Compressional Alfvén eigenmodes excited by runaway electrons

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Jan 25 2019
1. Introduction to runaway electrons and kinetic instabilities

2. Compressional Alfvén eigenmodes excited by runaway electrons
   - Frequencies and structures of CAE in DIII-D
   - Collisional damping of compressional Alfvén eigenmodes
   - Resonance of runaway electrons and CAE
   - Growth rate of CAE from kinetic simulation of runaway electrons

3. Highlights of my research work on runaway electrons in tokamaks

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Basic pictures of runaway electrons (REs)

- In plasma, drag force on electrons due to Coulomb collision is a non-monotonic function of $p$. For $v \gg v_{th}$, collision frequency $\sim v^{-3}$, collisional drag force decrease with $p$.

- With $E > E_{CH}$ (Connor-Hastie field), electrons with momentum larger than $p_{\text{crit}}$ can run away to higher energy.

- Knock-on collision of high energy electron with thermal electron can lead to avalanche growth of RE.
Runaway electron mitigation is an important issue in tokamak disruption studies

- In ITER disruption, a large population of energetic runaway electrons can be generated due to the strong inductive $E$ field.

- As most electrons get cooled due to collisions, electric field in disruptions can drive remnant of hot electrons from keV to MeV (hot-tail generation).
  - These electrons can become the seed of RE avalanche.

- RE beam can strike the first wall during final loss and cause damage to the device.
- Disruptions cannot be avoided 100%, so it is important to mitigate RE beam.
  - Suppress generation, limit energy, diffuse to the edge
Whistler waves excited by runaway electrons in flattop phase

• Runaway electrons can be generated in tokamak experiments in both flattop phase and post-disruption scenarios.

• Whistler waves excited by kinetic instabilities of REs have been observed in flattop phase of discharge.
  • The excited modes have discrete structure in spectrum and correlations with ECE signals.
  • Quasilinear simulations show that whistler waves are mostly driven by anisotropic distribution of RE beam.
  • Energy diffusion and pitch-angle scattering from whistler waves leads to a higher critical electric field than previously predicted, which is close to experimental observations.

Direct observation of kinetic instabilities in the current-quench phase

• In DIII-D disruption experiments, low-frequency kinetic instabilities are identified during current quench using ion cyclotron emission (ICE) diagnostic.
  • When low-frequency modes are strongly excited, the RE plateau will not build up.
  • Increase Ar density reduces the number of high-energy REs, suppress instabilities, help RE plateau survive.
• Compared to whistler waves, the low-frequency modes are easier to be excited since they are mostly ion-driven, and damping due to electron-ion collision is less significant.

Mode excitation and plateau dissipation have thresholds on RE energy

• Using gamma-ray imaging (GRI) to model RE energy spectrum, it is shown that excitation of modes and dissipation of RE plateau depend on the existence of high-energy REs.
  • Max $E_{RE} > 2.5 - 3$ MeV is required for the mode excitation.
  • RE plateau formation fails when max $E_{RE} > 6$ MeV.

• The modes spectrum shows discrete structures, with frequencies 0.1-2.4MHz with a spacing of 400kHz.
  • The frequencies are of the same order of Ar cyclotron frequency.
  • The discrete frequencies decrease during current-quench.
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Compressional Alfvén eigenmodes in spherical tokamaks

- Compressional Alfvén eigenmode (CAE) is a candidate explanation for the observed instabilities.
  - Strong $\tilde{B}_\phi$ components are observed at wall on low-field-side.
  - The mode is required to have a global structure in the poloidal plane, which can interact with REs at core and have strong field components at edge.
- Previous studies of CAE focus on cases in spherical tokamaks (ST), which can be excited by energetic ions in plasma.
  - Due to the inhomogeneity of $R^2$ and $v^2_A$, CAE in ST is strongly localized near the edge of low-field-side, and confined inside plasma.

$$\left( \nabla^2_{\text{pol}} - \frac{n^2}{R^2} + \frac{\omega^2}{v^2_A} \right) X = 0$$

