Effects of zonal flows on transport crossphase in Dissipative Trapped-Electron Mode turbulence

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Outline

General Intro

- Transitions to enhanced confinement regimes → key for future devices (ITER)
- H-mode regime well studied both experimentally and theoretically → L-H transition explained via 'flow-shear paradigm'[Biglari Diamond Terry '90, Moyer '95, Diamond, Itoh^2 Hahm '05]: flow shear suppresses transport by **shearing** apart turbulent eddies.
  → suppress both heat **and** particle transport the same way.
- For other regimes, e.g. I-mode (high energy confinement and low particle confinement), particle and heat transport **decouple** → the shearing paradigm **cannot apply** for these regimes!
  ↪ need to identify particle v.s. heat transport decoupling mechanisms
- One possible mechanism: nonlinear effects on the crossphase [Terry '01, An '17]. Here we show direct **effect of zonal flows on** the transport **crossphase**. (GAM ZF normally observed in I-mode)
- In this work, we only consider the effect on **particle transport**

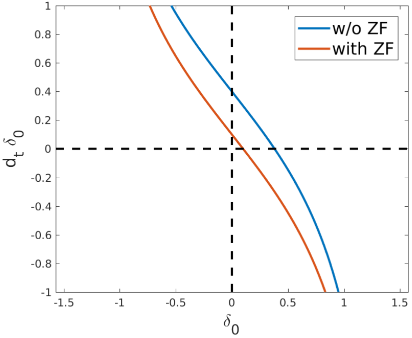
Turbulent flux and transport crossphase

- the turbulent particle flux can be written:

$$\Gamma_{turb} = \sqrt{\langle n^2 \rangle}\sqrt{\langle \phi^2 \rangle}\sin \delta_{n\phi}$$

- with $\delta_{n\phi}$: **transport crossphase**
- the particle flux can be suppressed by:
  - suppressing the turbulence amplitudes $\sqrt{\langle n^2 \rangle}, \sqrt{\langle \phi^2 \rangle} \searrow$ (eddy-shearing paradigm)
  - and/or
  - suppressing the crossphase $\delta_{n\phi} \searrow$

Basic Mechanism: $E \times B$ nonlinearity $\rightarrow$ nonlinear crossphase shift



- phase-locking diagram: ZFs nonlinear shift the crossphase (sketch)

evolution of **transport crossphase** (δ) between density and potential:

$$\frac{\partial \delta}{\partial t} = \omega_* - (\omega_k - \omega_E) - \frac{1}{\tau}\delta + \omega_E^{NL} + \omega_*^{NL}$$

- $\omega_E^{NL} \propto \mathrm{Im}\{\tilde{n}^* \, \tilde{\mathbf{v}}_E \cdot \nabla \tilde{n}\} \sim -V_{ZF}^2 \Delta\omega$: nonlinear crossphase shift
  $\Delta\omega$: frequency mismatch

Model

- fluid model for dissipative trapped-electron mode (DTEM), based on [Baver '02, Newman '94], including **zonal flows**:

$$
\begin{aligned}
\frac{\partial n}{\partial t} + v_E \cdot \nabla n + (1+\alpha\eta_e)\frac{\partial \phi}{\partial y} &= -\nu(\tilde{n} - \tilde{\phi}) \\
\frac{\partial}{\partial t}\left[(1-f_t)\tilde{\phi} - \nabla_\perp^2 \phi\right] + [1 - f_t(1+\alpha\eta_e)]\frac{\partial \phi}{\partial y} - v_E \cdot \nabla\nabla_\perp^2\phi &= f_t \nu(\tilde{n} - \tilde{\phi})
\end{aligned}
$$

$n = f_t n_{et} + n_{ep}$	:	effective density
ϕ	:	electric potential
$\nu = \nu_{ei}/\epsilon$	:	de-trapping rate
$f_t = \sqrt{\epsilon}$	:	trapping fraction
$\eta_e = L_n/L_{T_e}$		
$\alpha = 3/2$		
normalizations	:	space (ρ_s), time: (L_n/c_s)

Model (ct'd)

- This model has two nonlinearities ($\mathbf{v}_E = \hat{z} \times \nabla\phi$):

polarization nonlinearity

$$\hat{z} \times \nabla\phi \cdot \nabla\nabla_\perp^2\phi = \sum_{k=k'+k''} (k_\perp'^2 - k_\perp''^2)(\hat{\mathbf{z}} \times k') \cdot k'' \phi_{k'}\phi_{k''}$$

