Plasmoid Instability Mediated Current Sheet Disruption and Onset of Fast Reconnection

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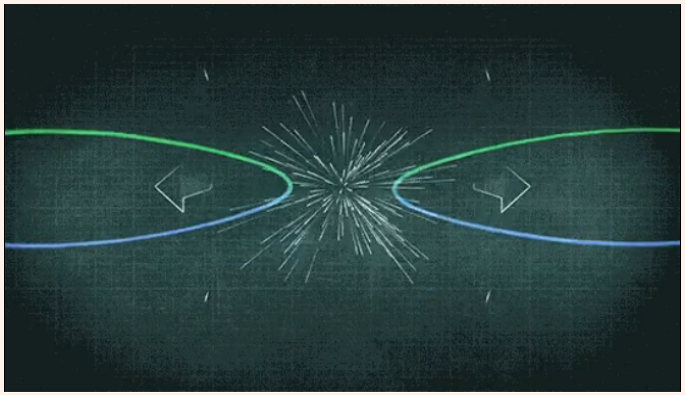
Outline of Today's Talk

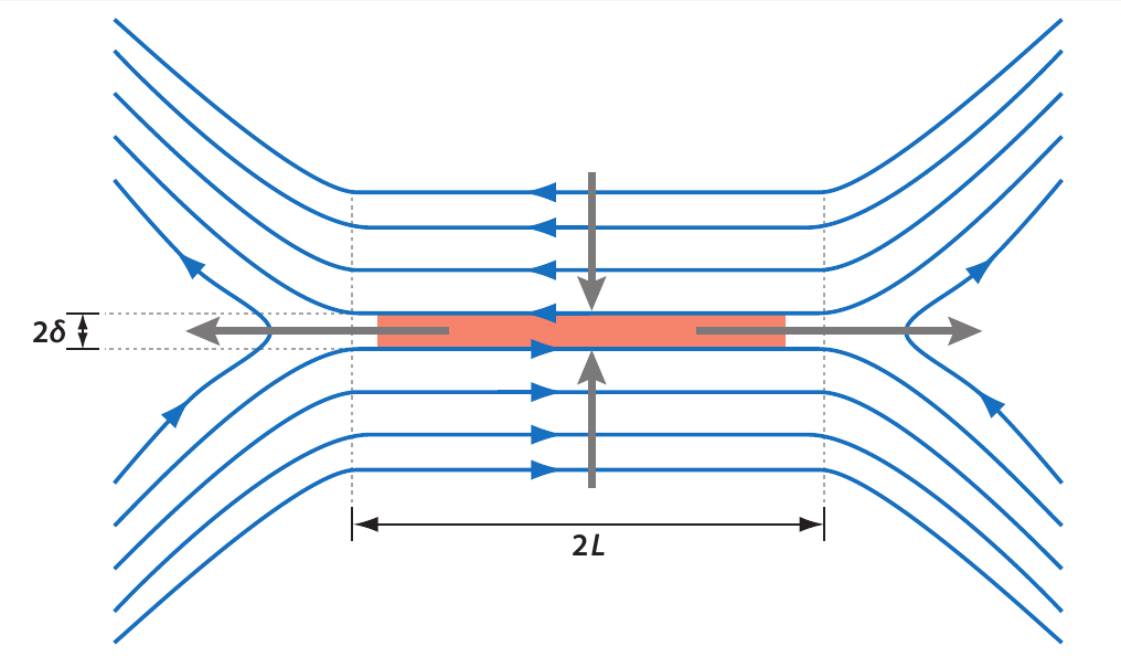
- Overview of the background
- Plasmoid instability in evolving current sheet and the condition for current sheet disruption
- Results from direct numerical simulations
  - Scalings of the current sheet width, linear growth rate, and dominant wavenumber at disruption
- A phenomenological model that reproduces observed scalings
- Critical Lundquist number for current sheet disruption
- Conclusion and future perspectives

What is Magnetic Reconnection



- Ideal Ohm's law $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ implies frozen-in condition, i.e. two fluid elements connected by a field line remain connected forever.
- Local breakdown of magnetic field line frozen-in condition in "diffusion region" leads to change of magnetic field line connectivity and subsequent release of magnetic energy.
- Reconnection events often exhibits a sudden onset of fast reconnection after an extended quiescent period — "trigger" problem.

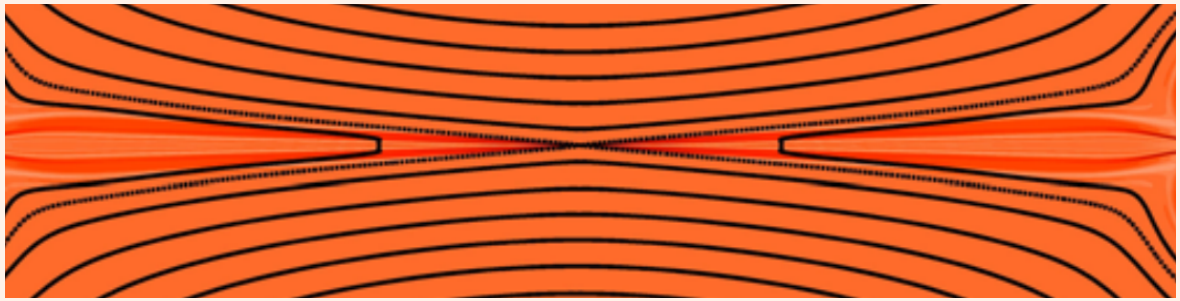
Classical Sweet-Parker Theory (1957)



Courtesy Zweibel & Yamada (2009)

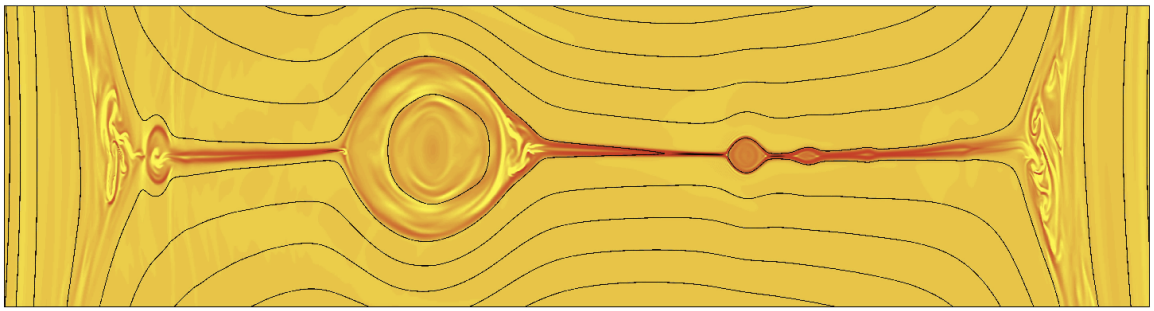
- Collisional, resistive MHD description
- Lundquist number $S = LV_A/\eta$
- $\delta \sim L/\sqrt{S}$, outflow $v_{out} \sim V_A$, inflow $v_{in} \sim V_A/\sqrt{S}$
- Solar corona: $S \sim 10^{12}$,
  $\tau_A = L/V_A \sim 1s \Rightarrow \tau_{SP} \sim L/v_{in} \sim 10^6 s \gg$ Solar flare time scales $10^2 - 10^3 s$.

Collisionless Reconnection



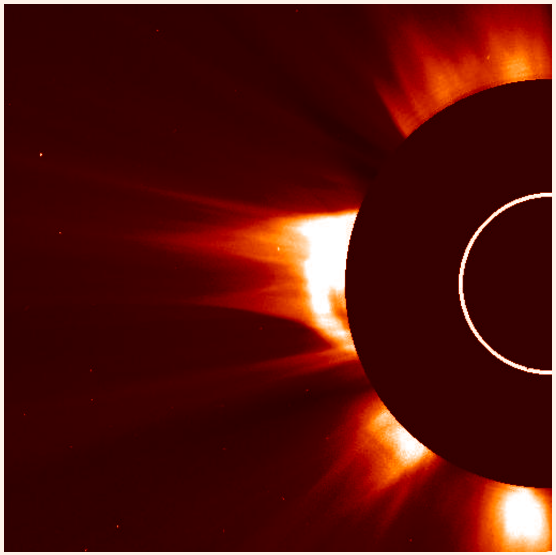
- Needs Hall MHD / Full Kinetic description
- Reconnection Rate $\sim 0.1 V_A B$, much faster than collisional Sweet-Parker reconnection
- Reconnection takes place at kinetic scales $\sim$ ion skin depth d_i or Larmor radius ρ_i
- c.f. chromosphere $d_i \sim 10^{-3} - 1$m, corona $d_i \sim 1 - 10$m.