CAE mode structures are similar to transverse-electric mode

- In DIII-D, $v_A$ is much larger than that in NSTX, so CAE with frequency of MHz will have much smaller $k_\perp$ value.
  - The mode can have a more global structure and even extend to vacuum region.
  - Tokamak boundaries, which can be regarded as conducting walls, play important roles in determining the eigenmode frequencies.
- If ignoring the $|B|$ variation and poloidal fields, the CAE is similar to the transverse electric (TE) mode in wave guides.
  - Replace $c$ with $v_A$ in dispersion calculation.
  - A more rigorous calculation can be done using MHD codes like NOVA.

\[
B_z = J_m(k_r r) \exp(im\theta)\exp(in\phi)
\]
\[
B_\theta = -\frac{mn}{RrK_r^2}J_m(k_r r) \exp(im\theta)\exp(in\phi) \\
B_r = \frac{in}{Rk_r}J'_m(k_r r) \exp(im\theta)\exp(in\phi)
\]
\[
E_r = -\frac{\omega}{c} \frac{m}{rk_r^2}J_m(k_\perp r) \exp(im\theta)\exp(in\phi) \\
E_\theta = \frac{\omega}{c} \frac{i}{K_r}J'_m(k_r r) \exp(im\theta)\exp(in\phi)
\]
\[
\frac{\partial J_m}{\partial r}\bigg|_{r=a} = 0 \quad \text{From conducting wall boundary condition}
\]
Eigenmode frequencies consistent with experiments

• In DIII-D current quench, plasma is mostly composed of Ar and electrons, frequencies of the first few eigenmodes can be calculated based on value of $v_A$.
  • For $n_e = 1 \times 10^{20} \text{ m}^{-3}$, $B = 2.1 \text{T}$, $Z_{\text{eff}} = 2$, $\omega_{TE11} = 0.44 \text{MHz}$, $\omega_{TE02} = 0.94 \text{MHz}$, $\omega_{TE12} = 1.30 \text{MHz}$, consistent with experimental observations.
  • During current quench, $T_e$ drops from 5eV to 2eV, thus $Z_{\text{eff}}$ of Ar drops from +2 to +1, which results in a decrease of $v_A$ and eigenmode frequencies.

• Modes have strong $B_z$ components at edge.
• $B_r$ components of CAE can lead to radial diffusion of runaway electrons.
  • Radial diffusion coefficients can be calculated following Rechester-Rosenbluth model.
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Collisional damping of CAE can be modeled using friction forces between electrons and ions

- In most literature about Alfvén eigenmodes in tokamaks, damping comes from kinetic effects such as Landau damping or collisions of resonant particles.
  - In post-disruptions, the kinetic effect is insignificant due to small $T_e$.
- For electron-driven modes like whistler modes, collisional damping rate can be calculated by adding a friction term in equation of motion of electrons (replace $m_e$ with $m_e(\omega + i\nu)/\omega$ in the dielectric tensor)
  - The CAE observed and post-disruption plasma with $T_e \sim$ a few eV, $\omega \ll \nu_{ei}$
  - However, Alfvén waves should not be strongly affected by electron-ion collisions, since they are mostly moving together.

- A more rigorous calculation of the collisional damping rate due to $\nu_{ei}$ can be done by adding resistivity and viscosity into multi-fluid equations.
  - Resistivity $\sim n/\tau \sim T^{-3/2}$, whereas viscosity $\sim nT\tau \sim T^{5/2}$, so for low temperature plasma the contribution from viscosity can be ignored.
  - Resistivity can be modeled using friction forces between electrons and ions.
Two-fluid equations with friction forces

\[
\begin{align*}
-i\omega V_{ix} &= eZ_i E_x/m_i + \omega_{ci} V_{iy} + \nu_{ie}(V_{ex} - V_{ix}) \\
-i\omega V_{iy} &= eZ_i E_y/m_i - \omega_{ci} V_{ix} + \nu_{ie}(V_{ey} - V_{iy}) \\
-i\omega V_{ex} &= -eE_x/m_e - \omega_{ce} V_{ey} + \nu_{ei}(V_{ix} - V_{ex}) \\
-i\omega V_{ey} &= -eE_y/m_e + \omega_{ce} V_{ex} + \nu_{ei}(V_{iy} - V_{ey})
\end{align*}
\]

\[
\begin{pmatrix}
V_{ix} \\
V_{iy} \\
V_{ex} \\
V_{ey}
\end{pmatrix} =
\begin{pmatrix}
i\omega - \nu_{ie} & \omega_{ci} & \nu_{ie} & 0 \\
-\omega_{ci} & i\omega - \nu_{ie} & 0 & \nu_{ie} \\
\nu_{ei} & 0 & i\omega - \nu_{ei} & -\omega_{ce} \\
0 & \nu_{ei} & \omega_{ce} & i\omega - \nu_{ei}
\end{pmatrix}^{-1}
\begin{pmatrix}
-eZ_i E_x/m_i \\
-eZ_i E_x/m_i \\
eE_x/m_e \\
eE_x/m_e
\end{pmatrix}
\]