ExB nonlinearity

$$\hat{z} \times \nabla\phi \cdot \nabla n = \frac{1}{2} \sum_{k=k'+k''} (\hat{\mathbf{z}} \times k') \cdot k'' (n_{k'}\phi_{k''} - \phi_{k'} n_{k''})$$

Linear analysis of the DTEM model

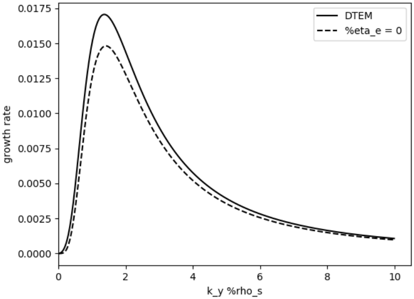


Figure: growth-rate v.s. k_y

- caveat: no linear threshold in this model

dispersion relation

$$\omega\left[1+\frac{k_\perp^2}{1-if_t\omega/\nu}\right]-k_y=0$$

DTEM frequency $\omega \ll \nu$

$$\omega_k = k_y/(1+k_\perp^2)$$

DTEM growth-rate

$$\gamma_k \simeq \frac{f_t}{\nu}\left[\frac{\alpha\eta_e k_y^2}{(1+k_\perp^2)^2}+\frac{k_\perp^2 k_y^2}{(1+k_\perp^2)^3}\right]$$

Zonal flows & zonal density

- zonal flows $V_{zon} = iq_x\phi_q e^{iqx} + c.c.$
  & zonal density $n_{zon} = n_q e^{iqx} + c.c.$
  are nonlinearly driven by DTEM turbulence:

$$
\begin{aligned}
q_x^2 \frac{\partial \phi_q}{\partial t} &= \sum_k (\hat{z} \times \mathbf{q}) \cdot \mathbf{k} \left(|\mathbf{k} + \mathbf{q}|^2 - k^2\right) \phi_k^* \phi_{k+q} \\
\frac{\partial n_q}{\partial t} &= \sum_k (\hat{z} \times \mathbf{q}) \cdot \mathbf{k} \, \frac{1}{2} \left(n_k^* \phi_{k+q} - \phi_k^* n_{k+q} \right)
\end{aligned}
$$

ϕ_q : zonal potential
n_q : zonal density

- zonal flows driven by the polarization nonlinearity & zonal density driven by $E \times B$ nonlinearity
- no dependence on $\nu \to$ not affected linearly by electron-ion collisions

Crossphase dynamics

- Write the Fourier modes in amplitude-phase form:

$$\begin{aligned} n_k &= |n_k| \exp(-i\delta_k) \\ \phi_k &= |\phi_k| \end{aligned}$$

- with δ_k: **crossphase** between density and potential

$$\frac{\partial \delta_k}{\partial t} = -\omega_k + (1 + \alpha\eta_e)k_y - \nu \tan\delta_k + N_k$$

- compared with [An '17] for Hasegawa-Wakatani model:

$$\partial_t \delta_{\vec{k}} = k_y \beta_{\vec{k}} \cos\delta_{\vec{k}} - \alpha\beta_{\vec{k}} \left(1 + \frac{1}{k^2 \beta_{\vec{k}}^2}\right) \sin\delta_{\vec{k}} + \mathcal{N}_{\vec{k}}$$

- we **explicitely** write the $E \times B$ nonlinearity N_k given by the **triplet correlation**:

$$N_k = \frac{1}{|n_k|^2} \mathrm{Im}\{n_k^* (\hat{z} \times \nabla\phi \cdot \nabla n)_k\}$$

