Stochastic acceleration of electrons in laser plasma interactions

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Outline

- **Introduction**
  - Background information
  - New Hamiltonian approach
- **Stochastic electron acceleration in laser-plasma interaction (LPI)**
  - Electron in colliding laser beams
  - Electron in laser and quasi-static periodic fields (e.g., undulators)
  - Electron in laser and quasi-static confining-fields (e.g., ion channels)
- **Conclusions**

Introduction

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  - Background information
  - Novel Hamiltonian approach
- Electron acceleration in laser-plasma interaction (LPI)
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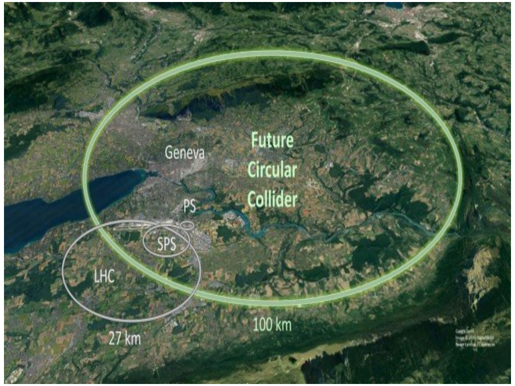
Electron acceleration in LPI

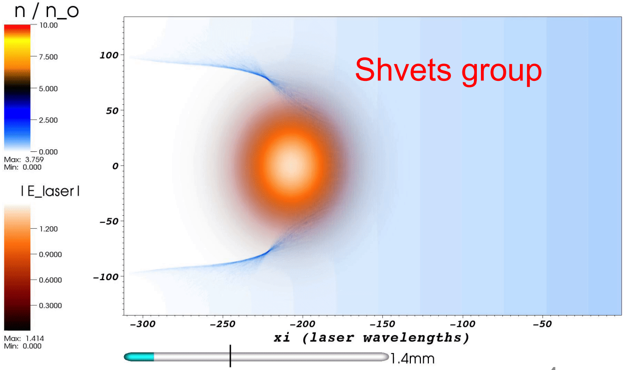
- LPI versus electron accelerator for energetic electrons
  - Electrons more responsive in LPI with immobile ions



- Many mechanisms of electron acceleration for relativistic laser radiation in under-dense plasma: $\omega_p < \omega$
  - Laser Wakefield acceleration (T. Tajima, and J. Dawson 1979)

$$\tau_{laser} < 1 / \omega_p$$

- Longitudinal electric field in bubble accelerates electrons



Electron acceleration in LPI (cont.)

- Direct laser acceleration (DLA)

  a. Stochastic electron acceleration in colliding lasers (J. Mendonca, *et al* 1983, Z.-M. Sheng et al, 2002)

  b. Electron in laser and quasi-static fields, e.g., in ion-channels $\tau_{\text{laser}} > 1/\omega_p$ or pre-plasma in front of target

  i. Transverse quasi-static EM fields:

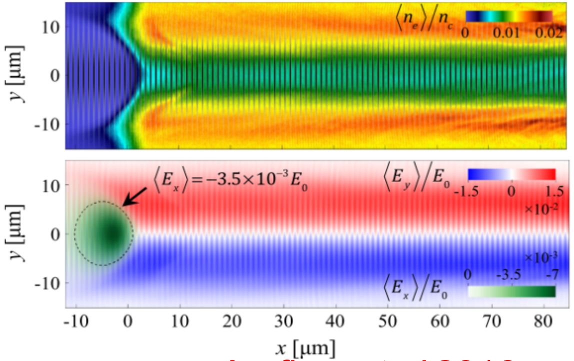
  Betatron resonance (Pukhov, *et al* 1999)

  Parametric amplification (Arefiev, *et al* 2012)

  ii. Longitudinal quasi-static electric field:

  Stochastic acceleration (Paradkar, *et al* 2012)

  iii. Stochastic acceleration in laser and transverse quasi-static fields, in laser and periodic quasi-static fields (Zhang *et al, PoP 2018, 2019; PPCF 2019*)



