Towards a quasi-dynamical model for 3D MHD based on energy minimisation and relaxation

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13th February 2020

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Funding acknowledgements:
Australian Gov. Dept. Education, Australian Research Council, Simons Foundation.

- **Motivation**
- **Background, overview and approach to constructing a quasi-dynamical model**
- **Characteristic features of MRxMHD: Current sheet interfaces**
- **Characteristic features of MRxMHD: Localised Taylor relaxation**
- **Next steps and on-going work**

- **Motivation**

Evidence of MHD activity has been observed in W7-X

Recent experiments have found evidence of MHD activity in stellarators, including the observation of sawtooth-like oscillations during electron cyclotron current drive (ECCD) experiments in the second Wendelstein 7-X campaign [1].

- For example [1], in #20171206.028 (constant 1800 kW total power) a large amplitude T_e crash is preceded by multiple smaller amplitude crashes over a multi-second ECCD discharge.
- The time between successive crashes was found to increase during the discharge.
- The hypothesised mechanism is due to current redistribution and accumulation near the axis, leading to modification of the ι-profile.
- The crashes are thought to be associated MHD instabilities resulting from ι crossing a low order rational.

Details in [1] M. Zanini et al., *EPJ Web of Conferences* 203, 02013 (2019).

There are two approaches to resolving the physics of sawtooth-like oscillations in stellarators

K. Aleynikova, P. Helander et al.

- A relaxation-based equilibrium model with full stellarator geometry is used to determine post-crash current profiles.
- Provides an explanation for the role of the small amplitude crashes in contributing to the large crashes.
- Numerical results reproduce key experimental measurements to $\sim 20\%$.
- Manuscript in preparation.

Q. Yu, E. Strumberger, S. Günter et al.

- Nonlinear growth of MHD modes studied using the 2-fluid code, TM1, with circular cross-section tokamak geometry and large aspect ratio.
- Low n modes proposed to explain T_e crashes.
- Non-monotonic q-profile: $q_0 \approx 1.08 - 1.15$, $q_{edge} \approx 1.03$, $q_{min} \approx 0.97$ at $r/a = 0.32$.
- Concludes that large T_e crash is similar to sawtoothing behaviour in tokamaks, i.e. due to $(n = 1, m = 1)$ internal kink mode.

How do we interpret these findings to derive new physics knowledge in a way that consistent with extended-MHD and, ultimately, to make quantitative predictions?

Current status of initial-value extended-MHD codes for stellarator geometry

Across multiple institutions, there are efforts to develop an initial-value extended-MHD code for stellarator geometry and perform physics studies.

For example:

M3D-C1 (PPPL) (Y. Zhou, N. Ferraro et al.)
Stellarator geometry proposed to be treated using conformal mapping to axisymmetric domain.

JOREK (IPP Garching) (R. Ramasamy, N. Nikulsin et al.)
Derivation and proposed implementation of a reduced MHD model that is consistent with extended-MHD. Simultaneously, implementation of a 'virtual current' to generate stellarator rotational transform in an axisymmetric domain.

NIMROD (UW-Madison) (T. Bechtel, C. Hegna, C. Sovinec et al.)
Studies of stellarator equilibrium β-limits performed of helically symmetric (straight) stellarator geometry by assuming a helical magnetic potential.

- **Motivation**

- **Background, overview and approach to constructing a quasi-dynamical model**

Our long-term goal is to develop a computationally efficient quasi-dynamical model for 3D MHD

Our long-term vision:

- To approximate extended-MHD* plasma evolution by a sequence of 3D MHD equilibria connected via re-equilibrating relaxation events.

Why?

To address the need for predictive and computationally efficient global modelling of macroscopic dynamics in stellarators.

General constraints and considerations:

- Consistency with the broader physics setting of extended-MHD* requires careful treatment of multiple time and length scales.
- The separation of timescales between the relaxation mechanism and imposed constraints, particularly in the resistive regime, is not guaranteed.
- This leads to a competition of timescales which must be resolved and is likely highly sensitive to nonlinearity.

* A single fluid model which may include more physics than resistive MHD (e.g. viscosity, anisotropic thermal conductivity, sources and sinks of heat, particles, and momentum).

Example of a cognate previous approach:

- Clemente et al. [1] applied the Grad-Hogan diffusion model [2] to show that, assuming a uniform but time dependent plasma temperature, the evolution of field reversed configurations (FRCs) could be approximated by a continuous sequence of equilibria.
- Assuming axisymmetry and spatially constant temperature, it is shown that solutions at any t can be parametrised by a single time-dependent parameter. The time evolution of this parameter is determined by imposing a constraint which must be satisfied at all t.
- In the Grad-Hogan model [2], assuming axisymmetry, two timescales are identified; a fast timescale in which ψ diffuses through approximately fixed p, and vice-versa for the slow timescale.

A particular challenge for our approach is to accommodate the variety of topological structures which can be supported in 3D MHD equilibria.

[1] R. Clemente et al., *Plasma physics and controlled nuclear fusion* 1988 V.2 (1989). [2] H. Grad & J. Hogan, *Physical Review Letters* 24.24 (1970).

Topological features of 3D MHD equilibria follow from the Hamiltonian nature of $\vec{B}$

- In general, magnetic field lines can be described by 1+1/2 DoF Hamiltonian.
- When $\partial_\zeta \to 0$ (i.e. axisymmetry), reduces to 1 DoF Hamiltonian. Known to be completely integrable.
- Completely integrable ⇒ continuously nested flux surfaces guaranteed.
- Correspondingly, 1+1/2 DoF Hamiltonian is known to be not completely integrable in general.

In 3D, magnetic fields can support a combination of magnetic islands, stochastic regions and some flux surfaces.

For a quasi-dynamical sequence that we eventually wish to construct, we require an equilibrium model which does not assume continuously nested flux surfaces.

Which 3D MHD equilibrium model/s meet our criteria?

(DoF = degree of freedom)

(Completely integrable = globally solvable in some sense, see [Kozlov, *Uspekhi Mat. Nauk,* 1983])

There are many mathematically valid 3D MHD equilibria, only a subset of which may be physically useful

We consider the set of static, non-dissipative MHD equilibrium equations:

$$\nabla p = \mathbf{J} \times \mathbf{B} \qquad (\text{we require } \mathbf{B} \cdot \nabla p = 0)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

which permit both smooth and non-smooth solutions.

- Smooth solutions have continuously nested flux surfaces.
- Non-smooth solutions: Proof by Bruno & Laurence [1] for globally non-uniform stepped pressure in toroidal domain.
- Non-smooth solutions: Pressure jumps occur at highly irrational surfaces, which are the most robust to perturbations away from axisymmetry (KAM invariant tori).
- Non-smooth solutions: Discontinuities satisfy the jump conditions: $[\![p + B^2/2\mu_0]\!] = 0$ and $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$.

Whether discontinuous mathematical solutions have a meaningful physical interpretation remains an open question.

[1] O. P. Bruno & P. Laurence, *Commun. Pure Appl. Math*, 49.7 (1996)

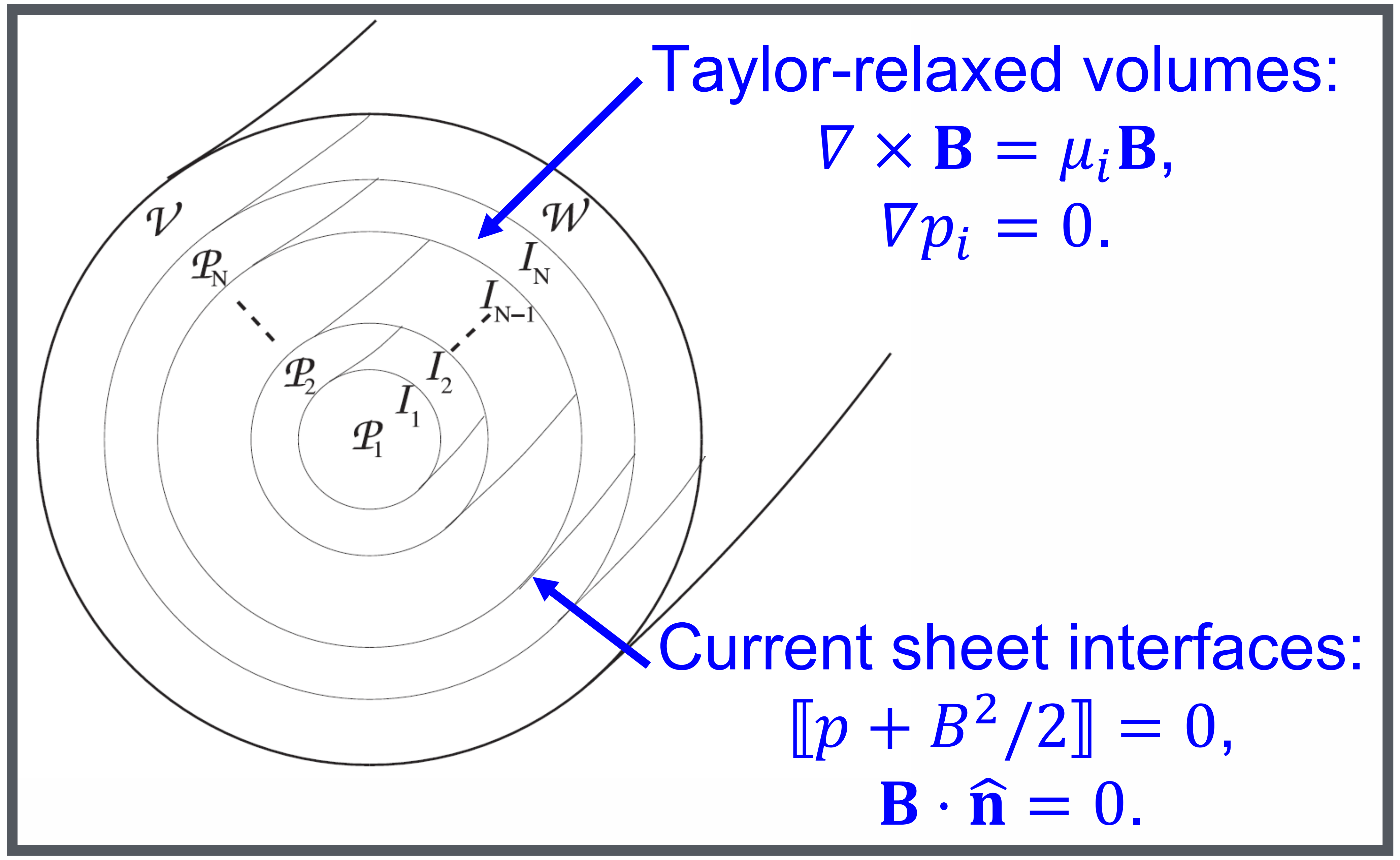
Multi-Region Relaxed MHD (MRxMHD): A discontinuous model based on energy minimisation

- Developed by R. L. Dewar and collaborators [1-3] to construct stepped-pressure equilibria.
- The plasma discretised into N volumes and the MRxMHD energy functional, F, is minimised subject to a finite set of constraints.
- The equilibrium equations are the Euler-Lagrange equations which follow from extremising F.

Potential energy (in each i).

$$F = \sum_{i}^{N} \left[\underbrace{\int_{V_i} \left(\frac{p_i}{\gamma - 1} + \frac{B^2}{2} \right) dv}_{\text{Potential energy}} - \underbrace{\frac{\mu_i}{2} \left(K_i - K_{0,i} \right)}_{\text{Helicity constraint}} \right]$$

Helicity constraint (in each i).



The MRxMHD formulation describes static equilibria. How can we use this theory as the basis for a future dynamical model?

[1] M. J. Hole et al., *JPP* 72 (2006); [2] M. J. Hole et al., *NF* 47 (2007); [3] R. L. Dewar et al., *JPP* 81 (2015).

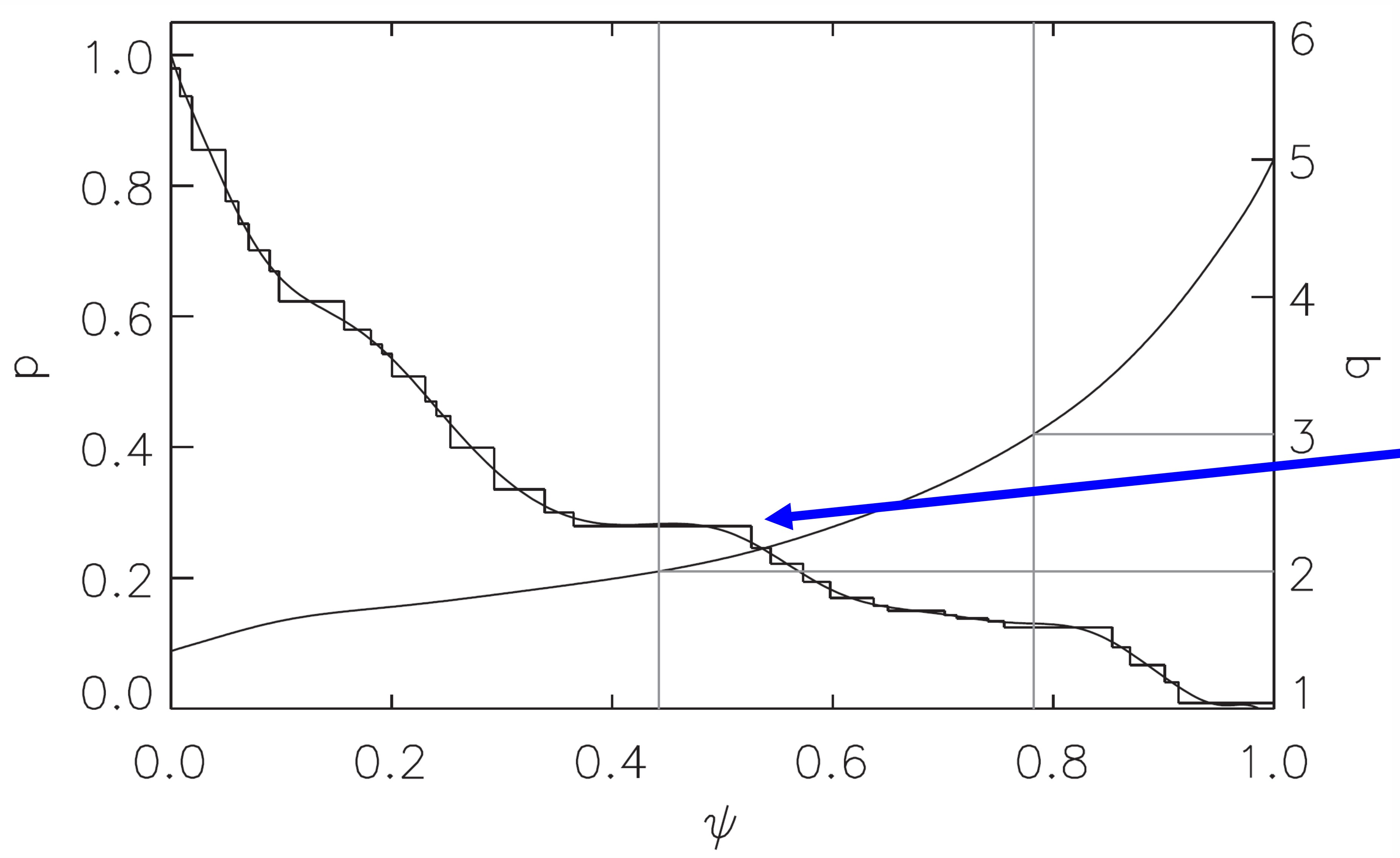
Stepped Pressure Equilibrium Code (SPEC)

- The MRxMHD model is the theoretical basis of the Stepped Pressure Equilibrium Code (SPEC) developed by S. Hudson (PPPL) [1].
- SPEC is used to study tokamak, stellarator, and reversed field pinch (RFP) configurations.
- There are currently users from multiple institutions (across 3 continents) and 9 active developers.