Parametric interaction analysis: four-wave interaction

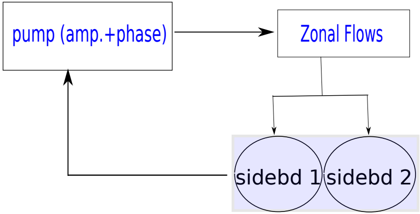
- A pump DTEM at $(\omega_0.\mathbf{k}_0)$ interacts with a seed zonal flow at $(\omega_q, \mathbf{q} = q_x\hat{x})$
  $\hookrightarrow$ generates two sidebands at frequencies $\omega_{1,2} = \omega_0 \pm \omega_q$ and wavenumbers $\mathbf{k}_{1,2} = \mathbf{k}_0 \pm \mathbf{q}$ (triad resonance condition)

the pump-wave is taken as:

$$\begin{bmatrix} n_P \\ \phi_P \end{bmatrix} = \begin{bmatrix} n_{k0} \\ \phi_{k0} \end{bmatrix} \exp[i\mathbf{k}_0 \cdot \mathbf{r} - i\omega_0 t]$$

the zonal flow is taken as:

$$V_{ZF} = iq_x\phi_q \exp[iq_x x - i\omega_q t]$$

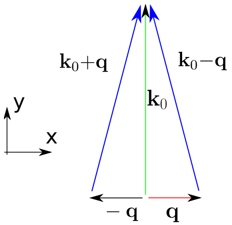


Parametric interaction analysis (ct'd)

- Decomposing into amplitudes n_0, ϕ_0 and crossphase δ_0:

the pump-wave is taken as:

$$\begin{bmatrix} n_P \\ \phi_P \end{bmatrix} = \begin{bmatrix} n_0 \exp(-i\delta_0) \\ \phi_0 \end{bmatrix} \exp[i\mathbf{k}_0 \cdot \mathbf{r} - i\omega_0 t]$$



the zonal flow is taken as:

$$V_{ZF} = iq_x\phi_q \exp[iq_x x - i\omega_q t]$$

four-wave interaction with approx: $k_{0x} = 0$

Zonal flows & zonal density

- zonal flows $V_{zon} = iq_x\phi_q e^{iqx} + c.c.$
  & zonal density $n_{zon} = n_q e^{iqx} + c.c.$
  For a four-wave parametric interaction:

$$\begin{aligned}
\frac{\partial \phi_q}{\partial t} &= q_x k_{0y} \left[\phi_{k_0}^* \phi_1 - \phi_{k_0} {\phi_2}^* \right] \\
\frac{\partial n_q}{\partial t} &= -q_x k_{0y} \left[\frac{1}{2}(n_{k_0}^* \phi_1 - \phi_{k_0}^* n_1) - \frac{1}{2}(n_{k_0} {\phi_2}^* - \phi_{k_0} {n_2}^*) \right]
\end{aligned}$$

ϕ_q : zonal potential
n_q : zonal density

- with ϕ_{k_0} : *pump*
- $\phi_{1,2} = \phi_{k_0 \pm q}$: potential sidebands
- $n_{1,2} = n_{k_0 \pm q}$: density sidebands

Zonal flows & zonal density (ct'd)

- using the amplitude/phase ansatz $\phi_{k_0} = \phi_0$ and $n_{k0} = \phi_0 \exp(-i\delta_0)$, this yields:

$$
\begin{aligned}
\frac{\partial \phi_z}{\partial t} &= q_x k_{0y} \ \mathrm{Re}\{\phi_0 \phi_1 + \phi_0 \phi_2\} \\
\frac{\partial n_z}{\partial t} &= -q_x k_{0y} \mathrm{Re}\{ \frac{1}{2}(\phi_0 e^{i\delta_0} \phi_1 - \phi_0 n_1 e^{-i\delta_1}) - \frac{1}{2}(\phi_0 e^{-i\delta_0} \phi_2 - \phi_0 n_2 e^{i\delta_2})\}
\end{aligned}
$$

$$
\begin{aligned}
\phi_z = |\phi_q| &\quad : \quad \text{zonal potential amplitude } (\sim \text{energy}) \\
n_z = |n_q| &\quad : \quad \text{zonal density amplitude}
\end{aligned}
$$

Parametric interaction (ct'd)

