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Description of Global EGAM in the maximum of local frequency during current ramp-up discharges in DIII-D

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Outline

- GAM / EGAM – brief introduction and relation to energetic particle transport.
- Dispersion Relation of GAM / EGAM.
- Theoretical description of global EGAM.
- Comparison to N=0 oscillation during ramp-up discharge on DIII-D – discharge #159243.

GAM – Geodesic Acoustic Mode

- GAM are a low frequency electrostatic oscillation, with toroidal N=0, poloidal M=0 combined with M=$\pm 1, \pm 2$.

- First predicted by [*N. Winsor, Phys. Fluids (1968)].*

- The eigenmode can appear with reverse q profile (close to the center), when the dispersion relation presents a maximum for thermal plasma [*H.L. Berk (2006) / V.P. Lakhin (2014)*].

- The presence of a similar mode exited by Energetic Particles (**E-GAM**), theoretically predicted [*G. Y. Fu, (2008)*] and experimentally observed [*R. Nazikian (2008)*] is possibly a cause of energetic ion losses.

GAM calculated by NOVA Code

- NOVA calculates the dispersion relation based on ideal MHD equations [*C.Z. Cheng, Phys. Fluids (1986)*]:

$$\omega^2\rho\frac{|\nabla\psi|^2}{B^2}\xi_s + (\boldsymbol{B}.\nabla)\frac{|\nabla\psi|^2}{B^2}(\boldsymbol{B}.\nabla)\xi_s + \gamma p k_s \nabla.\vec{\xi} = 0,$$

$$\left(\frac{\gamma p}{\boldsymbol{B}^2}+1\right)\nabla.\vec{\xi} + \frac{\gamma p}{\omega^2\rho}(\boldsymbol{B}.\nabla)\frac{\boldsymbol{B}.\nabla}{B^2}\nabla.\vec{\xi} + k_s\xi_s = 0. \qquad (1)$$

where ψ is the poloidal magnetic flux, p the plasma pressure, ρ density, $\xi_s \equiv \frac{\vec{\xi}[\boldsymbol{B}\times\nabla\psi]}{|\nabla\psi|^2}$, $\vec{\xi}$ the plasma displacement, $k_s \equiv \frac{2\boldsymbol{k}.[\boldsymbol{B}\times\nabla\psi]}{|\nabla\psi|^2}$, $\boldsymbol{k}$ and $\boldsymbol{B}$ are vector of magnetic curvature and field.

- The adiabatic index $\gamma = 5/3$, is a free parameter in the code, was changed to make the MHD [*N. Winsor, Phys. Fluids (1968)]* equal to kinetic result $\omega_{G,mhd}^2 = \frac{\gamma p}{\rho R^2}\left(2+\frac{1}{q^2}\right) = \omega_{G,th}^2 = \left[\frac{7}{4} + \tau_e + \frac{23+4\tau_e(4+\tau_e)}{2q^2(7+4\tau_e)}\frac{1}{q^2}\right]\frac{v_{Ti}^2}{R^2}$.

- NOVA calculates the N=0 dispersion relation with slightly lower frequency than the standard formula.

- GAM eigenmode can be calculated for strongly reverse q profile, when ω_G^2 produces maximum OFF-AXIS (accumulation point).

Kinetic equation and distribution function

Using the drift kinetic equation in the form:

$$\frac{\partial}{\partial t} f_\alpha(r,\vartheta,t) + \frac{v_\parallel}{qR_0}\frac{\partial}{\partial \vartheta} f_\alpha(r,\vartheta,t) - v_{dr}\frac{\partial}{\partial r} f_\alpha(r,\vartheta,t) = e_\alpha \frac{\partial}{\partial E} F_\alpha(r,\vartheta)\left(\frac{v_\parallel}{qR_0}\frac{\partial}{\partial \vartheta}\emptyset(r,\vartheta,t) - v_{dr}\frac{\partial}{\partial r}\emptyset(r,\vartheta,t)\right) \quad (2)$$

Radial drift velocity $v_{dr,\alpha} = sin\vartheta\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)/\omega_{c\alpha}R_0$

Maxwellian equilibrium distribution function for thermal particles (*th*) and slowing down like distribution function for (*ep*).

$$F_{th,\alpha} = \frac{n_{0\alpha}(r)}{\pi^{3/2}}\frac{e^{\left(-\frac{(v_\parallel^2+v_\perp^2)}{v_{T\alpha}^2}\right)}}{v_T^3} \quad , \quad F_{ep} = \frac{n_{0b}(r)\sqrt{1-\lambda_0}}{ln(U_b/U_c)\,\pi^{3/2}}\frac{e^{\left(-\frac{(\lambda-\lambda_0)^2}{\Delta\lambda_0{}^2}\right)}}{\Delta\lambda_0 U^3}$$