Example SPEC calculation of DIII-D equilibrium with RMP field applied [1]:

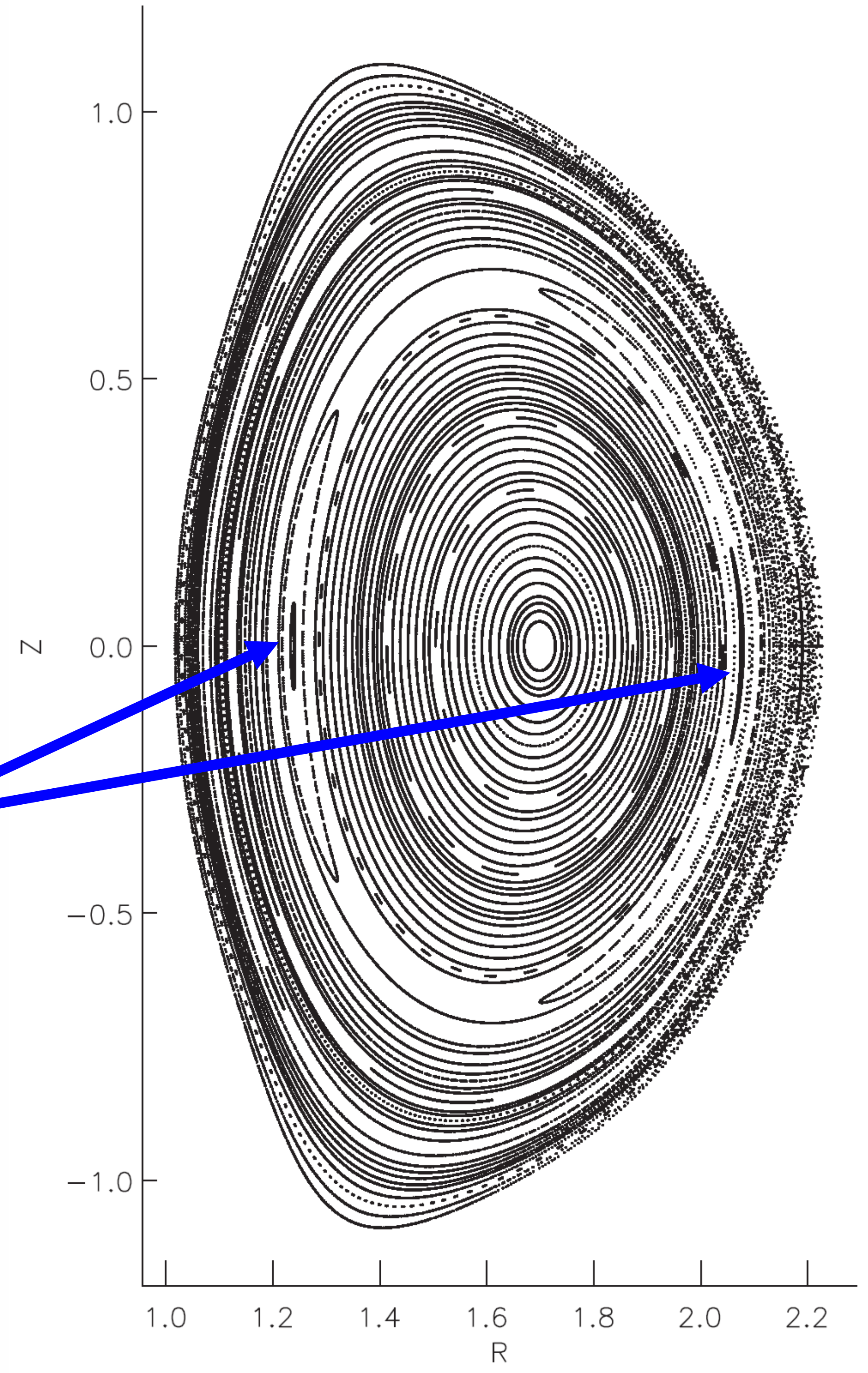
By choosing sufficiently large N, smooth pressure profiles can be approximated arbitrarily well. (RHS: $N = 32$)

Pressure profile taken from STELLOPT reconstruction and discretised for SPEC input with $N = 32$ volumes.



Large islands at $q = 2$ surface corresponding to significant flattening of pressure profile.

Poincaré plot of SPEC computed equilibrium



[1] S. R. Hudson et al., *PoP* 19 (2012)

Long term goal:
To develop a quasi-dynamical reduced model of 3D MHD which approximates extended-MHD evolution by a sequence of 3D MHD equilibria connected via relaxation events.

We seek to develop a model that is:
- Consistent with extended-MHD dynamics on $\boldsymbol{\tau} < \boldsymbol{\tau}_{\boldsymbol{transport}}$
- Quantitative
- Predictive
- Computationally efficient

Idea:
To use MRxMHD as the basis for such a quasi-dynamical reduced model.

Before we try and write down a formulation for this reduced model, we need first to determine under what conditions this is possible. This requires contextualising MRxMHD in the broader physics setting of extended-MHD.

To do this, we study two characteristic features of the MRxMHD equilibrium model within the framework of extended-MHD; **(i) current sheet interfaces** and **(ii) localised Taylor relaxation**.

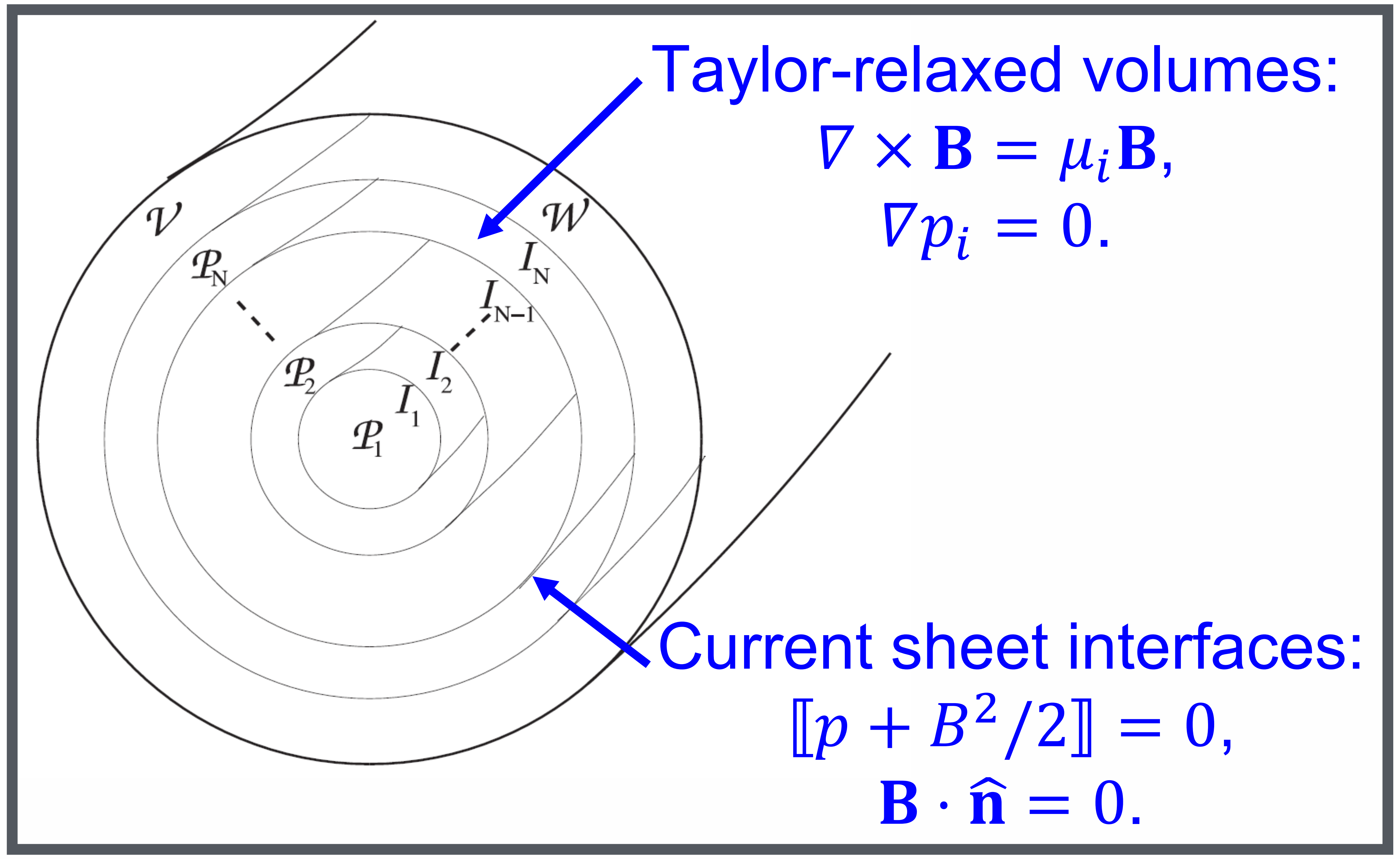
- **Motivation**

- **Background, overview and approach to constructing a quasi-dynamical model**

- **Characteristic features of MRxMHD: Current sheet interfaces**

Can MRxMHD interfaces be interpreted as highly localised (smooth) pressure gradients?

- The dynamical mechanism by which MRxMHD interfaces may form remains unclear.
- The robustness of MRxMHD interfaces to dissipation and extended-MHD dynamics remains to be fully determined.



By developing a new model, we address two key questions:

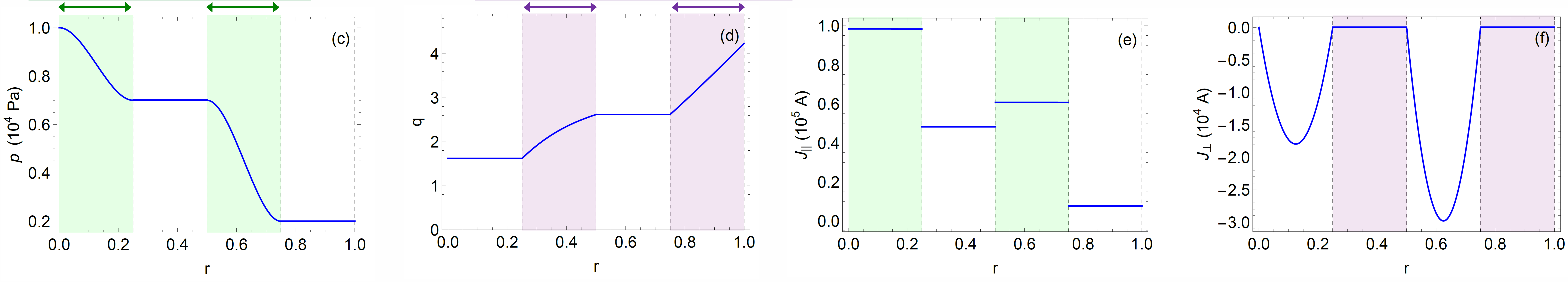
Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?

What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?

A new (continuous) cylindrical equilibrium model for studying stepped pressure states [1]

We develop a new equilibrium model to directly compare properties of continuous and discontinuous equilibria, where MRxMHD interfaces are extended to have finite volume.

'Ideal' regions ($\nabla p \neq 0$). 'Relaxed' regions ($\nabla p = 0$).



When $\nabla p \neq 0$:

- Equilibrium: $\mathbf{J} \times \mathbf{B} = \nabla p$.
- q is prescribed: constant and irrational, i.e. no resonances.

When $\nabla p = 0$:

- Equilibrium: $\nabla \times \mathbf{B} = \mu_i \mathbf{B}$.
- p is prescribed.

- Equilibria are constructed by matching p, ∇p, B_z and q at the internal boundaries, ensuring no current sheets.
- In 3D, **flux surfaces** would be localised to **ideal regions**. **Magnetic islands** and **chaotic fields** supported in **relaxed regions**.

- **Motivation**

- **Background, overview and approach to constructing a quasi-dynamical model**

- **Characteristic features of MRxMHD: Current sheet interfaces**

Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?

Stability analysis supports realisability of MRxMHD interfaces as highly localised pressure gradients

Finite-width MRxMHD interfaces must satisfy realisability conditions. We assess this analytically using linear stability analysis.

Key findings (full details in [1]):

- Recall resonant surfaces ($r = r_s$) occur whenever $q = m/n$ and are localised to 'relaxed' regions ($\nabla p = 0$).
- Ideally stable to internal modes when $\frac{B_\theta}{rB_z} \sim \epsilon$, $q \sim 1$, $\frac{\mu_0 p}{B_z^2} \sim \epsilon$ or ϵ^2 and $q(r = 0) > 1$ where $\epsilon = a/R$. Note: Suydam's criterion is satisfied and external modes precluded by construction.
- By varying the width of the ideal regions ($\nabla p \neq 0$) and analysing Δ', we find that the discontinuous pressure limit is robust against low and moderate m tearing modes.

Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?
Stability analysis suggests yes, provided q is irrational.

[1] A. M. Wright et al., *PoP* 26.6 (2019)

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Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?

What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?

Our continuous model can be used to study MRxMHD interface break-up dynamics directly with extended-MHD

What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?

- As MRxMHD interfaces break-up, the plasma in adjacent MRxMHD volumes can interact leading to redistribution of helicity, for example.

- MRxMHD interface dynamics have been studied within the framework of MRxMHD, e.g. ‘pressure jump Hamiltonian’ [1].

Using our continuous, finite-width interface model, we can study time-dependent interface dynamics directly with extended-MHD by:

- Interfacing directly with initial-value extended-MHD codes.
- Deriving analytic tools to study linear stability (e.g. Δ' and the energy principle).
- Investigating the effect of q-profile ‘irrationality’ on stability and dynamics.

[1] M. McGann et al., *Physics Letters A* 374.33 (2010)

A quasi-dynamical sequence of equilibria may be based on the successive break-up of interfaces

Key ideas:

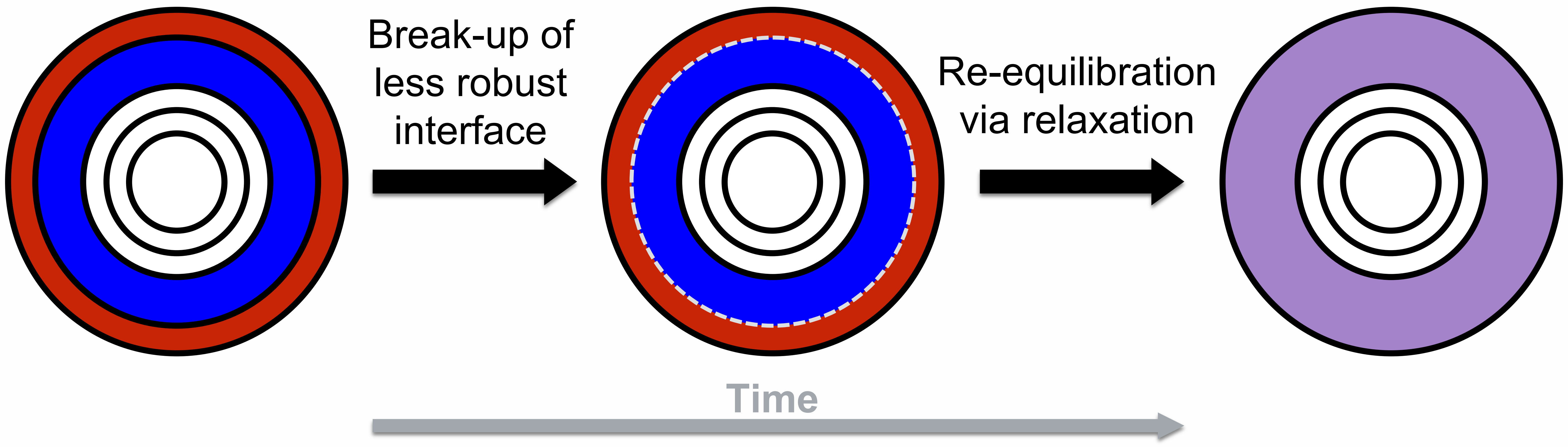
- Not all flux surfaces are equal.

