Investigating magnetic fluctuations in gyrokinetic simulations of tokamak SOL turbulence

Noah Mandell

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Why is SOL turbulence important?

- Plasma properties in the tokamak edge/scrape-off layer (SOL) constrain component lifetime and reactor performance

- Heat exhausted in SOL could damage divertor plates if heat flux width is too narrow
  - Can SOL turbulence broaden the heat flux width?
  - Can electromagnetic effects be important for SOL turbulence?
Modeling the edge/SOL is challenging

- Gyrokinetic (GK) theory and simulation are important first-principles tools for studying turbulence and transport in fusion plasmas, but most present codes optimized for core, small fluctuations (delta-f)

- Edge/SOL more challenging: large-amplitude fluctuations, open field lines, plasma-wall interactions, X-point geometry, atomic physics, transition from kinetic to fluid regimes → need specialized full-f GK codes

- Including electromagnetic effects (allowing magnetic field to fluctuate) also challenging → all GK SOL results to-date have been electrostatic (no magnetic fluctuations)

- Several GK codes making great progress in edge/SOL
  - XGC1, COGENT, ELMFIRE, GENE, Gkeyll, etc

- Essential to have several independent codes to attack from different perspective and cross-check on difficult turbulence problems!
Methods for solving gyrokinetic system

Particle-in-cell (Lagrangian)  Continuum (Eulerian)
Methods for solving gyrokinetic system

Particle-in-cell (Lagrangian)

- Sample phase space with ensemble of $N_p$ ‘superparticles’
- Fields on 3D grid, particles move through grid
- Historically, EM fluctuations challenging in GK PIC codes due to numerical “Ampere cancellation” problem
  - Codes like ORB5 and XGC1 have made good recent progress to mitigate issue

Continuum (Eulerian)
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Continuum (Eulerian)

- Discretize distribution function \( f(x, y, z, v_\parallel, \mu) \) on 5D phase space grid
- Can solve with standard PDE methods, e.g. spectral, finite volume, discontinuous Galerkin, etc.
- Continuum electromagnetic GK codes have mostly avoided the Ampere cancellation problem

Barnes et al, 2010
Methods for solving gyrokinetic system

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Gkeyll

- First successful continuum GK code on open field lines

- First electromagnetic GK on open field lines

https://github.com/ammarhakim/gkyl/
https://gkeyll.readthedocs.io
Full-\( f \) electromagnetic gyrokinetics

EMGK equation, \( f_s = f_s(R, v \parallel, \mu; t) \)

\[
\frac{\partial f_s}{\partial t} + \dot{R} \cdot \nabla f_s + \dot{v}_\parallel \frac{\partial f_s}{\partial v \parallel} = C[f_s] + S_s
\]

with nonlinear phase-space trajectories

\[
\dot{R} = \{R, H_s\} = \frac{B_0^* + \delta B_\perp}{B^*_\parallel} v \parallel + \frac{\hat{b}}{q_s B^*_\parallel} \times (\mu \nabla B + q_s \nabla \phi)
\]

\[
\dot{v}_\parallel = \{v \parallel, H_s\} - \frac{q_s}{m_s} \frac{\partial A \parallel}{\partial t} = -\frac{B_0^* + \delta B_\perp}{m_s B^*_\parallel} \cdot (\mu \nabla B + q_s \nabla \phi) - \frac{q_s}{m_s} \frac{\partial A \parallel}{\partial t}
\]

where \( B_0^* = B_0 + (m_s v \parallel / q_s) \nabla \times \hat{b} \) and \( \delta B_\perp = \nabla A \parallel \times \hat{b} \).