The perturbed frequencyf_α and electrostatic potential $\emptyset$ expanded in Fourier for ϑ, and eikoidal approximation for r and t:

$$X(r,\vartheta,t) = (X_0 + X_s sin\vartheta + X_c cos\vartheta + X_{2s} sin2\vartheta + X_{2c} cos2\vartheta)\ \exp(ik_r r - i\omega t) \quad (3)$$

Eikoidal approximation imply $\partial/\partial r \rightarrow ik_r$

Density Oscillation

- From quasineutrality, in the limit $\rho^2 k_r^2 \ll 1$

- Electron density oscillations:

$$
\begin{aligned}
n_{es} &= -\frac{2e_i n_0}{m_i v_{Ti}^2 \tau_e}\phi_s \\
n_{e2c} &= -\frac{2e_i n_0}{m_i v_{Ti}^2 \tau_e}\phi_{2c} \\
n_{ec} &= n_{e2s} = 0
\end{aligned}
\tag{4}
$$

- Electrostatic oscillations:

$$
\begin{aligned}
\phi_s &= -\tau_e\left[\left(\frac{v_{Ti}}{\Omega} + (2+\tau_e)\frac{v_{Ti}^3}{\Omega^3}\right) q\rho_{th}k_r + (27 + 14\tau_e + 2\tau_e^2)\frac{v_{Ti}^3}{\Omega^3} q^3\rho_{th}^3 k_r^3\right]\phi_0 \\
\phi_{2c} &= -\tau_e\left[(7+4\tau_e)\frac{v_{Ti}^2}{\Omega^2} + (161 + 92\tau_e + 20\tau_e^2)\frac{v_{Ti}^4}{16\Omega^4}\right] q^2\rho_{th}^2 k_r^2 \phi_0 \\
\phi_c &= \phi_{2s} = 0
\end{aligned}
\tag{5}
$$

- EP effects are negligible due to small density.

Thermal GAM dispersion relation

- From average of radial current $\langle j_\alpha \rangle = e_\alpha \oint d^3v\, v_{dr,\alpha} \sin\vartheta\, f_\alpha$.

- The resonance condition $\langle j_i \rangle + \langle j_e \rangle = 0$, disregarding $\rho^2 k_r^2$.

- Thermal GAM dispersion relation (D_{th} = 0):

$$D_{th} = 1 - \frac{7+4\tau_e}{4}\frac{v_{Ti}^2 \sigma_s^2}{\omega^2 R_0^2} + \frac{23+16\tau_e+4\tau_e^2}{8q^2}\frac{v_{Ti}^4 \sigma_s^4}{\omega^4 R_0^4 q^2} + iq^2 2\sqrt{\pi}\left[\frac{\omega^3 R_0^3 q^3}{v_{Ti}^3} + (1+\tau_e)\frac{\omega R_0 q}{v_{Ti}}\right]\exp\left(-\frac{\omega^2 R_0^2 q^2}{v_{Ti}^2}\right). \qquad (6)$$

σ_s is formally included, calculated by NOVA.

- From $\omega \to \omega + i\gamma$ with $\gamma \ll \omega$, the linear growth rate:

$$\gamma \approx -\frac{\omega R_0 q}{v_{Ti}} q^2 \sqrt{\pi}\left[\frac{\omega^2 R_0^2 q^2}{v_{Ti}^2} + (1+\tau_e)\right]\exp\left(-\frac{\omega^2 R_0^2 q^2}{v_{Ti}^2}\right)$$

EGAM dispersion relation

- EGAM Dispersion relation contribution ($\langle j_i \rangle + \langle j_e \rangle + \langle j_{ep} \rangle = 0 \rightarrow D_{th} + D_{ep} = 0$):

$$D_{ep} = -\frac{n_b}{n_i 16(1-\lambda_0)^2 \ln\left(\frac{V_b}{V}\right)}\left[\left((5\lambda_0 - 2)(2-\lambda_0) - \Delta\lambda_0^2 \frac{3(20-\lambda_0^2-4\lambda_0)}{16(1-\lambda_0)}\right)\ln\left(1-\frac{\omega_{tr}^2}{\omega^2}\right) + \frac{2\lambda_0(\lambda_0-2)\omega_{tr}^2}{\left(1-\frac{\omega_{tr}^2}{\omega^2}\right)\omega^2} - \Delta\lambda_0^2 \frac{3(2-\lambda_0)\omega_{tr}^2}{(1-\lambda_0)\left(1-\frac{\omega_{tr}^2}{\omega^2}\right)^2 \omega^2}\right] \quad (7)$$

- EP toroidal transit frequency $\omega_{tr} = V_b\sqrt{(1-\lambda_0)}/qR_0$.