The KAM theorem and properties of irrational numbers (number theory, Diophantine condition) predicts a hierarchy of surfaces, based on robustness to 3D perturbations. The ‘pressure jump Hamiltonian’ [1] is one approach to computing this hierarchy.

- The plasma can be partitioned into a finite number of volumes by the most robust flux surfaces.

We do not expect the local MRxMHD constraints to be equally well-conserved within each volume.

The successive break-up of surfaces, which exploits the Hamiltonian nature of magnetic fields, could be used to prescribe the sequence of equilibria which would comprise our quasi-dynamical model.



[1] M. McGann et al., *Physics Letters A* 374.33 (2010)

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Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?

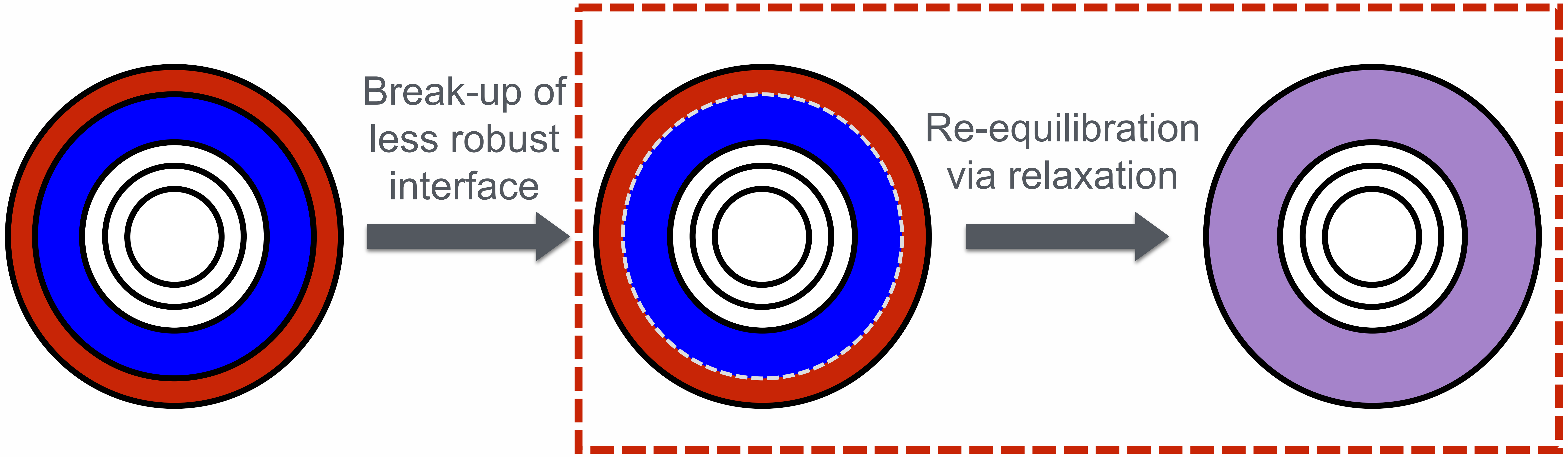
What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?

- **Characteristic features of MRxMHD: Localised Taylor relaxation**

Under what conditions can successive equilibria in the quasi-dynamical sequence be reached by the plasma in a way that is consistent with both MRxMHD and extended-MHD?

Dynamical accessibility is essential for a viable quasi-dynamical model

Assuming that an interface has broken-up, we now consider the dynamical process by which the plasma evolves to a new equilibrium.



In the MRxMHD model, this is due to Taylor relaxation.

Ultimately, we want to determine:

Under what conditions can successive equilibria in a quasi-dynamical sequence be reached by the plasma in a way that is consistent with both MRxMHD and extended-MHD?

To start addressing this complex question:

We examine in detail the nonlinear dynamics of Taylor relaxation using a simple model.

A simple force-free equilibrium model as a testbed for relaxation pathways

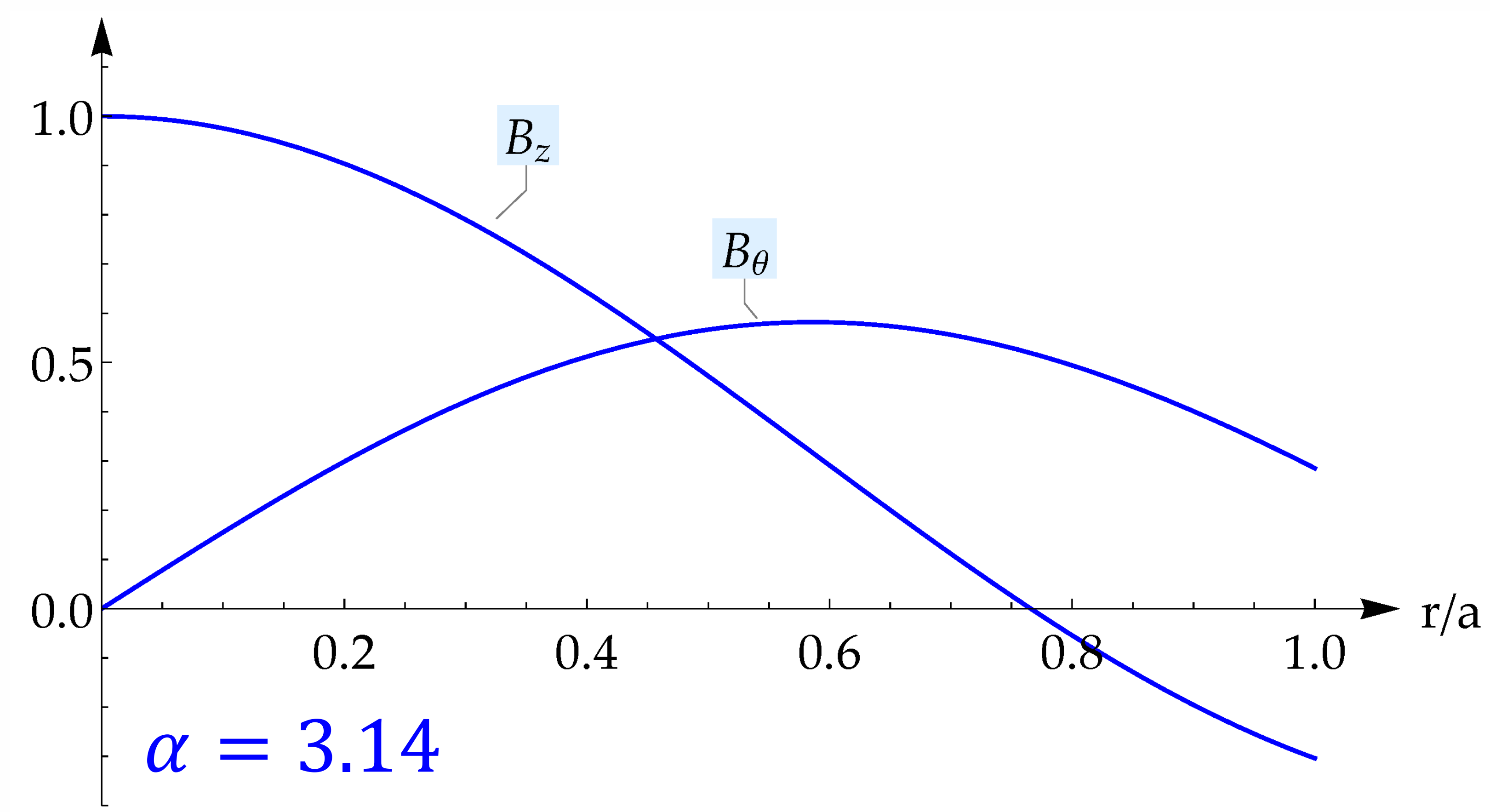
Linear force-free equilibrium $\vec{\mathrm{B}}$ in a periodic cylinder ($L = 2\pi R_0$) with inverse aspect ratio $\epsilon = a/R_0$ and boundary condition $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ at $r = a$. The equilibrium equation is:

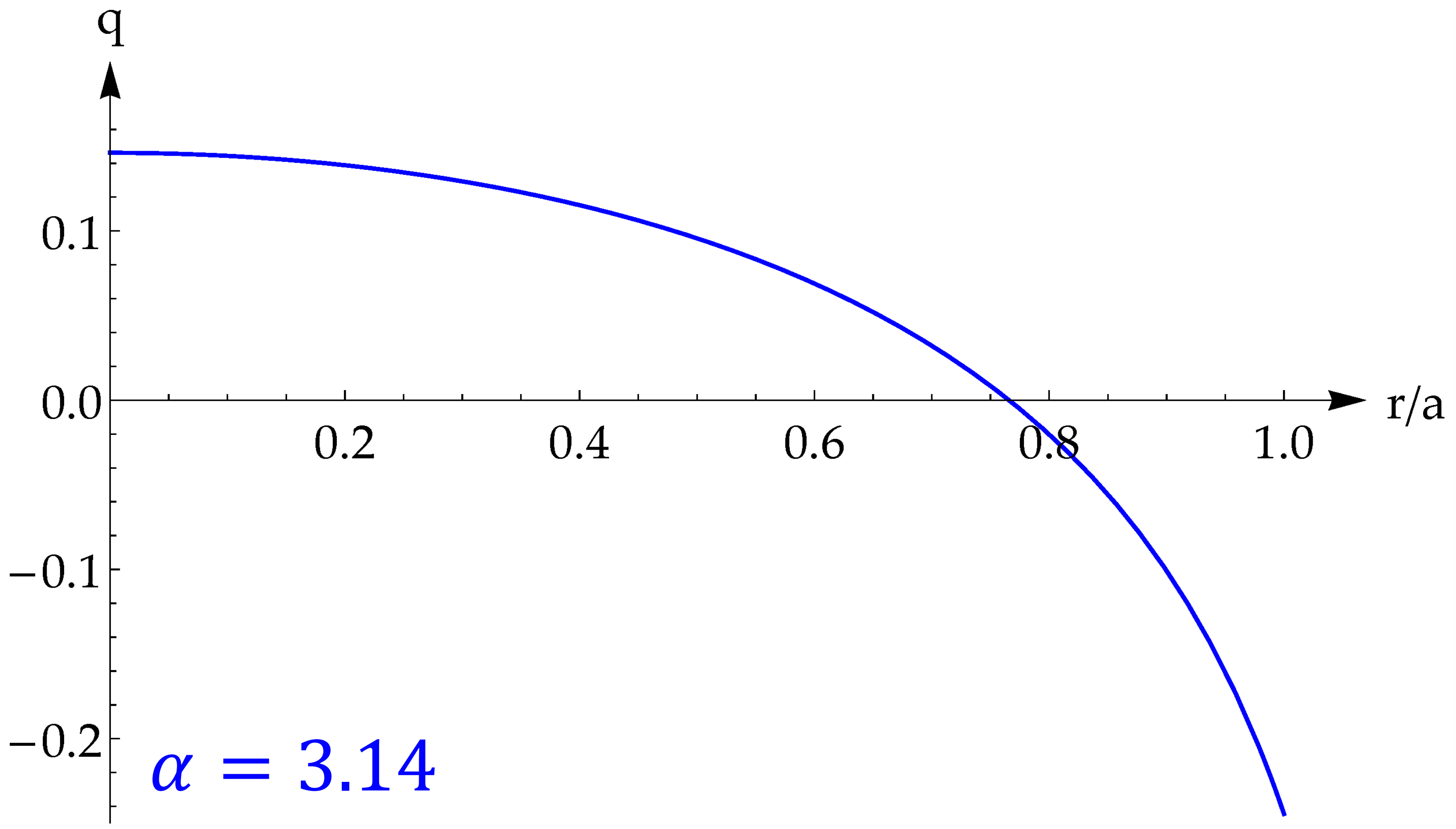
$$\nabla \times \mathbf{B} = \alpha a^{-1} \mathbf{B},$$

where $\alpha = a\mu_0 \mathrm{J}_{\parallel}/B^2 = constant$ and can have **axisymmetric** and **non-axisymmetric** solutions [1].

The **axisymmetric solution** is give by $\mathbf{B} = |\mathbf{B}(r=0)|\{0, J_1(\alpha r/a), J_0(\alpha r/a)\}$ where J_i are the standard Bessel functions, which satisfies $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ at $r = a$ for all α.

Typical axisymmetric profiles in the regime of interest ($2.405 < \alpha < 3.831$):





The q-profile (necessarily RFP-like) is given by:

$$\mathrm{q} = \frac{r B_z}{R_0 B_\theta}$$

Resonance condition:

$$q(r = r_s) = m/n$$

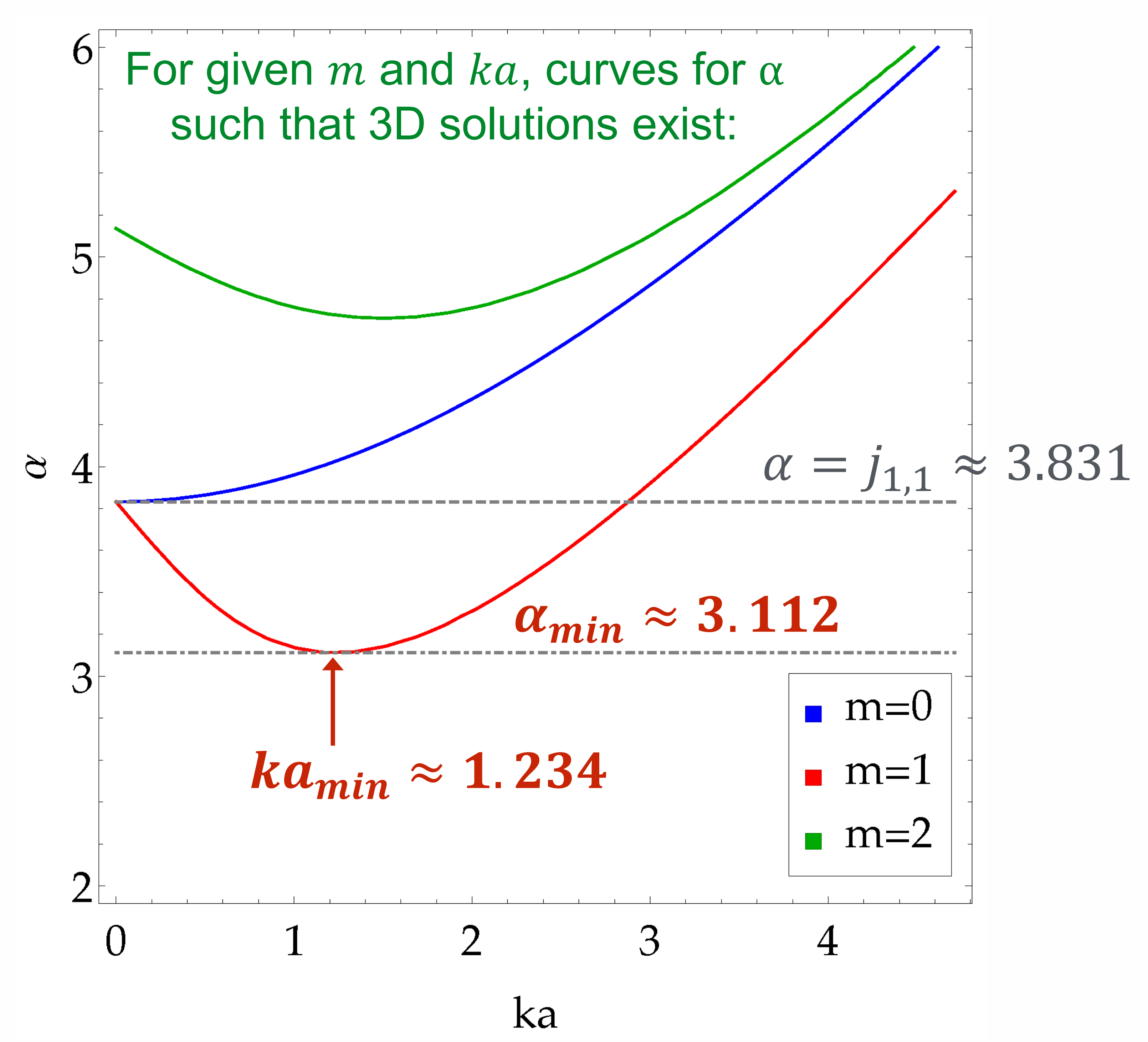
[1] S. Chandrasekhar & P. C. Kendall, *ApJ* 126 (1957)