- Parametric interaction analysis yields:

$$
\begin{aligned}
\frac{\partial \delta_0}{\partial t} &= (1+\alpha\eta_e)k_0 - \omega_0 - \nu\delta_0 + \Lambda[\frac{\phi_z n_1}{\phi_0}\Delta\delta_1 - \frac{n_z\phi_1}{\phi_0}\delta_0] + \mathrm{Sidb2} \\
(1+k_0^2)\frac{\partial \phi_0}{\partial t} &= f_t k_0(1+\alpha\eta_e)\phi_0\delta_0 - k_0^2\Lambda(\phi_1-\phi_2)\phi_z - f_t\Lambda(\phi_z n_1 - n_z\phi_1) \\
&\quad +\mathrm{Sidb2} \\
\frac{\partial \phi_z}{\partial t} &= \Lambda(\phi_1+\phi_2)\phi_0 - \mu\phi_z \\
\frac{\partial n_z}{\partial t} &= \Lambda(n_1-\phi_1)\phi_0 + \Lambda(n_2-\phi_2)\phi_0 \\
\frac{\partial \Delta\delta_{1,2}}{\partial t} &= \Delta\omega - \nu\Delta\delta_{1,2} - \frac{\Lambda}{2}[(\frac{\phi_0\phi_z - \phi_0 n_z}{n_{1,2}} - 2\frac{\phi_z n_{1,2}}{\phi_0})\Delta\delta_{1,2}] + \ldots
\end{aligned}
$$

δ_0 : pump crossphase , $\Delta\delta_{1,2}$: triad phase mismatch

ϕ_0 : pump amplitude

ϕ_z : zonal flow amplitude

Parametric interaction (ct'd)

- and the sidebands evolve as:

$$
\begin{aligned}
(1 + k_{1,2}^2 - f_t)\frac{\partial \phi_{1,2}}{\partial t} &= f_t \nu (n_{1,2} - \phi_{1,2}) \pm (k_0^2 - q_r^2)\Lambda \phi_0 \phi_z \\
\frac{\partial n_{1,2}}{\partial t} &= -\nu (n_{1,2} - \phi_{1,2}) - \frac{\Lambda}{2}(\phi_0 n_z - \phi_0 \phi_z)
\end{aligned}
$$

ϕ_1, n_1 : potential & density of sideband 1

ϕ_2, n_2 : potential & density of sideband 2

Parametric interaction (ct'd)

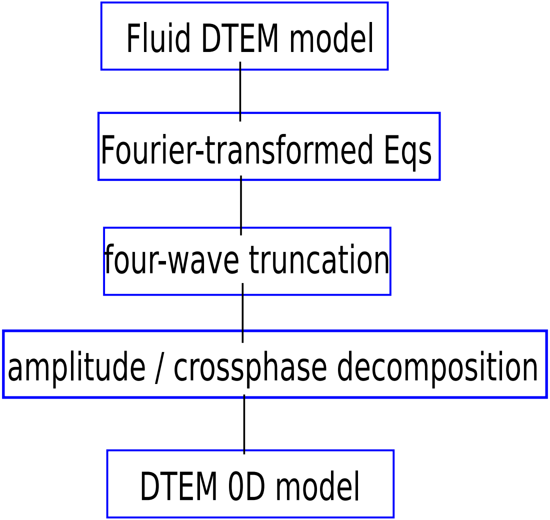
- For DTEM, the sidebands are nearly adiabatic $n_1 \sim \phi_1$ and $n_2 \sim \phi_2$ due to trapped/passing collisions (ν)
- Hence, negligeable NL drive for zonal density
- without zonal density, zonal flows play a major role for DTEM saturation
- For adiabatic sidebands, combining the Eqs for potential and density sidebands and using $n_{1,2} \sim \phi_{1,2}$ yields:

$$(1 + k_{1,2}^2)\frac{\partial \phi_{1,2}}{\partial t} + \gamma_d \phi_{1,2} = \pm(k_0^2 - q_r^2)\Lambda \phi_0 \phi_z$$

where we model the sideband dissipation by the damping rate γ_d [Chen '00]. For $\frac{1}{\phi}\frac{\partial \phi}{\partial t} \ll \gamma_d$, this yields the sideband response:

$$\begin{aligned} \phi_1 &\propto \frac{\Lambda}{\gamma_d}\phi_0\phi_z \\ \phi_2 &\propto -\frac{\Lambda}{\gamma_d}\phi_0\phi_z = \phi_1 e^{i\pi} \end{aligned}$$