Arefiev, *et al* 2016

New Hamiltonian approach

- Analytical treatment of stochastic electron motion complicated and limited
  - Nonlinearity of relativistic electrons dynamics
  - Multidimensional spatial-temporal laser beams

conventional wisdom:

$$\mathcal{H}(\vec{r},\vec{p},t)=\sqrt{m^2c^4+\left[\vec{P}+e\vec{A}(\vec{r},t)\right]^2c^2}$$

↓

  - Physics underlying stochastic acceleration unclear
  - Lack of potential to find new scenarios

- New Hamiltonian approach: proper canonical variables
  - **Property: new Hamiltonian time independent without perturbative field**
  - Merits: physical picture clearer and analysis simpler
  - Tackling fundamental laser-plasma interactions

Question: How?

- Electron acceleration in laser-plasma interaction (LPI)
  - Electron in colliding laser beams
  - Electron in laser and quasi-static periodic fields
  - Electron in laser and quasi-static confining-fields

    i. Transverse electric and magnetic fields

    ii. Longitudinal electric and transverse magnetic fields

    iii. Longitudinal magnetic and transverse electric field

Question: How?

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  - **Electron in colliding laser beams**
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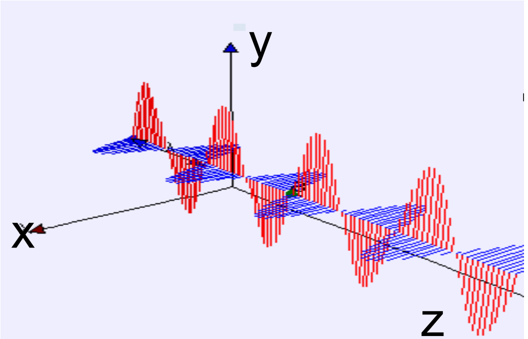
Free electron in laser field

- Electron can be accelerated by the laser pulse alone through the work done by laser electric field



Electron motion:

$$\frac{d\vec{p}}{dt} = -|e|\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)$$

$$\vec{p} = \gamma m\vec{v}$$

$$\gamma = \sqrt{1 + p^2/(mc)^2}$$

Laser:

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{A} = \vec{A}(\omega t - kz)$$

$$R = \gamma - p_z/mc$$

$$\gamma = \frac{A^2 + 1 + R^2}{2R}$$

$$p_x = a$$

- Electron gains energy from transverse laser electric field, which is converted into longitudinal motion by magnetic field
  - Relativistic case when $a = \frac{|e|E_0}{mc\omega} > 1$ $\quad E_{pond} = \gamma_{max}(R=1) = \frac{a^2}{2} + 1$

Electron in colliding lasers

- Setup: $\vec{A} = a\sin(v_p t - z)\vec{e}_x \quad \vec{A}_1 = a_1 \sin\left[k_1(v_p t + z)\right]\vec{e}_x$ $\boxed{a_1 \ll a}$

Effective time	$\tau = v_p t + z$
Canonical coordinate	$\chi = \gamma + v_p p_z$
Canonical momentum	$\eta = v_p t - z$
New Hamiltonian	$H = \gamma - v_p p_z$

$$\frac{d\chi}{d\tau} = \frac{\partial H}{\partial \eta}$$

$$\frac{d\eta}{d\tau} = -\frac{\partial H}{\partial \chi}$$

$$H(\chi, \eta, \tau) = \frac{2v_p}{v_p^2 - 1}\sqrt{\chi^2 + (v_p^2 - 1)P_\perp^2} - \frac{v_p^2 + 1}{v_p^2 - 1}\chi \qquad P_\perp^2 = 1 + \left[a\sin(\eta) + a_1 \sin(k_1\tau)\right]^2$$