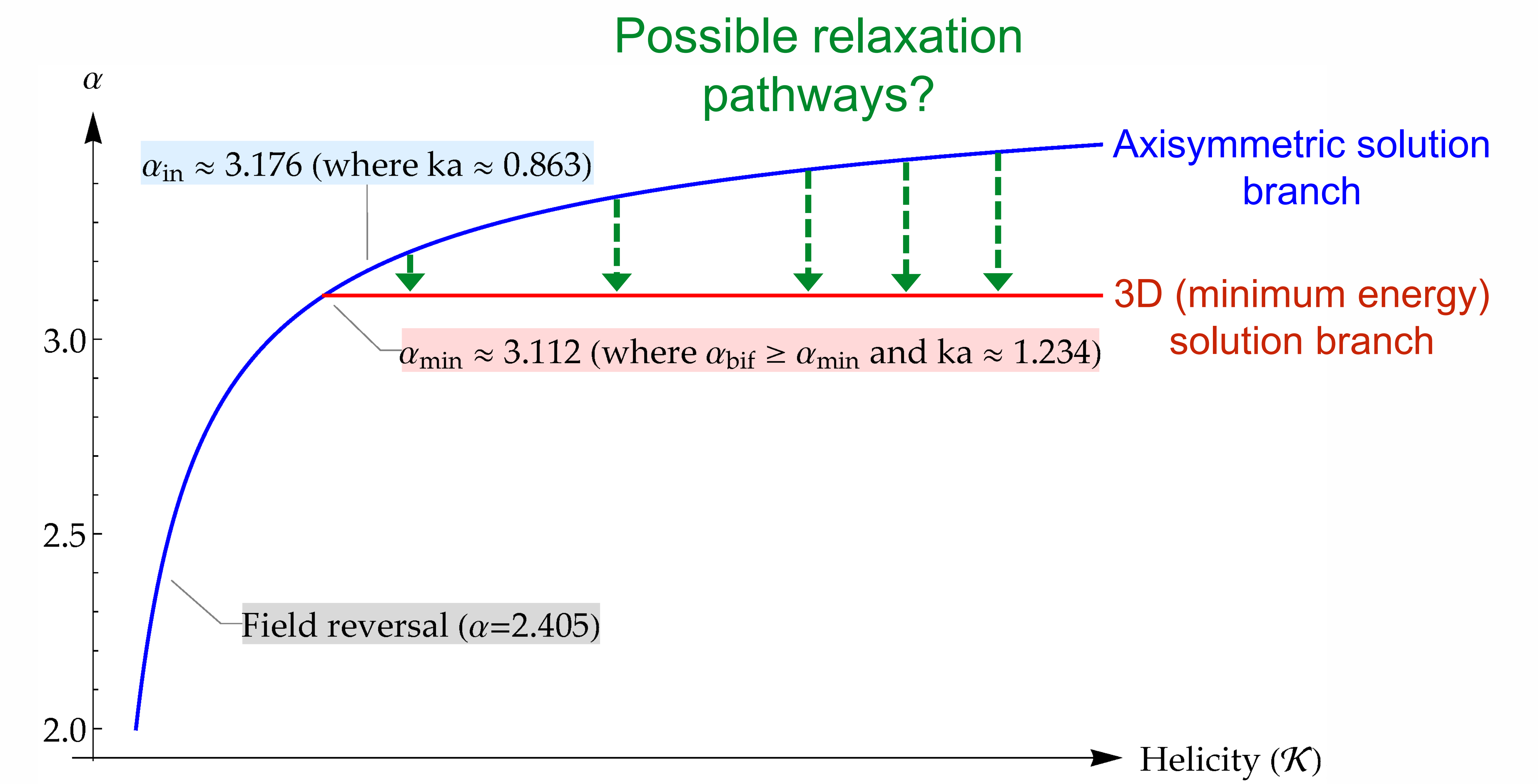
A simple force-free equilibrium model as a testbed for relaxation pathways [1]

Non-axisymmetric solutions [2] exist only for α satisfying:

$$-\frac{1}{y}\left[ka\, J_m'(y) + \frac{m\alpha}{y} J_m(y)\right] = 0$$

where $y^2 = \alpha^2 - (ka)^2$, $ka = -\epsilon n$, $\epsilon = a/R_0$ and n is the toroidal mode number.





[1] A. M. Wright et al., *PPCF* (submitted Dec. 2019); [2] J. B. Taylor, *Rev. Mod. Phys.* 58.3 (1986)

A simple force-free equilibrium model as a testbed for relaxation pathways is well-justified

Pros:

- The existence of a parameter space bifurcation is suited to testing dynamical accessibility and proposed relaxation pathways.
- Analytically tractable.

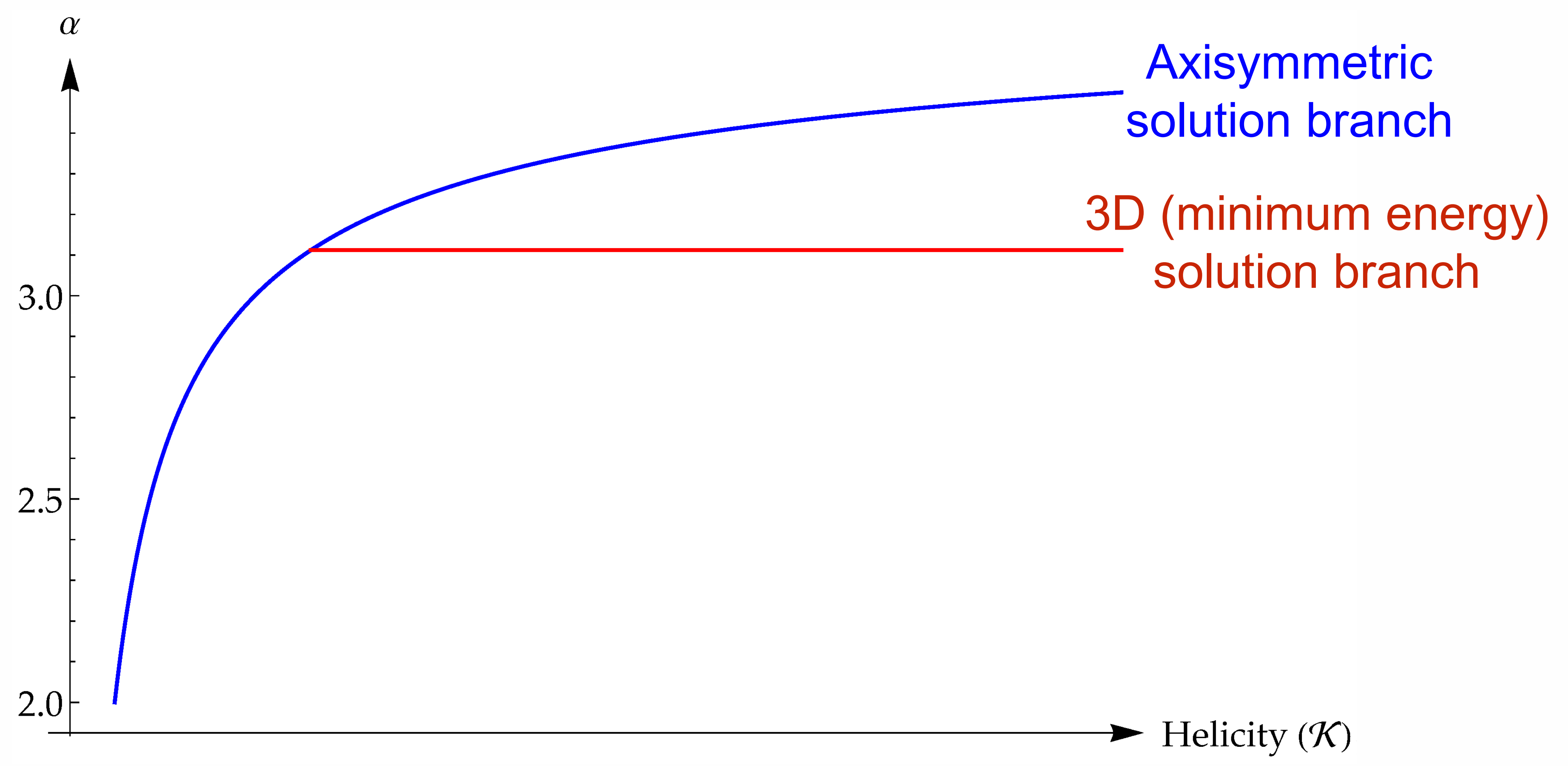
Apparent limitations:

- Necessarily RFP-like equilibrium profiles.

Nonetheless, we persist because:

- The model is equivalent to SPEC with $N = 1$ volumes.
- SPEC predicts the 3D solution to be energetically favourable.
- Setting $N > 1$ we can construct tokamak-like q-profiles.

Both solutions branches are recovered by SPEC



The $N = 1$ model is the first step to understanding dynamical accessibility of MRxMHD equilibria for fusion-relevant q-profiles (in tokamak and stellarator geometries).

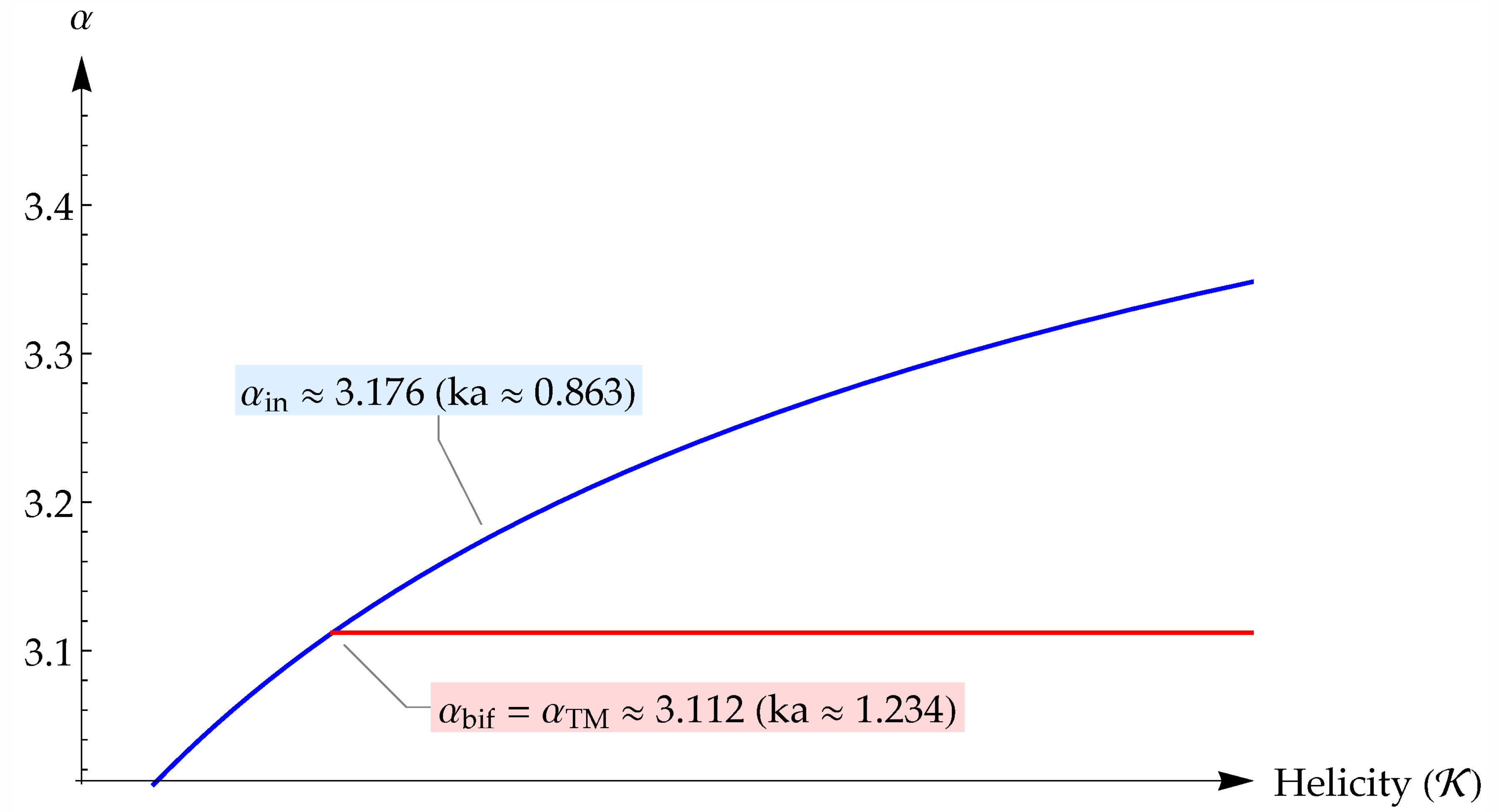
Linear stability analysis is one approach to classifying potential relaxation pathways

By performing linear stability analysis of the axisymmetric solution branch with Newcomb's criterion and Δ' we identify three critical values:

$\boldsymbol{\alpha_{bif}}$: bifurcation of axisymmetric and 3D solution branches.

$\boldsymbol{\alpha_{TM}}$: stability boundary of the tearing mode.

$\boldsymbol{\alpha_{in}}$: stability boundary for the least stable ideal mode.



- The bifurcation is associated with destabilisation of an $m = 1$ tearing mode.
- Each critical α is associated with a $(m = 1, n)$ mode but n depends on aspect ratio.

This suggests multiple possible relaxation pathways, leading to **3** topologically distinct states.

