The trigger problem of magnetic field reconnection and the “ideal” tearing

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Outline

• The trigger issue
• The tearing mode instability
• The “ideal” tearing mode
• Onset of fast reconnection in forming current sheets
• The problem of the critical Lundquist number
• Summary
Magnetic field reconnection plays a crucial role in solar activity from coronal heating to flares and Coronal Mass Ejections.

Flare: Rapid energy release
The largest flares can release $10^{25}$ Joules = 5 millions H-bombs in less than 1 hour!
Filaments can stand for \(\sim\) weeks in the corona, and then erupt in a \(\sim \) few tens of minutes.

- Ramp phase in saw-teeth \(\sim\) few ms, crash \(\sim\) 100\(\mu\)s
- Growing phase \((\sim 30\text{-}60\text{ min})\) and onset & disruption \((\sim 1\text{ min})\) in geomagnetic substorms
The trigger issue

How and under which conditions does magnetic reconnection occur on time scales compatible with the underlying ideal dynamics?

<table>
<thead>
<tr>
<th>Sources</th>
<th>Low Corona (Sun at ~ 1R☉)</th>
<th>Magnetotail (Central Plasma Sheet)</th>
<th>ITER</th>
<th>JET</th>
<th>MRX Device</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>10⁹ – 10¹⁰</td>
<td>10⁹ – 10¹⁰</td>
<td>300</td>
<td>80</td>
<td>10–20</td>
</tr>
<tr>
<td>nₑ</td>
<td>10⁹ – 10¹⁰</td>
<td>0.1</td>
<td>10¹⁴</td>
<td>10¹³</td>
<td>(2–6) × 10¹³</td>
</tr>
<tr>
<td>B</td>
<td>10–100</td>
<td>10⁻⁴</td>
<td>5.68 × 10⁴</td>
<td>3.45 × 10⁴</td>
<td>(1–3) × 10²</td>
</tr>
<tr>
<td>Tₑ</td>
<td>86</td>
<td>10³ – 10⁴</td>
<td>2 × 10⁴</td>
<td>3 × 10³</td>
<td>5 – 15</td>
</tr>
<tr>
<td>εₛ ≡ (S⁻¹)*</td>
<td>10⁻¹⁵ – 10⁻¹²</td>
<td>10⁻¹⁷ – 10⁻¹⁵</td>
<td>10⁻¹¹</td>
<td>10⁻⁹</td>
<td>8 × 10⁻⁴ – 3 × 10⁻²</td>
</tr>
<tr>
<td>εₛ ≡ (S⁻¹)*</td>
<td>10⁻¹¹ – 10⁻¹⁰</td>
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<td></td>
</tr>
<tr>
<td>εₑ ≡ (dₑ/L)²</td>
<td>10⁻¹⁹ – 10⁻¹⁶</td>
<td>10⁻⁸ – 10⁻⁶</td>
<td>10⁻⁸</td>
<td>10⁻⁶</td>
<td>10⁻⁵ – 10⁻⁴</td>
</tr>
</tbody>
</table>

(table from: Del Sarto et al. JGR 2016)
Initiation of “fast” reconnection in forming current sheets

Turbulence: current sheet formation in decaying turbulence [Biskamp (1993), Servidio et al. (2012), (Cerri+ 2017)] or in the field line tangling model of the nano-flare scenario [Rappazzo et al. (2013)]

Dynamics at X-points in general: (X-point collapse) [Sulem+(1985), Jemella+(2003), Loureiro+(2005), Cassak+ (2009)]

Magnetotail: central current sheet thinning [Sanny+(1994)]

Sun: CS at tip of helmet streamers/heliospheric current sheet/flares [Rappazzo et al. 2005, Viall et al. 2015, Shibata 1999]
“Spontaneous” reconnection as the outcome of an internal instability driven by the gradient of the current, the tearing mode instability (Furth, Kileen and Rosenbluth ’63).