- Luminal case $v_p = 1$

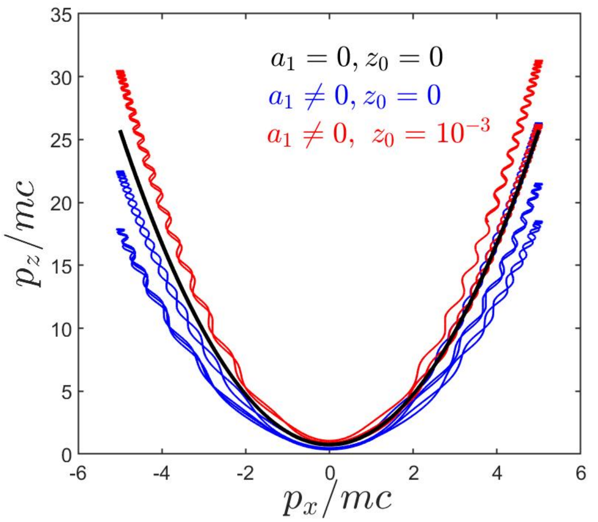
$$\boxed{H = \frac{P_\perp^2}{\chi}}$$

Physics of electron acceleration

- Unperturbed electrons in dominant laser $(a_1 = 0)$
  - Hamiltonian periodic in η $\qquad \omega(H) = \frac{2\pi}{T} = \frac{4H^2}{2+a^2}$
- Hamiltonian constant but electron kinetic energy varying

$$\gamma = \frac{\chi + H}{2} = \frac{1}{2}\left(\frac{1 + a^2 \sin^2\eta}{H} + H\right)$$

  - Hamiltonian H constant: limited electron energy

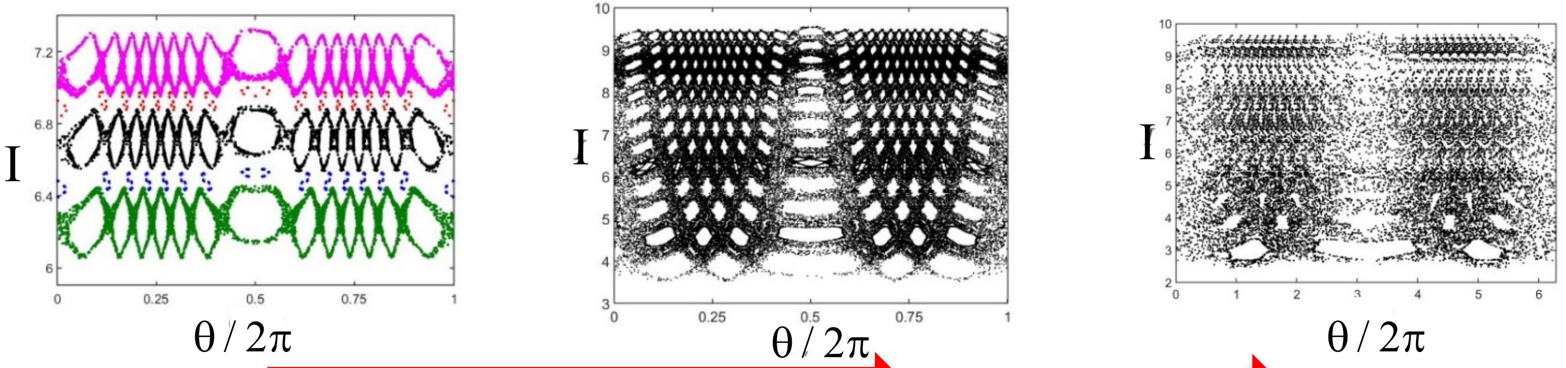


- Resonance between unperturbed oscillation and perturbative field frequencies: $\quad m\omega(H) = 1$
  - High harmonic resonance: $m \gg 1$ broadening and overlapping of resonances islands

$\bar{K} = (\Delta\omega + \Delta\omega')/2\delta\omega > 1$ → **Stochastic motion**

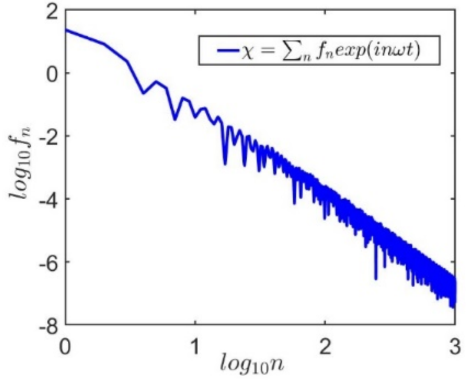
Physics of electron acceleration (cont.)