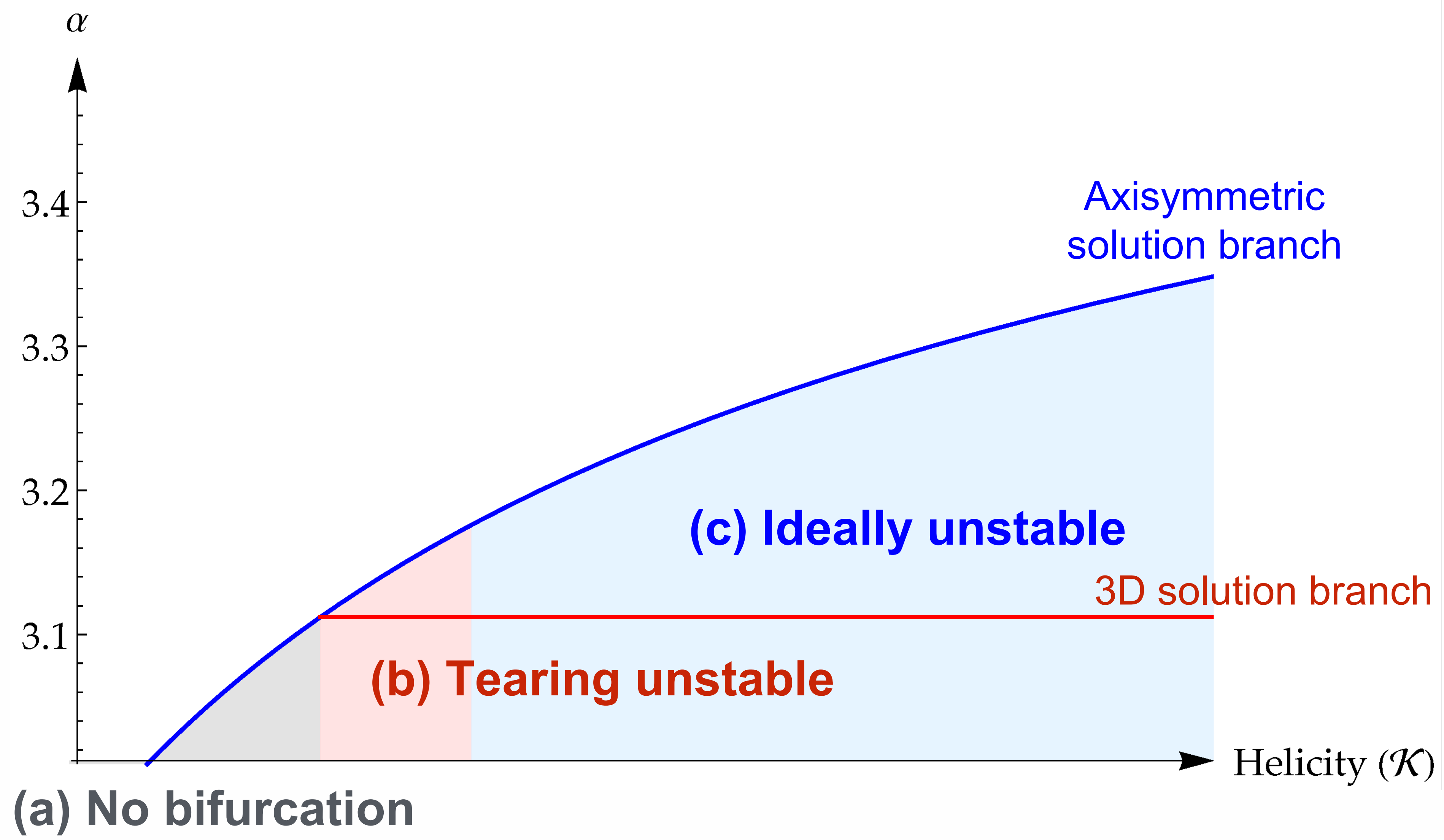
Linear stability allows identification and classification of potential relaxation pathways

- Linear stability analysis suggests 3 dynamical regimes:

a) $\alpha < \alpha_{bif}$: No 3D solution satisfying $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ at $r = a$.

b) $\alpha_{TM} < \alpha < \alpha_{in}$: Relaxation via non-ideal pathways.

c) $\alpha > \alpha_{in}$: Relaxation via ideal pathways.



- With 3 topologically distinct 3D states:

i. Reconnected state with stochastisation due to nonlinear interactions between multiple islands.

ii. Reconnected state dominated by a single $(m = 1, n)$ magnetic island.

iii. Topology-preserving helically deformed state.

Different models of relaxation have been proposed, for example:

Taylor relaxation [1] requires helicity to be sufficiently well-conserved in the presence of resistive processes (i.e. reconnection). This is postulated to occur when the dynamics are dominated by short wavelength modes so that magnetic energy decays faster than helicity:

$$\dot{K} \sim -2\eta \sum k B_k^2 \quad \text{compared to} \quad \dot{W} \sim -\eta \sum k^2 B_k^2$$

The relaxation model of Qin et al. [2] does not assume W decays faster than K and is valid for arbitrary k and low β.

Both relaxation models permit relaxation to a linear force-free state.

Linear analysis permits a simple classification of the parameter space and some insight into the preferred relaxation pathway.

Next, we use M3D-C1 to study the full nonlinear relaxation dynamics.

[1] J. B. Taylor, *Rev. Mod. Phys.* 58.3 (1986); [2] H. Qin et al., *Physical Review Letters* 109.23 (2012)

Numerical study of nonlinear relaxation dynamics with M3D-C1 [1]

- M3D-C1 is an initial-value extended-MHD code developed by PPPL (S. Jardin, N. Ferraro et al.).
- The dynamical model is the set of single fluid extended-MHD equations [1]:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$n m_i \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi + \mathbf{F}$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p + \Gamma(p \nabla \cdot \mathbf{u}) = (\Gamma - 1)[Q - \nabla \cdot \mathbf{q} + \eta J^2 - \mathbf{u} \cdot \mathbf{F} - \Pi : \nabla \mathbf{u}]$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J}$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

- With isotropic resistivity (η) and viscosity (Π), implicit external forces ($\mathbf{F}$) and heat sources (Q).
- Thermal conductivity model:

$$\mathbf{q} = -\kappa_t \nabla T - \kappa_\parallel \hat{\mathbf{b}}\hat{\mathbf{b}} \nabla T$$

where $\hat{\mathbf{b}} = \mathbf{B}/B$ and $T = T_i + T_e$.

[1] S. C. Jardin et al., *Comput. Sci. Discov.* 5 (2012)

Computational set-up and initialisation of M3D-C1

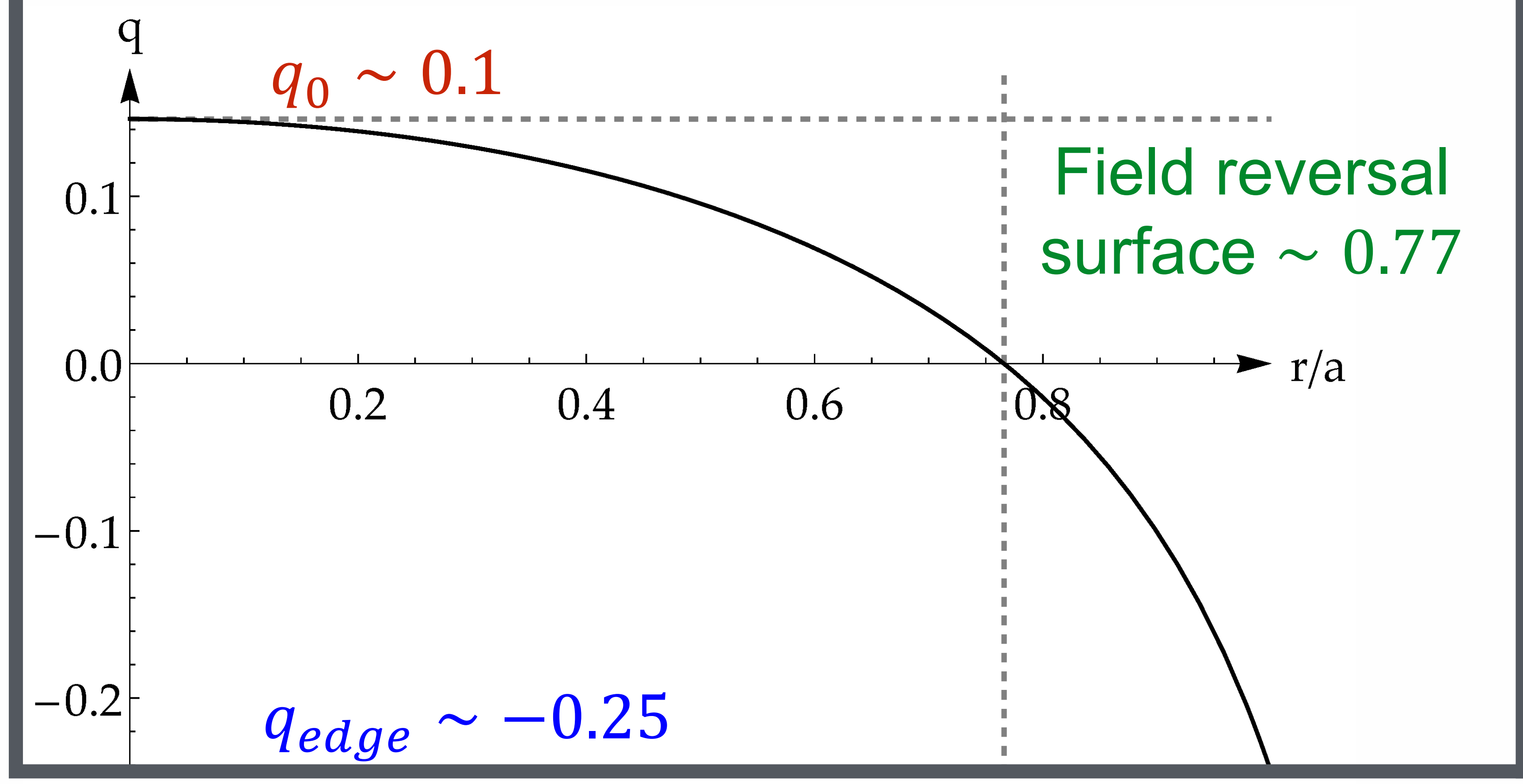
- In cylindrical coordinates (R, ϕ, Z) the vector potential $(\mathbf{A})$ and velocity $(\mathbf{u})$ are represented as:

$$\mathbf{A} = R^2 \nabla\phi \times \nabla f + \psi \nabla\phi$$

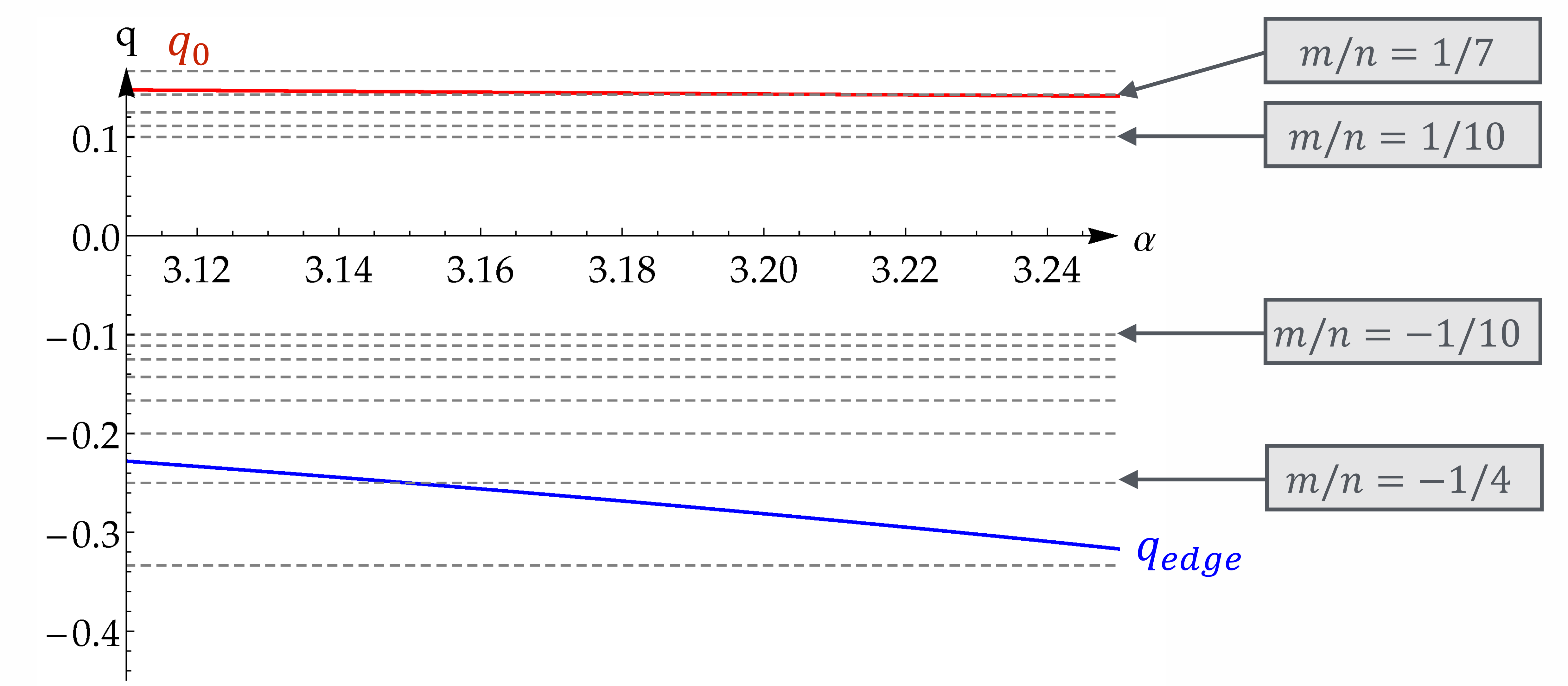
$$\mathbf{u} = R^2 \nabla U \times \nabla\phi + R^2 \omega \nabla\phi + \frac{1}{R^2} \nabla_\perp \chi$$

- Six scalar fields $(f, \psi, U, \omega, \chi, p)$ advanced with split implicit method. Further details in [1].
- Cylindrical geometry with RFX-mod parameters ($a = 0.459\text{m}$ and $R_0 = 2\text{m}$).
- Dirichlet and no-slip boundary conditions with $\mathbf{B} \cdot \hat{\mathbf{n}} = 0$ and $\mathbf{u} \cdot \hat{\mathbf{n}} = 0$ at $r = a$.