- The injection angle $\lambda_0 = V_{b\perp}^2 / V_b^2$.

- The linear growth rate $\gamma \propto (5-2\lambda_0)(2-\lambda_0) - \Delta\lambda_0^2 \frac{15}{4(1-\lambda_0)}$, for $\omega^2 < \omega_{tr}^2$.

- **Instability found when λ_0>0.43** for $\Delta\lambda_0 \sim 0.2$.

- Result similar to *[Z. Qiu, (2010)]* for $\Delta\lambda_0 = 0,\ \sigma_s = 1$.

Eigenmode Equation via Finite Larmor Radius (FLR)

- Keeping terms up to $\rho^2 k_r^2$:

$$\langle j_i \rangle + \langle j_e \rangle + \langle j_{ep} \rangle = 0 \quad \rightarrow \quad \left[\left(D_{th} + D_{ep}\right) + \left(L_{th}\rho_{th}^2 + L_{ep}\rho_{ep}^2\right)k_r^2\right]k_r\phi_0 = 0. \tag{8}$$

- Thermal FLR term:

$$L_{th} = \frac{3}{8} + \frac{747+481\tau_e+140\tau_e^2+\tau_e^3}{64}\frac{v_{Ti}^4}{\omega^4 R_0^4}. \tag{9}$$

- EP FLR-EP term:

$$L_{ep} = \frac{n_b(2-\lambda_0)^3}{n_i 16(1-\lambda_0)^2 \ln\left(\frac{V_b}{V_c}\right)}\left[\left(\frac{(9\lambda_0-2)}{1-\frac{\omega_{tr}^2}{\omega^2}} + \Delta\lambda_0^2 \frac{3\left(\lambda_0^2+12\lambda_0-76\right)}{16(1-\lambda_0)(2-\lambda_0)}\right)\left(\ln\left(1-\frac{\omega_{tr}^2}{\omega^2}\right) + \frac{1}{16}\ln\left(1-4\frac{\omega_{tr}^2}{\omega^2}\right)\right)\right.$$
$$\left. \frac{3\left(8\lambda_0^2+20\lambda_0-8\right)\omega_{tr}^4-5\omega_{tr}^2\omega^2(9\lambda_0-2)}{16\left(\omega^2-\omega_{tr}^2\right)\left(\omega^2-4\omega_{tr}^2\right)} - \Delta\lambda_0^2 \frac{600\omega_{tr}^6\omega^2-611\omega_{tr}^4\omega^4-112\omega_{tr}^4+190\omega_{tr}^2\omega^6}{16(2-\lambda_0)\left(\omega^2-\omega_{tr}^2\right)^2\left(\omega^2-4\omega_{tr}^2\right)^2}\right] \tag{10}$$

- FLR connect different magnetic surfaces
- In presence of EP, FLR is dominated by L_{ep}, $\quad \rho_{ep}^2 \gg \rho_{th}^2$

Eigenmode Equation

- Recovering the derivative form: $\partial/\partial r \sim ik_r$ in eq.8 ($[(D_{th}+D_{ep})+(L_{th}\rho_{th}^2+L_{ep}\rho_{ep}^2)k_r^2]k_r\phi_0=0$)

$$Q(r,\omega)\frac{d\phi_0}{dr}(r,\omega)-\frac{\partial^2}{\partial r^2}\frac{d\phi_0}{dr}(r,\omega)=0 \tag{11}$$

$$\mathrm{Q}(r,\omega)=\frac{D_{th}+D_{ep}}{F_{th}\rho_{th}^2+F_{ep}\rho_{ep}^2}$$

- The solution is calculated numerically between the axis and some point far from $Q\sim 0$, then matched with the WKB approximation:

$$\frac{d\phi_0}{dr}=\frac{C_1 e^{i\int\sqrt{Q}dr}+C_2 e^{-i\int\sqrt{Q}dr}}{Q^{1/4}} \tag{12}$$

- A physical solution is found only around the maximum of dispersion relation.

- $Im(Q)$ induces convergence runaway [LeVeque (2007)], physical solution is found for C_2=0.

- Disregarding $Im(F_{ep})$ preserves the eigenmode structure, with minimum convergence runaway solution.