- Physical picture from action-angle description



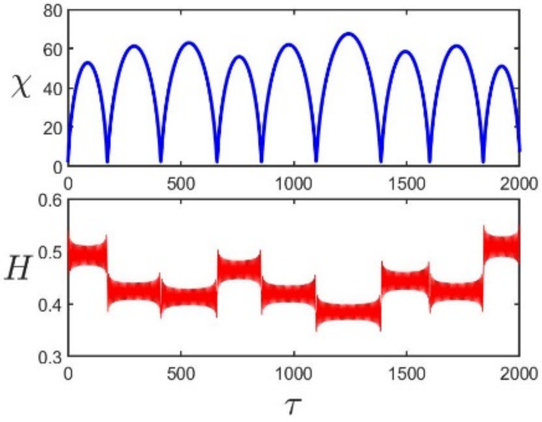
Increasing perturbation a_1

- Consider relativistic laser $a > 1$
  - Unperturbed χ having zig-zag time-dependence
  - PDF of its harmonic's amplitude: long tail



Method: Chirikov-like mapping

- Adiabatic H except "kicks" in short time near local minimum χ
  - **Small denominator of H:** $H = P_\perp^2 / \chi$
  - Adiabatic motion except kicks due to large: $\chi = \gamma + p_z$
  - Chirikov-like mapping



- Hamiltonian and time recurrence relation passing through nonadiabatic region

$$H_{n+1} - H_n = \Delta H(H_n, \tau_n) = \int_{\tau \approx \tau_n} \frac{\partial H}{\partial \tau} d\tau \qquad \tau_{n+1} - \tau_n = 2\pi / \omega_n (H_{n+1})$$

  - Hamiltonian variation using the unperturbed electron trajectories

- Stochastic condition (Sagdeev, *et al* 1990)

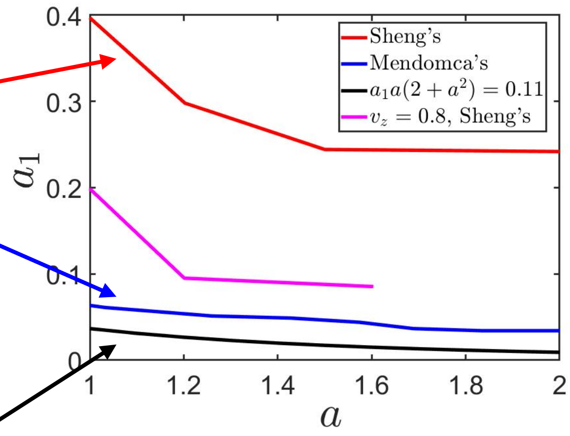
$$K = \left| \frac{d\Delta\tau_n}{dH_{n+1}} \frac{d\Delta H_n}{d\tau_n} \right| > 1 \quad \longleftrightarrow \quad \begin{cases} I_{n+1} = I_n + K \sin \psi_n \\ \psi_{n+1} = \psi_n + I_n \end{cases}$$

Stochastic condition

- Numerical thresholds for stochastic motion

  $a_1 > a_s \approx 1/2a$ Z.-M. Sheng *et al*, 2002

  $a_1 > a_s \approx 1/16a$ J. Mendonca, *et al* 1983



- Threshold for stochasticity

$$K = 4\pi^2 a a_1 (2+a^2)\beta^2 \,|A_i^{'}(\beta)|$$

$$\beta = (k_1 / H^2 a)^{2/3}$$

$$a_1 > a_s \approx \frac{0.11}{a(2+a^2)}$$

  - Threshold value smaller
  - Most stochastically unstable region in H space: $H_s \approx 0.68\left(k_1 / a\right)^{1/2}$
  - For $a_1 \approx a_s$, stochastic region near H_s requires pre-acceleration of electron

