Theory R&R Seminar - March 2018

From Gyrokinetics to MHD

W. W. Lee
PPPL

1. Physics of Plasmas Special Issue on Fusion GPS, Aug. 2017

2. Gyrokinetic MHD and the Associated Equilibria

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PPPL Pioneers

Gyrokinetics
Ed Frieman

Particle Simulation
John Dawson

Over the years, many PPPL researchers have worked and made important contributions in these areas

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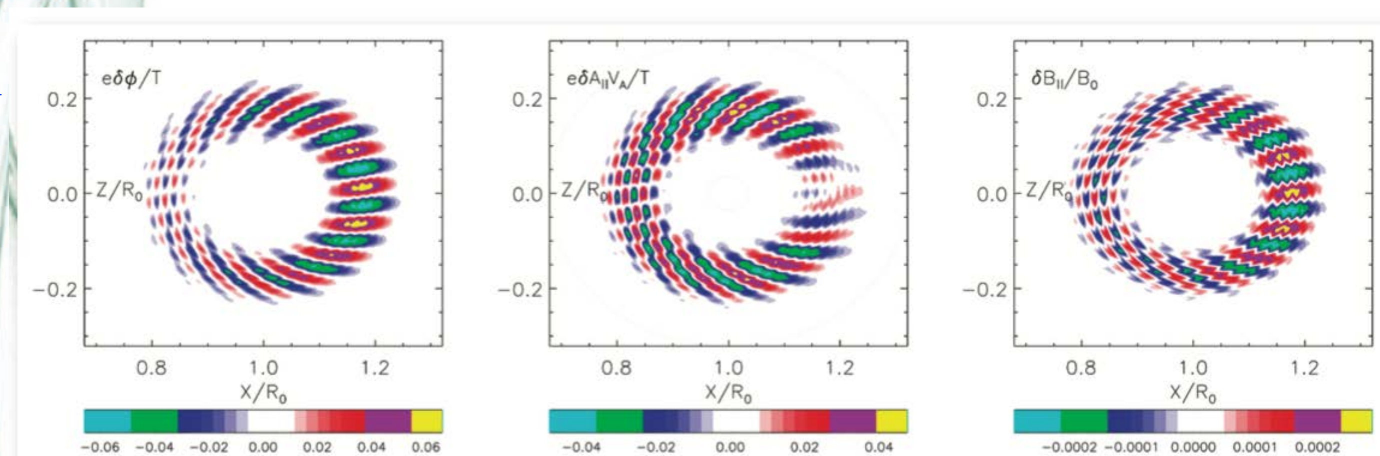
AIP | Physics of Plasmas



SPECIAL TOPIC:
Gyrokinetic Particle Simulation: A Symposium in Honor of Wei-li Lee

Gyrokinetic particle simulations of the effects of compressional magnetic perturbations on drift-Alfvenic instabilities in tokamaks
by G. Dong, J. Bao, A. Bhattacharjee, A. Brizard, Z. Lin, and P. Porazik

Preface to Special Issue Containing Invited Papers Presented at Gyrokinetic Particle Simulation: A Symposium in Honor of Wei-li Lee (University of California, Irvine, July 18–22, 2016)

Z. Lin[a)]
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…..Twenty-eight papers were presented at the symposium to discuss the current status and future directions in the gyrokinetic particle simulation, followed by technical discussions on extending the gyrokinetic simulation to new parameter regimes and for applications including the key area of energetic particle dynamics under burning plasma conditions. These papers provide a snapshot of a vibrant and fruitful research area of gyrokinetic simulation that is effectively leveraging the remarkable advances in physics models, numerical methods, and computing power to improve our understanding of instability, turbulence, and transport in magnetized plasmas……

Variational principle for the parallel-symplectic representation of electromagnetic gyrokinetic theory

Alain J. Brizard
Department of Physics, Saint Michael' s College, Colchester, Vermont 05439, USA

….. In the full-f formulation of the gyrokinetic Vlasov-Maxwell theory presented here, the gyrocenter parallel Ampere equation contains a second-order contribution to the gyrocenter current density that is derived from the second-order gyrocenter ponderomotive Hamiltonian…….