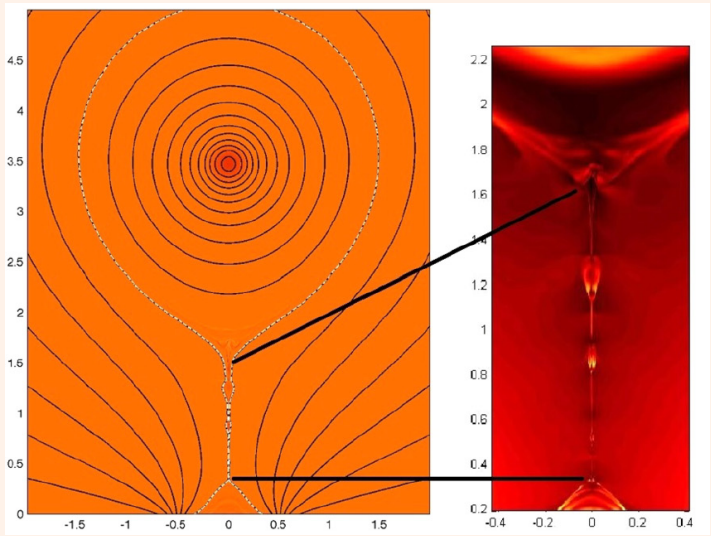
Plasmoid Instability Brings New Perspectives to Reconnection



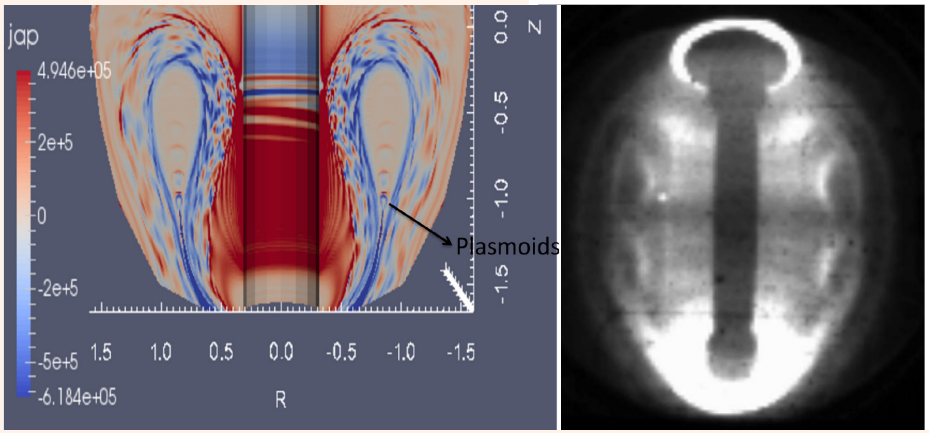
- The reconnection current sheet is unstable to secondary tearing instability at high S — current sheet fragmentation and formation of plasmoids
- Fast reconnection even in resistive MHD, with reconnection rate $\sim 0.01 V_A B$, nearly independent of S (Bhattacharjee et al. 2009; Huang & Bhattacharjee 2010)
- Can trigger even faster collisionless/Hall reconnection if the secondary current sheets become small than ion skin depth d_i or Lamor radius ρ_i (Daughton et al. 2009, Shepherd & Cassak 2010, Huang et al. 2011)

Plasmoids in Nature and Laboratories

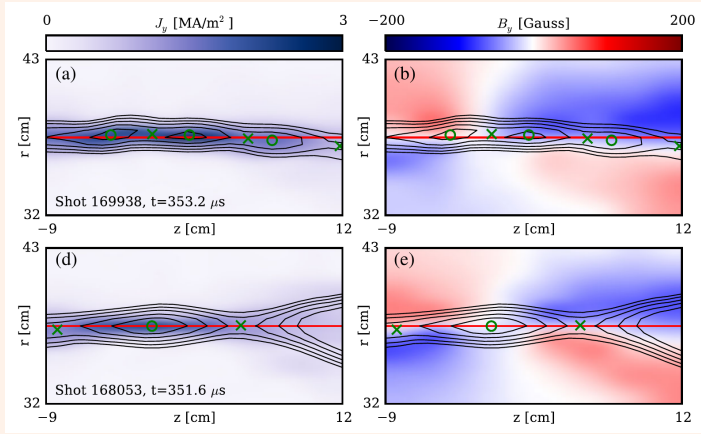




Guo et al. APJL (2013)



Ebrahimi & Raman 2015, 2016



Jara-Almonte et al. PRL 2016

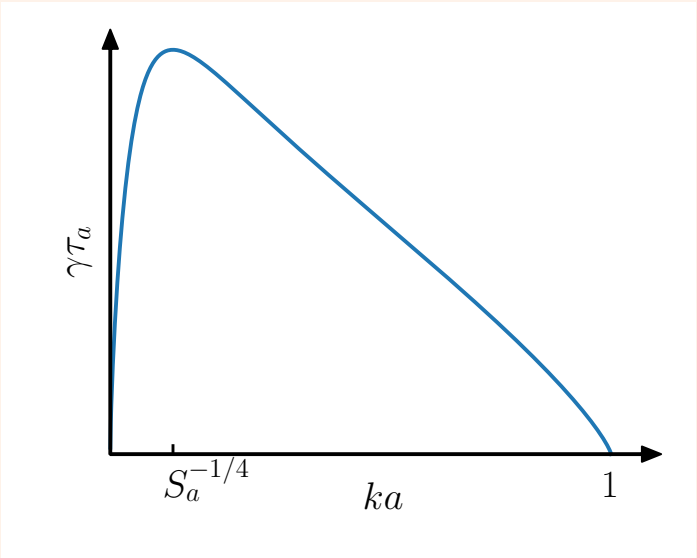
Linear Tearing Instability

Harris sheet profile $\mathbf{B} = B_o \tanh(x/a)\hat{\boldsymbol{y}}$

$$\gamma\tau_a \sim \begin{cases} S_a^{-3/5}(ka)^{-2/5}(1-k^2a^2)^{4/5}, & ka \gg S_a^{-1/4} \\ S_a^{-1/3}(ka)^{2/3}, & ka \ll S_a^{-1/4} \end{cases}$$

Coppi et al. 1976

- Here the Lundquist number $S_a = aV_A/\eta$ and the Alfvén time scale $\tau_a = a/V_A$ are defined with the current sheet **thickness** a.
- Fastest growth rate $\gamma_{max}\tau_a \sim S_a^{-1/2}$ at $k_{max}a \sim S_a^{-1/4}$.



Tearing Instability in Reconnection Current Sheet

- Consider tearing mode in a reconnection current sheet — it is more convenient to define the Lundquist number $S = LV_A/\eta$ and the Alfvén time scale $\tau_A = L/V_A$ in terms of the current sheet **length** L.
- For a Sweet-Parker current sheet, $a \sim L/\sqrt{S}$ — $S_a \sim S^{1/2}$, $\tau_a \sim S^{-1/2}\tau_A$

$$\gamma_{max}\tau_a \sim S_a^{-1/2} \implies \gamma_{max}\tau_A \sim S^{1/4}$$

$$k_{max}a \sim S_a^{-1/4} \implies k_{max}L \sim S^{3/8}$$

(Tajima & Shibata 1997 "Plasma Astrophysics", Loureiro et al. 2007)

- More generally, if $a \sim LS^{-\alpha}$ — $S_a \sim S^{1-\alpha}$, $\tau_a \sim S^{-\alpha}\tau_A$

$$\gamma_{max}\tau_a \sim S_a^{-1/2} \implies \gamma_{max}\tau_A \sim S^{(3\alpha-1)/2}$$

$$k_{max}a \sim S_a^{-1/4} \implies k_{max}L \sim S^{(5\alpha-1)/4}$$

Plasmoid Instability in Evolving Current Sheet

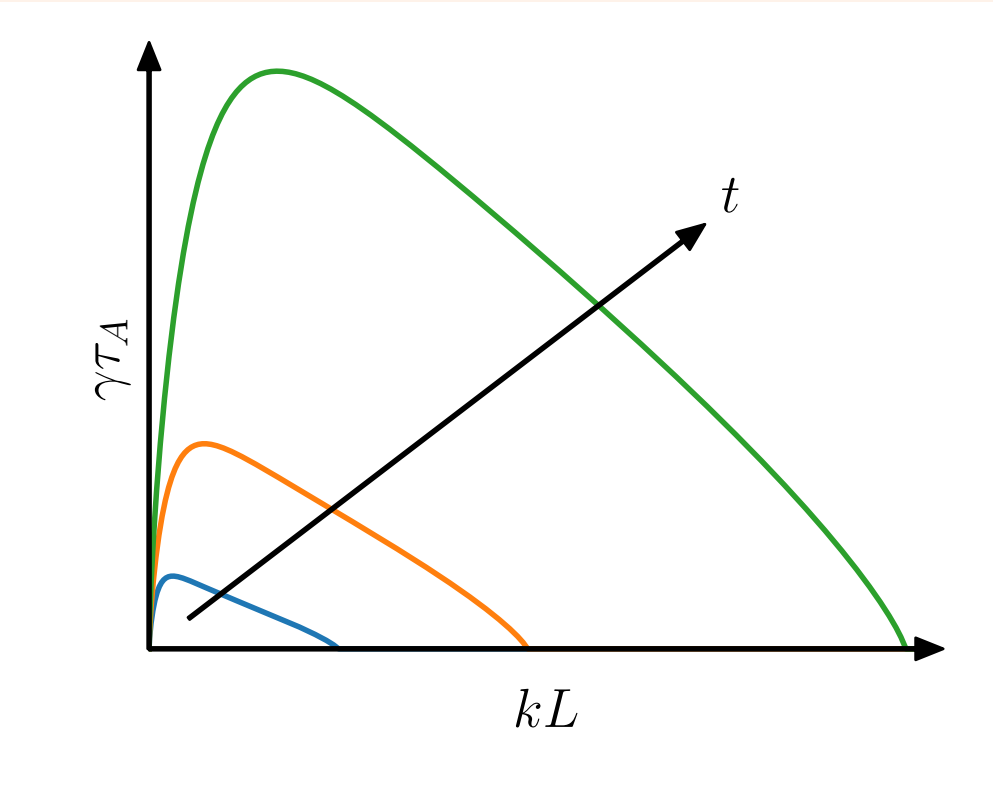
- Sweet-Parker current sheet $a/L \sim S^{1/2}$ — $\gamma_{max}\tau_A \sim S^{1/4}$
- Because a Sweet-Parker sheet must be realized dynamically over time, the fact that $\gamma_{max}\tau_A \sim S^{1/4}$ diverges as $S \to \infty$ suggests that the current sheet will break apart before it reaches the Sweet-Parker width for a high-S system. (Pucci & Velli 2014)
- **Plasmoid instability & current sheet disruption must be studied in the context of an evolving current sheet.**
- When will the current sheet disrupt? How do current sheet width, growth rate, dominant wavenumber at disruption scale with S?