- \(\nu_{ei}\) and \(\nu_{ie}\) are slowing-down collision frequency.
  - Conservation of momentum requires \(n_e m_e \nu_{ei} = n_i m_i \nu_{ie}\)
- With \(\nu_{ei} = \nu_{ie} = 0\), the equations just give the standard plasma dielectric tensor.
- The full inverse matrix becomes very complicated, but can be solved numerically.
Collisional damping rate of CAE for low-temperature plasmas

\[ n_e = 1 \times 10^{20} \text{m}^{-3}, \ T_e = 5 \text{eV}, \ \text{Ar with } Z_{\text{eff}} = 2 \]

- Damping rate of CAE due to electron-ion collision \( \gamma_c \sim \nu_{ei}k^2 \)
- For the first eigenmode (TE11), \( \gamma_c/\omega \approx 5 \times 10^{-4} \).
- As \( \omega \) gets larger and becomes close to \( \omega_{ci} \), the collisional damping rate increase.
  - As ions become less magnetized, the motion of electrons and ions have larger separation.

- A more rigorous calculation of collisional damping rate can be done by solving first-order kinetic equation including the Landau collision operator.
  - Similar to Braginskii, but with nonzero frequency.
  - Collisions between the same species can also modify the damping (as affecting resistivity).
Outline

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4. Summary
CAE can have resonances with transit and precession motion of high-energy electrons

In order to have wave-particle interactions, runaway electrons must have resonances with CAE.

- $\omega_{ce} \approx 58\text{GHz} \gg \omega_{CAE}$, so Doppler resonance ($\omega = n\omega_{ce}$) is unlikely.
- $\omega_{CAE}/(1/R) \approx 0.01c$, so a runaway electron ($v \sim c$) satisfying Cherenkov resonance is almost certainly a trapped particle.
- Transit and bounce frequencies of relativistic electrons ($\sim 13\text{MHz}$) are large compared to $\omega_{CAE}$.
  - For passing electron near a rational surface, $\omega_{CAE} = n\omega_{\phi} - m\omega_{\theta}$ can be satisfied.
- Precession frequency of trapped runaway electrons is about $0.3\text{MHz}$, so the resonance condition $\omega = n\omega_{d}$ can be satisfied.
  - This mechanism was used to explain the excitation of beta-induced Alfvén eigenmodes (BAE) driven by energetic electrons.
  - Unlike transit and bounce frequencies, precession frequency is proportional to the relativistic factor $\gamma$.
Toroidal frequency affected by electron drift motion

- For trapped particles,
  \[ \omega_\phi = \frac{v_d q}{r} \langle \cos \theta \rangle \]

- For passing particles, the drift motion can also contribute to toroidal frequency
  \[ \omega_\phi = \frac{v}{R T} + \frac{v_d q}{r} \langle \cos \theta \rangle \]

  where
  \[ v_d = \frac{p^2_\parallel + p^2_\perp/2}{\gamma m e B} B \times \nabla B \]

  \[ T = \frac{1}{\theta_{\text{max}}} \int_{0}^{\theta_{\text{max}}} d\theta \frac{1}{\sqrt{1 - (1 - \xi^2)b(\theta)}} \]

  \[ \langle \cos \theta \rangle = \frac{1}{T \theta_{\text{max}}} \int_{0}^{\theta_{\text{max}}} d\theta \frac{\cos \theta}{\sqrt{1 - (1 - \xi^2)b(\theta)}} \]

- Unlike transit and bounce frequencies, precession frequency is proportional to the relativistic factor \( \gamma \). For high-energy electrons, the contributions can not be ignored.

