Collisionless mechanisms of plasma transport in the presence of stochastic open magnetic field lines

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Motivation & Introduction
- Thermal quench and open stochastic magnetic fields
- New 3-D kinetic capabilities for simulating plasma transport in stochastic fields

Roles of stochastic open magnetic field lines
- 3-D topology of the open stochastic magnetic field lines
- Magnetically passing and trapped particles

Roles of self-consistent electric fields
- $E_\parallel$: Ambipolarity of plasma transport
  Passing-trapping condition of electrons
- $E_\perp$: ExB mixing effects on the plasma transport

Summary
The plasma disruption is a major challenge of tokamak fusion plasma

- Thermal Quench (TQ) and Current Quench (CQ)
- Rapid release of thermal and magnetic energy can damage to PFCs

Causes of TQ depend on causes of disruption

- Intentional plasma shutdown for machine protection
  - Impurity pellet injection or massive gas puffing
  - Radiative cooling of bulk thermal plasma
- Disruptive MHD instabilities
  - Break magnetic surface $\rightarrow$ magnetic stochasticity
  - Vertical displacement events
- Duration of TQ $\sim$ a few milliseconds $\rightarrow$ huge heat load to PFCs

Plasma transport mechanism in the open stochastic magnetic field lines is the key!
Plasma transport mechanism in stochastic magnetic fields is a long-standing research subject

- **Parallel transport along stochastic field lines with collisional cross-field decorrelation process**
  - Spatial diffusion of stochastic field lines: $\langle (\Delta r)^2 \rangle \sim 2LD_m$  
  - Particle motion along and decorrelation from a given field line
  
  [Rechester & Rosenbluth (PRL’1978), Krommes (JPP’1983)]

- **$\nabla B$ and curvature drift effects due to the toroidal geometry**
  - Trapped electrons should not be stochastic
  - Long confinement of runaway electrons

  [Mynick & Krommes (1979, 1980)]

- **Ambipolar electric fields for quasi-neutrality**
  - Simplified 1-D radial transport model with stochastic magnetic diffusion coefficient
  - “Working model” for a gyrokinetic simulation on stochastic heat flux

  [Harvey (PRL’1980)]
  [Wang (POP’2011)]

- **Intensive researches for external Resonant Magnetic Perturbation (RMP)**
  - RMP produces a thin stochastic layer at plasma edge to mitigate/suppress the Edge Localized Modes (ELMs)
  - Plasma transport in stochastic fields + edge physics

  [Evans (Nat.Phys’2006), Park (Nat.Phys’ 2018)]
  [Park (POP’2010), Hager (NF’ 2019)]
This work focuses on the 3-D topology of open magnetic field lines and ambipolar electric fields.

- **Previous studies have mostly focused on:**
  - Infinite length of stochastic magnetic field lines (internal stochastic layer)
  - Characterized by 0-D or 1-D stochastic diffusivity of magnetic fields ($D_{m,st}$)
  - Dynamics of passing particle along the stochastic field line with collisions

- **Key effects essential for understanding the Thermal Quench physics**
  - 3-D topology of the stochastic *open* magnetic field lines
  - *Ambipolarity* of the plasma transport with self-consistent potential in the stochastic layer
  - Dynamics of *trapped particles* (magnetic mirror + electric potential well)
  - Cross-field decorrelation by $E \perp \times B$ transport and mixing effects

A comprehensive picture of the relation between the plasma dynamics and the 3-D topology of the stochastic layer is needed.
New 3-D kinetic capabilities have been developed for simulating plasma transport in stochastic fields

- GTS (Gyrokinetic Tokamak Simulation)
  - A global gyrokinetic $\delta f$ particle simulation code to study micro turbulence physics of the fusion plasma in tokamaks

- New 3-D kinetic capabilities have been developed to study the plasma transport in the stochastic open magnetic field lines
  - High-resolution Vacuum Field Analysis
  - 3-dimensional Poisson solver
  - Novel delta-f particle method for plasma-wall boundary
  - GPU acceleration using OpenACC (x5 speed-up for total performance)
  - Improved numerical schemes to overcome numerical challenges
**Governing equations of system**

- **Prescribed magnetic perturbation**

\[ \alpha \equiv \frac{\delta A_\parallel}{B_0} \quad \delta B = \nabla \times (\alpha B_0) = \nabla \alpha \times B_0 + \alpha (\nabla \times B_0) \]

