Regimes of weak ITG/TEM modes for transport barriers without velocity shear

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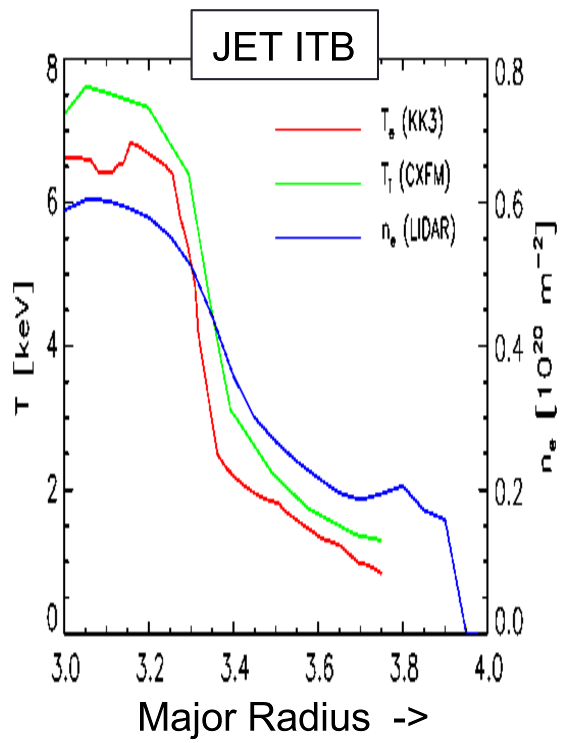
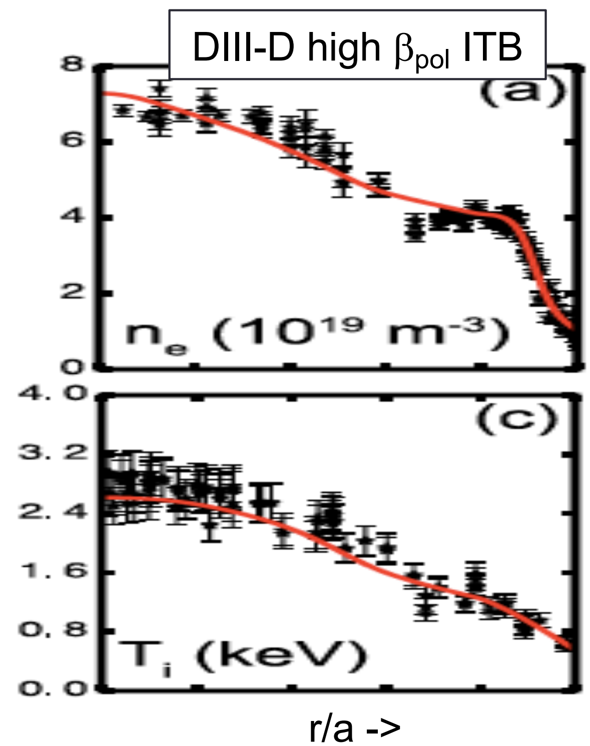
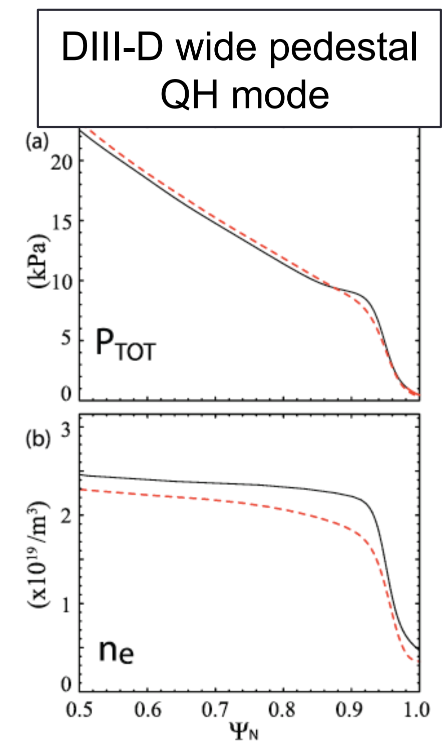
Some Transport Barriers (TBs) don't need velocity shear