Recent Theoretic Works on The Disruption Condition

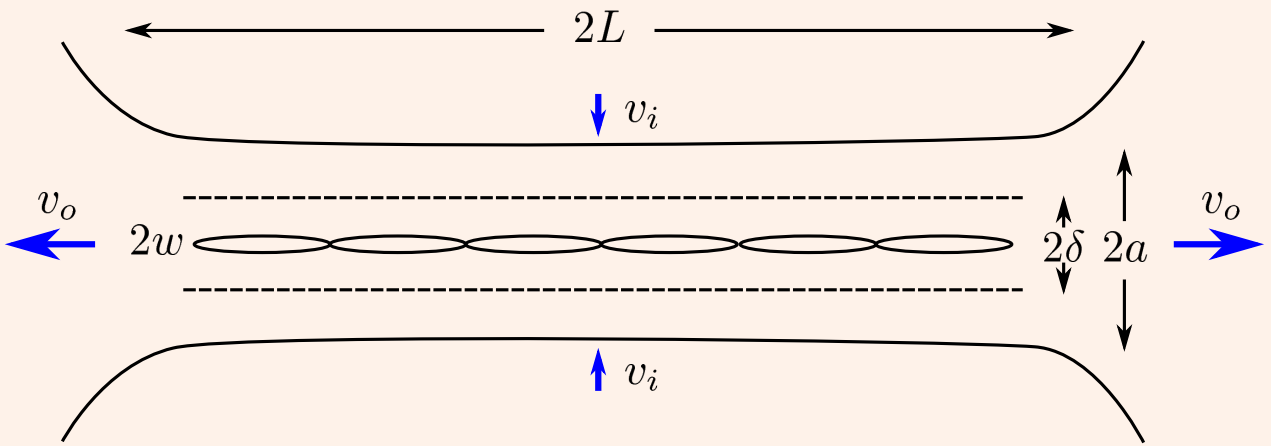
- Pucci & Velli (2014) hypothesize $\gamma_{max}\tau_A$ should become independent of S. Assuming $a/L \sim S^{-\alpha}$, this condition gives $\gamma_{max}\tau_A \sim O(1)$ and the scaling relations $a/L \sim S^{-1/3}$, $k_{max}L \sim S^{1/6}$ when the current sheet breaks apart.
- Uzdensky & Loureiro (2016) assume that the current sheet is essentially frozen when $\gamma\tau_{dr} \simeq 1$, and the fastest mode at the time will be the one that disrupts the current sheet.
- Comisso et al. (2016) employ a principle of least time — "the dominant mode to disrupt the current sheet is the one that takes the least time to do that"
  - Scalings are power laws multiplied by logarithmic factors — $S^{\alpha}(\log f(S, \epsilon, ...))^{\beta}$
  - Dependence on noise

Growth Rate in Thinning Current Sheet

- As $a(t)$ decreases in time, the growth rate γ increases and more modes become unstable.



Condition For Disruption



- Inner layer half-width

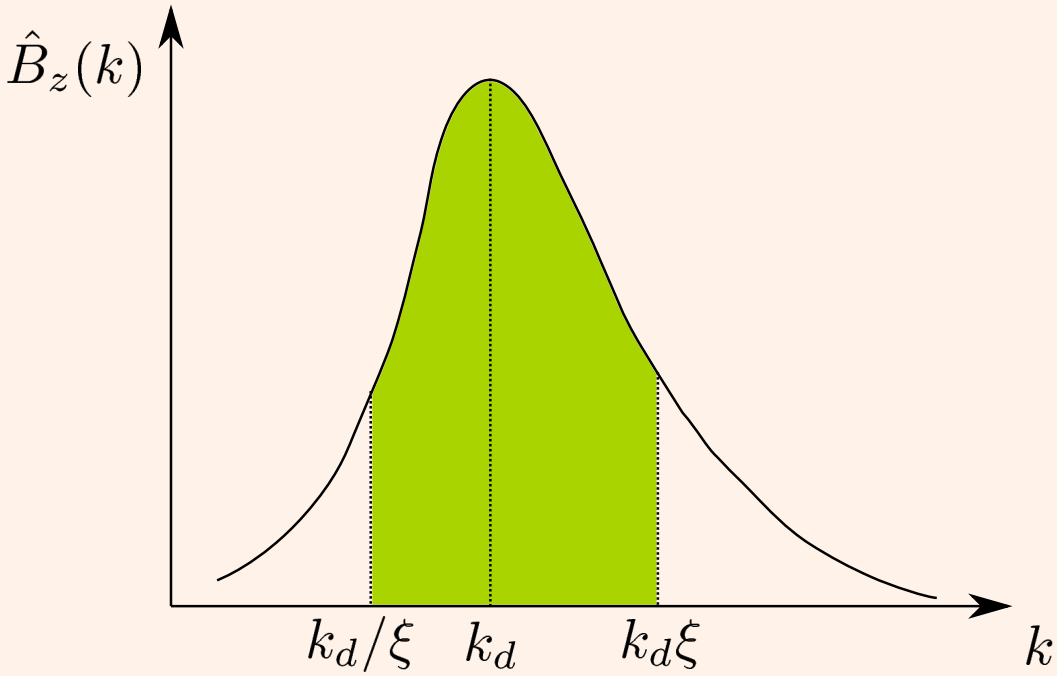
$$\delta = \left(\frac{\gamma}{V_A/a} \frac{1}{(ka)^2 S_a} \right)^{1/4} a$$

- Island half-width

$$w = 2\sqrt{\frac{a\tilde{B}}{kB_x}}.$$

- Tearing instability becomes nonlinear when $w = \delta$. At this time $\tilde{J} \sim J$ and the current sheet is "disrupted".

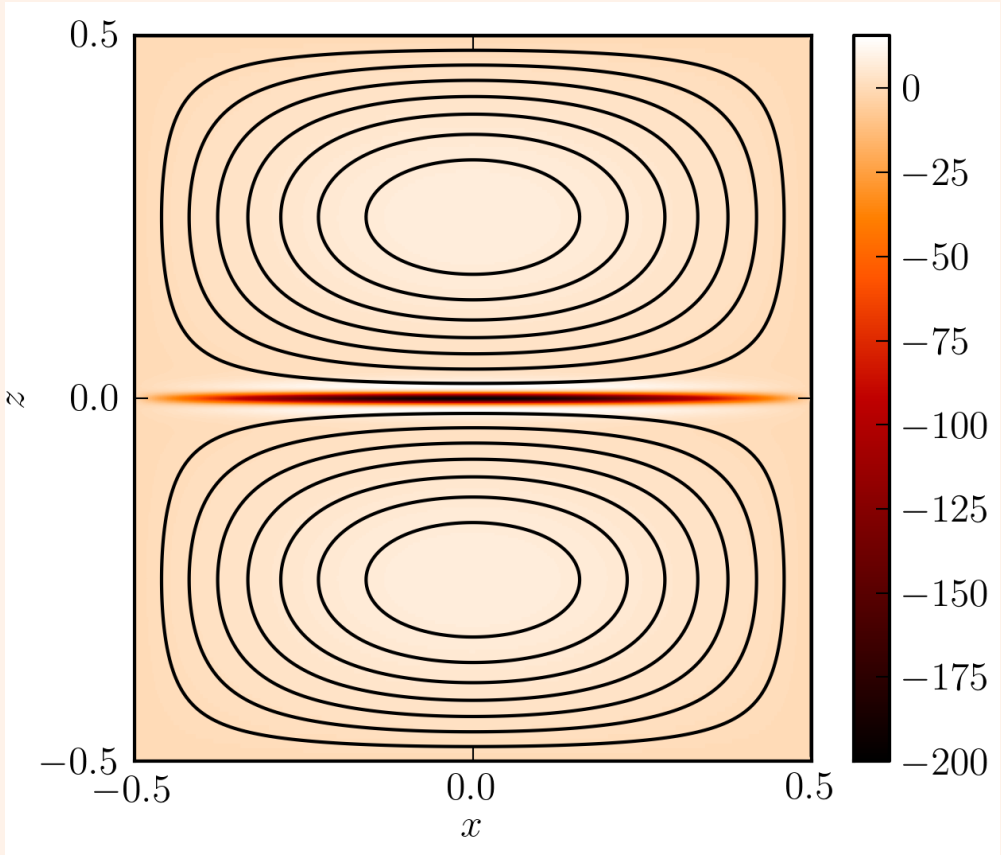
Estimating Island Size with Superposition of Modes



$$\tilde{B} = \left(\frac{1}{\pi L} \int_{k_d/\xi}^{k_d\xi} |\hat{B}_z(k')|^2 dk' \right)^{1/2}$$