Parallel ion compressibility effects on kinetic ballooning mode for different magnetic shears

Y. Li and Y. Xiao[a)]
Institute for Fusion Theory and Simulation, Department of Physics, Zhejiang University, Hangzhou 310027, People's Republic of China

….. In this work, the gyrokinetic code GTC is employed to investigate the KBM's sensitivity to equilibrium plasma profiles. An outward radial shift of the radial mode is found for the normal magnetic shear case, but there is no shift if the shear is negative. The simulation results are explained by a (local) linear eigenmode theory. It is found that the observed phenomenon is an effect of the parallel ion compressibility.

Comprehensive magnetohydrodynamic hybrid simulations of fast ion driven instabilities in a Large Helical Device experiment

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Alfven eigenmodes (AEs) destabilized by the neutral beam injection (NBI) in a Large Helical Device experiment are investigated using multi-phase magnetohydrodynamic (MHD) hybrid simulation, which is a combination of classical and MHD hybrid simulations for fast ions….. We use the MEGA code,[19] in which the bulk plasma is described by the nonlinear MHD equations and the energetic ions are simulated with the gyrokinetic particle-in-cell (PIC) method. Several hybrid simulation models have been constructed[19–25] to study the evolution of Alfven eigenmodes destabilized by energetic particles.

Equilibrium potential well due to finite Larmor radius effects at the tokamak edge

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We present a novel mechanism for producing an equilibrium potential well near the edge of a tokamak. Briefly, because of the difference in gyroradii between electrons and ions, an equilibrium electrostatic potential is generated in the presence of spatial inhomogeneity of the background plasma, which, in turn, produces a well associated with the radial electric field, E_r, as observed at the edge of many tokamak experiments. We will show that this theoretically predicted E_r field,….., agrees well with recent experimental measurements……

Gyrokinetic particle simulations of the effects of compressional magnetic perturbations on drift-Alfvenic instabilities in tokamaks

Ge Dong,[1] Jian Bao,[2] Amitava Bhattacharjee,[1] Alain Brizard,[3] Zhihong Lin,[2,a)] and Peter Porazik[1]
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The compressional component of magnetic perturbation $\delta B_\parallel$ can play an important role in drift-Alfvenic instabilities in tokamaks, especially as the plasma β increases. In this work, we have formulated a gyrokinetic particle simulation model incorporating $\delta B_\parallel$, and verified the model in kinetic Alfven wave simulations using the GTC code in slab geometry. Simulations of drift-Alfvenic instabilities in tokamak geometry shows that the kinetic ballooning mode (KBM) growth rate decreases more than 20% when $\delta B_\parallel$ is neglected for β_e = 0.02, and that $\delta B_\parallel$ has stabilizing effects on the ion temperature gradient instability, but negligible effects on the collisionless trapped electron mode. The KBM growth rate decreases about 15% when equilibrium current is neglected.

Mitigation of the cancellation problem in the gyrokinetic particle-in-cell simulations of global electromagnetic modes

Alexey Mishchenko,[1,a)] Alberto Bottino,[2] Roman Hatzky,[2] Eric Sonnendrücker,[2] Ralf Kleiber,[1] and Axel Könies[1]
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Electromagnetic gyrokinetic particle-in-cell simulations have been inhibited for long time by numerical problems. This paper discusses the origin of these problems. It also gives an overview and summary of the mitigation techniques.

A conservative scheme of drift kinetic electrons for gyrokinetic simulation of kinetic-MHD processes in toroidal plasmas

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A conservative scheme of drift kinetic electrons for gyrokinetic simulations of kinetic-MHD processes in toroidal plasmas has been formulated and verified. Both vector potential and electron perturbed distribution function are decomposed into adiabatic part with analytic solution and non-adiabatic part solved numerically. The adiabatic parallel electric field is solved directly from the electron adiabatic response, resulting in a high degree of accuracy. The consistency between electrostatic potential and parallel vector potential is enforced by using the electron continuity equation. Since particles are only used to calculate the non-adiabatic response, which is used to calculate the non-adiabatic vector potential through Ohm's law, the conservative scheme minimizes the electron particle noise and mitigates the cancellation problem. Linear dispersion relations of the kinetic Alfven wave and the collisionless tearing mode in cylindrical geometry have been verified in gyrokinetic toroidal code simulations, which show that the perpendicular grid size can be larger than the electron collisionless skin depth when the mode wavelength is longer than the electron skin depth.