- High α ("Shafranov") stabilization leads to Transport Barriers in tokamak experiments without requiring velocity shear
  - Multiple experimental examples of TB are not attributed to high velocity shear
  - They have high α and/or negative magnetic shear
- ITG/TEM modes were evidently considerably weakened in these barriers







Understanding how TBs arise without velocity shear is crucial, since burning plasmas have much less of it

- After extensive examination:
  - High α is **neither** necessary nor sufficient for the fundamental physics
  - Optimized stellarators could likely access the relevant physics even better than tokamaks
  - Density gradients are crucial to allow high T gradients
- Here we examine these TBs from the point of view of general concepts of Non-Equilibrium Thermodynamics (NET)
  - We find these to be considerably better than existing interpretations
- We start with tokamak TBs with high α
- **Once we understand the tokamak cases from an NET perspective, generalizing to other geometries is straightforward**

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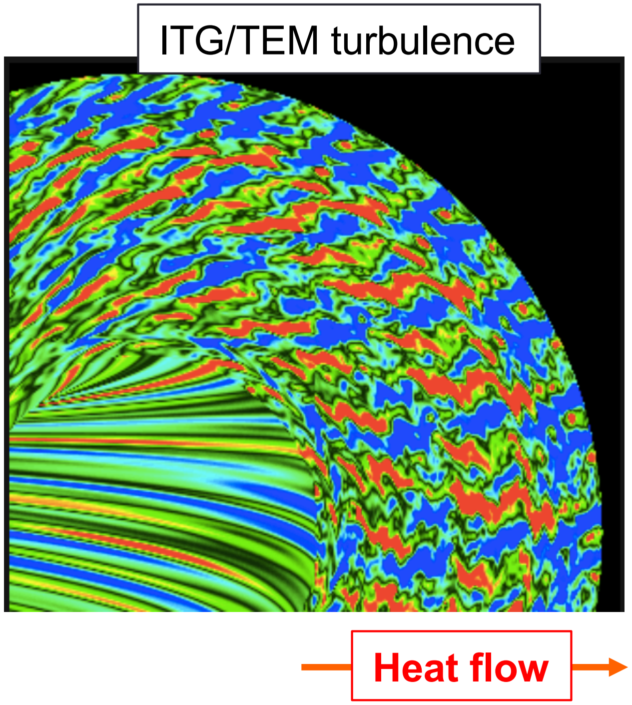
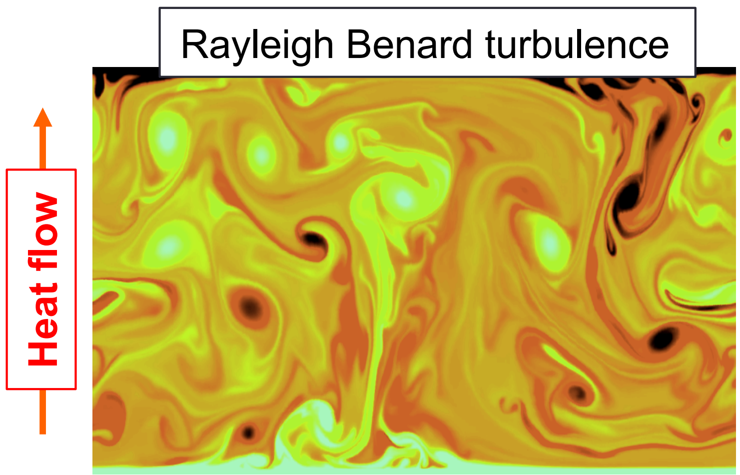
Typical behaviors in Non-Equilibrium Thermodynamics (NET)

- Very often in NET, the fluctuations are highly *adaptive*
  - Adjust to produce a high thermodynamic flux
  - Qualitatively, Maximize Entropy Production (MEP)
- Fluctuations often behave as Self-Organized States (SOS)
  - Adaptivity is one signature characteristic of these
- Conventional fluids can have similar ingredients to ITG/TEM - turbulent fluctuations with T gradients and gravity (= curvature):
  - Rayleigh Benard cells
  - **Considered paradigmatic cases of SOS, MEP**
- We'll see: ITG/TEM have many properties common in NET ,e.g., highly adaptive to maximize heat flux:
  - So why do they allow a TB with high dT/dx but low heat flux ????