- No assumption of scale separation between background and fluctuations
- Taking long-wavelength (drift-kinetic) limit, neglecting gyroaveraging for now
- Using symplectic \( (v \parallel) \) formulation of EMGK, so \( \frac{\partial A \parallel}{\partial t} \) appears explicitly
Full-$f$ electromagnetic gyrokinetics

Quasineutrality equation (long-wavelength):

$$- \nabla \cdot \sum_s \frac{m_s n_{0s}}{B^2} \nabla_{\perp} \phi = \sum_s q_s \int d^3 v \ f_s$$  \hspace{1cm} (1)

Parallel Ampère equation:

$$- \nabla_{\perp}^2 A_\parallel = \mu_0 \sum_s q_s \int d^3 v \ v_\parallel f_s$$  \hspace{1cm} (2)

Can take $\frac{\partial}{\partial t}$ to get an exact Ohm’s law:

$$- \nabla_{\perp}^2 \frac{\partial A_\parallel}{\partial t} = \mu_0 \sum_s q_s \int d^3 v \ v_\parallel \frac{\partial f_s}{\partial t}$$  \hspace{1cm} (3)

Writing GK eq. as

$$\frac{\partial f_s}{\partial t} = \frac{\partial f_s}{\partial t}^* + \frac{q_s}{m_s} \frac{\partial A_\parallel}{\partial t} \frac{\partial f_s}{\partial v_\parallel}$$  \hspace{1cm} (4)

where $\frac{\partial f_s}{\partial t}^*$ denotes all the terms in the gyrokinetic equation except the $\frac{\partial A_\parallel}{\partial t}$ term, can write Ohm’s law as

$$\left( - \nabla_{\perp}^2 + \sum_s \frac{\mu_0 q_s^2}{m_s} \int d^3 v \ f_s \right) \frac{\partial A_\parallel}{\partial t} = \mu_0 \sum_s q_s \int d^3 v \ v_\parallel \frac{\partial f_s}{\partial t}^*$$  \hspace{1cm} (5)
Ampère cancellation problem

• In $p\|\$ formulation, Ampère’s law:

$$\left(-\nabla^2_\perp + C_n \sum_s \frac{\mu_0 q_s}{m_s} \int d^3 p \ f\right) A_\| = C_j \mu_0 \sum_s \frac{q_s}{m_s^2} \int d^3 p \ p_\| f$$

• “Cancellation problem” arises when there are small errors in the calculation of the integrals, represented by $C_n$ and $C_j$ (which should be exactly 1 in the exact system)

• Recall $v\|$ formulation Ohm’s law... same problem...

$$\left(-\nabla^2_\perp + C_n \sum_s \frac{\mu_0 q_s^2}{m_s} \int d^3 v \ f_s\right) \frac{\partial A_\|}{\partial t} = C_j \mu_0 \sum_s q_s \int d^3 v \ v_\| \frac{\partial f_s}{\partial t}$$

• The simplest Alfvén wave dispersion relation (slab geometry, uniform Maxwellian background with stationary ions) becomes (with $\hat{\beta} \equiv \frac{\beta_i}{2} \frac{m_i}{m_e}$)

$$\omega^2 = \frac{k_\| v^2_A}{C_n + k^2_\perp \rho_s^2 / \hat{\beta}} \left[ 1 + (C_n - C_j) \frac{\hat{\beta}}{k^2_\perp \rho_s^2} \right]$$

• This reduces to the correct result if integrals calculated consistently, so that $C_n = C_j$, but if not there will be errors $\sim \omega_H$ for modes with $\hat{\beta}/k^2_\perp \rho_s^2 \gg 1$.

Gkeyll’s DG scheme computes integrals consistently so that errors cancel exactly (appendix of Mandell et al, JPP 2020 shows numerical dispersion relation calculation)
Linear benchmark: kinetic Alfvén wave

\[ \omega^2 = \frac{k_{||}^2 v_A^2}{C_n + k_{||}^2 \rho_s^2 / \hat{\beta}} \left[ 1 + (C_n - C_j) \frac{\hat{\beta}}{k_{||}^2 \rho_s^2} \right] \]

\[ (k_{\perp} \rho_s \ll 1) \]

\[ \hat{\beta} / k_{\perp}^2 \rho_s^2 = 10^5 \]

Gkeyll results match theory very well, even for case with

\[ \frac{\hat{\beta}}{k_{\perp}^2 \rho_s^2} = \frac{\beta_e / 2}{m_i / k_{\perp}^2 \rho_s^2 m_e} = 10^5 \]