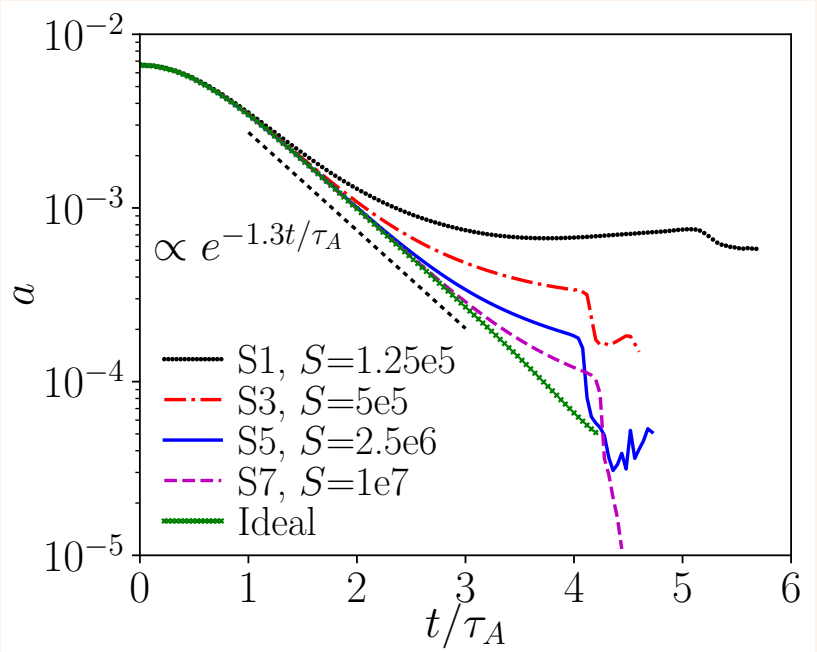
$$w = 2\sqrt{\frac{a\tilde{B}}{k_d B_x}}.$$

Simulation Setup (Huang & Bhattacharjee 2010)



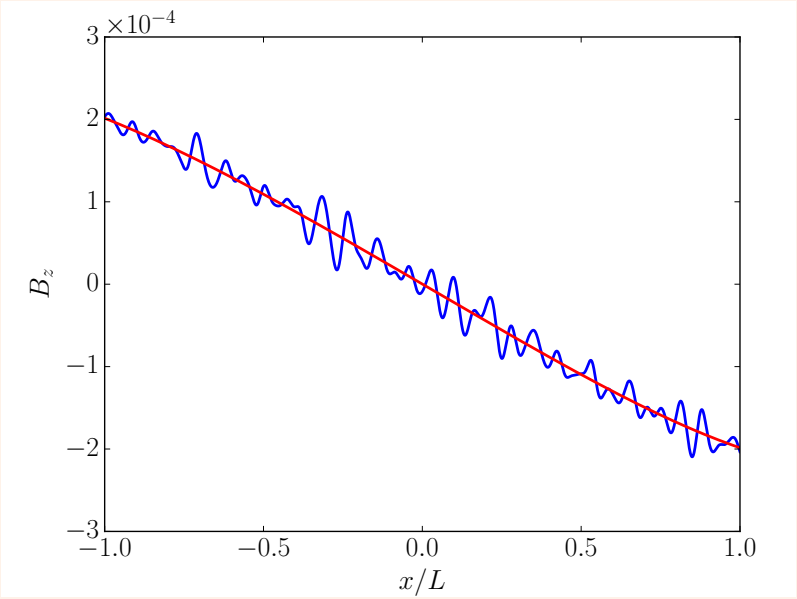
- The merger of the two islands drives reconnection
- Initial current sheet width $a = 6.67\text{e}{-3}$.
- Initial velocity field is seeded with a random noise with amplitude ϵ.

Current Sheet Thinning in Simulations



- After a short period of initial transient time, the current sheet width a decays exponentially at a time scale $\sim \tau_A$.
- The exponential decay then slows down due to resistivity.
- The current sheet width measurement is no longer applicable when the current sheet is "disrupted".

Separate Fluctuations from Background



- Take B_z at the midplane and make a polynomial fitting to obtain the background

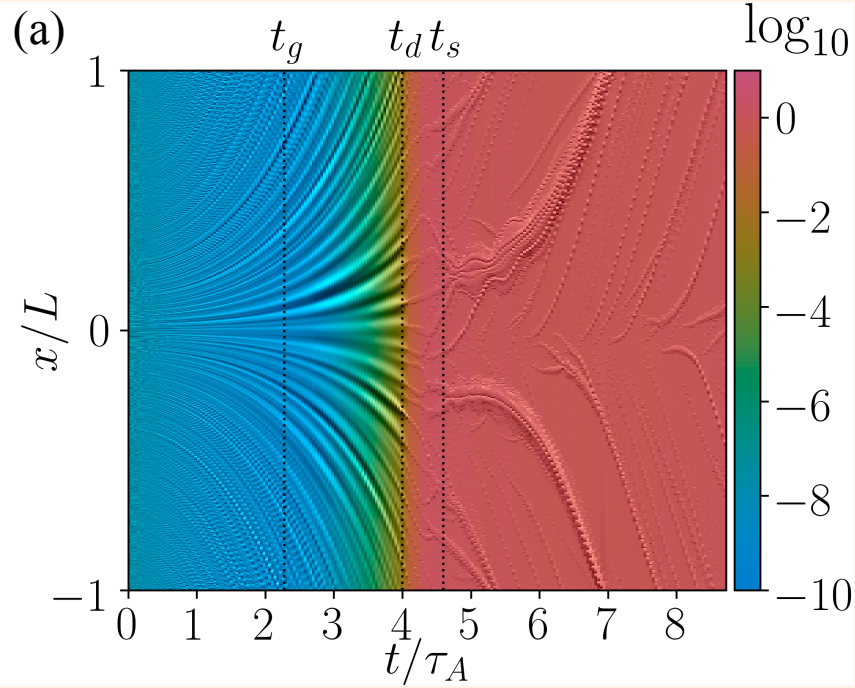
$$B_z(x) = \sum_{n=0}^{m} a_n T_n(x/L) + \tilde{B}_z(x)$$

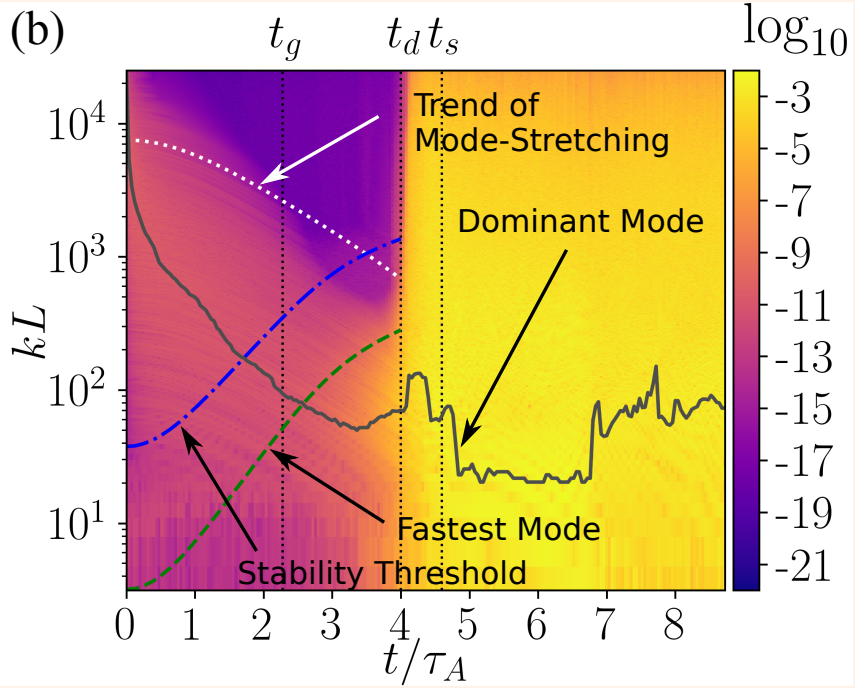
- The degree of polynomial is determined adaptively by increasing the number of Chebyshev basis functions $T_n(x/L)$ until the fluctuation part stabilizes.