Parameter values:	
Aspect ratio	4.3573
$\alpha_{TM} = \alpha_{bif}$	3.11557 $(m = 1, n = -5)$
α_{in}	3.15794 $(m = 1, n = -4)$



Low-order resonant surfaces:

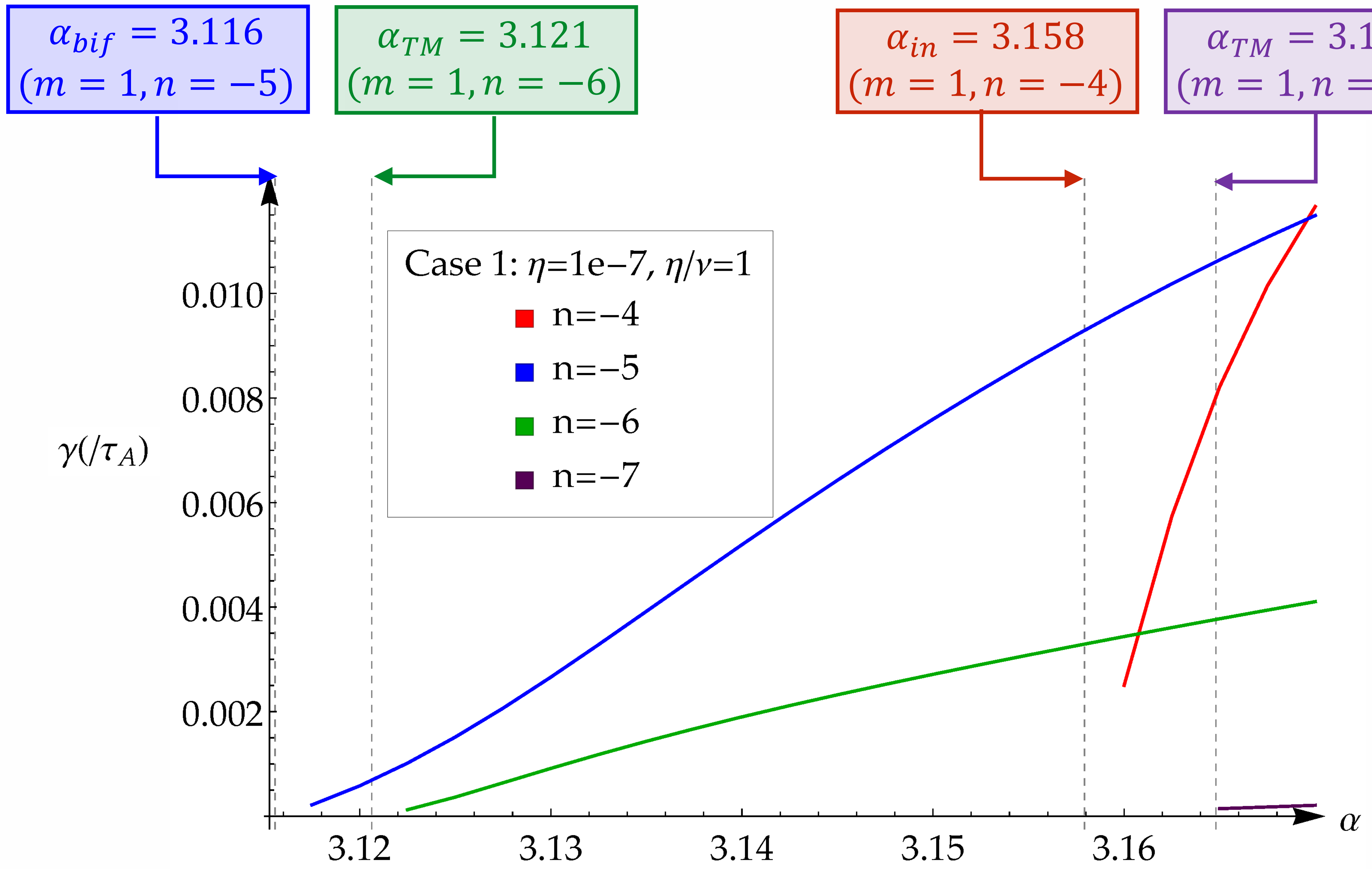


[1] S. C. Jardin et al., *Comput. Sci. Discov.* 5 (2012)

Linear studies performed to verify analytic stability boundaries and calculate growth rates

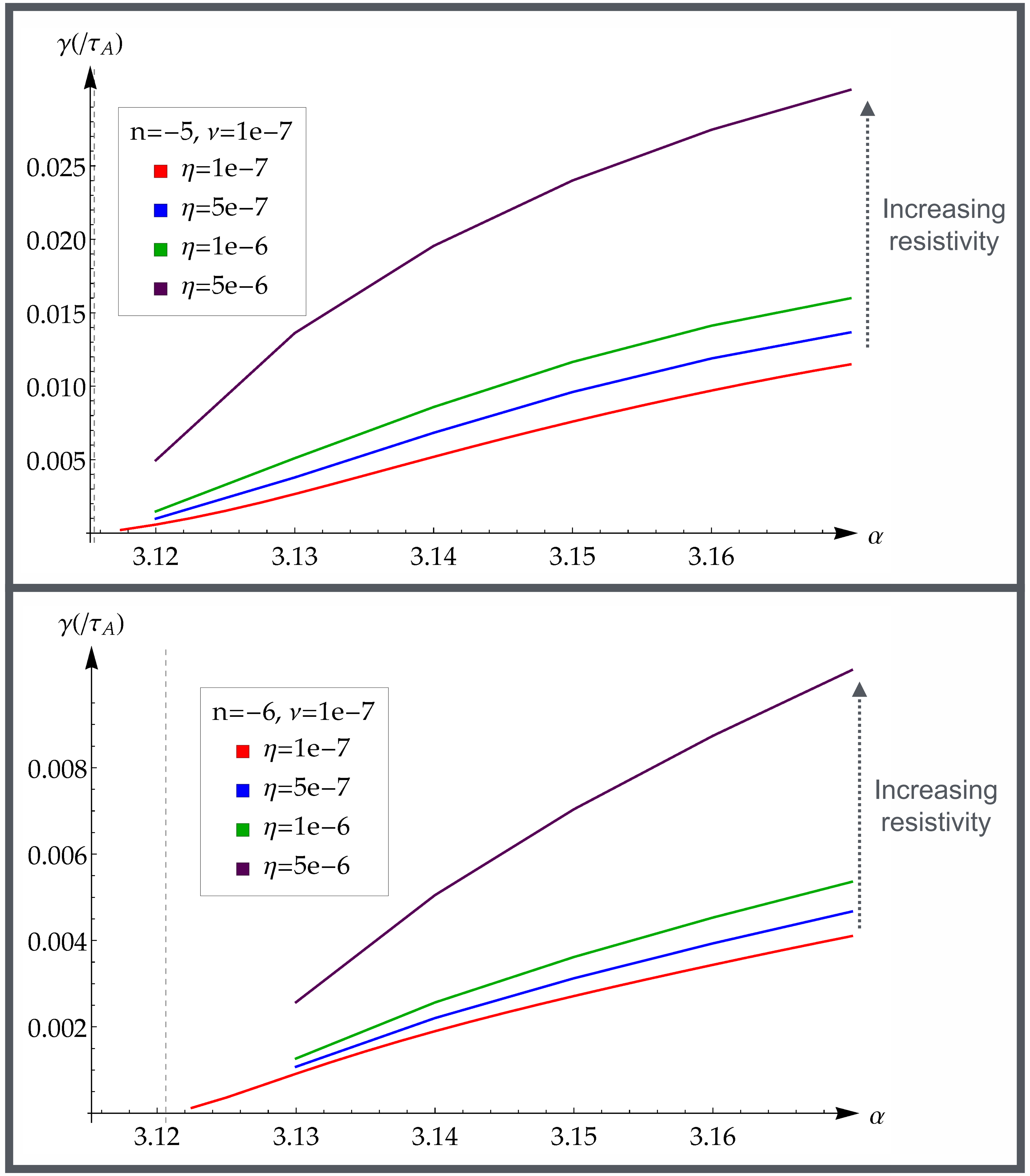
We perform linear parameter scans in the α regime of interest to:

- Verify analytically calculated stability boundaries.
- Calculate linear growth rates.
- Verify η dependence of resistive modes.



(Grey lines denote analytic stability boundaries.)

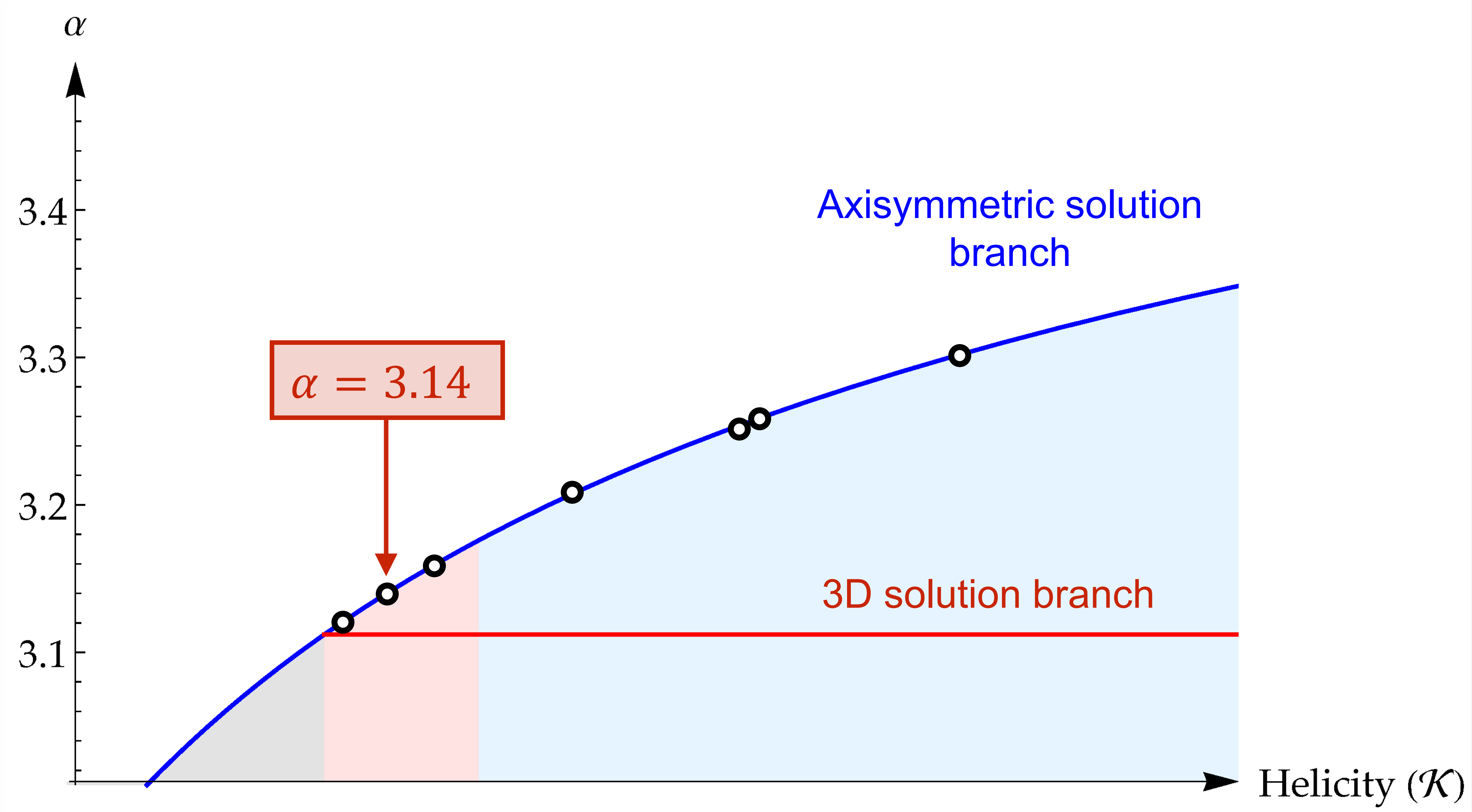
Verifying η-dependence of $m = 1, n = -5$ and -6 modes:



21k triangular elements (unstructured mesh): $S = \tau_A/\tau_R \sim 10^6$, $P_m = \nu/\eta = 1$, $\kappa_t/\kappa_\parallel = 10^{-7}$, $\beta = \beta_p\beta_t/(\beta_p + \beta_t) \sim 0.4$.

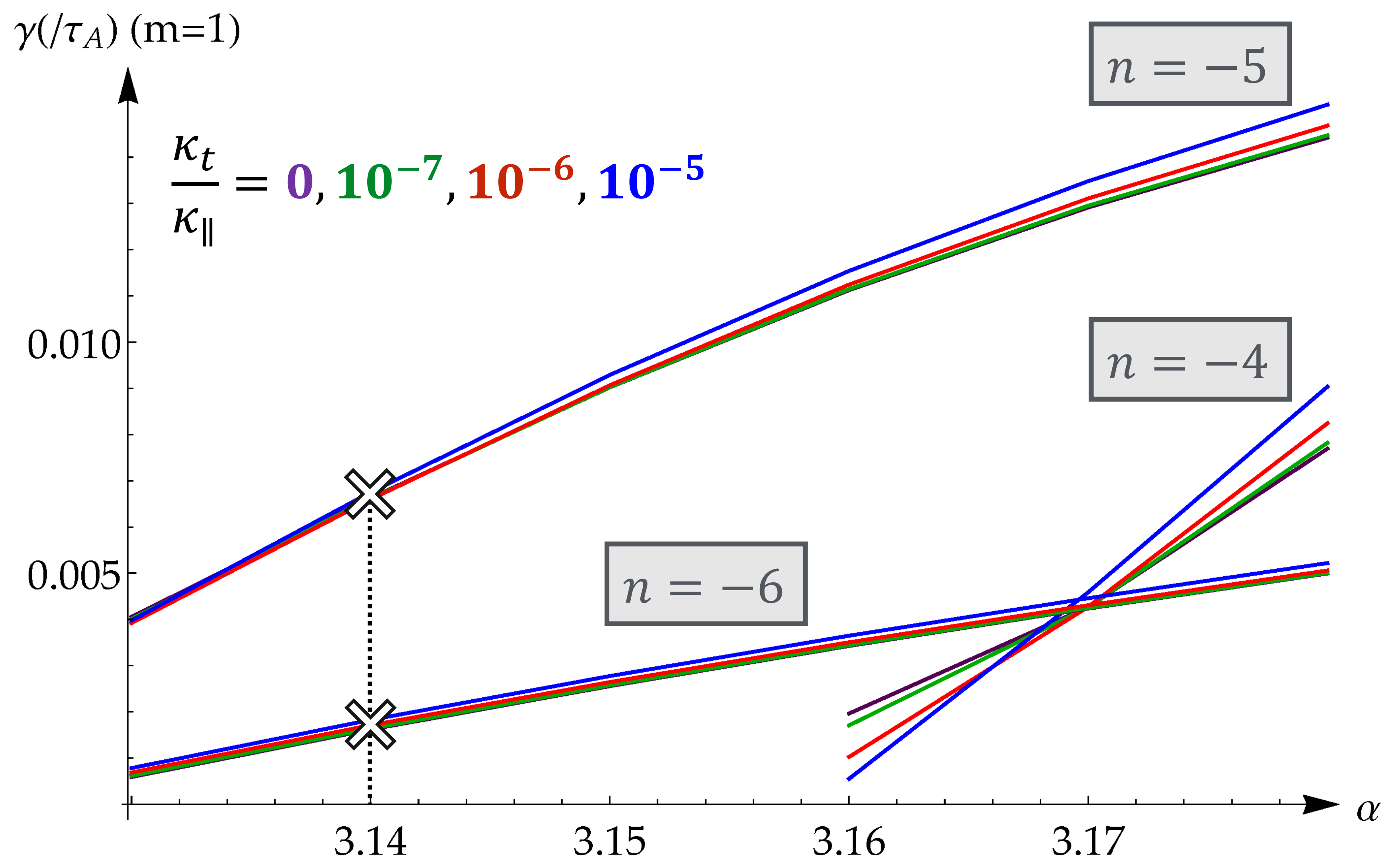
Multiple nonlinear simulations find plasma evolution is qualitatively different to linear predictions