Low frequency fully kinetic simulation of the toroidal ion temperature gradient instability

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A fully kinetic ion model is useful for the verification of gyrokinetic turbulence simulations in certain regimes, where the gyrokinetic model may break down due to the lack of small ordering parameters. Here, a fully kinetic ion model is formulated with weak gradient drive terms and applied to the toroidal ion-temperature-gradient (ITG) instability for the first time. Implementation in toroidal geometry is discussed, where orthogonal coordinates are used for particle dynamics, but field-line-following coordinates are used for the field equation allowing for high resolution of the field-aligned mode structure. Variational methods are formulated for integrating the equation of motion allowing for accuracy at a modest time-step size. Linear results are reported for both the slab and toroidal ITG instabilities. Good agreement with full Vlasov and gyrokinetic theory is demonstrated in slab geometry. Good agreement with global gyrokinetic simulation is also shown in toroidal geometry.

Gyrokinetic Magnetohydrodynamics and the Associated Equilibria

[W. W. Lee, S. R. Hudson, and C. H. Ma, Phys. Plasmas 24, 124508 (2017)]

The gyrokinetic magnetohydrodynamics (MHD) equations, related to the recent paper [1] and their associated equilibria properties are discussed. This set of equations is consisted of the time-dependent gyrokinetic vorticity equation, the gyrokinetic parallel Ohm's law, and the gyrokinetic Ampere's law as well as the equations of state, which are expressed in terms of the electrostatic potential, Φ, and the vector potential, A, and support both spatially varying perpendicular and parallel pressure gradients and the associated currents. The corresponding gyrokinetic MHD equilibria can be reached when $\Phi = 0$ and A becomes constant in time, which, in turn, gives $\nabla\cdot(\mathbf{J}_\perp+\mathbf{J}_\parallel) = 0$ and the associated magnetic islands, if they exist. Examples in simple cylindrical geometry are given. These gyrokinetic MHD equations look quite different from the conventional MHD equations and their comparisons will be an interesting topic in the future.

[1] W. W. Lee, "Magnetohydrodynamics for collisionless plasmas from the gyrokinetic perspective," Phys. Plasmas **23**, 070705 (2016)].

Gyrokinetic MHD

- Fully Electromagnetic Gyrokinetic Vlasov Equation:

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\parallel \mathbf{b} - \frac{c}{B_0} \nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \times \hat{\mathbf{b}}_0 \right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m} \left[\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \cdot \mathbf{b} + \frac{1}{c}\frac{\partial \overline{A}_\parallel}{\partial t} \right] \frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

- Associated Gyrokinetic Field Equations:

$$\nabla^2 \phi + \frac{\omega_{pi}^2}{\Omega_i^2} \nabla_\perp^2 \phi = -4\pi \sum_\alpha q_\alpha \int \overline{F}_\alpha dv_\parallel d\mu$$ -- for $k_\perp^2 \rho_i^2 \ll 1$

$$\nabla^2 \mathbf{A} - \underline{\frac{1}{v_A^2} \frac{\partial \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c} \sum_\alpha q_\alpha \int \mathbf{v}\overline{F}_\alpha dv_\parallel d\mu$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$\mu_B \equiv \mu/B \approx const.$ $\quad \mu = v_\perp^2/2$

$$\mathbf{v}_p^L = -(mc^2/eB^2)(\partial \nabla_\perp \phi/\partial t)$$

$$\mathbf{v}_p^T = -(mc/eB^2)(\partial^2 \mathbf{A}_\perp/\partial^2 t)$$

- Energy Conservation: $\quad \Phi \equiv \phi - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c$

$$\frac{d}{dt} \left\langle \int (\frac{1}{2}v_\parallel^2 + \mu)(m_e F_e + m_i F_i) dv_\parallel d\mu + \frac{\omega_{ci}^2}{\Omega_i^2} \frac{|\nabla_\perp \Phi|^2}{8\pi} + \frac{|\nabla A_\parallel|^2}{8\pi} \right\rangle_{\mathbf{x}} = 0$$

- Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic and Lin, PoP (2010); Startsev et al. APS (2015)]

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt}\frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times \nabla lnB_0 - \frac{c}{B_0}\nabla\bar{\Phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2}\mathbf{b}^* \cdot \nabla lnB_0 - \frac{q_\alpha}{m_\alpha}\left(\mathbf{b}^* \cdot \nabla\bar{\Phi} + \underline{\frac{1}{c}\frac{\partial \bar{A}_\parallel}{\partial t}}\right)$$