Evolution of Fluctuations, $S = 2.5e6$

Real Space

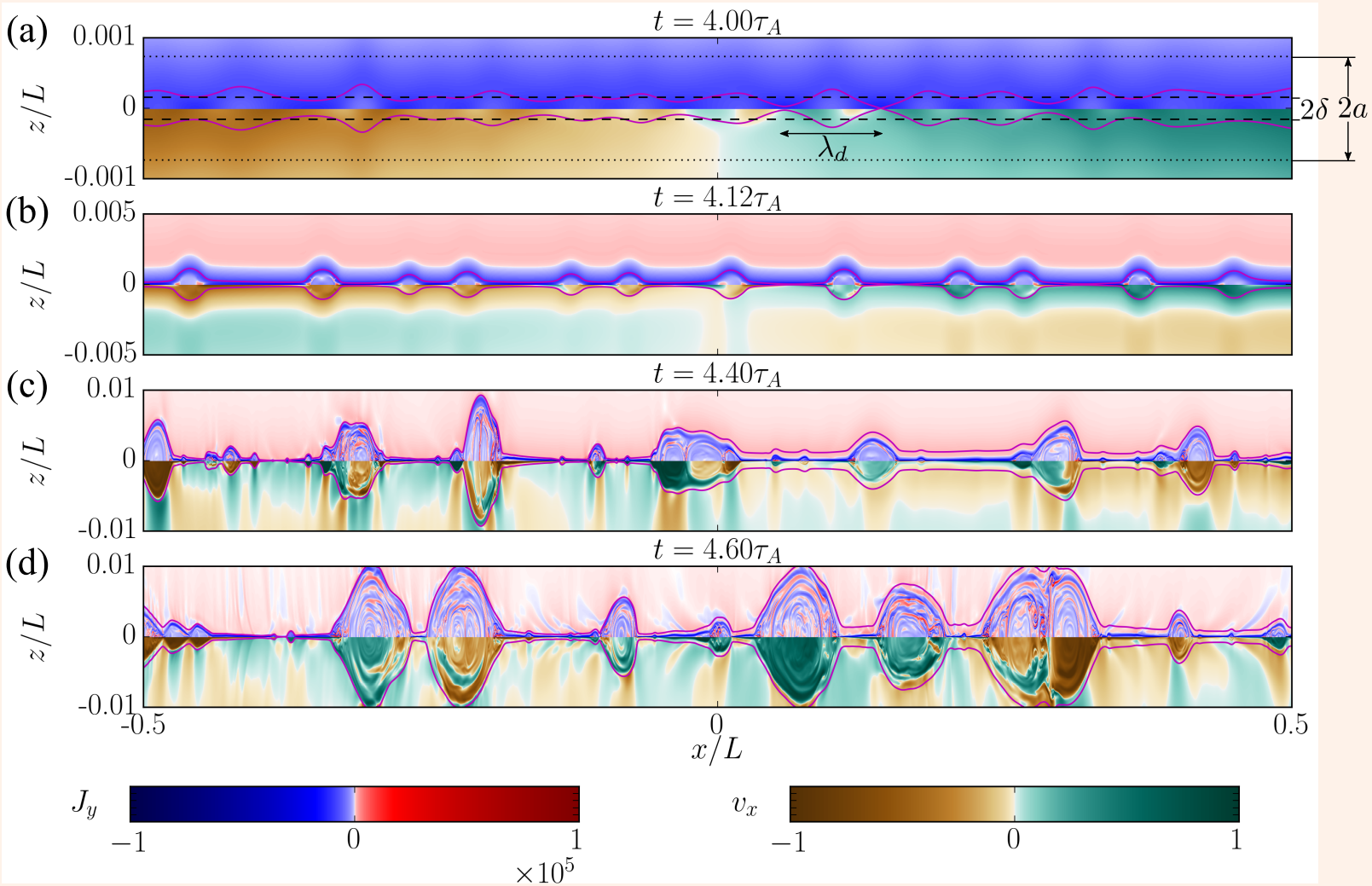
Fourier Space



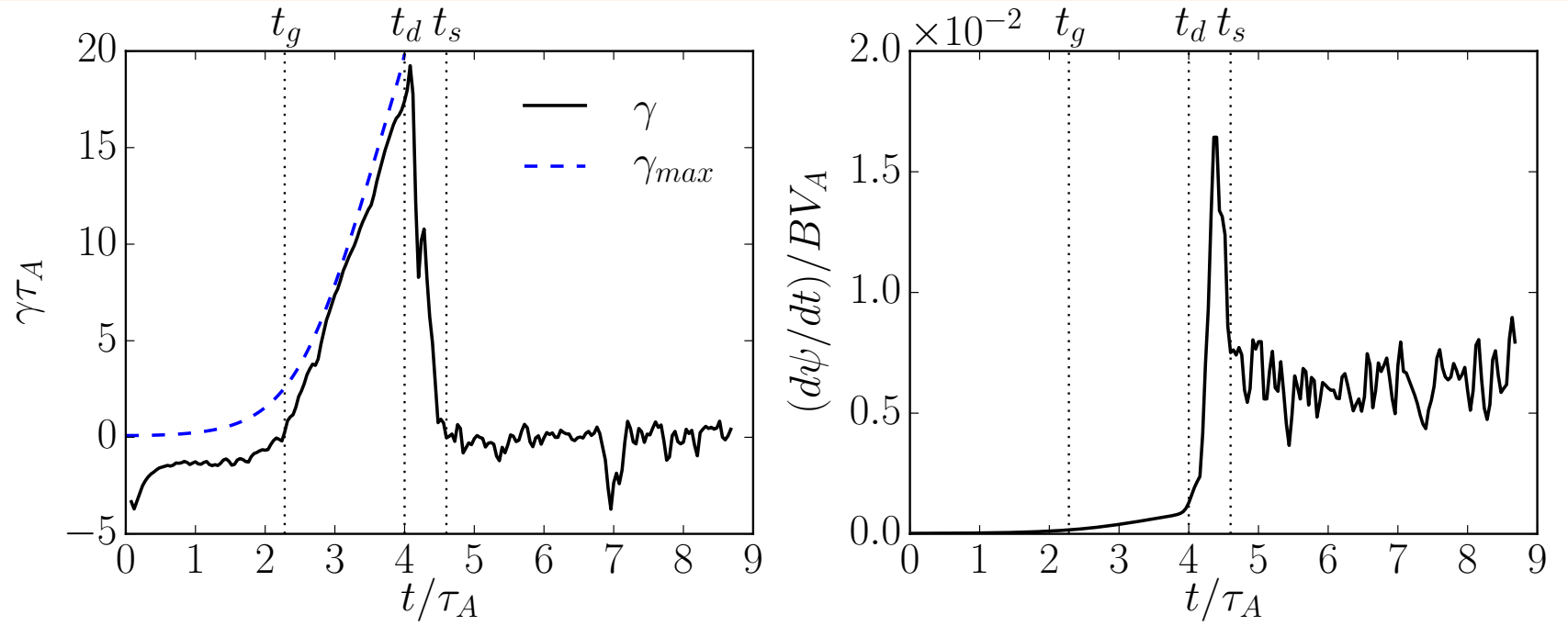


- Fluctuations are stretched along the x direction by the outflow jets: $dk/dt = -kv'_x$
- t_g: amplitude starts to grow; t_d: disruption; t_s: saturation