No cancellation problem!
Modeling the NSTX SOL with Gkeyll
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- Open-field-line region only
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- Simplified helical geometry with vertical flux surfaces, const curvature and no shear (no X point)
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- Model flux of heat and particles across separatrix with source
Modeling the NSTX SOL with \texttt{Gkeyll}

- Open-field-line region only
- Simplified helical geometry with vertical flux surfaces, constant curvature and no shear (no X point)
- Model flux of heat and particles across separatrix with source
- Boundary conditions:
  - perfectly conducting walls ($\phi = A_\parallel = 0$) in radial direction, $x$
  - periodic in binormal direction, $y$
  - conducting sheath model BC along field line, $z$
Conducting-sheath boundary condition

- Need to model non-neutral sheath using BCs (GK assumes quasi-neutrality, cannot resolve sheath)

- Sheath potential should reflect low energy electrons

- Solve Poisson equation on $z$ boundary to get $\phi_{sh}(x, y) \doteq \phi(z = z_{sh})$, then use $\Delta \phi = \phi_{sh} - \phi_w$ to reflect electrons with $mv^2/2 < |e| \Delta \phi$

\[-\nabla_\perp \cdot \sum_s \frac{m_s n_{0s}}{B^2} \nabla_\perp \phi(z = z_{sh}) = \sum_s q_s \int d^3v f_s(z = z_{sh})\]

- Potential self-consistently relaxes to ambipolar-parallel-outflow state, and allows local currents in and out of wall (unlike “logical” sheath model)
Sheath boundary condition for electrons

(a) Outgoing electrons with $v_\parallel > v_{cut} = \sqrt{2e\Delta \phi/m}$ are lost into the wall

(b) Rest of outgoing electrons $0 < v_\parallel < v_{cut}$ are reflected back into plasma

Ions: Assuming positive sheath potential (relative to wall), all ions are lost
Modeling the NSTX SOL with Gkeyll

- Simple helical model of NSTX SOL
  - Field-aligned simulation domain that follows field lines from bottom divertor plate, around the torus, to the top divertor plate
  - Length along field line ~ connection length $L_\parallel = 8$ m (constant, no shear for now)
  - All bad curvature → interchange instability, blob dynamics
  - Real deuterium mass ratio, Lenard-Bernstein collisions
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NSTX GPI vs Gkeyll

D\alpha signal \sim \text{density at midplane}

 Ion density at midplane

F: 514 T: 5.1400e-04

Quasi-separatrix

Source region

\[ \vec{B}_0 \]

S. Zweben, NSTX GPI 412.843 ms

Separatrix
NSTX GPI

$D\alpha$ signal $\sim$ density at midplane

$\vec{B}_0$

Source region

Quasi-separatrix

Ion density at midplane

F: 514 T: 5.1400e-04

$\alpha \sim$ Separatrix
Demonstrating Gkeyll’s EMGK capabilities: tracing magnetic field lines

Straighten out flux-tube domain

$\mathbf{L_z} = 8 \text{ m}$

(length along $\mathbf{B}_0$)
Demonstrating Gkeyll’s EMGK capabilities: tracing magnetic field lines

\[ \vec{B}_0 \]

Straighten out flux-tube domain

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(length along \( \vec{B}_0 \))

\[ \text{Demonstrating Gkeyll’s EMGK capabilities: tracing magnetic field lines} \]
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\[ \mathbf{B}_0 + \delta \mathbf{B}_\perp \]

(length along \( \mathbf{B}_0 \))
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(length along $\vec{B}_0$)
Demonstrating Gkeyll’s EMGK capabilities: tracing fluctuating magnetic field lines

Project on x-z plane

\[ \vec{B}_0 + \delta \vec{B}_\perp \]

(length along \( \vec{B}_0 \))
Demonstrating Gkeyll’s EMGK capabilities: tracing *fluctuating* magnetic field lines

\[ \vec{B}_0 + \delta \vec{B}_\perp \]