Startsev et al.

$$\Omega_{\alpha 0} \equiv q_\alpha B_0/m_\alpha c$$

$$\bar{\Phi} \equiv \bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c \qquad \underline{\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} = -\frac{1}{2\pi}\frac{eB_0}{mc}\int_0^{2\pi}\int_0^\rho \delta B_\parallel r dr d\theta}$$

Porazic and Lin

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla)\hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

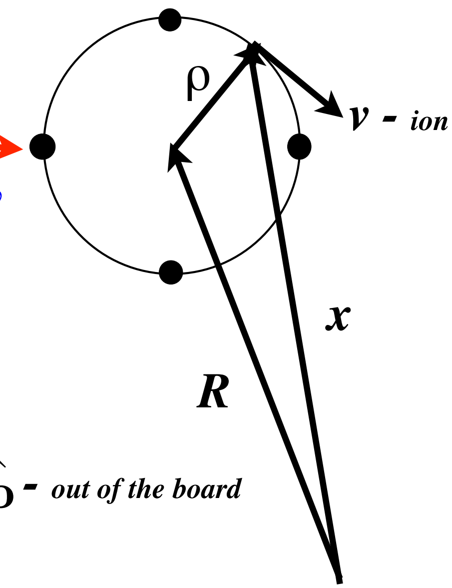
$$F_\alpha = \sum_{j=1}^{N_\alpha} \delta(\mathbf{R} - \mathbf{R}_{\alpha j})\delta(\mu - \mu_{\alpha j})\delta(v_\parallel - v_{\parallel \alpha j})$$

Gyrokinetic Current Densities & Pressure Balance

[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \boxed{\mathbf{J}^{M}_{\perp gc}(\mathbf{x})} + \mathbf{J}^{d}_{\perp gc}(\mathbf{x}) + \mathbf{J}^{E\times B}_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d + \mathbf{v}_{E\times B})\delta(\mathbf{R} - \mathbf{x} + \rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi$$



FLR calculation

$$\mathbf{v}_d = \frac{v_\parallel^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$

$$\mathbf{J}^{M}_{\perp gc}(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp} \qquad p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2) F_{\alpha gc}(\mathbf{x}) dv_\parallel d\mu$$

$\hat{\mathrm{b}}$ - *out of the board*

$$\mathbf{J}^{d}_{\perp gc} = \frac{c}{B}\sum_\alpha \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB)\right] \qquad p_{\alpha\parallel} = m_\alpha \int v_\parallel^2 F_{\alpha gc}(\mathbf{x}) dv_\parallel d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^{M}_{\perp gc} + \mathbf{J}^{d}_{\perp gc} = \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp\right]$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla_\perp p_{\alpha\perp}} \qquad \text{for} \qquad (\nabla \times \hat{\mathbf{b}})_\perp \approx 0$$

For externally imposed potential:

$$F = F_m(\mathbf{v}) exp(\frac{q_\alpha \phi}{T_\alpha}) \longrightarrow \boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times (\nabla_\perp p_{\alpha\perp} - n_\alpha q_\alpha \nabla_\perp \phi)}$$

• Gyrokinetic MHD Equations: a reduced set of equations but in full toroidal geometry

-- For $k_\perp^2 \rho_i^2 \ll 1$ $\quad \bar{F} \to F \quad \bar{\phi} \to \phi \quad \bar{A}_\parallel \to A_\parallel \quad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \to 0$

-- Ampere's law

$$\boxed{\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} J_\parallel}$$

$$\boxed{\nabla_\perp^2 \mathbf{A}_\perp - \cancel{\frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\mathbf{J}_\perp}$$

Negligible if $\omega^2 \ll k_\perp^2 v_A^2$

$$\delta \mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Perpendicular Current due to the ions:

$$\boxed{\mathbf{J}_\perp = \frac{c}{B}\mathbf{b} \times \nabla_\perp p_\perp} \qquad p_\perp \equiv p_{\perp i} + p_{\perp e}$$

--Vorticity Equation:

$$\boxed{\frac{d}{dt}\nabla_\perp^2 \phi - 4\pi \frac{v_A^2}{c^2} \nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0} \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla$$

-- Ohm's law:

$$\boxed{E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla \phi \approx -\frac{1}{en_e}\frac{\partial p_{\parallel e}}{\partial x_\parallel} + \eta J_\parallel}$$