CAE can be driven by resonant REs due to gradients in momentum space and radial direction.

\[
\gamma_L = \frac{4\pi^2 e^2}{E_{\text{CAE}}} \int \frac{|\langle G \rangle|^2}{\omega} \delta(\omega - n\omega_d) \left( \omega \frac{\partial}{\partial E} + n \frac{\partial}{\partial P_\phi} \right)_\mu f d^3p
\]

\[
G = E \cdot v_d + E_y v_\perp J_1(k_\perp \rho), \quad P_\phi = p_\parallel R - \psi
\]

\[
(\omega \frac{\partial}{\partial E} + n \frac{\partial}{\partial P_\phi})_\mu = \frac{\omega}{v_\parallel} \left( \frac{\partial}{\partial p_\parallel} \right)_{p_\perp, \psi} + \left( \frac{\omega R}{v_\parallel} - n \right) \left( \frac{\partial}{\partial \psi} \right)_{p_\perp, p_\parallel}
\]

- Ware pinch of trapped electrons lead to a peaked profile.
- For trapped electrons, \( \omega < (n/R)v_\parallel \), so a peaked profile of RE leads to a positive growth rate.

Resonances between RE and CAE for $q \approx 1$

\[ G = \mathbf{E} \cdot \mathbf{v}_d + E_y v_{\perp} J_1(k_{\perp \rho}) \]

- For runaway electron plateau, $q$-profile near core is flat and above 1 due to internal kink mode.
  - For CAE with $m = 1$, $n = 1$, the phase of the mode is almost fixed along electron trajectory, thus the TTMP term gives the dominant resonance.
  - For CAE with $m = 0$, $n = 1$, $E$ from CAE resonate with electron’s drift velocity.

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Bounce-average kinetic equation equation of runaway electrons

\[ C_1(\xi) \frac{\partial f}{\partial t} + E\sigma \left( \frac{1}{p^2} \frac{\partial}{\partial p} p^2 - \frac{1}{p} \frac{\partial}{\partial \xi} (1 - \xi^2) \right) f \quad \text{electric force} \]

\[-C_1(\xi) \frac{1}{p^2} \frac{\partial}{\partial p} p^2 \left( C_F + C_A \frac{\partial}{\partial p} \right) - \frac{1}{p^2} \frac{\partial}{\partial \xi} (1 - \xi^2) C_B C_2(\xi) \frac{\partial}{\partial \xi} f \quad \text{collision operator} \]

\[-C_1(\xi) \frac{1}{p^2} \frac{\partial}{\partial p} p^2 C_R p \gamma (1 - \xi^2) f + \frac{1}{p^2} \frac{\partial}{\partial \xi} (1 - \xi^2) C_R C_2(\xi) \frac{p^2 \xi}{\gamma} f \quad \text{synchrotron radiation} \]

\[ + S(f) = 0 \quad \text{avalanche source} \]

where

\[ C_1 = \xi \int \frac{d\theta/2\pi}{\sqrt{1 - (1 - \xi^2) b(\theta)}}, \quad C_2 = \frac{1}{\xi} \int d\theta/2\pi \sqrt{1 - (1 - \xi^2) b(\theta)} \]

- \( C_1 \) plays the role of Jacobian for distribution function
- The avalanche source term is also bounce-averaged.

Finite-method to solve bounce-average kinetic equation

- For trapped electrons, the distribution is symmetric with respect to $\xi = 0$.
- Fluxes crossing the passing-trapped boundaries is shared between two boundaries, to satisfy the particle number conservation.
Effects of partially screened nuclei on electron collisions

• Pitch-angle scattering is weak for high-energy electrons in classical theories, as scattering coefficients drops as $1/p^2$.

• With partially-ionized high-$Z$ impurities, the slowing-down and pitch-angle scattering of REs in high energy regime is significantly enhanced due to partially-screening.
  • High-energy electron can penetrate into electron cloud and get closer to the nuclei, interacting with bounded electrons and naked nuclei.
  • Slowing-down enhanced by factor of $Z$, while scattering is enhanced by $Z^2$.

• Enhancement due to partially-screening can be calculated using Thomas-Fermi model or density function theory (DFT).