TRANSP profile of #159243

- Time evolution:
  - 500ms – solid line
  - 900ms – dot line

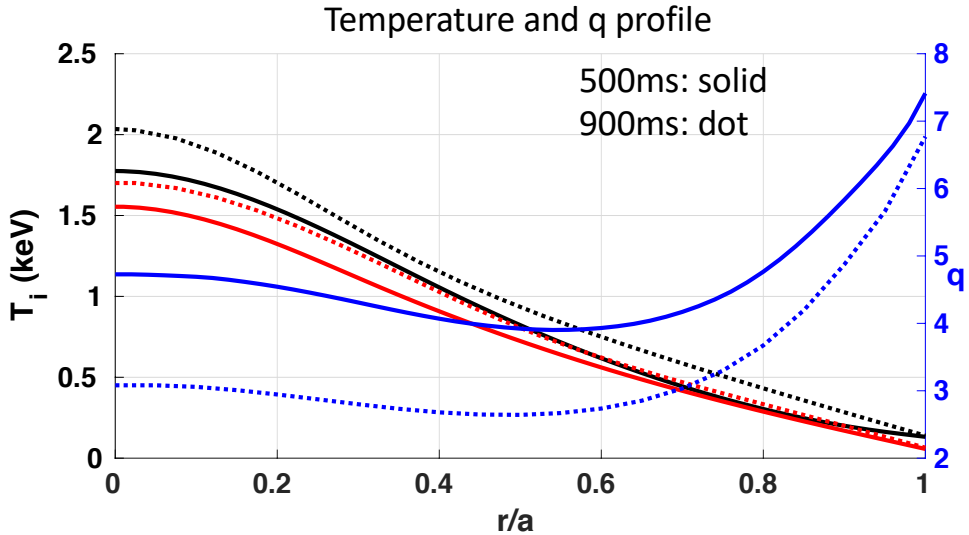
- No strong radial variation in temperature, and the ratio remains around $\tau_e \sim 0.86$.

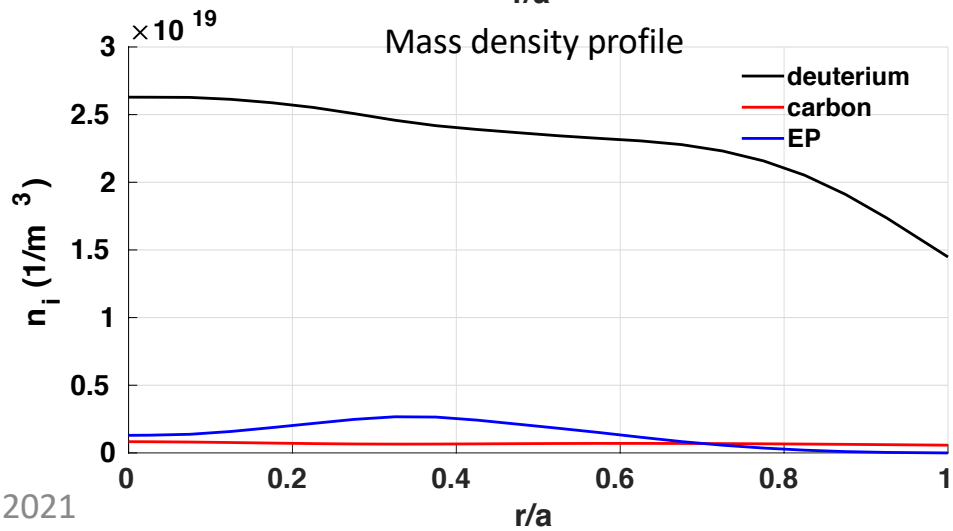
- q at axis decrease from ~ 4.7 to 3.

- mostly co-tangencial discharge with $\lambda_0 = 0.5 \sim 0.6$, $\Delta\lambda_0 = 0.2$.

- Kick model for EP is used [M. Podesta, (2014)].

- Carbon impurities correction are included via effective mass.





Local dispersion relation

- The local dispersion relation for 500ms and 900ms, injection angle $\lambda_0 = 0.5$ and $n_b/n_i \approx 4\%$ for $r/a < 0.45$, smoothly decrease for $r/a > 0.45$, for 900ms EP is limited to $r/a < 0.35$.