Instability parameter: depends only on equilibrium and $k$

$$\Delta'(k) = a \left( \frac{b'(0^+) - b'(0^-)}{b(0)} \right) > 0$$

$$ka < 1 \quad \eta/a^2 < \gamma < va/a$$
The tearing mode instability (FKR ‘63)

\[ \gamma \bar{T} \]

\[ \bar{\tau} = \frac{a}{v_a} \]

\[ \bar{S} = \frac{av_a}{\eta} \]

\[ \gamma_m \bar{\tau} \sim (\bar{S})^{-1/2} \quad \text{SLOW!} \]
The tearing mode in thin current sheets

If the width of the current sheet is not the macroscopic length of the system used to define the dynamical time-scale, a proper renormalization of time scales is required to translate the tearing mode results to a “thin” (on a macroscopic scale of the dynamical system) current sheet, viz. a Sweet–Parker sheet

\[ \bar{\tau} = \tau_A \left( \frac{a}{L} \right) \]

\[ \bar{S} = S \left( \frac{a}{L} \right) \]

The resistive tearing mode maximum growth rate, measured on the macroscopic Alfvén time \( L/v_A \), scales as [Tajima–Shibata 1997, Loureiro+ 2007, Battacharjee+2009, Pucci & Velli 2014]:

\[ \gamma_m \bar{\tau} \sim \bar{S}^{-1/2} \Rightarrow \gamma \tau_A \sim S^{-1/2} \left( \frac{a}{L} \right)^{-3/2} \]
Towards $S \to \infty$: the “ideal” tearing mode

\[ \gamma_m \tau_A \sim S^{-1/2} \left( \frac{L}{a} \right)^{3/2} \]

\[ \frac{L}{a} \sim S^\alpha \]

\[ \tau_A = \frac{L}{v_a} \quad S = \frac{Lv_a}{\eta} \]

\[ \alpha < \frac{1}{3} \quad \lim_{S \to \infty} \gamma_m \tau_A \to 0 \]

“ideal tearing”

\[ \alpha = \frac{1}{3} \quad \lim_{S \to \infty} \gamma_m \tau_A \to O(1) \]

Way too fast!

\[ \alpha > \frac{1}{3} \quad \lim_{S \to \infty} \gamma_m \tau_A \to \infty \]
Towards $S \to \infty$: the “ideal” tearing mode

There exists a critical aspect ratio $(L/a)_i$ at which the growth rate becomes independent from $S$. It provides a “sup” for current sheets that can naturally form (Pucci & Velli 2014; Tenerani et al 2015a,b; 2016)

\[ \frac{a}{L} \sim S^{-1/3} \]
\[ \frac{\delta}{L} \sim S^{-1/2} \]
\[ kL \sim S^{1/6} \]
Effect of the equilibrium profile on the ideal tearing scaling

\[ \Delta' \sim (ka)^{-p} \]

The critical exponent \( \alpha \) is not universal and depends on the equilibrium profile (Del Sarto et al. 2016, Pucci et al. 2018).

\[ \alpha = \frac{1}{2} \left( \frac{1 + p}{1 + 2p} \right) \]

\[ \frac{1}{4} < \alpha < \frac{1}{3} \]

\[ p \geq 1 \]

Harris profile: \( p=1 \); bounded field, \( p=2 \)

Bound field case

The critical exponent \( \alpha \) is not universal and depends on the equilibrium profile (Del Sarto et al. 2016, Pucci et al. 2018).
Kinetic effects (electron inertia)

\[ k_M^* \simeq (d_e^*)^{\frac{1}{p}} \left( \frac{L}{a} \right)^{\frac{1}{p}} \gamma_M^* \tau_A^* \simeq (d_e^*)^{\frac{1+p}{p}} \left( \frac{L}{a} \right)^{\frac{1+2p}{p}} \]

Harris profile, \( p=1 \)

\[ d_e^* = \frac{c}{(\omega_{pe} L)} \quad \alpha = \frac{1}{3} \quad \Rightarrow \tau \sim \tau_A \]

Collisionless MHD (Del Sarto et al. 2016)
Kinetic effects (Hall MHD)

Hall effect when:

\[
\delta/d_i \approx 1
\]

\[
\frac{d_i}{L} \approx S^{-1/2}
\]

(Pucci et al. 2017)
Kinetic effects (Hall MHD)

\[ \gamma m \tau_A \sim S^{-1/2} \left( \frac{d_i}{L} S^{1/4} \right)^{2/3} \]

\[ \frac{a}{L} \sim S^{-1/3} \left( \frac{d_i}{L} S^{1/2} \right)^{\beta} \]

\[ \beta \approx 0.29 \]