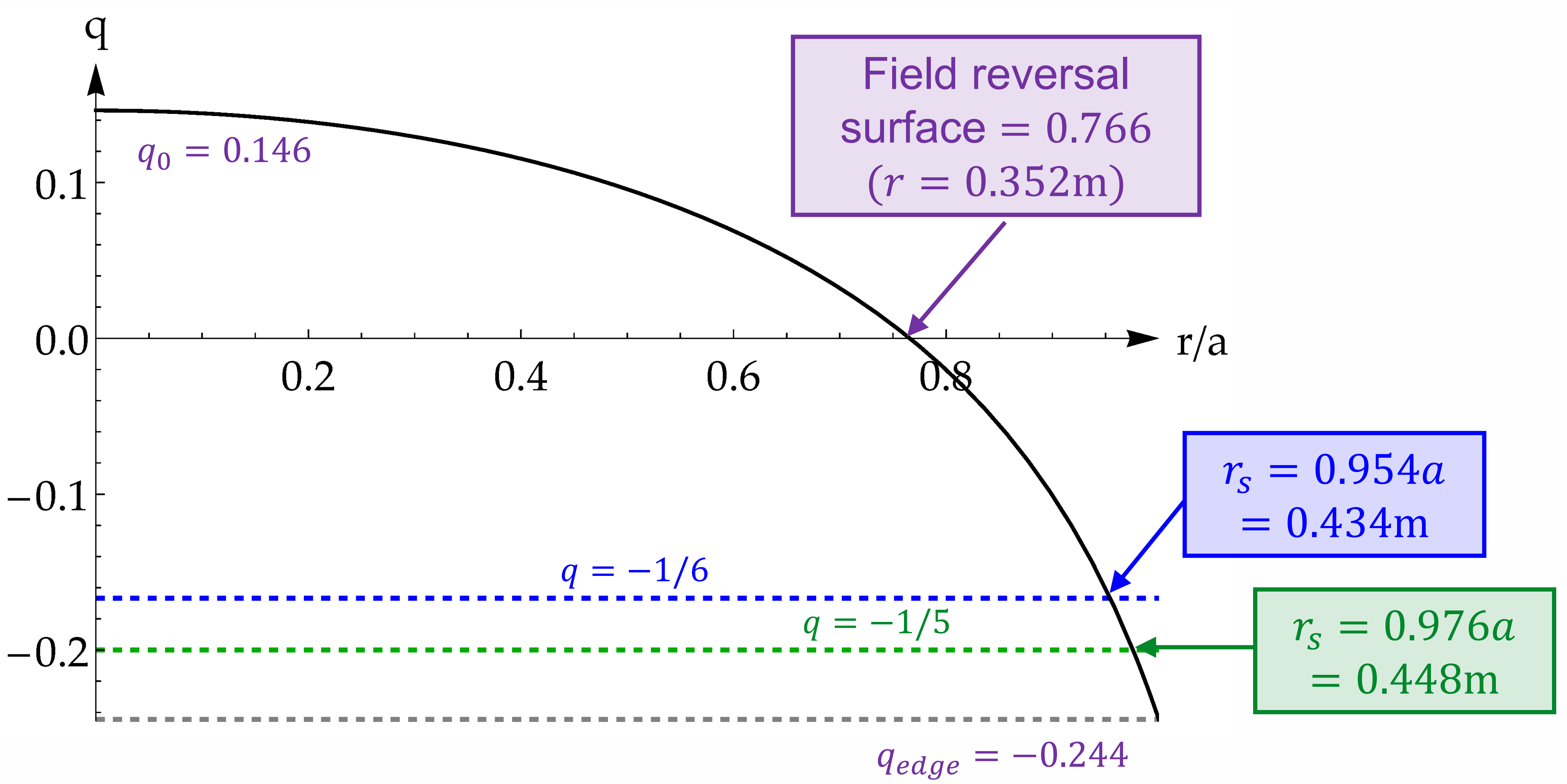
Nonlinear simulations for **7** different initial states with $\mathbf{3.12 \leq \alpha \leq 3.30}$ (between 1k, 5k and 10k resolution) have found plasma evolution is dominated by nonlinear dynamics and is qualitatively different from linear predictions.



We examine a specific case ($\alpha = 3.14$) which is predicted by linear analysis to be tearing unstable regime dominated by a single (m=1,n) resistive mode.

Key parameters for $\alpha = 3.14$ nonlinear relaxation study:





21k triangular elements (unstructured mesh): $S = \tau_A/\tau_R \sim 10^6$, $P_m = \nu/\eta = 1$, $\beta = \beta_p\beta_t/(\beta_p + \beta_t) \sim 0.4$.

Implicit sources allows for study of nonlinear relaxation dynamics in isolation

To investigate the relaxation dynamics in isolation, we introduce implicit sources to maintain initial profiles by assuming the decomposition:

$$\mathrm{f} = \mathrm{f}(\mathrm{equilibrium}) + \mathrm{f}(\mathrm{time{-}dependent})$$

Within the quasi-dynamical model we would like to construct, there are multiple competing timescales to account for:

i. Evolution of the instantaneous plasma state which is being approximated by an equilibrium.

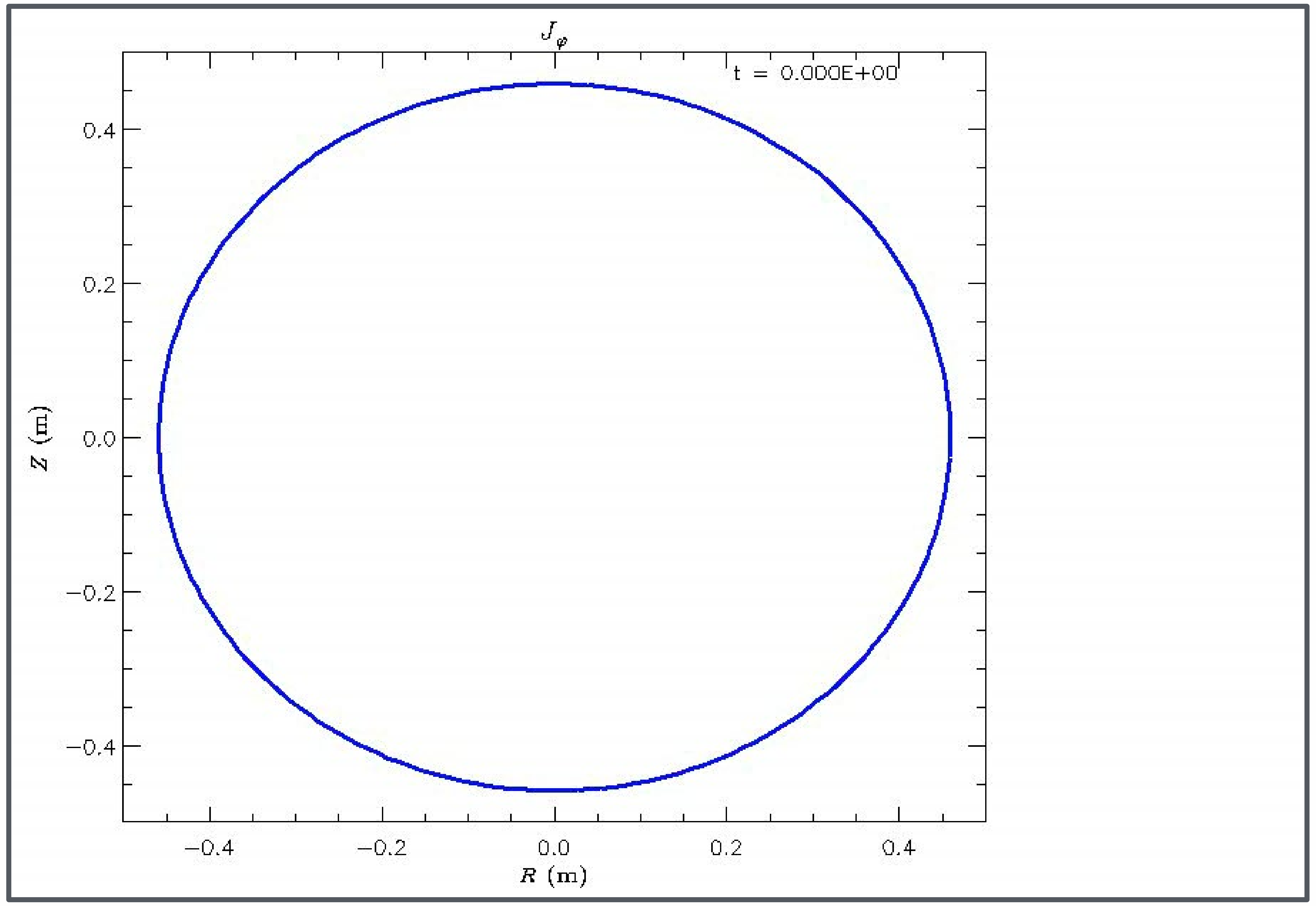
ii. Re-equilibration via Taylor relaxation.

Note for future work:

Ensuring sufficient separation of timescales between **(i)** and **(ii)** is important for dynamical accessibility.

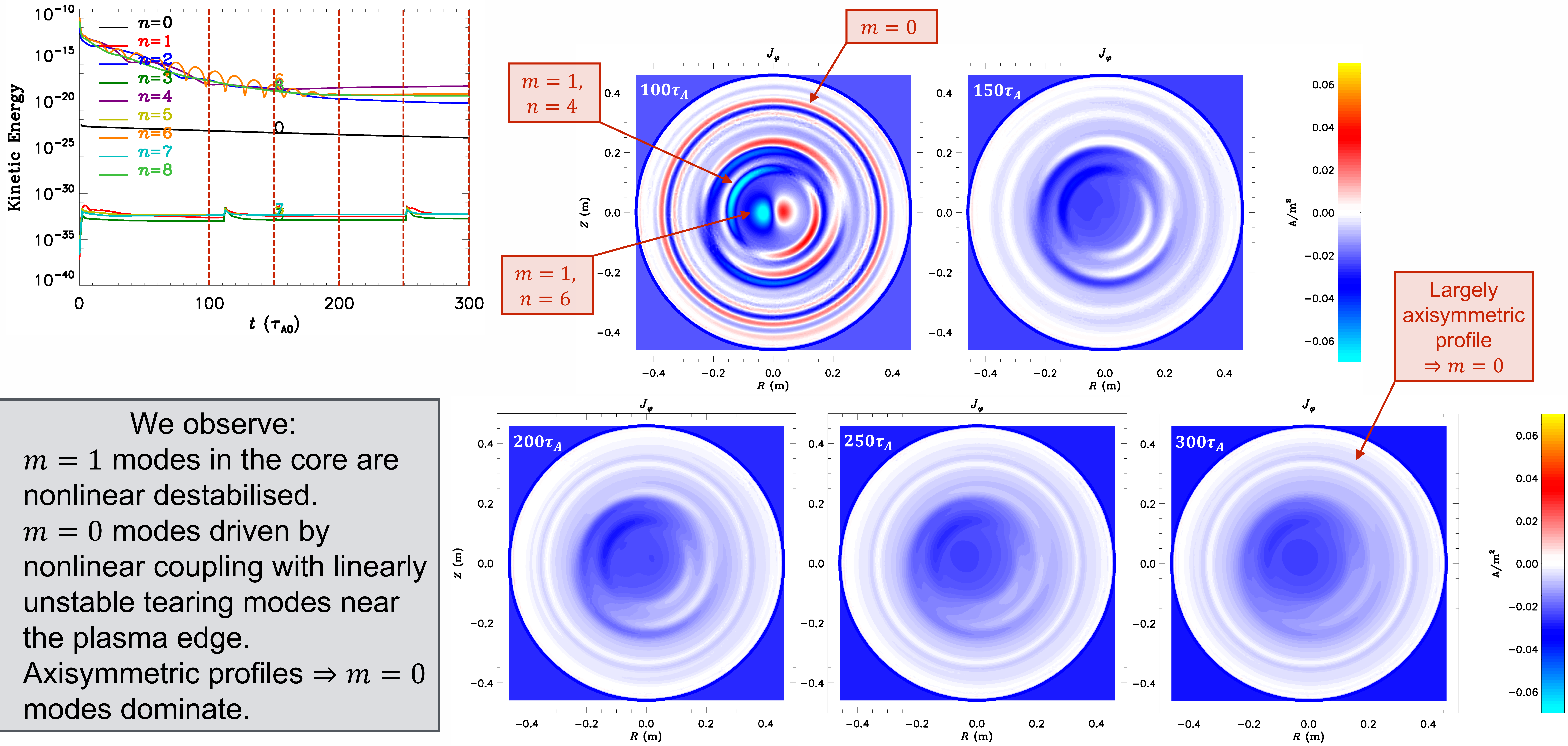
Relaxation dynamics on τ_{ideal}

Perturbed toroidal current density from $t = 0$ to $t = 300\tau_A$ shows multiple modes are unstable. Since linear stability analysis predicts ideal stability, this is due to nonlinear destabilisation.



10k triangular elements in $R - Z$ plane, 16 toroidal planes: $S = 9.2 \times 10^4$, $P_m = 1$, $\kappa_t/\kappa_\parallel = 10^{-6}$, $\beta \sim 0.4$, $\alpha = 3.14$.

Relaxation dynamics on τ_{ideal}: Nonlinearly destabilised $m = 0$ and $m = 1$ modes