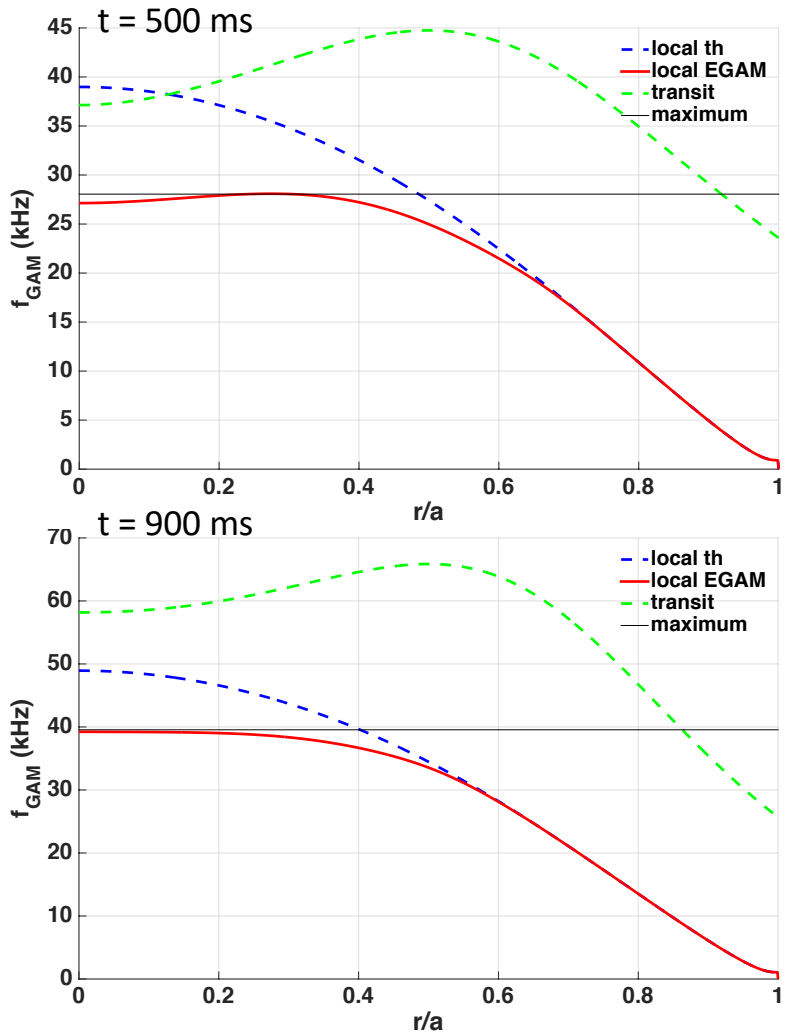
- **Local th GAM** is calculated by NOVA.

- The interaction between **EP transit** frequency and **Local th GAM** produces the **Local EGAM.**

- **Maximum OFF-AXIS** in the dispersion relation is found for 500ms, around r/a=0.3.

- **Maximum ON-AXIS** is found for 900ms.

- Both have positive growth rate $\gamma > 4\ \%$.



Global E-GAM radial structure

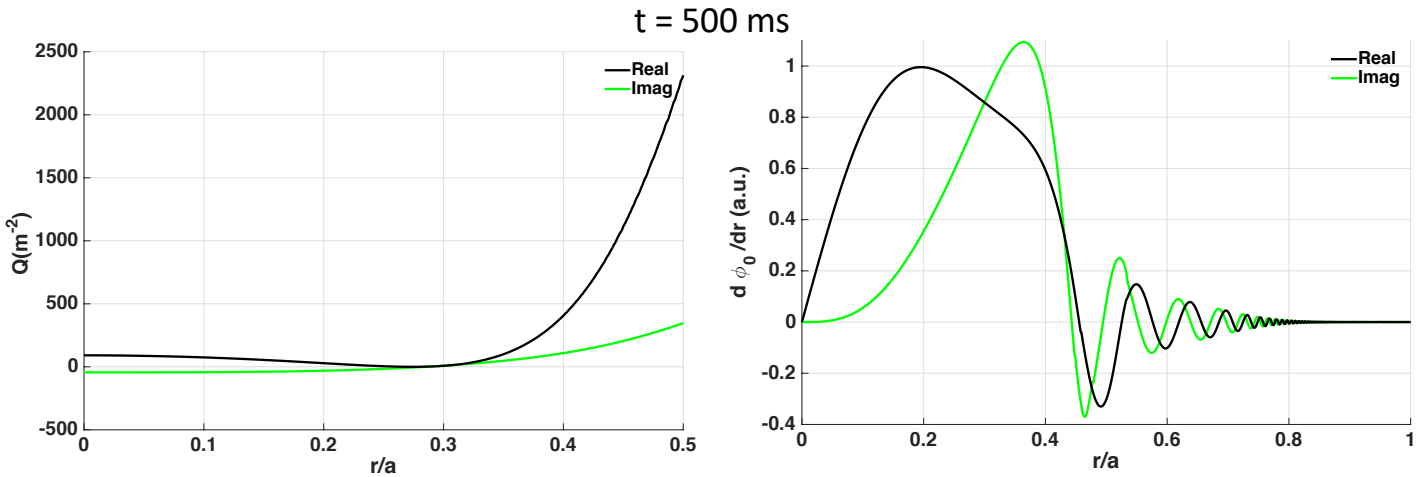
- The global behavior is much more pronounced for 500ms, with Airy function for r/a = 0 - 0.5, and a outgoing wave is find for r/a > 0.5.