Ceiling of stochastic acceleration

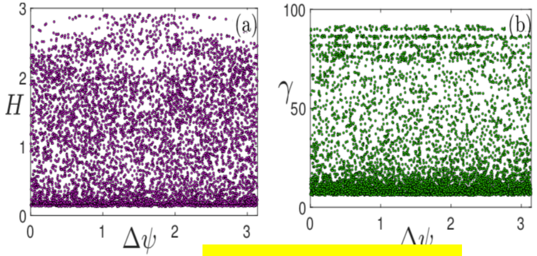
- $a_1 \gg a_s$ efficient stochastic acceleration in H and thus in electron energy space
- Maximum electron kinetic energy at the lower boundary of stochastic regions in H space

$$H_{min} \approx \frac{H_s}{\sqrt{1.6 + 0.69\ln(a_1/a_s)}}$$

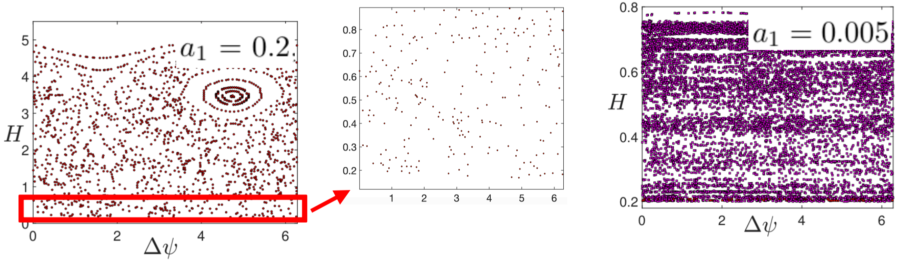
$$\gamma_{max} \approx (1 + a^2)/2H_{min}$$

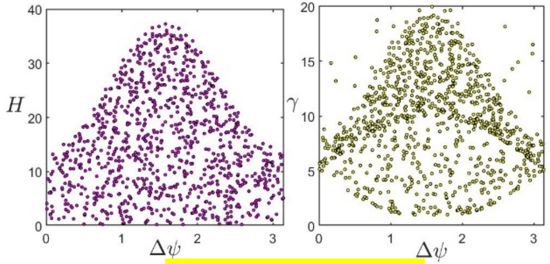
  - **Weak dependence on the amplitude of perturbative laser**

- Perturbative laser leading electron to small Hamiltonian (moving with dominant laser) but not energy source
  - **Two comparable lasers resulting in lower energy as no preferable direction, but both provide energy**



$a = 5, a_1 = 0.1$

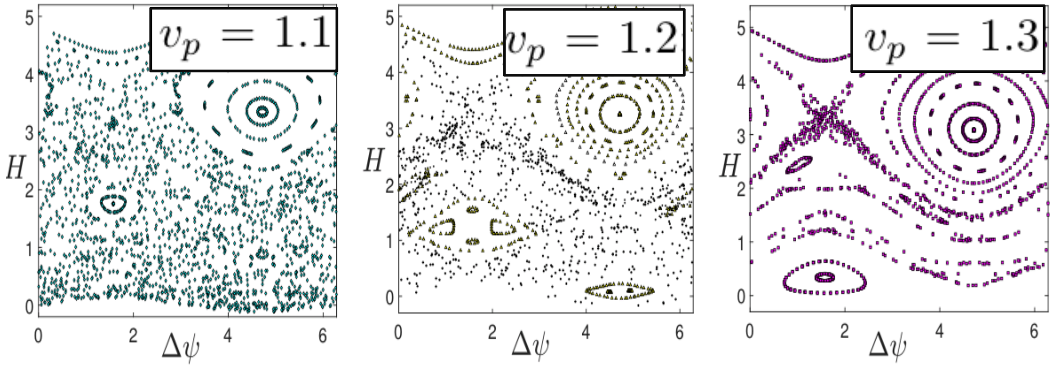




$a = 5, a_1 = 5$

Superluminal phase velocity

- Threshold for onset of stochasticity: $v_p < v_{thre} = 1 + a_1^{3/4}$



$a_1 = 0.2$

$v_{thre} = 1.3$

- Stochasticity in H space slightly changes with phase velocity, but the energy significantly decreases

$$\gamma_{max} = \frac{v_p}{v_p^2 - 1}\sqrt{H^2 + (v_p^2 - 1)(1 + a^2)} - \frac{H}{v_p^2 - 1}$$

$$\gamma_{max} \approx v_p\sqrt{(1 + a^2)/(v_p^2 - 1)}$$

Intermediate conclusion

- New Hamiltonian (dephasing rate between electron and dominant laser) constant without perturbative field
  - Perturbative laser affecting the dephasing rate but not as energy source
  - Electron energy in the longitudinal direction → lower energy for two comparable lasers
  - Adiabatic oscillation of electron except localized minimum p_z , small denominator effect
  - Weak dependence of upper (lower) boundary of stochastic region in γ (H) on the perturbative laser amplitude
  - Lower energy for superluminal phase velocity