(Pucci et al. 2017)
Disruption of a forming current sheet: numerical simulations

When current sheets approach the critical aspect ratio $L/a$ they disrupt by the onset of a rapid tearing mode instability. This suggests a possible route to the triggering of catastrophic phenomena such as flares, geomagnetic substorms, etc. (Tenerani et al. 2015b)

MHD simulation of a shrinking current sheet that transitions towards a rapidly unstable state.

$$\delta b(t) \sim \exp \left[ \int_0^t \gamma_m(t') dt' \right]$$
Disruption of a forming current sheet: numerical simulations

\[ \mathbf{B}_0(x) = \tanh(x/a(t))\hat{y} + \frac{1}{\cosh^2(x/a(t))}\hat{z} \]

\[ a(t) = a_0 e^{-t/\tau_c} + a_\infty (1 - e^{-t/\tau_c}) \]

Nonlinear evolution: the “fractal reconnection” revised

Effect of flows can’t be neglected at “low” $S$

$$\alpha_c = \frac{2 \log \mu + \log S_n}{3 \log S_n}.$$
Evidence of self-similar profiles of the reconnecting magnetic field

Nonlinear evolution: “fractal reconnection” revised

(1) \( a_n / L_n \sim S_n^{-1/3} \)

(2) \( a_n / L_{n-1} \sim S_{n-1}^{-1/2} \)

\[ \rightarrow L_n / L = S^{-1+(3/4)^n}, \quad S_n = S^{(3/4)^n} \]

(1) Threshold for instability growth
(2) The \( n \)th unstable layer is the diffusion region of layer \((n-1)\)th

Since the formation and disruption of new current layers occurs on (locally) Alfvénic timescales, the “ideal” tearing mode scenario with proper renormalization can provide a basis for a heuristic model which explains the nonlinear evolution and the transition to a fully turbulent state. We therefore modify the original fractal model, first introduced for flares by Shibata (2001), inspired by the “ideal” tearing scenario. We assume that each \( n \)th CS destroys at the local critical threshold until the marginally stable CS at \( S_c \sim 10^4 \) is reached, and at which we expect a spectral break. In particular, for \( S=10^6 \) then \( S_c \sim (L_c V_a) / \nu = L_c S \) and \( L_c \sim 0.01 \).

Previous models: Shibata & Tanuma 2001, Uzdensky et al. 2010, Ji & Daughton 2011
The critical Lundquist number and the effect of flows

The Sweet–Parker can persist as a stable configuration below a Lundquist number $S \sim 10^4$ (Biskamp 1986).

It was suggested that flows along the current sheet provide the main stabilizing effect for the growth of the tearing mode (Bulanov 1979).

$$V_0 = \Gamma(x\hat{x} - y\hat{y})$$

$$B_0(y) = \frac{\bar{B}_0}{0.54} \exp\left[ -\left( \frac{y}{\sqrt{2a}} \right)^2 \right] \int_0^{y/\sqrt{2a}} e^{s^2} \, ds$$
The critical Lundquist number and the effect of flows

Growth rates at increasing $S$ in a Sweet–Parker like current sheet

The critical Lundquist number and the effect of flows

The perturbation has to grow rapidly enough to disrupt the current sheet before plasmoids are evacuated.

\[
\frac{1}{\lambda} \frac{d\lambda}{dt} = \frac{dV_{0x}}{dx} \Rightarrow k(t) = k_0 e^{-\Gamma t}
\]

The “ideal” tearing onset determines the thickness at which a forming current sheet breaks apart for $S \gg S_{\text{critic}}$ and provides a general framework to determine “how much small” small-scales have to be to trigger a fast reconnection.

The “ideal” tearing has been extended to various equilibria and models including kinetic effects: the critical thickness is not universal.

Onset of reconnection in turbulence (Mallet et al. 2017, Boldyrev & Loureiro 2017)
- Need improvement to consider effects of turbulent flows and different current profiles.

Dynamics in three-dimensions and fractal reconnection in 3D.

Kinetic regime: stationary vs turbulent reconnection.