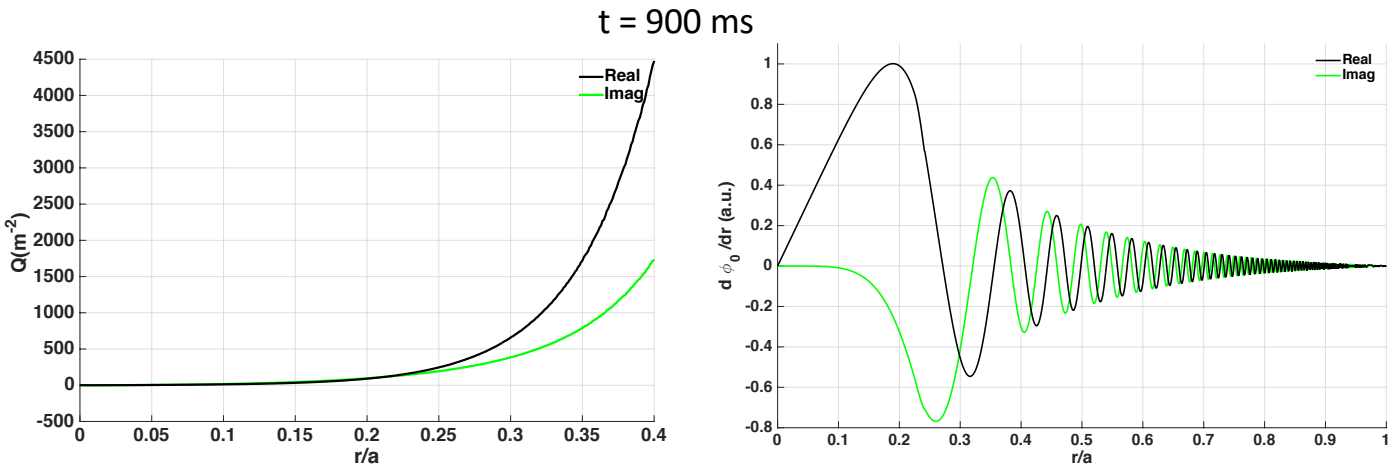
- For 900ms is found the outgoing wave from r/a=0.

- In both cases $\rho^2 k_r^2 \approx 0.6$

- Radial structure similar to obtained by kinetic, numerical [G. Y. Fu, (2008)] and analytically [Z. S. Qu, (2017)] for EGAM.

- MHD numerically [*H.L. Berk, 2006*] and analytically *[Lakhin, 2014]*, present Dirac for $d\phi_0$/dr, in the maximum of dispersion relation, M=1,2 are global.





Consistency with N=0 oscilations observed in DIII-D

- NOVA simulation of TRANSP kick model profiles.

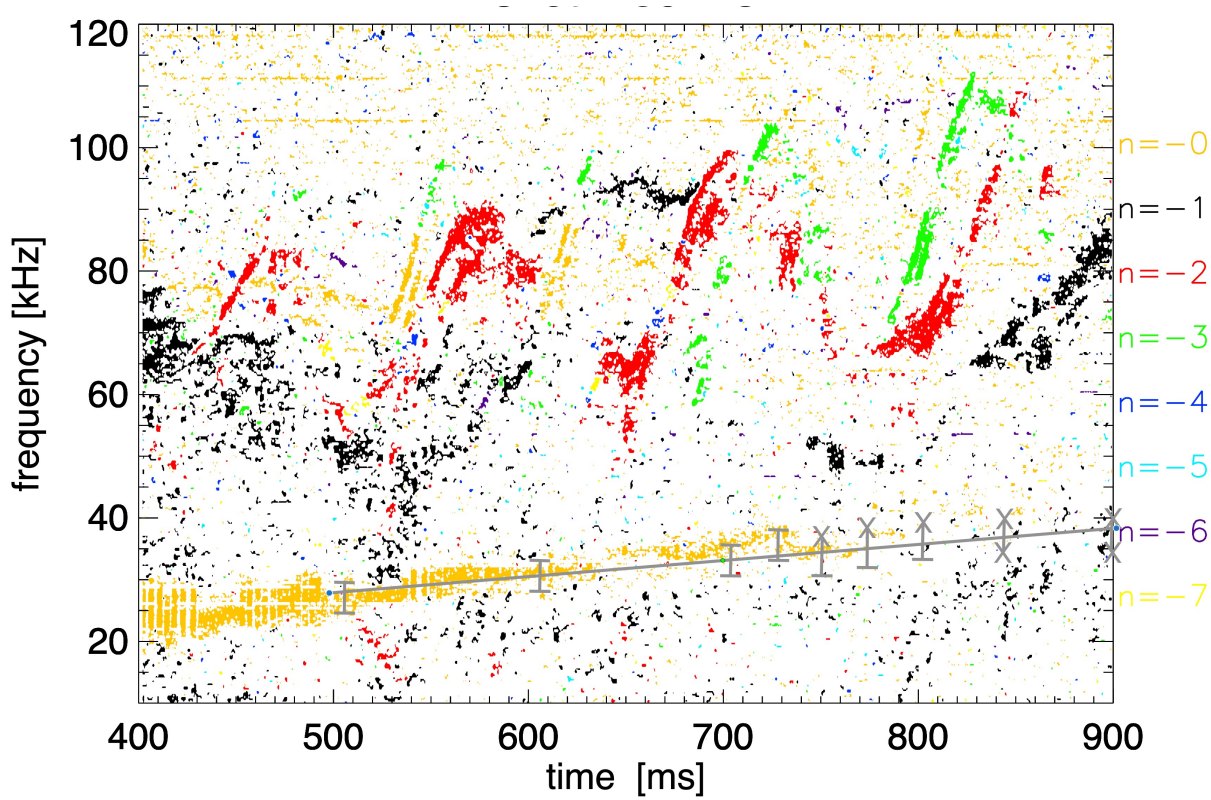
- The results match the observed N=0 mode by Mirnov coils on #159243 in DIII-D. The observed frequency in yellow, and the matched points in gray.