Snapshots from Disruption to Saturation

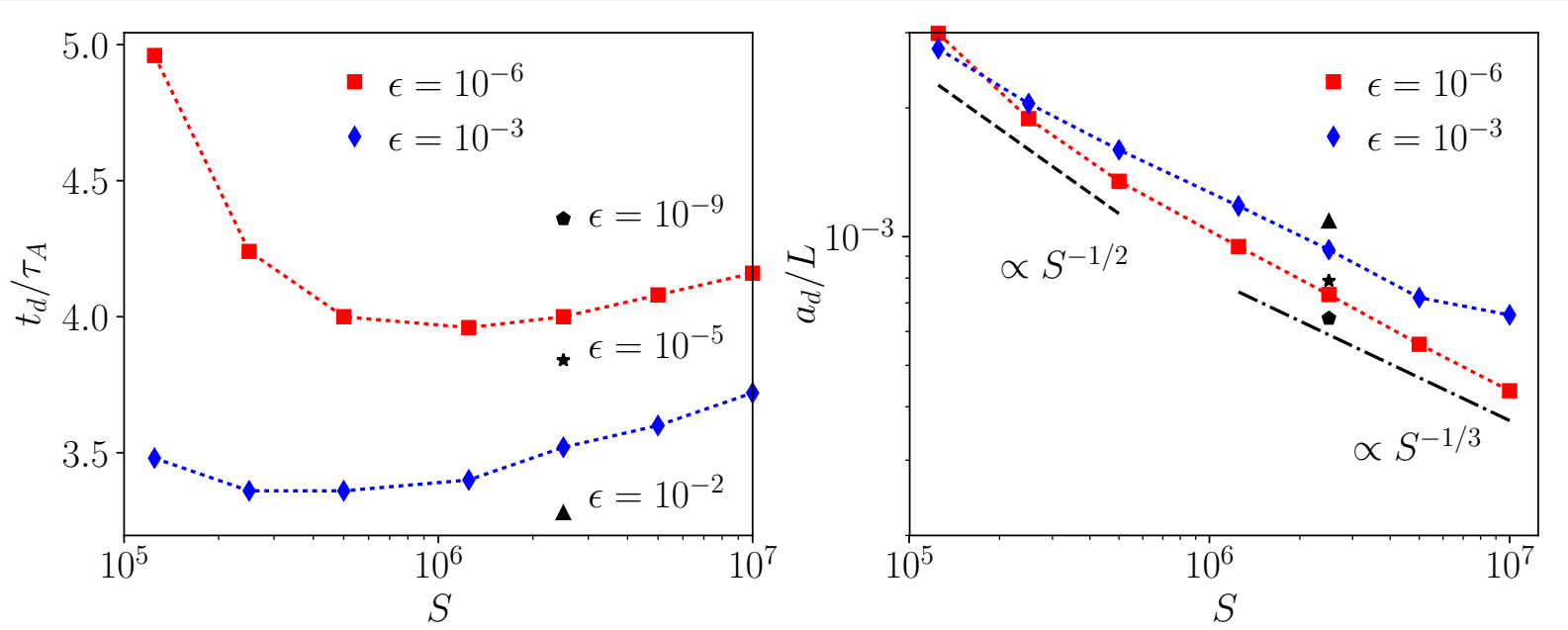


Time History of Growth Rate and Reconnection Rate



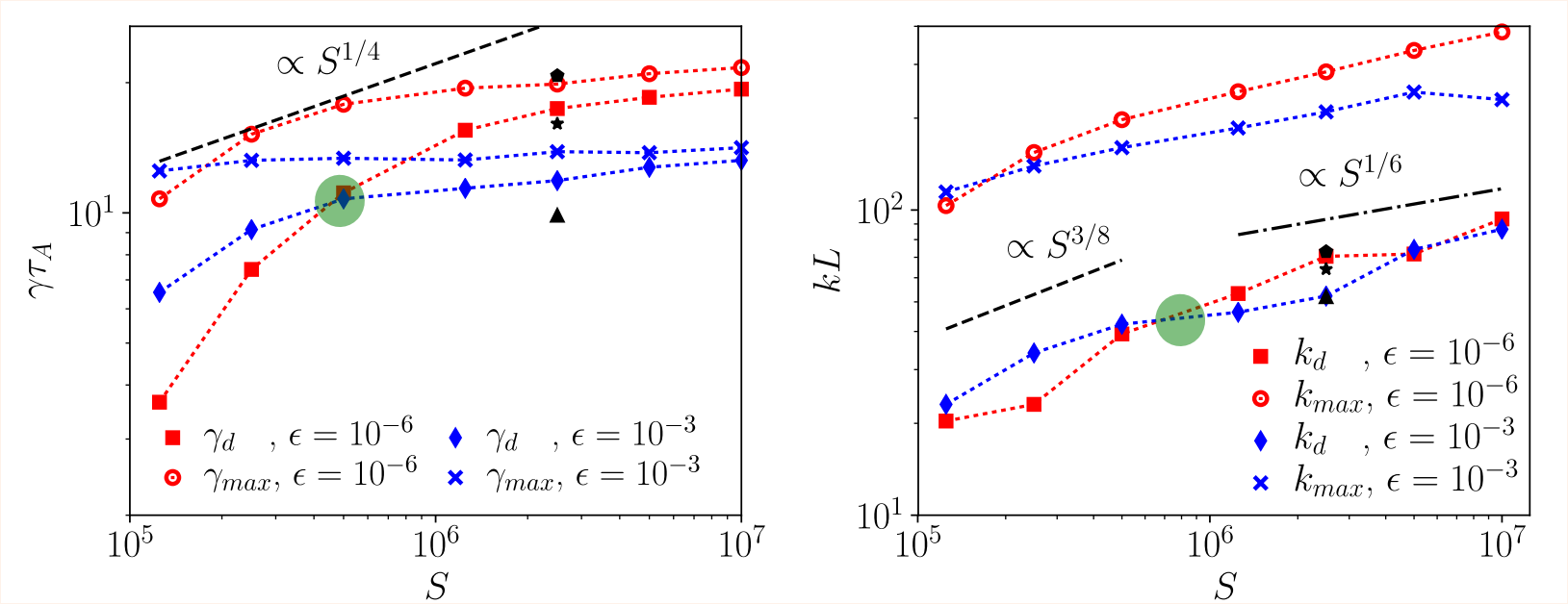
- The "total" perturbation amplitude $||\tilde{B}_z|| \equiv \left(\int_{-L}^{L} \tilde{B}_z^2 dx\right)^{1/2}$ typically starts to grow when $\gamma\tau_A \simeq O(1)$, and $\gamma\tau_A \gg 1$ at the disruption
- Onset of fast reconnection at $t = t_d$

Scalings from Simulations: Disruption Time and Width



- Disruption time t_d is a non-monotonic function of S
- Disruption width scales as $S^{-1/2}$ at low S, but is close to $\propto S^{-1/3}$ at high S.
- Dependence on the initial noise level.

Scalings from Simulations: Growth Rate and Dominant Mode



- Scaling of the fastest growth rate γ_{max} is close to $S^{1/4}$ at low S but γ_d is significantly lower than γ_{max}.
- The dependence of γ_d on S flattens at high S.
- **The dominant mode is not the fastest growing mode.** The dominant wavenumber is approximately 3 – 6 times smaller than the fastest growing wavenumber.

A Phenomenological Model

- Mode-stretching by outflow jets

$$\frac{dk}{dt} = -kv_x'$$

- Evolution of the fluctuation spectrum $f(k) \equiv |\hat{B}_z(k)|/B_0 L_0$

$$\frac{df}{dt} = \partial_t f - kv_x' \partial_k f = \left(\gamma(k, a(t)) - \frac{v_x'}{2} + \frac{1}{2L}\frac{dL}{dt} \right) f.$$

Stretching | Linear Growth | Advection Loss | Length Evol.

- Only consider the domain $k \geq \pi/L$.
- Disruption takes place when island size = inner layer width of the dominant mode

Solving the Model Eq. by Method of Characteristics

Assuming $dL/dt = 0$ and $v'_x = V_A/L$, the equation becomes

$$\boxed{\frac{df}{dt} = \partial_t f - \frac{k}{\tau_A}\partial_k f = \left(\gamma(k, a(t)) - \frac{1}{2\tau_A}\right) f}$$

Along a characteristic

$$k = k_0 e^{-t/\tau_A},$$

the solution is

$$f(k,t) = f_0(k_0) \exp\left(\int_0^t \gamma(k(t'), a(t'))dt' - \frac{t}{2\tau_A}\right),$$

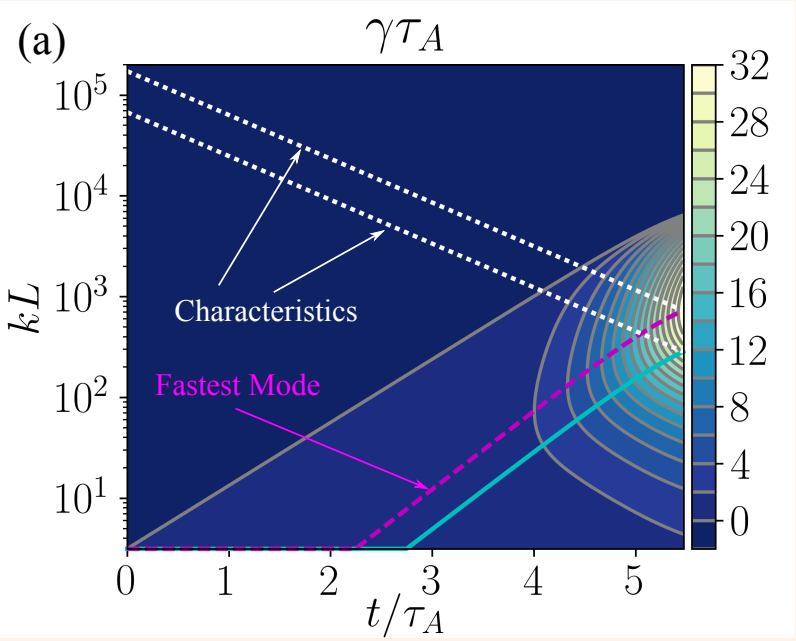
where $f_0(k_0)$ is the initial condition.

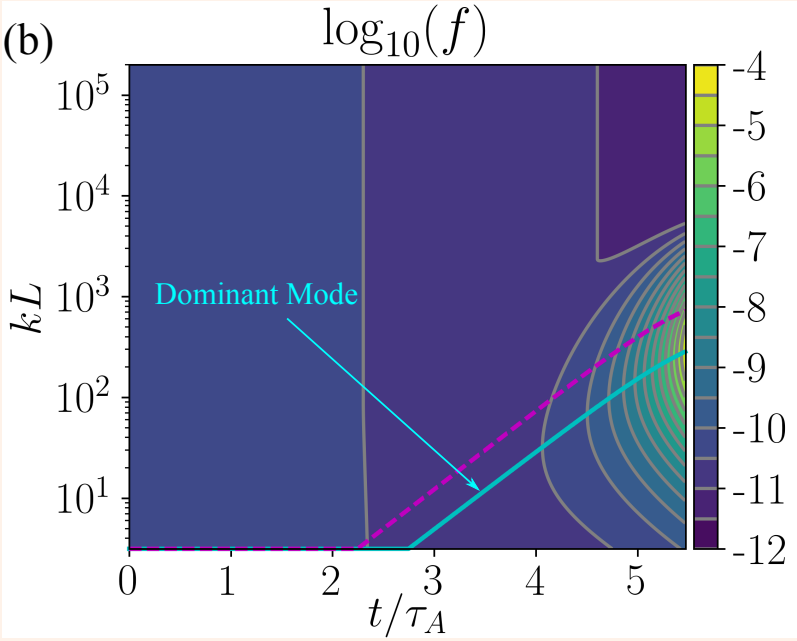
Solution of the Model Eq.

- Current sheet width $a(t)$ is an arbitrary function in the model, which must be determined by global conditions. Here we assume

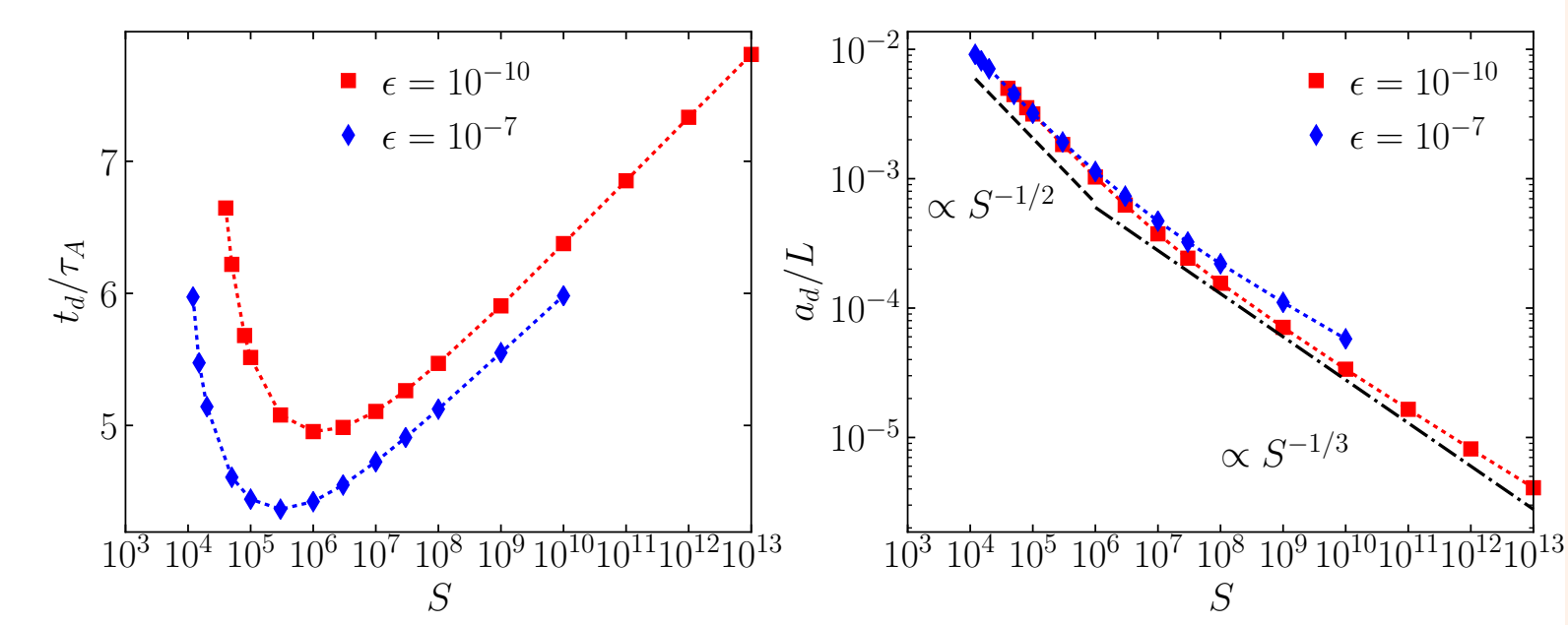
$$a^2 = a_{SP}^2 + (a_0^2 - a_{SP}^2)\exp(-(2/\log 2)(t/\tau_A))$$