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Stationary periodic static fields

- Perturbative laser ⟶ stochastic force ⟷ quasi-static periodic longitudinal electric and transverse magnetic fields

$$U = E_1 \sin(k_1 z)\vec{e}_z \qquad \vec{A}_B = B_1 \sin(k_1 z)\vec{e}_x$$

  - Quasi-static periodic fields like electric and magnetic undulators (Avetissian *et al*, 1978), wiggler magnetic field (Mehdian *et al*, 2007), and plasma wave (Mangles *et al*, 2005)

- Canonical transformation: $\vec{A} = a\sin(v_p t - z)\vec{e}_x$

Effective time	$\tau = t + z$
Canonical coordinate	$\chi = \gamma + p_z - 2U$
Canonical momentum	$\eta = t - z$
New Hamiltonian	$H = \gamma - p_z$

$$\frac{d\chi}{d\tau} = \frac{\partial H}{\partial \eta}$$

$$\frac{d\eta}{d\tau} = -\frac{\partial H}{\partial \chi}$$

Physics of stochastic acceleration

$$H = \frac{1+\left[a\sin(\eta)+A_B(\tau/2-\eta/2)\right]^2}{\chi+2U(\tau/2-\eta/2)}$$

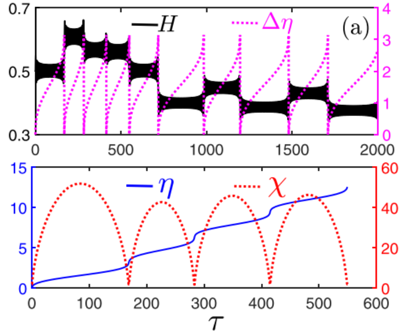
- Quasi-static fields as perturbation

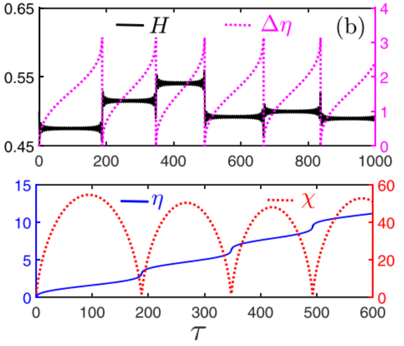
  $B_1 \ll a \qquad E_1 \ll \chi_{min} \approx 1/H \qquad H=\gamma-p_z$ usually small

- "Kicks" in short time near local minimum χ
  - **Small denominator of H**
  - Small χ and thus small p_z and γm

- Threshold for stochasticity

$$B_1 > B_s \approx \frac{0.11}{a(2+a^2)} \qquad E_1 > E_s \approx \frac{0.24}{(2+a^2)(ak_1)^{1/2}}$$





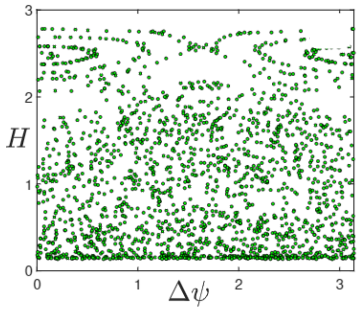
Stochastic acceleration

- Weak dependence of lower (upper) boundary of H (γ) on the amplitude of perturbative fields

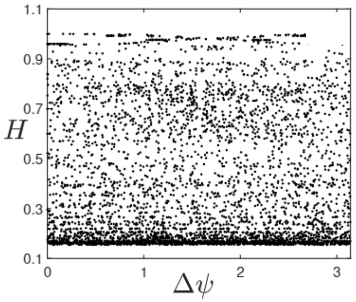
$$H^b_{min} \approx \frac{H^b_s}{\sqrt{1.6 + 0.69 \ln(B_1 / B_s)}} \qquad H^e_{min} \approx \frac{H^e_s}{\sqrt{1.8 + 0.62 \ln(E_1 / E_s)}}$$