Project on x-z plane

Project on x-y plane

(length along \( \vec{B}_0 \))
Demonstrating Gkeyll’s EMGK capabilities:

dancing field lines
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dancing field lines
Demonstrating Gkeyll’s EMGK capabilities:

**dancing field lines**

Length along $B_0 (m)$

$\vec{B}_0 = B_0 \hat{z}$

Time 0 $\mu$s
Demonstrating Gkeyll’s EMGK capabilities: dancing field lines

Blobs ($\beta \sim 1\%$) bend/stretch magnetic field lines
First EMGK simulations of SOL ✓
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- We’ve used simple helical geometry and parameters from NSTX H-mode SOL, but with 10x $n_0$ to stress-test EM effects (could happen locally in ELM?)
  - Results in magnetic fluctuations $\delta B_\perp / B_0 \sim 1\%$
  - \texttt{Gkeyll} can handle this strong magnetic turbulence robustly ✓
  - Mandell et al, JPP 2020; Hakim et al, PoP 2020
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Do EM fluctuations affect SOL turbulence dynamics?

• Now going to do side-by-side comparison of electrostatic and electromagnetic cases. Things to look for:
  • Changes in the blob structures, dynamics, frequency, etc
Electrostatic/electromagnetic comparison: midplane ion density

Time 500 $\mu$s

$n_i$ (m$^{-3}$) (ES) $\times 10^{20}$

$n_i$ (m$^{-3}$) (EM) $\times 10^{20}$

$\vec{B}_0 = B_0\hat{z}$
Electrostatic/electromagnetic comparison: midplane ion density

Time 500 μs

\[ \vec{B}_0 = B_0 \hat{z} \]
Electrostatic/electromagnetic comparison: density fluctuations statistics

EM has larger, more intermittent density fluctuations
Electrostatic/electromagnetic comparison: radial particle flux (near midplane)

- Might expect larger density fluctuations means more transport, but…

Radial particle transport reduced in EM case by ~ 40%
The radial $E \times B$ particle flux is defined as

$$\Gamma_r = \langle \tilde{n}_e \tilde{v}_r \rangle = n_{e, rms} v_{r, rms} \cos \alpha$$

In this case, using $n_{rms}$ as a surrogate diagnostic for transport is not sufficient!
Because EM case has less radial transport, heat flux to divertor is more peaked in EM case

ES case over-predicts transport, over-predicts heat flux width
Modest simulation cost (even for EM!)

<table>
<thead>
<tr>
<th></th>
<th>Electrostatic</th>
<th>Electromagnetic</th>
<th>only ~25% more</th>
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<tbody>
<tr>
<td>Total simulation time</td>
<td>1 ms</td>
<td>~20 ion transit times</td>
<td></td>
</tr>
<tr>
<td>(Nx, Ny, Nz, Nv, Nm)</td>
<td>(32, 64, 20, 20, 10)</td>
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<tr>
<td>Total wall-clock time</td>
<td>2.7 days</td>
<td>3.4 days</td>
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<td>(128 cores)</td>
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Electrostatic

Electromagnetic

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Towards more realistic SOL geometry

- We know magnetic shear can be important in SOL, especially near X point
- All Gkeyll results to date (NSTX and Helimak) have used simplified helical geometry
  - Neglected most geometrical factors, no magnetic shear
- Even in SMT configuration (e.g. Helimak) with const vertical field, there should be some magnetic shear because toroidal field $\sim 1/R$

$$
\mathbf{B} = \frac{B_0 R_0}{R} \hat{\phi} + B_v \hat{Z}, \quad q(R) = \frac{H B_0 \phi}{2\pi R B_v} = \frac{B_0 R_0 H}{2\pi R^2 B_v}, \quad \hat{s} = \frac{R}{q} \frac{dq}{dR} = -2
$$

- Keeping helical configuration, can adjust shear by making $B_v = B_v(R) = B_v(0)(R/x_0)^n$, so that

$$
\hat{s} = -2 - \frac{R}{B_v} \frac{dB_v}{dR} = -2 - n
$$

- Take field-aligned helical coordinate system with

$$
x = R, \quad z = Z, \quad y = x_0 \left( \phi - \frac{2\pi q Z}{H} \right),
$$

$$
\mathbf{B} = \frac{R B_v}{x_0} \nabla x \times \nabla y
$$
Helical geometry with magnetic shear