- **Particle motion in the presence of \( \delta B \)**

\[ \mathcal{L} = q_s \left( A_0^* + \delta A \right) \cdot \dot{\mathbf{R}} - \left( m_s / q_s \right) \dot{\xi} - \mathcal{H} \]

\[ \mathcal{H} = \left( q_s^2 \rho_\parallel^2 B_0^2 / 2m_s \right) + \mu B_0 + q_s \bar{\Phi} \]

\[ \frac{d\rho_\parallel}{dt} = \frac{B_0^* + \delta B}{B_0 \cdot (B_0^* + \delta B)} \left[ \frac{1}{q_s} \nabla \mathcal{H} \right] \]

\[ \frac{d\mathbf{R}}{dt} = \frac{1}{B_0 \cdot (B_0^* + \delta B)} \left[ \frac{1}{q_s} \frac{\partial \mathcal{H}}{\partial \rho_\parallel} (B_0^* + \delta B) + \frac{1}{q_s} B_0 \times \nabla \mathcal{H} \right] \]

- **\( \delta B \) effects on the particle motion**

  - particle streaming \( (\propto \rho_\parallel \delta \mathbf{B}) \)
  - magnetic mirror force \( (\propto \mathbf{B} \cdot (-\nabla B_0)) \)
  - electric force \( (\propto \mathbf{B} \cdot (-\nabla \Phi)) \)

- **3-D field equation**

\[ -\nabla \left[ \varepsilon_0 \mathbf{g} + \sum_s \frac{n_s m_s}{B_0^2} \left( \mathbf{g} - \frac{B_0 B_0}{B_0^2} \right) \right] \cdot \nabla \Phi = e (\delta n_i - \delta n_e) \]
Prescribed $\delta B$ applied on “Cyclone base case” Equilibrium

- Equilibrium magnetic configuration
  - “Cyclone base case”

- Circular shape limiter wall at $\sqrt{\psi_t} = 0.9$
  - Absorbing particle wall
  - Grounded conductor ($\Phi = 0$ at the wall)

- Magnetic perturbations with multiple harmonics
  \[
  \alpha = \sum_{m,n} \alpha_{(m,n)} = \Gamma(r) \cos(n\phi - m\theta - \omega t + \xi_0)
  \]
  \[
  (m, n) = [(2, 1), (3, 2), (4, 2), (5, 2), (5, 3), (6, 3), (7, 3), (8, 3)]
  \]
  \[|\delta B / B_0| \leq 10^{-2}\]

- Stochastic layer is produced from $\sqrt{\psi_t} \sim 0.45$

- Auto-correlation length of stochastic fields
  \[
  L_{auto} = \pi R_0 / \ln(0.5\pi C) \approx 5.6 \text{ m when } C = 3
  \]
Roles of stochastic open magnetic field lines

- **Vacuum Field Analysis in high-resolution**
  - Connection length of open magnetic field lines
    - Passing particle dynamics
  - Effective magnetic mirror ratio
    - Trapped particle dynamics

- **Test particle simulation without electric fields**
  - Temporal evolution of plasma profile in the stochastic layer
  - Characteristics of magnetically passing and trapped particles
3 stochastic open magnetic field lines started from the same $\sqrt{\psi_t}$ but different $\theta$ position

- Starting point
- Wall endpoint ($+\zeta$ direction)
- Wall endpoint ($-\zeta$ direction)

Each field line can have a different connection length between two wall endpoints
Connection length of open field lines

“High-resolution vacuum field analysis” enables understanding 3-D magnetic topology

1-D averaged Connection Length

\[
\langle L_c \rangle
\]

3-D Connection Length

\[
L_c @ \zeta = 0
\]

\[
L_c @ \zeta = \frac{\pi}{2}
\]

Confinement time of **passing particles** is proportional to \( L_c \)

\[
\tau_{||} \sim \frac{0.5 \ L_c}{v_{th}}
\]
Trapped particle motion under strong 3-D $\delta B$

$\Delta r = \text{electron banana width} \ll \Delta r = \text{determined by stochastic field structure}$

- $\Delta r$ of trapped particle trajectory is much larger than the electron banana width
- The toroidal precession is one of the **cross-field decorrelation mechanism**
  - Electrons can slowly move to different magnetic field lines and positions
  - Collisionless detrapping by moving electrons from magnetic uphill ($M_{\text{eff}} > 1$) to downhill ($M_{\text{eff}} < 1$)
Magnetic mirror effect along field line trajectory