  - Same order with electron in colliding lasers

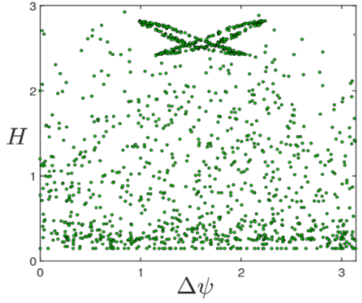
- All the results are confirmed by numerical simulations



B_1=0.1



B_1=0.01



E_1=0.1

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    i. **Transverse electric and magnetic fields**

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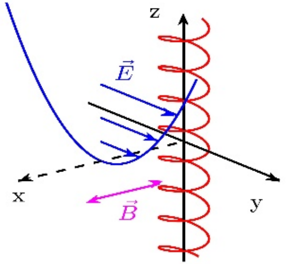
Quasi-static transverse EM fields

- Electron motion is confined in the quasi-static electric and magnetic fields

$$U(y) = \kappa_u y^2 / 2 \qquad A_B(y) = \kappa_b y^2 / 2$$



$$\vec{A} = a_0 \sin(t - z)\vec{e}_{x,y}$$

- Confining-EM fields dominant, while laser perturbative
- Electron energy as new Hamiltonian

Effective time	$\xi = t - z$
Canonical coordinate	y
Canonical momentum	$\tilde{P}_y = p_y - \tilde{A}_y$
New Hamiltonian	$H = \gamma + U$

$$\frac{dy}{d\xi} = \frac{\partial H}{\partial \tilde{P}_y}$$

$$\frac{d\tilde{P}_y}{d\xi} = -\frac{\partial H}{\partial y}$$

$$H(v_p = 1) = \frac{1}{2}\left[\frac{1 + \left(\bar{P}_x + \tilde{A}_x\right)^2 + \left(\tilde{P}_y + \tilde{A}_y\right)^2}{C_\perp - \left(U + A_B\right)} + U - A_B\right]$$

$$C_\perp \equiv \gamma - p_z + U(y) + A_B(y)$$

Physics of electron acceleration

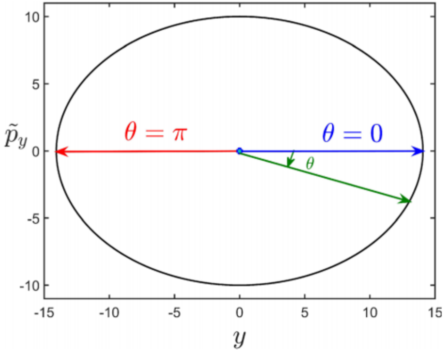
- Strongest impact of laser on electron motion occurs at small denominator of Hamiltonians $C_\perp - U - A_B = \gamma - p_z$
  - **Static fields enhance the electron-laser interaction**

$$U + A_B \to C_\perp$$

  - Static magnetic field effect absorbed in electric field $A_B = 0$
  - Superluminal phase velocity decreases the electron-laser interaction

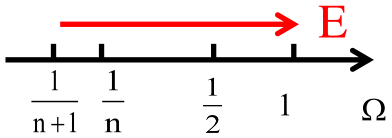
$$\gamma - P_z \equiv C_\perp - U + (v_p - 1)p_z$$

- Nonadiabatic regions: $y_{max,min} \approx \pm\sqrt{2C_\perp / \kappa_u}$



Universal parameter

- Unperturbed electrons oscillation frequency: $\Omega = 2\sqrt{E\kappa_u} / C_\perp$



$$E_{abs}^{max} = C_\perp^2 / 4\kappa_u$$

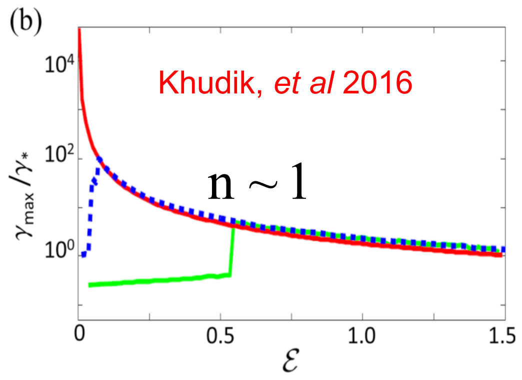
  - Lowest harmonic resonance for an upper limit of electron energy.