Large pitch-angle passing and trapped runaway electrons can be generated by enhanced pitch-angle scattering

- We solve the bounce-averaged kinetic equation of runaway electrons to obtain the evolution of $f$.
  - $n_e = 1 \times 10^{20} m^{-3}$, $T_e = 5$eV, $B = 2.1$ T and $E = 2$V/m($\approx 25E_{CH}$)
  - $r = 0.15$m, $R = 1.67$m, $q = 1.08$
  - By applying a large initial $T_e$ and a large $E$ at early time, runaway electron tail are generated from hot-tail generation, and dragged to high energy by $E_\parallel$.

- As REs tail moves to higher energy, the enhanced pitch-angle scattering from Ar can scatter electrons into large pitch angles, and even become trapped electrons.
  - For both kinds of electrons, distribution function satisfies $\partial f / \partial p_\parallel > 0$. 

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**RE distribution at r=0.15m, t=5ms**

![Graph showing RE distribution](image-url)
The linear growth rate can be calculated from distribution function:

- Radial gradient of trapped RE is calculated by assuming $\frac{\partial f}{\partial r} = -\frac{f}{r_0}$, where $r_0 = 0.4\text{m}$.

For the first few eigenmodes, the growth rate given by RE can become larger than collisional damping:

- Higher frequency modes are excited later than lower frequency ones, consistent with experiments.

To calculate the mode amplitudes self-consistently, a quasilinear model including the diffusion in both momentum space and radial direction is required:

- Currently the bounce-averaged kinetic equation is only solved in momentum space with fixed $r$ and $q$.
- It is promising to use NOVA-K to solve the problem using resonance-broadening quasilinear (RBQ) approximation.
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Excitation of whistler wave by REs in flattop scenario

Simulation based on DIII-D flattop RE experiments shows that whistler waves can be excited during the RE avalanche in flattop phase.

- Low frequency whistler waves (LFWW, 1GHz-10GHz) first get excited, and scatter RE in high energy regime ($15 < \gamma < 20$, fan instability).
  - Stop RE from going into higher energy regime
- High frequency whistler waves (HFWW, 10GHz-40GHz, close to resonance cone) get excited later, and scatter RE in low energy regime ($2 < \gamma < 5$).
• The excited waves can cause both energy diffusion and pitch angle scattering of REs.
• For $E \gg E_{CH}$, energy diffusion enhances the RE avalanche growth.
  • Resonance region of excited waves overlaps with the runaway-loss separatrix, so energy diffusion causes more electrons entering the runaway region.
• For smaller $E$ field, overlapping does not happen and pitch-angle scattering suppresses the avalanche growth.
• Competition of two effects results in a higher critical electric field than predicted by Rosenbluth-Putvinski theory, which is much closer to experimental observations.

Prompt growth of ECE signals are observed in RE experiments

- In both quiescent runaway electron (QRE) experiments and post-disruption cases, fast growth of ECE signals (beyond the electron temperature value) are observed.
- This growth of ECE signals comes from fast increase of RE pitch angle, associated with kinetic instabilities.
• We develop a new ECE synthetic diagnostic tool for runaway electrons, in order to benchmark with experiments.

• ECE signals from REs start to grow abruptly after the high frequency whistler wave are exited, and can overwhelm the ECE from thermal electrons.

• Signals at higher frequencies is more enhanced than lower frequencies.

• Most of the ECE enhancement are from RE in lower energy regime (<2MeV) with very large pitch angle.
  • These electrons are generated by the pitch-angle scattering from high frequency whistler waves

• The results match well with DIII-D ECE diagnostic results.
Magnetic moments of relativistic runaway electrons

• Full-orbit simulation of relativist runaway electrons show strong variations of magnetic moment $\mu$.

$$\mu = \mu_0 + \mu_1 + \mu_2$$

$$= \left( \frac{|p_\perp|^2}{2mB} \right) + \left( \frac{p_\parallel^2 p_\perp \cdot \kappa \times b}{qmB^2} \right) + \left( \frac{p_\parallel^4 |\kappa \times b|^2}{q^2B^2 2mB} \right)$$

$$= \frac{|p_\perp + p_\parallel^2 \kappa \times b/(qB)|^2}{2mB}$$

• This new expression of $\mu$ depends on both $p_\parallel$ and $p_\perp$, and the curvature of magnetic field.

• In the standard guiding-center ordering, the second term is one order of magnitude smaller than the first one. But in the new ordering, the two terms are of the same order of magnitude.
The new magnetic moment shows much better conservation property than the standard one.