- $|B|$ is toroidally asymmetric along field line trajectory

\[ B^{+\zeta}_{\text{max}}(x) \neq B^{-\zeta}_{\text{max}}(x) \quad \Rightarrow \quad B^{\text{eff}}_{\text{max}}(x) = \min(B^{+\zeta}_{\text{max}}(x), B^{-\zeta}_{\text{max}}(x)) \]

- $B^{\text{eff}}_{\text{max}}(x)$ determines the passing-trapping condition

- $B^{\text{eff}}_{\text{max}}(x)$ depends on the position $x$ even in the same field line
• If one of the trajectories in \( \pm \zeta \) directions has a very short connection length, the effective magnetic mirror becomes weaker, and particle can more easily exit to the wall.

Less trapping

No trapping
Effective magnetic uphill and downhill

- Effective magnetic mirror ratio: \( M_{\text{eff}} = \min(M^{+\zeta}, M^{-\zeta}) \)

\[
M^{\pm \zeta}(x) = \begin{cases} 
\frac{B_{\text{max}}^{\pm \zeta}}{B(x)}, & \text{if } B_{\text{max}}^{\pm \zeta} > B(x) \\
\frac{B_{\text{wall}}^{\pm \zeta}}{B(x)}, & \text{if } B_{\text{max}}^{\pm \zeta} = B(x) 
\end{cases}
\]

Effective magnetic uphill
\( (M_{\text{eff}} = \frac{B_{\text{max}}^{\text{eff}}}{B_0} > 1) \)

- Trapped by the magnetic mirror force
  if \( \left| \frac{v_\parallel(x_0)}{v_\perp(x_0)} \right| < \sqrt{M_{\text{eff}}(x) - 1} \)

Effective magnetic downhill
\( (M_{\text{eff}} = \frac{B_{\text{wall}}^{\text{eff}}}{B_0} < 1) \)

- No magnetic trapping
  \( \text{Particle is accelerated to the wall} \)
Magnetic mirror ratio and Trapped Particle Fraction

- Effective magnetic uphills \((M_{\text{eff}} > 1)\)
- Effective magnetic downhills \((M_{\text{eff}} < 1)\)

- Trapped particle fraction of Maxwellian distribution

\[
F_{\text{trap}}(x) = \begin{cases} 
\sqrt{1 - M_{\text{eff}}(x)^{-1}}, & \text{if } M_{\text{eff}}(x) \geq 1 \\
0, & \text{if } M_{\text{eff}}(x) < 1 
\end{cases}
\]

- A considerable amount of electrons \((\leq 60 \%)\) can be trapped in the device

- The electron trapped at the uphill can move to the downhill by the toroidal precession and exit to the wall

→ Collisionless detrapping

**Trapped electron dynamics** is critical to understand the **electron thermal transport**
Particle simulation setup

- Uniform density & temperature with Maxwellian distribution
  \[ T_e = T_i = 5 \text{ keV} \]
  \[ n = 1.6 \times 10^{19} \text{ m}^{-3} \]
  Focusing on how the plasma collapses to the wall

- Collisionless plasma due to long mean-free-path
  \[ \lambda_{mfp} \gg L_c \gg L_{auto} \]
  10 km  1 km  10 m

- Three types of particle simulations
  1. Test particle simulation (\textit{without E fields})
  2. Ambipolar transport simulation (\( E_\parallel \) only; ignoring \( \text{ExB} \))
  3. Full simulation with consistent potential and \( \text{ExB} \) (\( E_\parallel + E_\perp \times B \))
2D Evolution of Electron and Ion Density (without E fields)

\(n_i\)

\(n_e\)

Wall

\(n_i @ t = 0.0 \mu s\)

Wall

\(n_e @ t = 0.0 \mu s\)
Electron density collapse is 60 times faster than ion collapse \( \frac{v_{th}^e}{v_{th}^i} = \sqrt{m_i/m_e} \approx 60 \)

Electron density quickly saturates to the level of the trapped particles
Electron temperature is anisotropic ($T_{e,\perp} \gg T_{e,\parallel}$) due to the remained trapped electrons.

Edge temperature is slowly decreasing because of the detrapping by toroidal precession.