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Demonstration of adaptive behavior of ITG/TEM

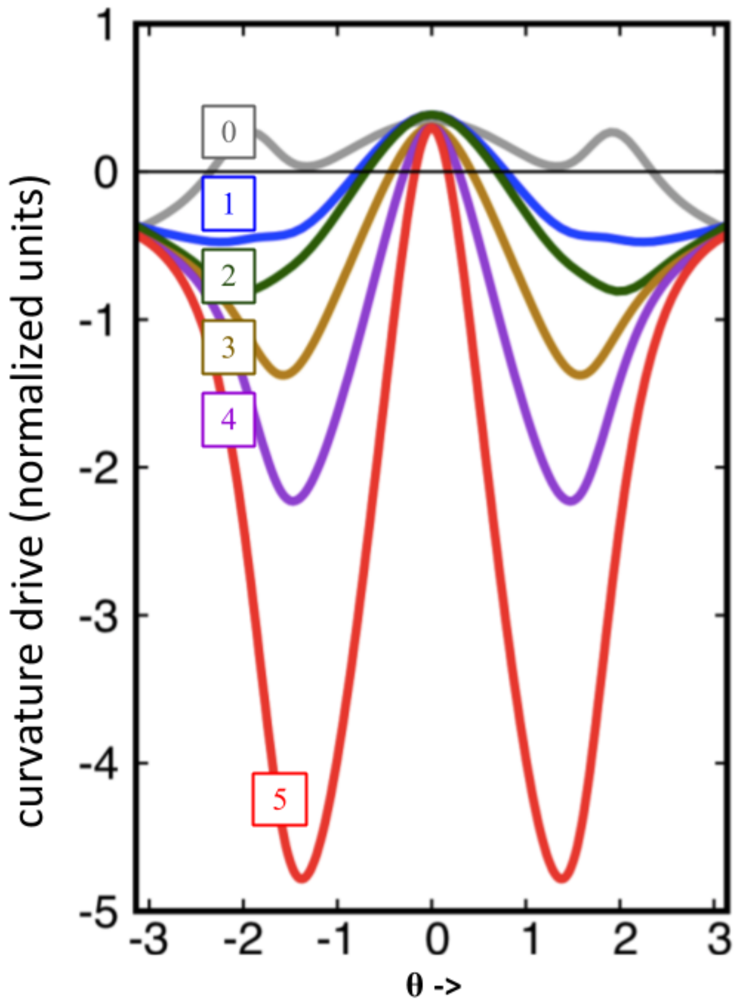
- The ITG/TEM adjust their spatial structure to grow despite huge variations in the magnetic geometry

- The growth rate γ only decreases by ~ 40% going from case 0 to case 5 if only dT/dr is present
  - We'll present many details of this scan a little later

- Eigenfunctions *strongly* concentrate in the bad curvature region, avoiding the stabilizing region

- So why is the heat flux low enough to allow a TB ????

Equilibrium scan with enormous changes in the curvature drive in the gyrokinetic eqn.



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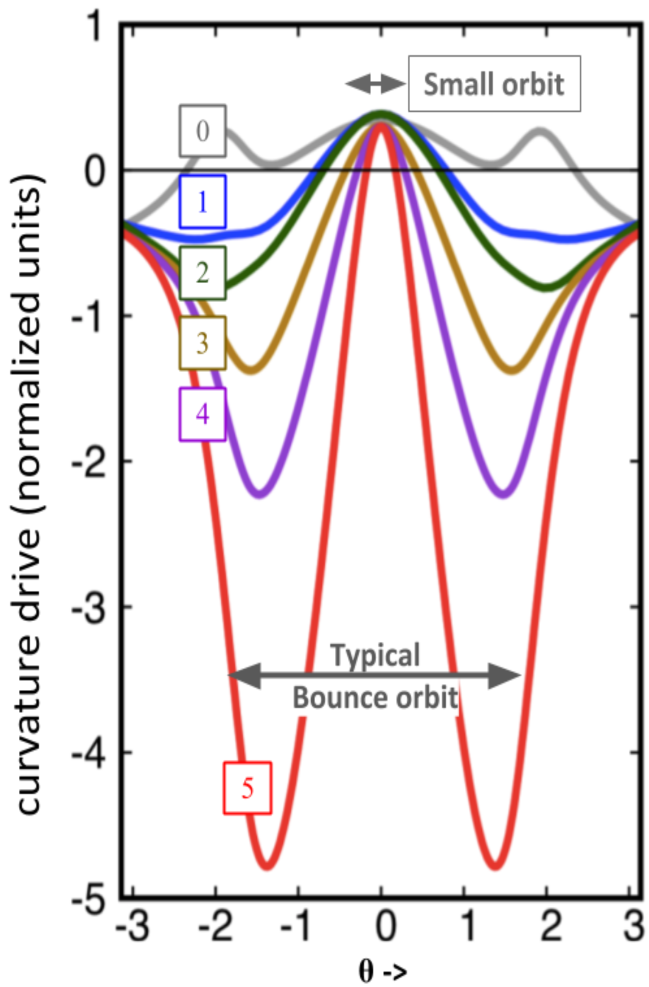
Why a TB is possible: ITG/TEM become subject to a thermodynamic constraint