Schematic derivation of the model



Predator-prey like DTEM model

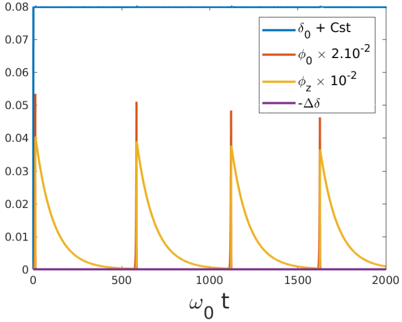
- Using the sideband response and $n_z \simeq 0$, parametric interaction analysis leads to the **predator-prey like model** :

$$
\begin{aligned}
\frac{\partial \delta_0}{\partial t} &= (1+\alpha\eta_e)k_0 - \omega_0 - \nu\delta_0 - \frac{\Lambda^2}{\gamma_d}\phi_z^2\Delta\delta \\
(1+k_0^2)\frac{\partial \phi_0}{\partial t} &= f_t k_0(1+\alpha\eta_e)\phi_0\delta_0 - (2k_0^2+f_t)\frac{\Lambda^2}{\gamma_d}\phi_z^2\phi_0 \\
\frac{\partial \phi_z}{\partial t} &= \frac{2\Lambda^2}{\gamma_d}\phi_0^2\phi_z - \mu\phi_z \\
\frac{\partial \Delta\delta}{\partial t} &= \Delta\omega - \nu\Delta\delta - \frac{\Lambda^2\gamma_d}{2}\left[1 - \frac{2\phi_z^2}{\gamma_d^2}\right]\Delta\delta
\end{aligned}
$$

δ_0	:	pump crossphase between density and potential
ϕ_0	:	pump amplitude
ϕ_z	:	zonal flow amplitude
$\Delta\delta$	:	triad phase mismatch

Results: Dynamics of the model without back-reaction on crossphase

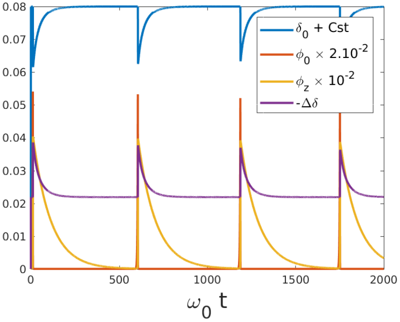
- **without back-reaction** ($\Delta\delta = 0$):



- Typical Limit-Cycle Oscillations between turbulence amplitude and zonal flows
- The transport crossphase is **phase-locked** to its linear value, after a short transient

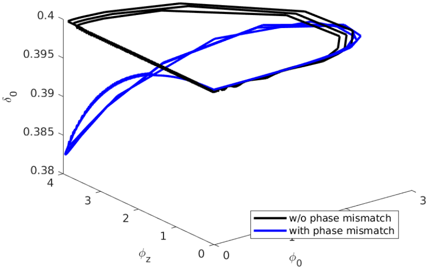
Results: Dynamics of the model with back-reaction on crossphase

- **with back-reaction ($\Delta\delta \neq 0$):**



- Limit-cycle oscillations between turbulence amplitude, zonal flows and **transport crossphase**
- The transport crossphase is transiently **suppressed by zonal flows**

- **with back-reaction ($\Delta\delta \neq 0$):**



- Limit-cycle oscillations between turbulence amplitude, zonal flows and **transport crossphase**
- The transport crossphase is transiently **suppressed by zonal flows**

Summary for this part

- A four-wave interaction analysis predicts that zonal flows can suppress the transport crossphase by nonlinearly shifting the crossphase

Wave kinetic approach

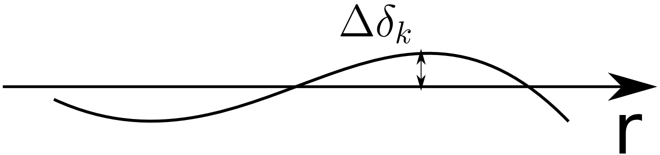
- Disclaimer: This is work in progress, not fully rigorous
- The turbulent particle flux at wavenumber k can be written:

$$\Gamma_k \propto N_k \sin \delta_k$$

- with $N_k = (1 + k^2)|\phi_k|^2 / \omega_k$ the wave action density for DTEM.
- write $N_k = \langle N_k \rangle + \Delta N_k(r,t)$ and $\delta_k = \langle \delta_k \rangle + \Delta \delta_k(r,t)$
- for small crossphase $\delta_k \ll 1$, the **nonlinearly modified particle flux** is:

$$\langle \Gamma_k \rangle \propto \Gamma_0 + \langle \Delta N_k \Delta \delta_k \rangle$$