- Error bars due to variation of $\lambda_0 = 0.5$ - 0.6.

- The frequency increment in time is due to time evolution of temperature and q.

- X means that there is no maximum in the continuum and no eigenmode should be observed at that point.

- Good numerical agreement between theorical prediction and experimental data.



Summary

- Slightly reverse *q* profile is a source for global EGAM.
- The instability is dominated by injection angle λ_0 and absolute value of *q*, due to relation of toroidal EP transit frequency and and GAM frequency.
- The condition for instability is the presence of OFF-AXIS maximum in the EGAM local dispersion relation.
- Good quantitative agreement with DIII-D N=0 oscillation on #159243 discharge.

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Profiles used slides 12 and 13

Consistent with TRANSP

$$T_{500}(J) = 1.8\left(1-(r/a)^2\right)^{2.5}\ keV$$

$$T_{900}(J) = 2.0\left(1-(r/a)^2\right)^{2.5}\ keV$$

$$q_{500} = 4.7 - 10(r/a)^2 + 14.8(r/a)^3 - 2(r/a)^4$$

$$q_{900} = 3 - 1.8(r/a) \quad -4(r/a)^2 + 9{,}6(r/a)^4$$

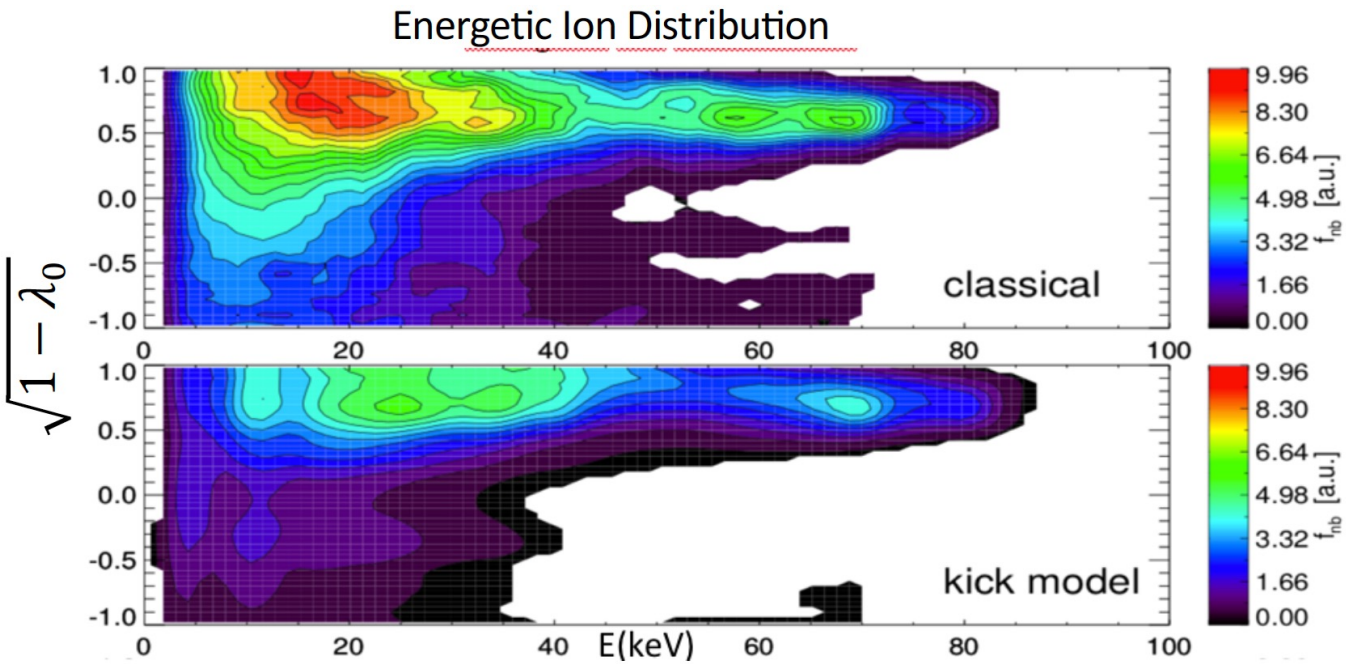
$$\tau_e = 0{,}86$$

$\sigma_{mhd} = 0.7$ (NOVA gives a profile 0.7 in center 0.75 near edge)

$$R_0 = 1{,}67m$$

$$a = 0.64$$

Classical EP profile – TRANSP reconstruction

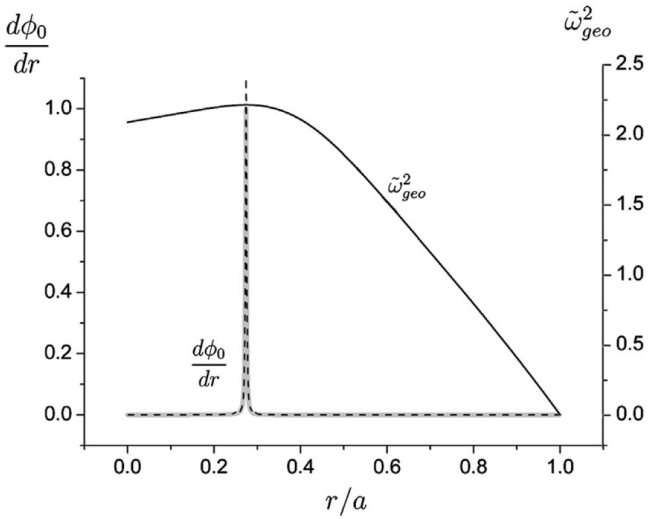


N.N. Gorelenkov et al (2018)

Injection starts with $\lambda_0 = 0.6$, and moves to parallel motion as slow down

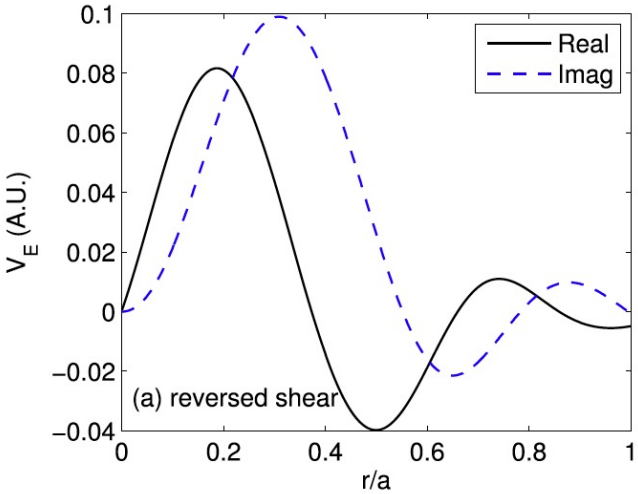
Eigenmode – previews work comparision

MHD, thermal GAM



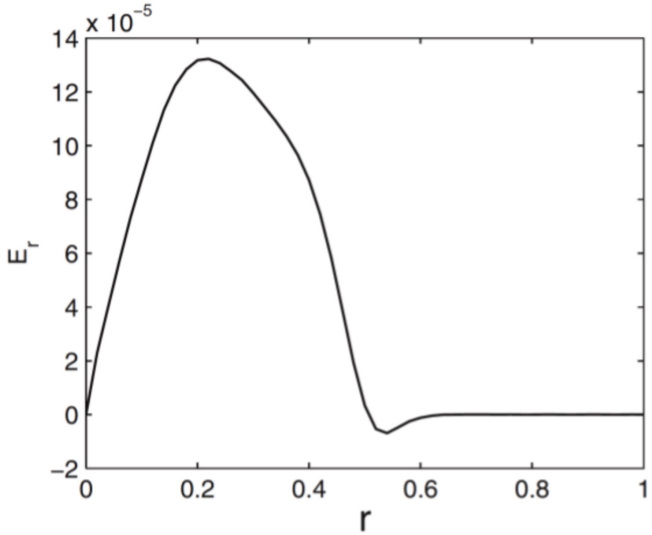
[Lakhin, 2014]

Kinetic – analytical
Single energy EP



[Z. S. Qu, (2017)]

Kinetic – numerical
Slowing Down EP



[G. Y. Fu, (2008)]