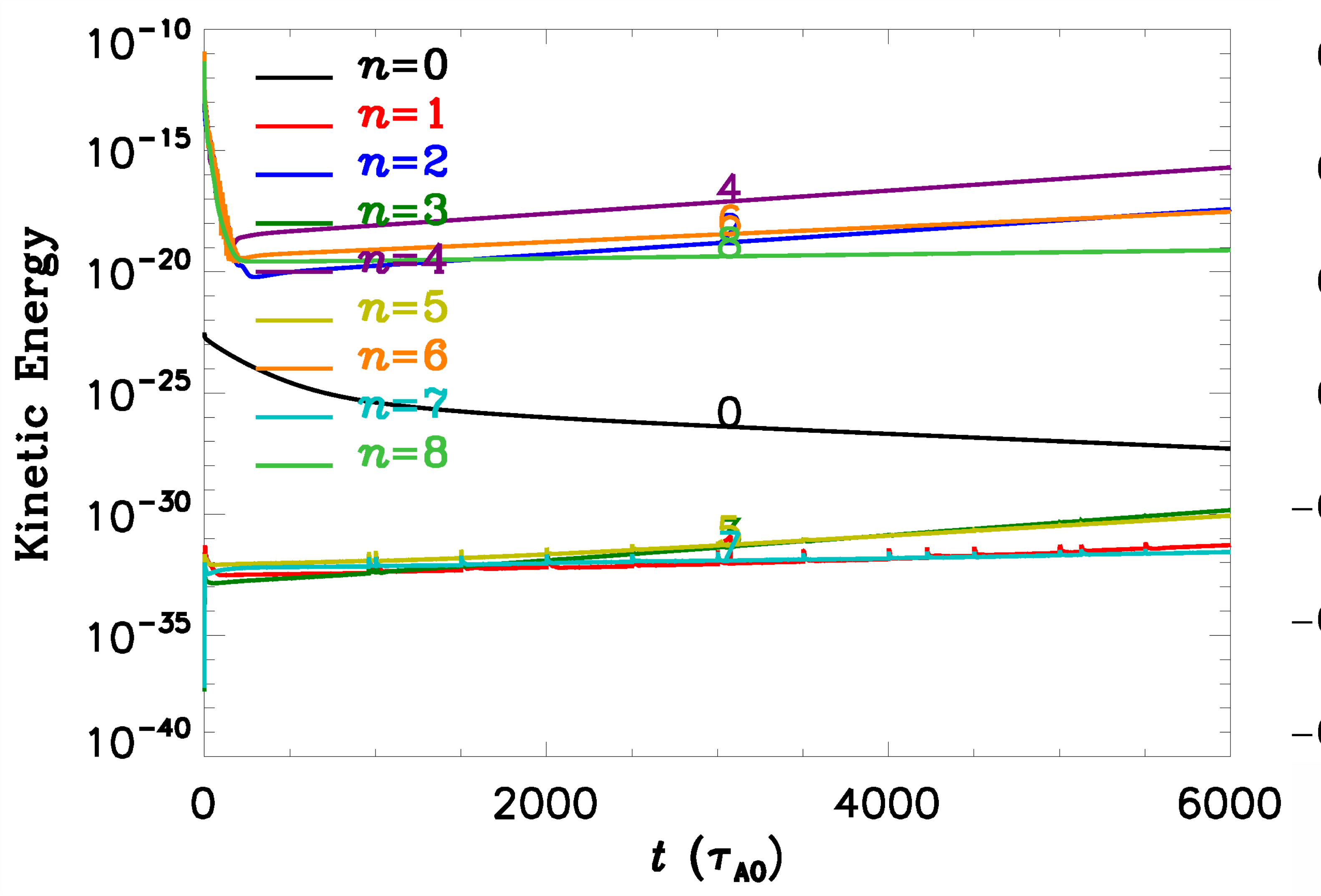


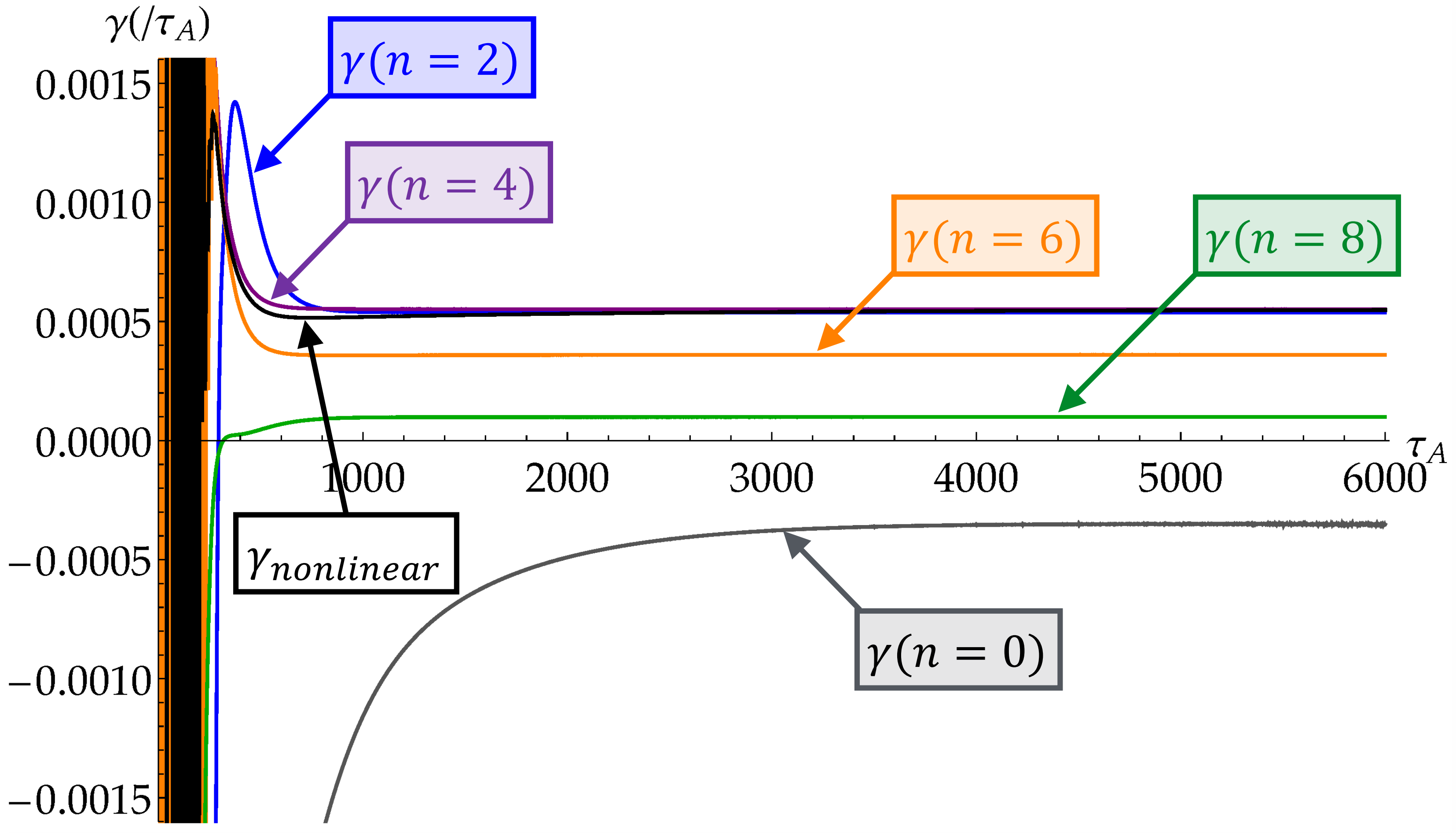
We observe:

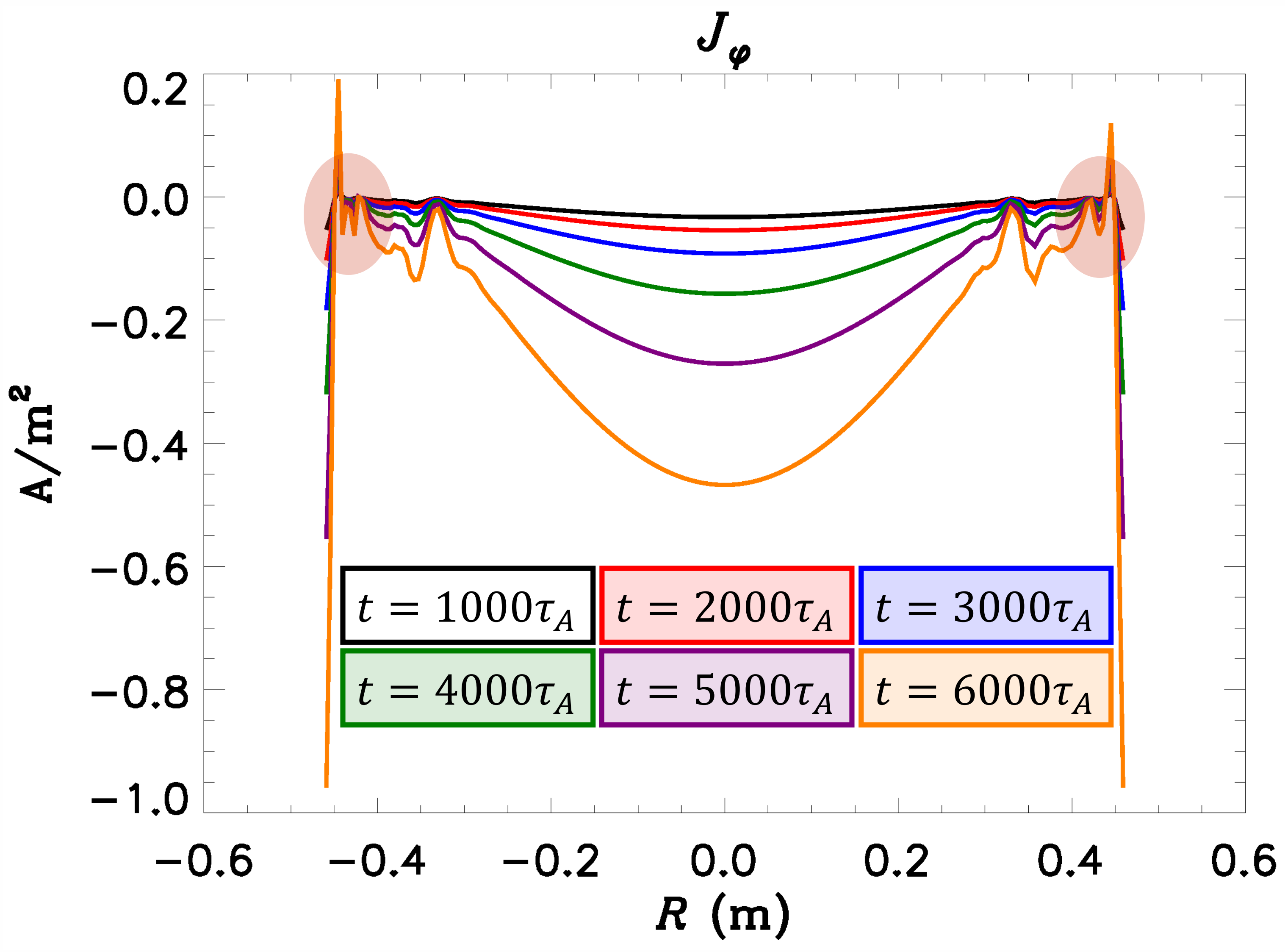
- $m = 1$ modes in the core are nonlinear destabilised.
- $m = 0$ modes driven by nonlinear coupling with linearly unstable tearing modes near the plasma edge.
- Axisymmetric profiles ⇒ $m = 0$ modes dominate.

10k triangular elements in $R - Z$ plane, 16 toroidal planes: $S = 9.2 \times 10^4$, $P_m = 1$, $\kappa_t/\kappa_\parallel = 10^{-6}$, $\beta \sim 0.4$, $\alpha = 3.14$.

Competition between nonlinear effects may determine preferred relaxation pathway







- Nonlinear coupling to linearly unstable $\boldsymbol{m = 1}$ modes destabilises $\mathbf{m = 0}$, $\boldsymbol{n = 2, 4, 8}$ modes.

- Early signatures in perturbed toroidal current density may be consistent with the $(\boldsymbol{m = 1, n = -5, -6})$ tearing modes predicted by linear analysis.

- On intermediate timescales $(10^3 - 10^4\tau_A)$, the plasma evolution consists primarily of competition between $\boldsymbol{n = 0}$ diffusion and nonlinearly destabilised $(\boldsymbol{m = 0, n = 2, 4})$ modes which may determine the preferred relaxation pathway.

⇒ Constraining the conditions which would favour volume-localised Taylor relaxation requires quantitative analysis of nonlinear dynamics. Work towards this is on-going.

5k triangular elements in $R - Z$ plane, 16 toroidal planes: $S = 9.2 \times 10^4$, $P_m = 1$, $\kappa_t/\kappa_\parallel = 10^{-6}$, $\beta \sim 0.4$, $\alpha = 3.14$.

- **Motivation**

- **Background, overview and approach to constructing a quasi-dynamical model**

- **Characteristic features of MRxMHD: Current sheet interfaces**

Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?

What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?

- **Characteristic features of MRxMHD: Localised Taylor relaxation**

Under what conditions can successive equilibria in the quasi-dynamical sequence be reached by the plasma in a way that is consistent with both MRxMHD and extended-MHD?

- **Next steps and on-going work**

Summary

Motivation:
Developing a computationally efficient global model of macroscopic dynamics to explain experimental observations of MHD activity in stellarators.

Long-term goal:
To develop a quasi-dynamical reduced model which approximates extended-MHD evolution by a sequence of 3D MHD equilibria connected via relaxation events.

Planned approach:
To use MRxMHD as the basis for such a quasi-dynamical reduced model.

This work:
As a first step, achieving the long-term goal requires detailed study of the two characteristic features of MRxMHD: current sheet interfaces and localised Taylor relaxation.

Lessons learnt so far

In this work, we have tried to address three fundamental questions.

1) **Can we approximate MRxMHD interfaces as highly localised (continuous) pressure gradients?**

   Provided q is irrational, our newly developed finite-width interface model suggests yes.

2) **What properties must the MRxMHD interfaces/localised pressure gradients satisfy in order for us to propose a quasi-dynamical model?**

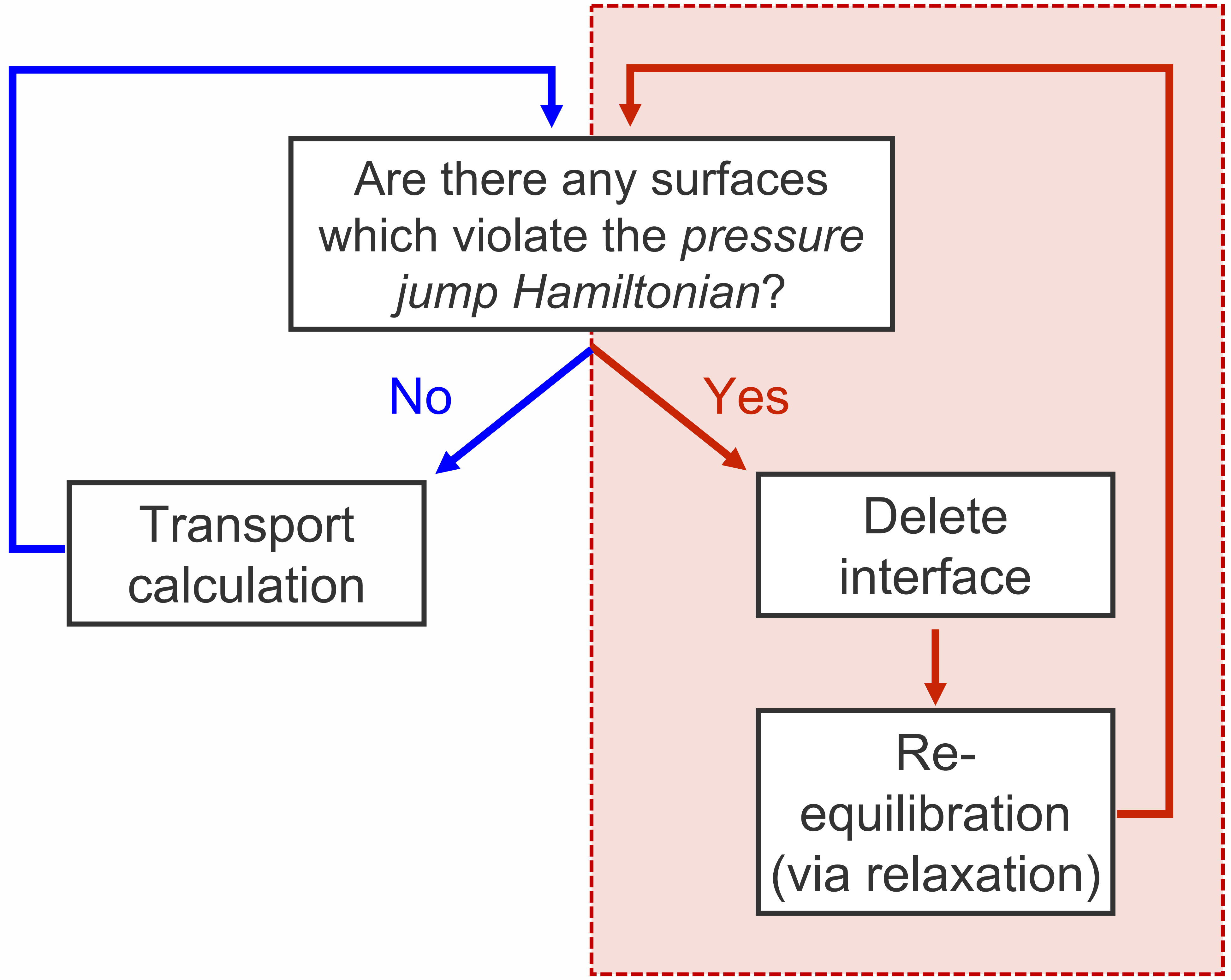
   With our finite-width interface model, we can study time-dependent interface dynamics directly with extended-MHD.

3) **Under what conditions can successive equilibria in a quasi-dynamical sequence be reached by the plasma in a way that is consistent with both MRxMHD and extended-MHD?**

   We have found that understanding the nonlinear dynamics is essential which makes the task particularly challenging.

Outlook: Towards an algorithm for predicting partial and global relaxation events

Could an avalanche-scenario provide a unified explanation for partial and global macroscopic relaxation events in fusion plasmas?



Potential applications:

- **Sawtoothing:**
Can we predict the post-crash current profile width?

- **Computational efficiency:**
Could we generate large training sets for machine learning?

- **Edge-Localised Modes:**
How does edge physics affect localised macroscopic relaxation?

- **Other suggestions?**