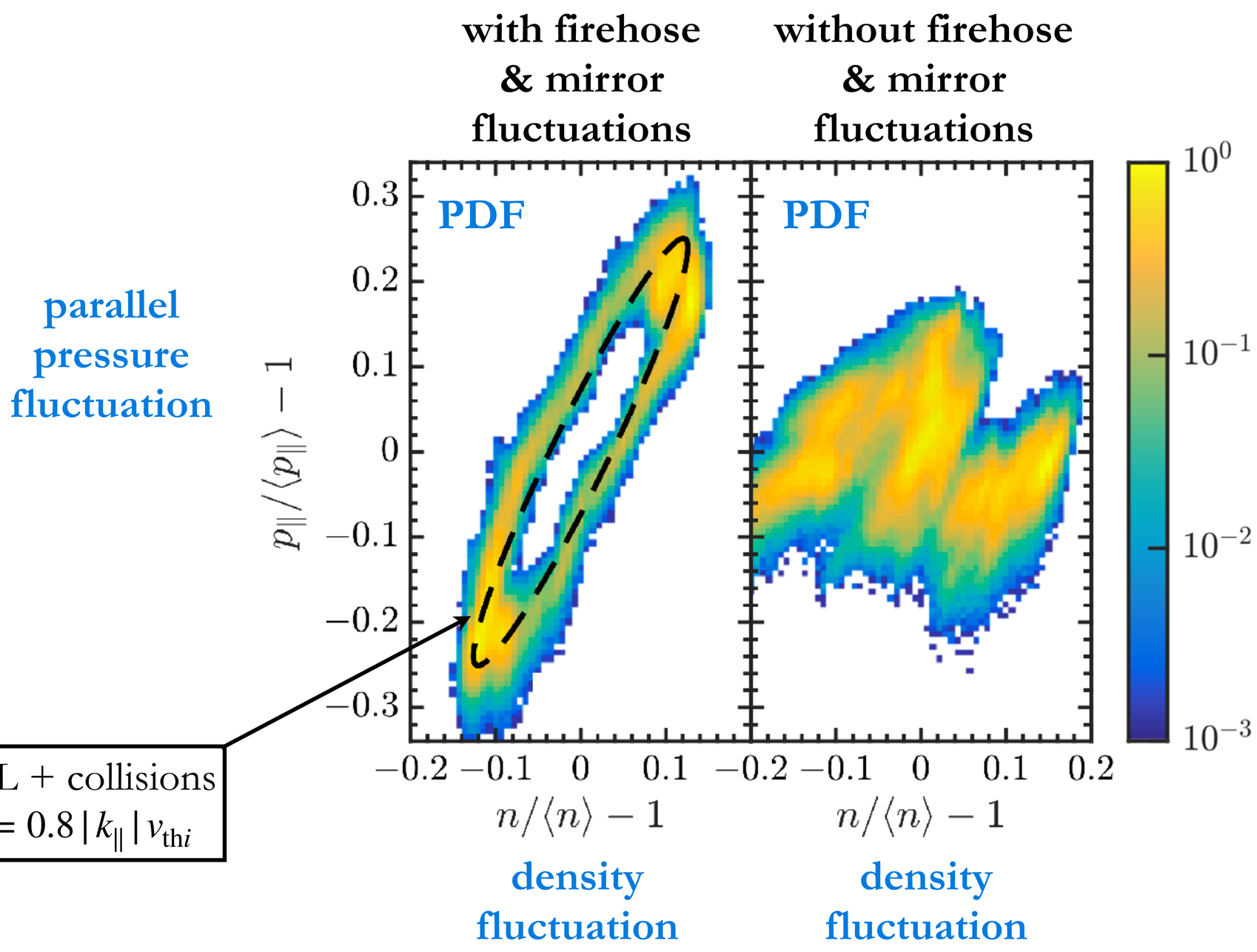
time

measured equation of state of plasma — firehose and mirror make plasma behave more like a fluid



CGL + collisions
$\nu = 0.8\,|k_\parallel|\,v_{\mathrm{th}i}$

measured equation of state of plasma — firehose and mirror make plasma behave more like a fluid



ion-acoustic waves do not efficiently
Landau damp for $\delta n/n \gtrsim \beta^{-1}$.

(Triggered firehose/mirror scatter/trap particles, frustrating resonances.
Changes equation of state and compressive fluctuations become fluid-like.
See also Verscharen *et al.* 2017 for solar-wind context.)

"self-sustaining sound" at high β

(Kunz, Squire, Schekochihin & Quataert, 2020, JPP)

Implications and Wild Speculation

In a high-β low-collisionality plasma…

- Firehose and mirror instabilities regulate the pressure anisotropy, and thus set the effective plasma viscosity (important for dynamo, heating, waves)
- There should be a β-dependent maximum amplitude for different polarizations of Alfvén waves (testable prediction in solar wind)
- Compressive fluctuations with amplitudes above a β-dependent threshold should live longer than they would otherwise (and be MHD-like)
- Direct energy transfer from macroscales to microscale fluctuations and thermal energy, w/o customary scale-by-scale cascade ($\mathrm{Re}_{\parallel} \sim 1$? turb. sims. seem to say so)
- Modern theories of Alfvén-wave turbulence (e.g., GS95) may not apply at sufficiently high β. New theory of turbulence? (with Lev Arzamasskiy, Archie Bott)

Thank you for spending yet another half-hour on Zoom!

if you're interested in high-beta plasma physics:

see US plasma decadal white paper by Kunz & Squire, et al.

arXiv:1903.04080

Balbus, Bale, Chen, Churazov, Cowley, Forest, Gammie, Quataert, Reynolds, Schekochihin, Sironi, Spitkovsky, Stone, Zhuravleva, Zweibel