A possible fluid-kinetic framework for runaway electron simulation

A self-consistent fluid-kinetic framework for runaway electrons is needed for disruption simulation.

- Current coupling is used in previous studies, but the generation of REs from the bulk population is missing.
- Analytical form of RE source (Dreicer, hot-tail) can be inaccurate for fast-evolving scenarios.

Method: one-fluid equations and kinetic equation for runaway electrons coupled in terms of transition probability between the kinetic tail and bulk fluid.

\[
\begin{align*}
\partial_t \varrho + \nabla \cdot (\varrho \mathbf{u}) &= - \int m_e l_e d\mathbf{v} \quad E + \mathbf{u} \times \mathbf{B} = \eta (\mu_0^{-1} \nabla \times \mathbf{B} + e n_{e1} \mathbf{v_{e1}}) \\
\varrho (\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) &= - \nabla \cdot \mathbf{p} + \mu_0^{-1} \nabla \times \mathbf{B} \times \mathbf{B} - \sum_\alpha \mathbf{F_{e1,\alpha}} - \int m_e (\mathbf{v} - \mathbf{u}) l_e d\mathbf{v} \\
&\quad + e n_{e1} (\mathbf{E} + \mathbf{v_{e1}} \times \mathbf{B}) \\
\frac{df_{e1}}{dt} &= \sum_\alpha C_{e\alpha} [f_{e1}, f_\alpha] + l_e
\end{align*}
\]
Compute $l_e$ using backward Monte-Carlo method

The time-dependent transition probability ($l_e$) depends on the runaway probability function, which can be computed efficiently using the backward Monte-Carlo method, by following the stochastic trajectories of particles.

$$l(z, t) = \frac{f_0}{\tau} (1 - \mathbb{E}[\mathbf{1}_{\Omega_0}(Z_{t+\tau})|Z_t = z])$$
$$- \frac{f_1}{\tau} \mathbb{E}[\mathbf{1}_{\Omega_0}(Z_{t+\tau})|Z_t = z]$$

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- Analyzing the frequencies and mode structures of compressional Alfvén eigenmodes in DIII-D indicates that it can explain the kinetic instabilities observed in post-disruption.
- Calculations based on two-fluid models show that damping due to electron-ion collision for CAE is relatively weak compared to whistler waves.
- CAE can have resonance with the transit frequencies of passing electrons or precession frequencies of trapped electrons, and driven by gradient of RE distribution function in both momentum space and radial directions.
- Bounce-average kinetic simulation shows that large pitch-angle runaway electrons can be generated by the enhanced pitch-angle scattering from ions due to partially-screening effects, which can resonate with CAE.
  - Ware pinch of trapped REs can enhance radial gradient and drive the modes.
Future work

• Solve the eigenmode frequencies and structure with correct wall boundary condition using a full-wave or MHD code.
  • For $\omega \sim \omega_{ci}$, Hall term is necessary in the MHD equations
  • Collisional damping and mode growth rate can be more rigorously calculated using mode structure.

• Extend the kinetic simulation of $f$ evolution from 2D to 3D, to simulation runaway electrons at different flux surfaces.
  • Include radial transport and Ware pinch effect.

• Add quasilinear diffusion term into the kinetic simulation to self-consistently calculate the evolution of mode amplitudes and RE distribution function.
  • For discrete eigenmodes, a resonance-broadening quasilinear (RBQ) model may be required.

• Use a test-particle tracing code to calculate the spatial diffusion of REs with excited CAEs, and compare the average loss time with experiments.
Future work (continued)

• Use a time-dependent MHD simulation code to study the macroscopic instabilities (like internal kink) with RE current.
  • To better calculate RE and $q$ profiles, which are important for CAE excitation.
  • Understand MHD instabilities happening during RE plateau and find ways to stabilize and attenuate RE beam
• Build a better model to calculate RE generation during thermal quench and current quench.
  • Investigate the possibility to generate a large population of suprathermal electrons before or during TQ, to avoid strong avalanche during CQ.
• Apply the current model to explain the electron driven Alfvén modes found in other experiments.
  • Post-disruption scenario provides a new regime to test wave-particle interaction models, like RBQ.
Thank you