Vertical (~poloidal) flux: \( \Psi(R, Z) = R^2 B_v / 2 \sim R^{-\hat{s}} \)

Connection length:

\[
L_c = \frac{HB}{B_v} \approx \frac{HB_0 R_0}{B_v R} \sim \frac{1}{R^{n+1}} \sim \frac{1}{R^{-\hat{s}-1}}
\]

NSTX SOL connection length
Boedo et al, PoP 2014
\( \hat{s} \) scan: radial particle flux (near midplane)

- Transport decreases as \( |\hat{s}| \) increases
- EM cases again have less transport
- This is at ~ experimental \( \beta \sim 0.1\% \) (no more 10x)
$\hat{s}$ scan: divertor heat flux profiles

- Heat flux profile gets narrower as $|\hat{s}|$ increases
- EM effects more important as $|\hat{s}|$ increases
Summary

- **Gkeyll** is being used to study SOL turbulence in tokamaks like NSTX (only handles open field lines right now).

- **Gkeyll** has produced the first nonlinear electromagnetic gyrokinetic simulations in the SOL, can handle strong magnetic turbulence with $\delta B_\perp/B_0 \sim 1\%$

- In high $\beta$ regime, including electromagnetic fluctuations results in larger, more intermittent fluctuations in SOL but less transport.

- Moving towards more realistic SOL geometry, including magnetic shear.
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Acknowledgements

Gkeyll team:
Ammar Hakim
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Jimmy Juno
Rupak Mukherjee
Liang Wang
Eric Shi
... and others!

https://github.com/ammarhakim/gkyl/
https://gkeyll.readthedocs.io
Gkeyll GK references


Gkeyll GK references


Back-up slides
Dancing field lines at $\sim$ experimental $\beta$ (Time 0 $\mu$s)

**Diagram:**

- Vertical axis labeled "Length along $\vec{B}_0$ (m)" with values ranging from -4 to 4.
- Horizontal axis labeled "$x$ (m)" from 1.3 to 1.4.
- Field lines illustrated showing the dynamic behavior.

Mathematical Expression:

$$\vec{B}_0 = B_0 \hat{z}$$
Dancing field lines at ~ experimental $\beta$

Time 0 $\mu$s

$\vec{B}_0 = B_0 \hat{z}$

Length along $\vec{B}_0$ (m)
But increasing connection length, adding magnetic shear can also increase magnetic fluctuations and affect transport.
\( \beta_e \sim 0.5 \% \), \( \beta_i \sim 1 \% \) in SOL region (\( \sim 10 \times \) NSTX SOL)

• Profiles are steeper in source region, shallower in SOL region in EM case
\( \hat{s} \) scan: midplane profiles (EM)

- \( \beta \sim 0.1 \% \) (~ NSTX SOL)
- Profiles drop off more quickly as \( |\hat{s}| \) increases
- Previous simulations with simplified geometry similar to \( \hat{s} = -2 \) case
Current/Future Work

• Generalizing the magnetic geometry to include magnetic shear, non-constant curvature, closed field line regions, X-point
  • Non-orthogonal field-aligned coordinate system with magnetic shear now implemented
  • X-point is a singularity in these coordinates, challenging!

• More studies of EM effects on blobs/ELMs
  • Comparisons with magnetic fluctuation measurements in experiments?
Imaging the SOL with GPI

- GPI = Gas-puff imaging diagnostic (S. Zweben)
- Real-time turbulence movies in NSTX SOL
- Data taken using fast camera (400,000 fr/s)
- $D\alpha$ intensity proportional to some combination of $n$ and $T$
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Linear benchmark: KBM instability (local limit)

\[ k_{\perp} \rho_s = 0.5, \quad k_{\parallel} L_n = 0.1, \quad R/L_n = 5, \quad R/L_{Ti} = 12.5, \quad R/L_{Te} = 10, \quad \tau = 1 \]