Passing-trapping boundary:

$$|v_{\parallel}(x)/v_{\perp}(x)| < \sqrt{M_{\text{eff}}(x) - 1}$$
Different confinement times of Passing and Trapped particles

\[ \tau_{\parallel, \text{pass}} \approx 0.5 \frac{L_c}{v_{th}} \]

- **Passing electron density** quickly decays with a short confinement time
- **Trapped electron density** very slowly decays due to the collisionless detrapping by the toroidal precession
  - The outer radial surface has more magnetic downhill regions → faster detrapping
“Vacuum Field Analysis” well predicts the dynamics of test particles

- The passing particle density is higher at longer connection length regions

\[ \tau_{\parallel,\text{pass}} \sim 0.5 \frac{L_c}{v_{th}} \]

- The trapped particle density ratio is the same as the trapped particle fraction

\[ F_{\text{trap}}(x) = \begin{cases} \sqrt{1 - M_{\text{eff}}(x)^{-1}}, & \text{if } M_{\text{eff}}(x) \geq 1 \\ 0, & \text{if } M_{\text{eff}}(x) < 1 \end{cases} \]
Roles of the electric fields

- $E_\parallel$: acceleration of charged particles
  - Impedes the fast electron loss
  - Ambipolar plasma transport (quasi-neutrality)
  - Determines the passing-trapping condition of electrons (combined with the magnetic potential)

- $E_\perp$: ExB drift motion across the magnetic field lines
  - Deforms the plasma structure (mixing effect)
  - Direct cross-field transport in the radial direction
  - Enhances the collisionless detrapping of high-$v_\perp$ trapped particle
  - The steady decrease of the electron temperature
The ambipolar electric field impedes the fast electron loss

The positive ambipolar potential \((e \Phi \sim T_e)\) builds up for the ambipolar transports \((\Gamma_i \approx \Gamma_e)\) 

\[\frac{v_{th}^e}{v_{th}^i} = \sqrt{\frac{m_i}{m_e}} \sim 60\]

- The electron loss is 60 times faster than the ion loss without the ambipolar electric field
- The positive ambipolar potential builds up for the ambipolar transports
  - Impedes the fast electron loss to match with the ion loss (for quasi-neutrality)
1-D model of ambipolar electric fields

Radial electric fields from the simulation agree with the analytic model except the edge.

At the edge, the ion flux is not negligible, and the distribution function is deformed from Maxwellian.

The ambipolar potential has 3-D structure associated with the topology of the stochastic layer.

1-D amipolar radial electric fields (Harvey PRL’ 1981)

\[ E_r^A(r) = -\left( \frac{\langle T_e \rangle}{e} \right) \frac{\partial}{\partial r} \left[ \ln(\langle n_e \rangle \langle T_e \rangle^{1/2}) \right] \]

- Maxwellian Distribution
- Zero flux assumption \( \Gamma_i = \Gamma_e = 0 \)
Plasma ambipolar transports by electric fields

\[ \mathbf{E} \parallel \mathbf{B} \quad \text{only} \]

\[ \mathbf{E} \parallel \mathbf{B} + \mathbf{E}_\perp \times \mathbf{B} \quad \text{(Yes ExB)} \]

\( n_i \quad \text{No ExB} \quad n_i \mathbf{B} \mathbf{n} \mathbf{i} \mathbf{n} \mathbf{e} \mathbf{b} \)
Temporal evolution of the plasma (only $E_\parallel$ case)

1. Ambipolar potential builds-up with electron thermal speeds

2. Plasma collapses with ion sound speeds
   - Ambipolar transport
   - Lower density at shorter $L_c$ regions

3. The potential becomes smaller again at the edge due to lower pressure gradient

$$E^A_r(r) = -\frac{(T_e/e)}{e} \frac{\partial}{\partial r} \left[ \ln(\langle n_e \rangle \langle T_e \rangle^{1/2}) \right]$$

$$\nabla_\parallel (n_e T_e^{1/2}) \downarrow \rightarrow \ E^A_\parallel \downarrow \rightarrow \Phi \downarrow$$

The plasma has 3-D structures correlated to $L_c$ structure
• Total potential energy for charged particles

\[ \mathcal{V}(x) = q\Phi(x) + \mu B(x) \]

• Exact Trapping condition

\[ \varepsilon_{||}(x) + \mathcal{V}(x) < V_{\text{max}}^{\text{eff}}(x) \]

- It is hard to find exact \( V_{\text{max}}^{\text{eff}}(x) \) due to fluctuating \( \Phi \)