-- Equations of State:

$$\boxed{\frac{dp_\perp}{dt} = 0 \qquad \frac{dp_{\parallel e}}{dt} = 2E_\parallel J_\parallel}$$

— 3rd order (and higher) velocity moments are ignored, otherwise one needs to include toroidal effects

-- Normal modes: $\quad \omega = \pm k_\parallel v_A \quad$ for $\quad E_\parallel \approx 0 \quad$ and $\quad \mathbf{J}_\perp \approx 0$

• MHD Equilibrium

1. For a given perpendicular pressure profile and a given $\boldsymbol{B}$, we should obtain the following

$$\boxed{\nabla\cdot\mathbf{J}_\perp=\frac{c}{B}\nabla_\perp p_\perp\cdot\nabla\times\mathbf{b}=\frac{c}{B}\nabla_\perp p_\perp\times\mathbf{b}\cdot\nabla lnB}$$

2. We then solve the equations of the form

$$\boxed{\begin{aligned}&\frac{d}{dt}\nabla_\perp^2\phi+\frac{v_A^2}{c}(\mathbf{b}\cdot\nabla)\nabla_\perp^2A_\parallel-4\pi\frac{v_A^2}{c^2}\nabla\cdot\mathbf{J}_\perp=0\\&\frac{1}{c}\frac{\partial A_\parallel}{\partial t}+\mathbf{b}\cdot\nabla\phi\approx\frac{1}{en_e}\frac{\partial p_{\parallel e}}{\partial x_\parallel}+\eta\frac{c}{4\pi}\nabla_\perp^2A_\parallel\end{aligned}}$$

$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}-\frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

$$\frac{d}{dt}\nabla_\perp^2\phi-4\pi\frac{v_A^2}{c^2}\nabla\cdot(\mathbf{J}_\parallel+\mathbf{J}_\perp)=0$$

$$J_\parallel\approx\frac{c}{4\pi}\nabla_\perp^2A_\parallel$$

3. for a given parallel pressure profile $p_{\parallel e}$ and look for a solution for

$$\phi\rightarrow 0\qquad\text{and}\qquad E_\parallel\rightarrow 0\ \text{ i.e. }\ \frac{\partial A_\parallel}{\partial t}\rightarrow 0$$

to obtain the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla\cdot(\mathbf{J}_\perp+\mathbf{J}_\parallel)\approx 0}$$

Gyrokinetic MHD Equilibria

From
$$\begin{cases} \mathbf{J}_\perp = \dfrac{c}{B}\mathbf{b}\times\nabla_\perp p_\perp \\ \eta\mathbf{J}_\parallel = \dfrac{1}{en}\nabla_\parallel p_{\parallel e} \end{cases}$$

$$p_\perp = n(T_i + T_e)$$
$$p_{\parallel e} = nT_e$$
$$\mathbf{b} \equiv \mathbf{B}/B$$

or
$$\begin{cases} \dfrac{\mathbf{J}_\perp}{enc_s} \approx \mathbf{b}\times\dfrac{\rho_s\nabla p_\perp}{nT_e} \\ \dfrac{\nu}{\Omega_e}\dfrac{\mathbf{J}_\parallel}{enc_s} = \dfrac{\mathbf{b}}{p_{\parallel e}}\mathbf{b}\cdot\rho_s\nabla p_{\parallel e} \end{cases}$$
in normalized units

Using Amperes' Law
$$\nabla\times\mathbf{B} = \frac{4\pi}{c}(\mathbf{J}_\parallel + \mathbf{J}_\perp)$$

or
$$\boxed{\nabla\times\bar{\mathbf{B}} = \beta(\bar{\mathbf{J}}_\parallel + \bar{\mathbf{J}}_\perp)}$$
in normalized units

$$\bar{\mathbf{J}} \equiv \frac{\mathbf{J}}{enc_s} \quad \bar{\mathbf{B}} \equiv \frac{\mathbf{B}}{cT_e/ec_s\rho_s} \quad \beta = (\frac{c_s\rho_s}{c\lambda_{De}})^2 \quad \bar{\mathbf{J}}_\perp \approx \mathbf{b}\times\frac{\bar{\nabla}p_\perp}{nT_e} \quad \bar{\nu}\bar{\mathbf{J}}_\parallel = \frac{\mathbf{b}}{p_{\parallel e}}\mathbf{b}\cdot\bar{\nabla}p_{\parallel e} \quad \bar{\nu} \equiv \frac{\nu}{\Omega_e}$$