- Initial condition $f_0(k_0) = \epsilon$
- The solution for $\epsilon = 10^{-10}$, $S = 10^8$:



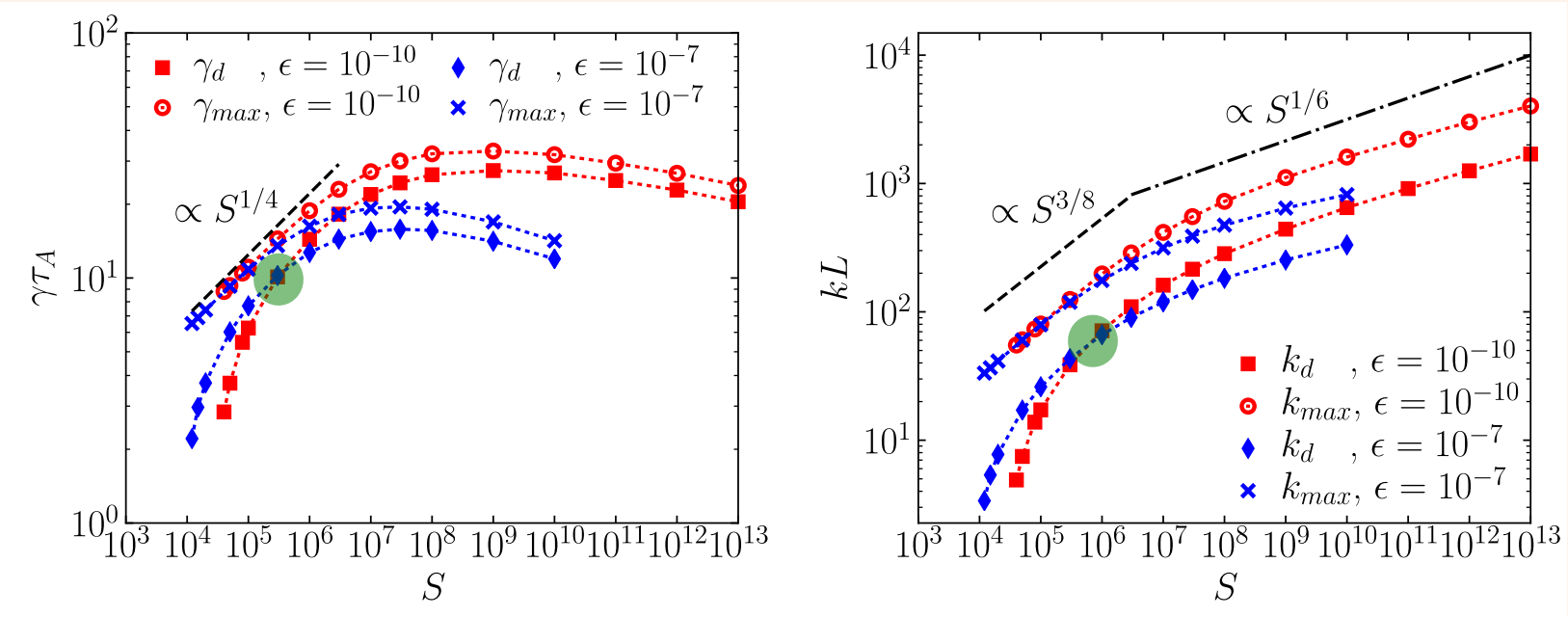


Scalings of Disruption Time and Width



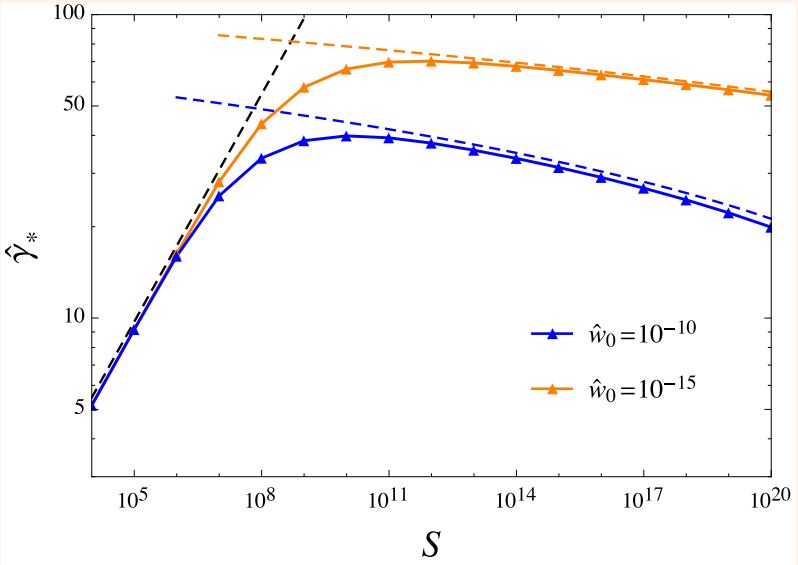
- The model qualitatively reproduces the dependences of t_d and a_d on S and ϵ from simulations.
- The scaling of a_d is close to, not exactly $\propto S^{-1/3}$ at high S.

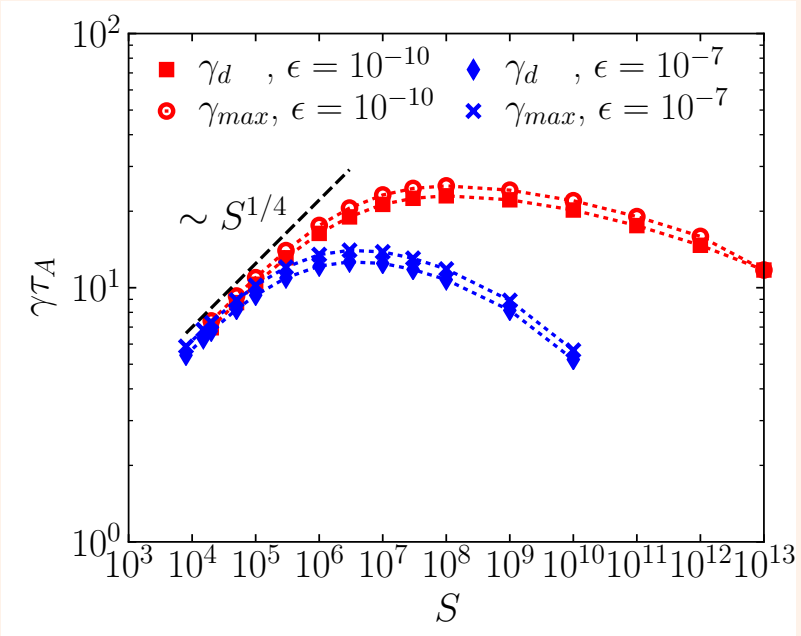
Scalings of Growth Rate and Dominant Mode



- The $\gamma_{max}\tau_A \propto S^{1/4}$ and $k_{max}L \propto S^{3/8}$ scalings are approximately realized at low S. At high S, the scaling of k_dL is close to, but not exactly $\propto S^{1/6}$.
- "Crossing" of curves with different ϵ because the critical S_c depends on ϵ.