• Simplified trapping conditions

Trapping for high \( \mu B \) particles (\( \mu B \gg q\Phi \))

\[ \begin{align*}
\varepsilon_{||}(x) &< V_{\text{max}}^{\text{eff}}(x) - \mathcal{V}(x) \approx \mu \left( B_{\text{max}}^{\text{eff}}(x) - B(x) \right) \\
v_{||}^{2}(x) &< \left( B_{\text{max}}^{\text{eff}}(x)/B(x) - 1 \right) v_{\bot}^{2}
\end{align*} \]

The same as the pure magnetic mirror condition

Global trapping with respect to wall endpoints \((\Phi_{\text{wall}} = 0)\)

\[ \begin{align*}
\varepsilon_{||}(x) + \mathcal{V}(x) &< \mu B_{w}^{\text{eff}} \\
v_{||}^{2}(x) + \left( 1 - \frac{B_{w}^{\text{eff}}}{B(x)} \right) v_{\bot}^{2}(x) &< -\frac{2q}{m}\Phi(x)
\end{align*} \]
Global trapping with respect to wall endpoints

- At the magnetic uphill ($\Delta B > 0$), more particles are trapped due to additional magnetic mirror effects.
- At the magnetic downhill ($\Delta B < 0$), the high $v_\perp$ particles can be passing particles.
ExB mix the plasma across the field line

Simulation including only $E_\parallel$

- Strong $E_\perp$ across different $L_c$ regions
  - Fast ExB transport and mixing

Full Simulation including both $E_\parallel$ and $E_\perp$

- ExB mixing deforms the plasma structure
  - Radial eddies + poloidal flow
ExB mix the plasma across the field line

- Non-zonal potential $\delta \Phi$ at a specific radial surface ($\sqrt{\psi_t}=0.77$)

**Diagram:**

(a) $(E_{||}$ only) case

(b) $(E_{||} + E_{\perp})$ case

(c) $(k_y \Phi_{k_y})^2$

- $E_{||}$ only case
  - Field-aligned structures
  - A sharp difference across field line
  - Peak energy spectrum at $k_y \sim 8$

- $(E_{||} + E_{\perp} \times B)$ case
  - Field-aligned structures
  - Higher $k_y$ modes become more important
  - Dynamic finer structures by ExB mixing
ExB mixing enhances the collisionless detrapping of high-$v_\perp$ particle

- **Confinement of high $v_\perp$ particles (parallel dynamics)**
  - They are well-trapped at magnetic uphill regions
  - They can exit to the wall at magnetic downhill regions by overcoming the electric reflective force

- **ExB transport carries and mixes the electrons radially and poloidally**

  - ![ExB mixing diagram](image)

ExB mixing enhances the collisionless detrapping in average
ExB contributes considerable amount of electron fluxes

Electron Radial Fluxes @ $\sqrt{\psi_t} = 0.65$

- **ExB transport** contributes about (30~40)% of particle flux and (50~60)% of heat flux
Comparison of Electron temperatures with 3 different physical models

- **Trapped particle transport** is critical for determining the electron thermal transport and temperature

- (No E field case) and (only $E_\parallel$) cases have saturated electron temperature with higher gradients

- ($E_\parallel + E_\perp$) case shows that the electron temperature steadily decrease in the time scale of milliseconds
  
  - High-$v_\perp$ trapped electrons can be detrapped by ExB mixing and toroidal precession
  
  - At $\sqrt{\psi_t} = 0.7$ surface, the temperature decreasing rate is about (-500 eV/ms)
First-principles-based calculation of plasma transport in stochastic magnetic fields has been developed for a global gyrokinetic code GTS.

We found that self-consistent electric fields for plasma transport ambipolarity play critical roles in determining plasma transport associated with the 3-D topology of stochastic layer:

- $E_\parallel$ makes ambipolar plasma transport that propagates along stochastic fields with ion sound speed.
- $E_\parallel$ and 3-D magnetic mirror ratio determines the passing-trapping condition of the particles.
- $E_\perp \times B$ radial transport is considerable (particularly for the trapped particles).
- $E_\perp \times B$ mixing across the stochastic fields enhances the collisionless detrapping of high-$v_\perp$ trapped particle.

We observed a considerable degradation of the global plasma profile and electron temperature within the timescale of milliseconds that agrees with the typical time scale of the thermal quench.

Future works:
- Collisional transports
- Recycling particles
- More realistic plasma profile and magnetic perturbations