To obtain
$$\nabla\cdot(\mathbf{J}_\perp + \mathbf{J}_\parallel) \approx 0$$

or
$$\boxed{\bar{\nabla}\cdot(\bar{\mathbf{J}}_\parallel + \bar{\mathbf{J}}_\perp) \approx \frac{1}{p_0}\bar{\nabla}p_\perp\cdot\mathbf{b}\times\frac{\bar{\nabla}\bar{B}}{\bar{B}} + \frac{1}{\bar{\nu}p_0}\frac{\partial}{\partial\bar{x}_\parallel}(\frac{\partial p_e}{\partial\bar{x}_\parallel}) = 0}$$

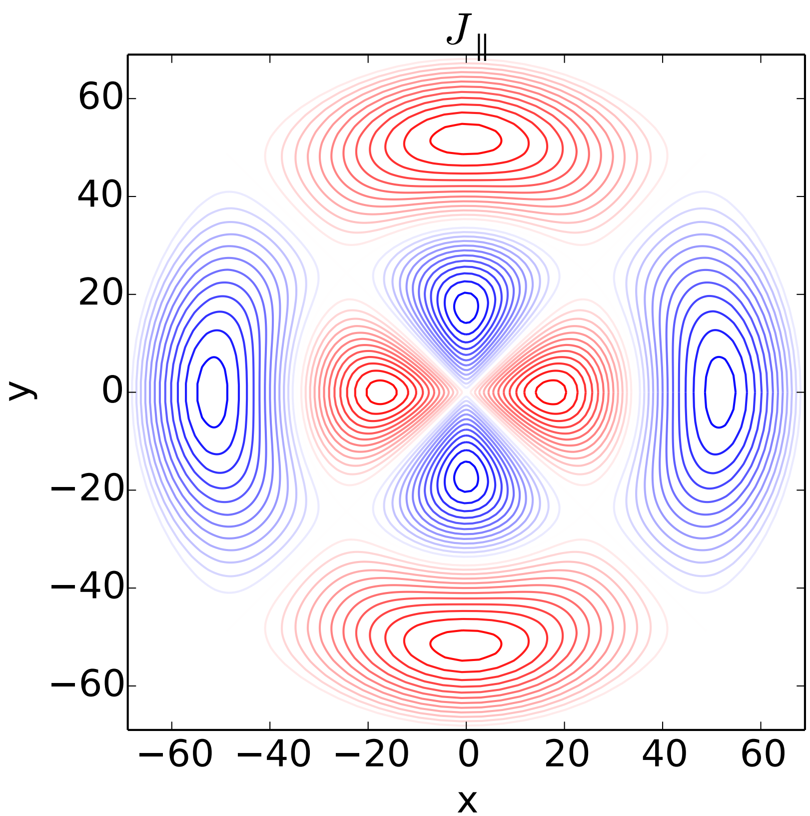
$$p_{\parallel e} = p_0 + p_e$$
$$p_0 \approx nT_e$$