- Consider electron acceleration beyond ponderomotive scaling

$$E_{pond} < E < E_{max}^{abs}$$

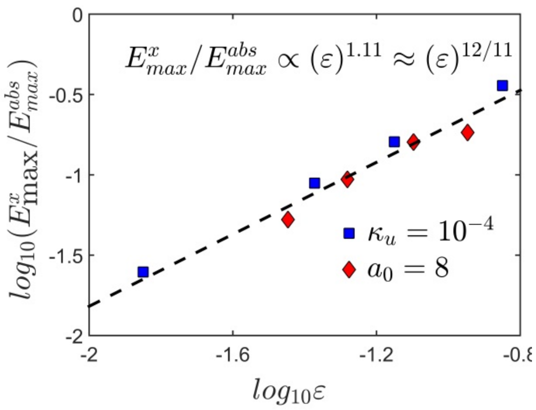
  - A small parameter for transverse case:

$$\varepsilon \equiv \sqrt{2} a_0 \kappa_u^{1/2} C_\perp^{-3/2} < 1 \rightarrow E_{max}^{abs} = E_{pond}\varepsilon^{-2} \rightarrow$$



Stochastic condition

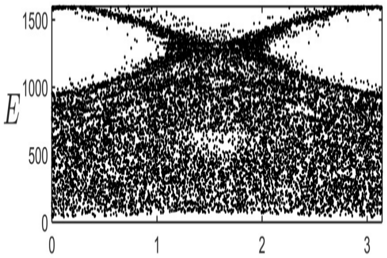
- Stochastic acceleration begins $(\kappa_x \sim \kappa_y \sim 1)$
  - For: $\tilde{\vec{A}} = a_0 \sin(\xi)\vec{e}_y \quad K_y = \kappa_y \left(E_{max}^{abs} E^{-1}\right)^{7/6} \quad \varepsilon > 1$
  - For: $\tilde{\vec{A}} = a_0 \sin(\xi)\vec{e}_x \quad K_x = \kappa_x \left(E_{max}^{abs} E^{-1}\right)^{11/6} \quad \varepsilon^2 > 1$

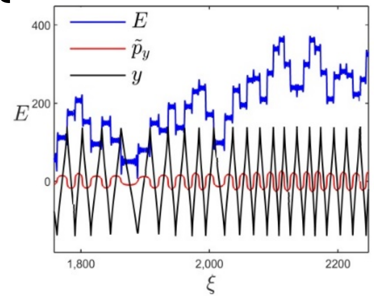


- Upper limit of energy $\varepsilon \ll 1$

$$E_{max}^{x} \approx E_{max}^{abs}\varepsilon^{12/11} = E_{pond}\varepsilon^{-10/11} < E_{max}^{y} \approx E_{max}^{abs}\varepsilon^{6/7} = E_{pond}\varepsilon^{-8/7}$$

- The maximum energy is above the ponderomotive scaling but below the absolute boundary set by $\Omega \sim 1$
  - Set by the universal parameter ε





Conclusions

- Introduction
  - Background information
  - Novel Hamiltonian approach
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Conclusions

- New Hamiltonian approach to electron dynamics in different configurations of lasers and quasi-static fields
  - Proper canonical variables
  - Time independent Hamiltonian with no perturbation
- Physical pictures clearly revealed
  - Nonadiabatic interaction ("kick") at small denominators (dephasing rate between electron and perturbative field and effective electron mass)
  - Assistant fields affect the dephasing rate between the electron and dominant laser but not as energy source
- Stochastic conditions and upper limits of electron energy via Chirikov-like mappings

Discussions

- Limitations:
  - Single electron model, where plasma impact are only mimicked by superluminal phase velocity
  - Superluminal phase velocity impact is analytically complicated and analyzed qualitatively
  - Cut-off energy due to short laser pulse duration
  - Plane laser waves
- Questions based on universality:
  - Particle in other multiple fields?
  - Other LPI other than stochastic acceleration?

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Thank you!