With $v'_x \to 0$, Scalings are Similar to Comisso et al. 2016, 2017

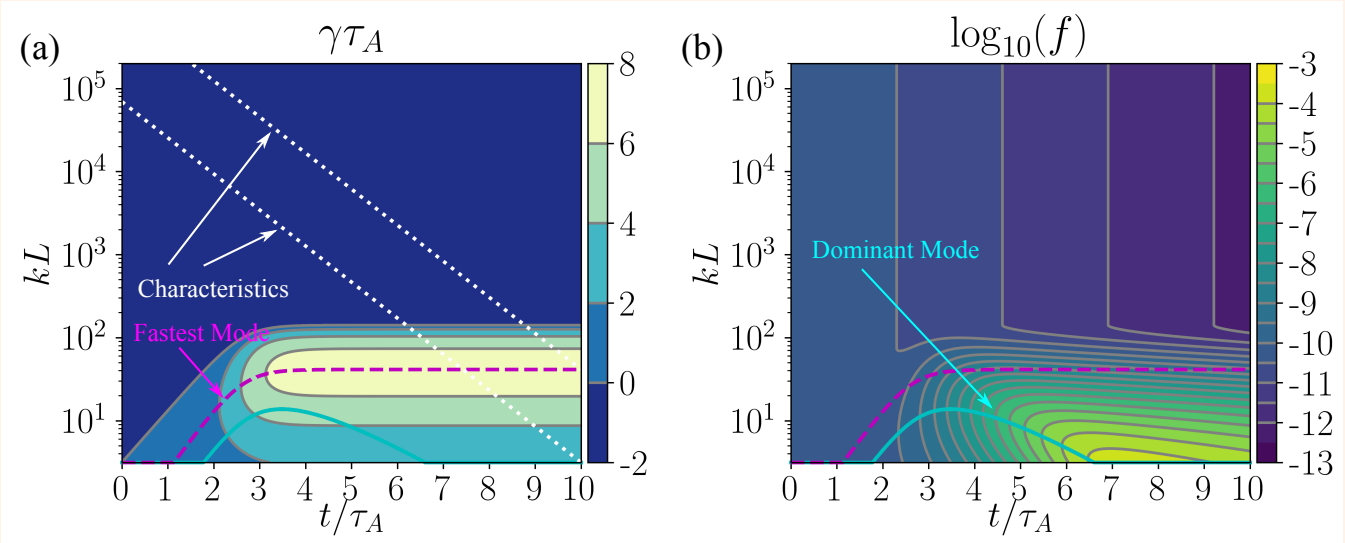




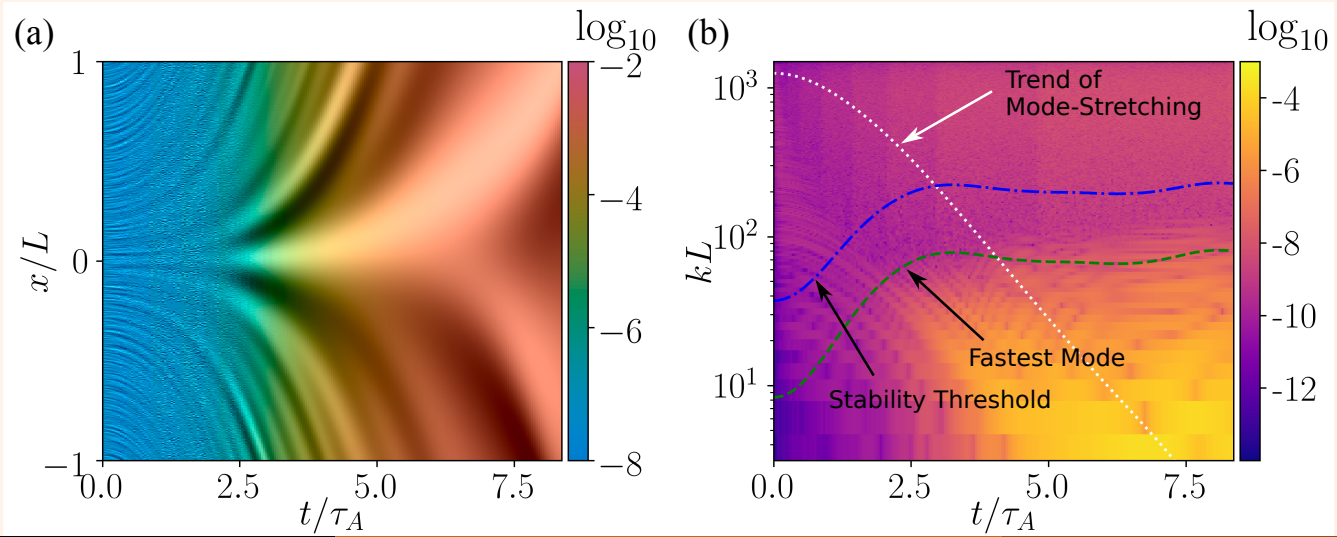
- Qualitative similarities can be made precise by a proper "translation" of the languages.
- The effect of v'_x is more significant in the low-S limit.

Critical Lundquist Number S_c

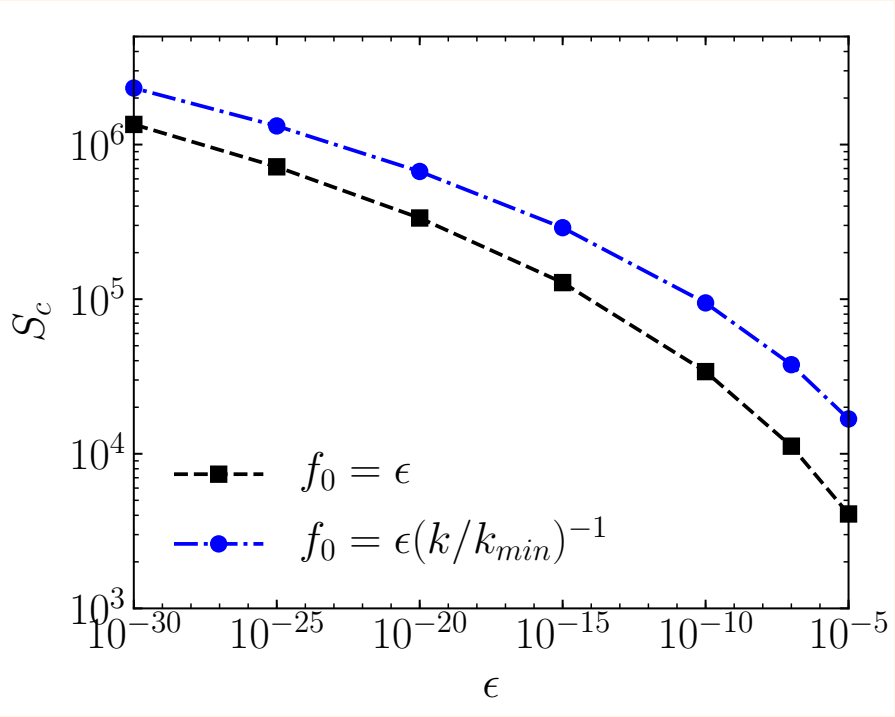
When $S < S_c$, disruption does not occur.



From simulation ($S = 5 \times 10^4$, $\epsilon = 10^{-6}$):

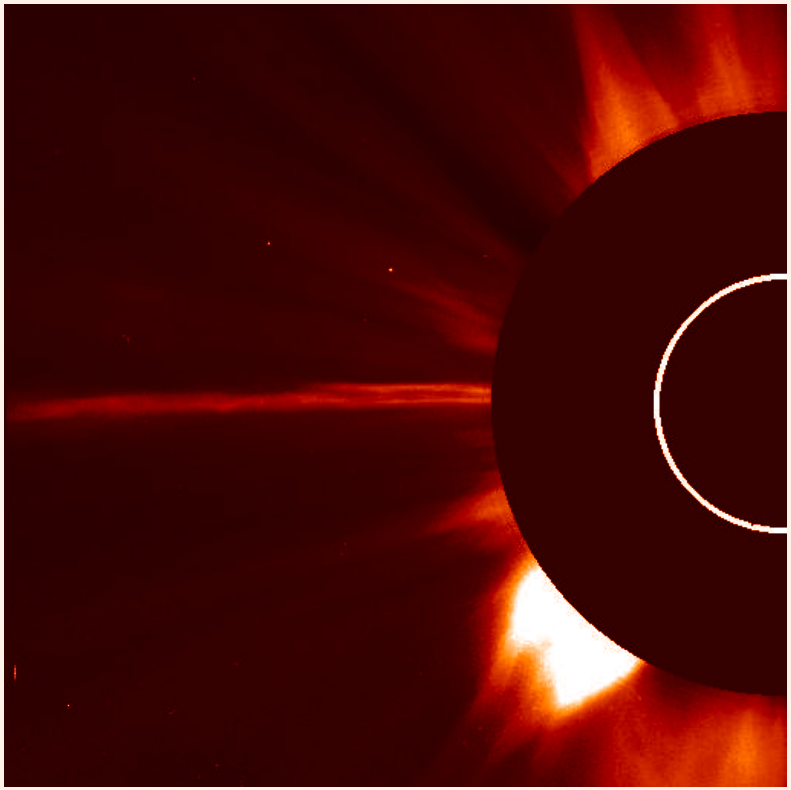


Critical S_c Weakly Depends on Noise



- **Dependence on the noise is weak:** 25 orders of difference in ϵ $\Longrightarrow$ less than 3 orders of difference in S_c.
- The often quoted value of S_c is 10^4, but instability at $S \simeq 10^3$ and stability at $S = 10^5$ have been reported

Application: Post-CME Current Sheet



- $L \simeq 3 \times 10^{11}\,\mathrm{cm}$
- $V_A \simeq 2 \times 10^{7}\,\mathrm{cm/s}$
- $T \simeq 10^{6}\,\mathrm{K}$
- $n = 10^{10}\,\mathrm{cm}^{-3}$
- $S = 3 \times 10^{14}$
- Assume $\gamma_{max}\tau_A \simeq 20$ at disruption — $a_d \simeq 4.5 \times 10^{5}\,\mathrm{cm}$.
- Dominant wavenumber $k_d \simeq k_{max}/4$ — $k_d L \simeq 1500$
- Inner layer width $\delta \simeq 5000\,\mathrm{cm} \gg d_i \simeq 200\mathrm{cm}$ — disruption occurs in MHD regime

Conclusions and Future Perspectives

- Current sheet disruption triggers the onset of fast reconnection.
- The disruption time, growth rate and dominant mode at disruption depend on many factors, e.g. the thinning process, noise level, and the spectrum of the noise, etc.
  - The Sweet-Parker current sheet can form before disruption only at relatively low S — transition of scaling behaviors from low to high S regimes
  - Fluctuations start to grow when $\gamma\tau_A \sim O(1)$; typically $\gamma\tau_A \gg 1$ at disruption.
  - Dominant mode is not the fastest growing mode.
- The critical S_c is not a fixed value, but weakly dependent on noise.
- Future Perspectives:
  - The theoretical framework can be applied to models beyond resistive MHD.
  - Predictions from theories/simulations may be tested with new experiments, e.g. FLARE.