- with $\Gamma_0 = \langle N_k \rangle \delta_k^{lin}$



Wave kinetic approach

Wave Kinetic equation **[Malkov, Diamond, Rosenbluth '01]**

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega}{\partial k_r}\frac{\partial N_k}{\partial r} - \frac{\partial \omega}{\partial r}\frac{\partial N_k}{\partial k_r} = \hat{\gamma}_{NL} N_k$$

- with the non-linear growth-rate operator satisfying:

$$\hat{\gamma}_{NL} N_k = \gamma_k N_k - \Delta\omega N_k^2$$

- we extend the wave-kinetic equation, to include the dependence of growth-rate on crossphase

$$\gamma_k = C\delta_k$$

- the linearized response, noting $\gamma_k N_k \simeq C\delta_k^{lin}\Delta N_k + C\langle N_k\rangle\Delta\delta_k$, is given by:

$$\left[\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial x} + \gamma_k^{lin}\right]\Delta N_k = k_\theta V'_{zon}\frac{\partial\langle N_k\rangle}{\partial k_r} - C\langle N_k\rangle\Delta\delta_k$$

Back-reaction

- the back-reaction on the turbulence is described by:

$$\frac{\partial\langle N_k\rangle}{\partial t} = \frac{\partial}{\partial k_r}\Big\langle k_\theta V'_{zon}\Delta N_k\Big\rangle + C\Big\langle \Delta\delta_k \Delta N_k\Big\rangle$$

- and we take the modulations in the form:

$\Delta N_k = N_q \exp(iq_r r - i\Omega t) + c.c.$, and $V'_{zon} = V'_q \exp(iq_r r - i\Omega t) + c.c.$ and $\Delta\delta_k = \delta_{k,q} \exp(iq_r r - i\Omega t) + c.c.$

- The linear response N_q is then:

$$N_q \simeq \frac{1}{\gamma_k^{lin}}\left[k_\theta V'_q \frac{\partial\langle N_k\rangle}{\partial k_r} - C\delta_{k,q}\langle N_k\rangle\right]$$

Wave Kinetic Equation

- and we obtain the quasilinear equation for $\langle N_k \rangle$:

$$\frac{\partial \langle N_k \rangle}{\partial t} = \frac{\partial}{\partial k_r} \frac{k_\theta^2 |V_q'|^2}{\gamma_k^{lin}} \frac{\partial \langle N_k \rangle}{\partial k_r} - \frac{C^2}{\gamma_k^{lin}} |\delta_{k,q}|^2 \langle N_k \rangle + \text{off} - \text{diagonal terms}$$

- 1st term on RHS : shearing effect due to zonal flows
- 2nd term: additional damping term due to **radial modulation of the crossphase** (also due to zonal flows)

Wave kinetic approach (c'td)

- In addition the nonlinearly modified particle flux is:

$$\begin{aligned} \langle \Gamma_k \rangle &= \Gamma_0 + \left\langle \Delta N_k \Delta \delta_k \right\rangle \\ &= \langle N_k \rangle \delta_k^{lin} - \frac{C \langle N_k \rangle}{\gamma_k^{lin}} |\delta_{k,q}|^2 \end{aligned}$$

- Direct **suppression of the particle flux** due to the crossphase modulation

Wave kinetic approach (c'td)

- In this picture, the crossphase modulation is driven by ZFs as:

$$\left[\frac{\partial \delta_{k,q}}{\partial t} + \nu \sin \delta_{k,q}\right] = k_\theta V_q$$

- i.e after phase-locking:

$$\delta_{k,q} \simeq \frac{k_\theta V_q}{\nu}$$

- The particle flux thus takes the form:

$$\langle \Gamma_k \rangle \simeq \langle N_k \rangle \delta_k^{lin} - \frac{C\langle N_k \rangle}{\gamma_k^{lin} \nu^2} k_\theta^2 |V_q|^2$$

Summary and conclusions

- In the framework of a fluid DTEM model, we showed that zonal flows can directly affect the crossphase
- A four-wave interaction analysis predicts that zonal flows can suppress the transport crossphase by nonlinearly shifting the crossphase
- This is confirmed in the wave-kinetic picture, where this stabilization is interpreted as a **radial modulation** of the transport crossphase, due to zonal flows.

- Open Questions
  - What is the effect of zonal flows on trapped electron temperature?

THANK YOU

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