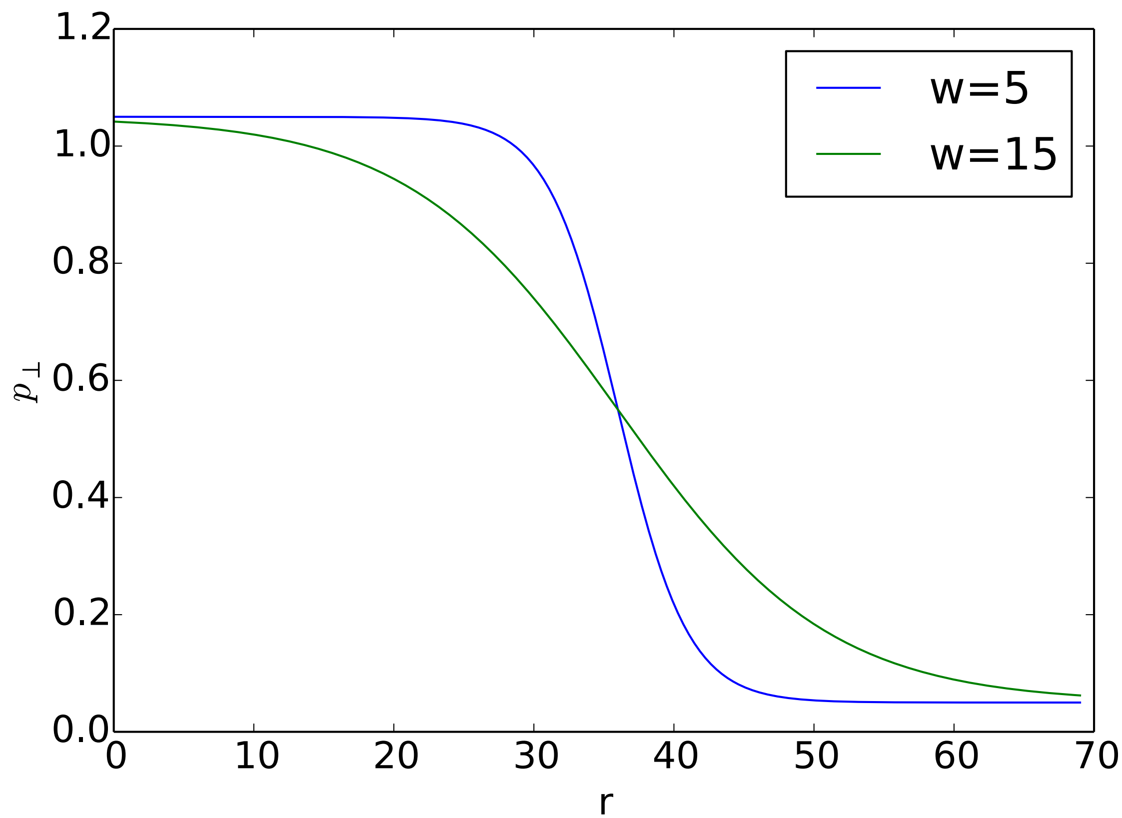
Gyrokinetic MHD Equilibria - cont.

- Governing equations: $\bar{\nabla} \cdot (\bar{\mathbf{J}}_{\|} + \bar{\mathbf{J}}_{\perp}) \approx 0$ and $\nabla \times \bar{\mathbf{B}} = \beta(\bar{\mathbf{J}}_{\|} + \bar{\mathbf{J}}_{\perp})$ — ignorable for low-β

- In cylindrical geometry: $\bar{\mathbf{J}}_{\perp} = \frac{1}{p_0} \frac{\partial p_{\perp}(\bar{r})}{\partial \bar{r}} \mathbf{e}_{\theta}$ $\quad \frac{\partial \delta \bar{B}}{\partial \theta} = \frac{\bar{B}_0}{\bar{\nu}} \left[\frac{\partial}{\partial \bar{x}_{\|}} (\frac{\partial p_e}{\partial \bar{x}_{\|}}) \right] / (\frac{\partial p_{\perp}}{\partial \bar{r}})$

$$(\partial p_e / \partial \bar{x}_{\|}) / p_0 = A \sin(2\pi r/a) \cos m\theta \sin(2\pi x_{\|}/L_{\|}) \qquad \frac{p_{\perp}(r)}{p_0} = \frac{1}{2} - \frac{\tanh[(r - r_0)/w]}{2}.$$

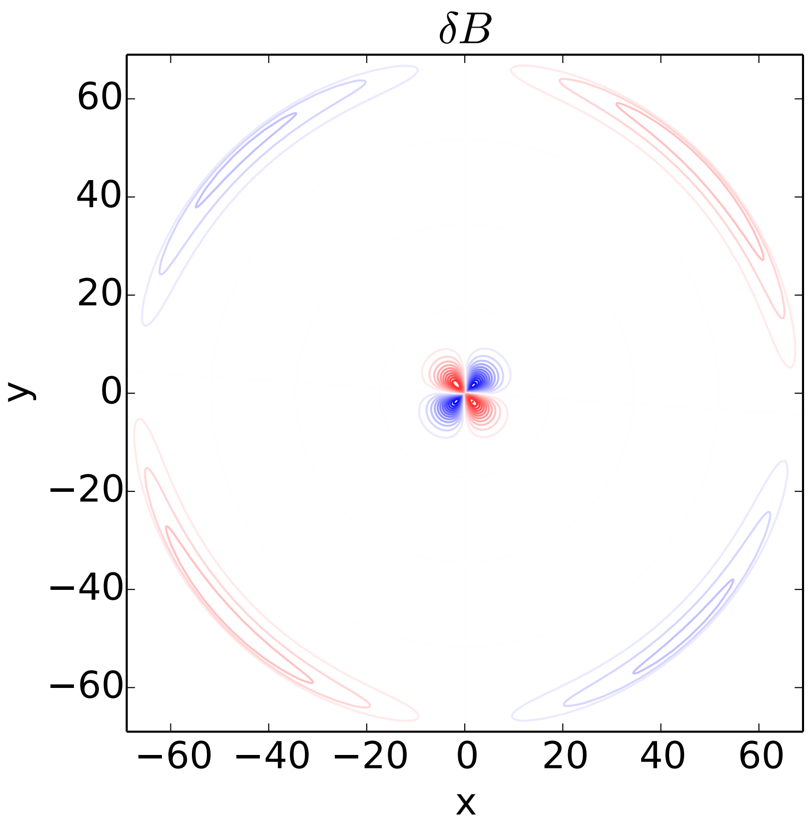




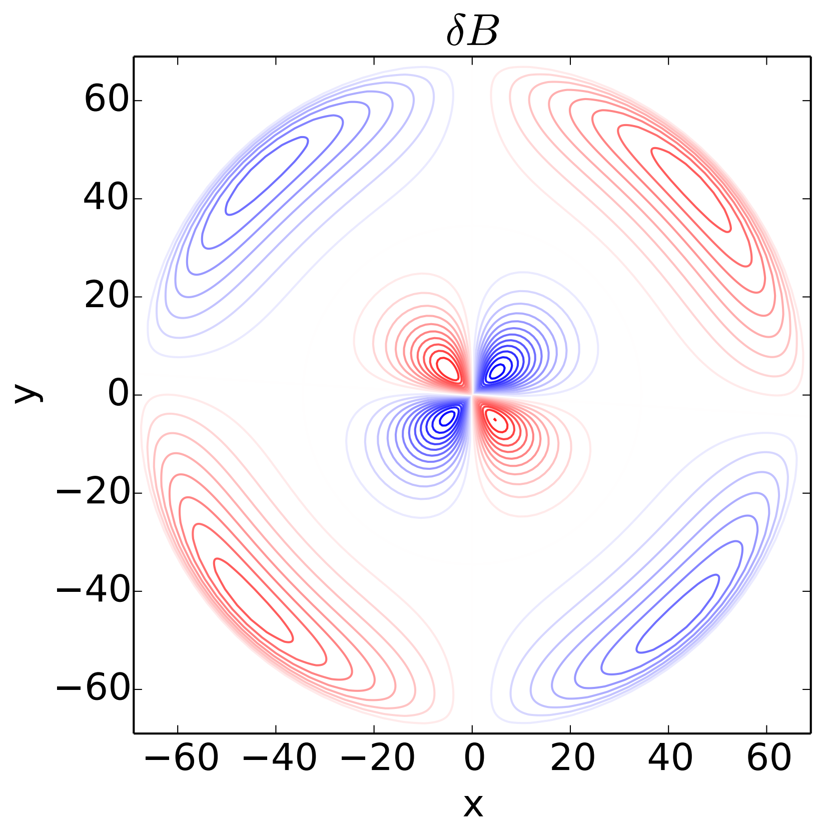
Gyrokinetic MHD Equilibria - cont.

- Magnetic Islands for m = 2, $x_\parallel = L_\parallel/4$, $a = 69$, and $r_0 = 36$

$w = 5$



$w = 15$



Summary

- This set of gyrokinetic equations can indeed be used to study steady state electromagnetic turbulence.

- It can also recover the equilibrium MHD equations in the absence of fluctuations.

- In the presence of spatially varying parallel current, we have observed magnetic islands in the steady state.

- It will be interesting to couple a 3D global EM PIC code, e.g., GTS [Wang et al., 2006] with a 3D MHD equilibrium code, e.g., SPEC [Hudson et al., 2012] for transport minimization purposes.

- In fact, GTS has observed non vanishing parallel current in the steady state.

- Unfortunately, a SciDAC proposal, "First Principles Based Transport and Equilibrium Module for Whole Device Modeling and Optimization," based on these two codes along with the theory presented here was rejected by DoE in 2017.