- The Maximum Entropy Production principle: *constraints* on entropy production (=heat flux) of fluctuations are crucial
  - The constraints are a crucial aspect of the maximization
  - The only way to restrict highly adaptive fluctuations
- High α geometries engender such a constraint: the electrons become approximately adiabatic
- The eigenfunctions become narrow so they can't couple to most trapped particles, which bounce average over a much larger extent
- Stellarator geometries engender the same constraint, via different geometric routes



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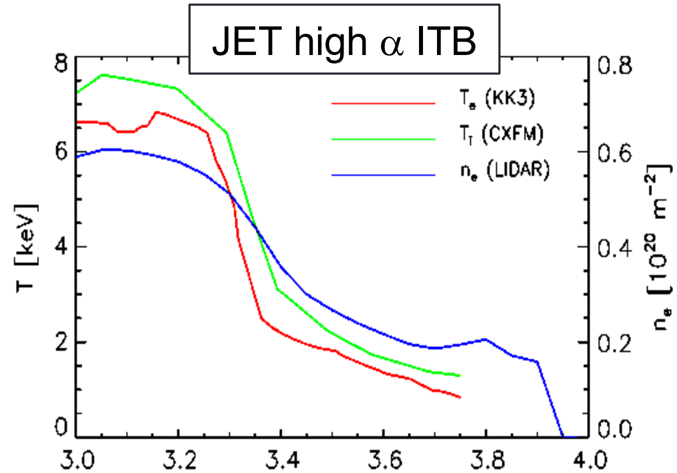
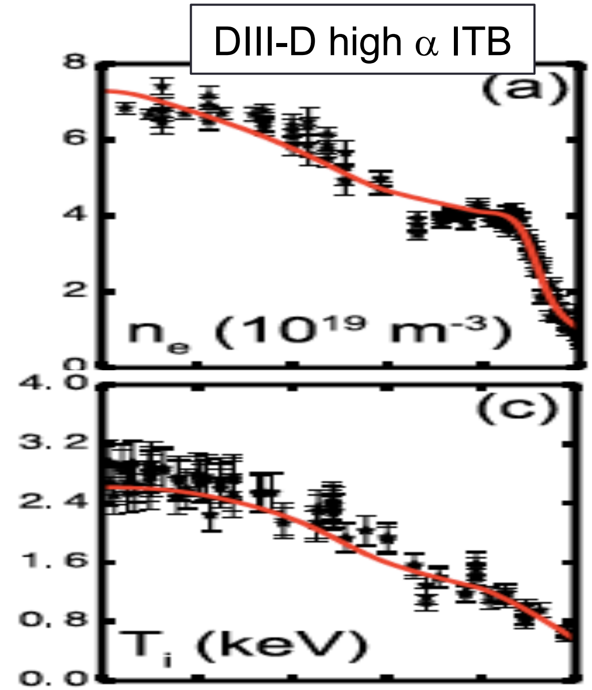
The constraint is only binding when density gradients are a significant fraction of the pressure gradient

- F_p = fraction of pressure gradient ∇p from density gradient = $T \nabla n / \nabla p$
- As F_p increases, the free energy available to drive instability transitions from all ∇T to a mix of ∇n and ∇T
- Instability cannot feed on density gradients because:
  - poor coupling of eigenfunction to trapped electrons =>
  - Electrons react essentially adiabatically =>
  - no particle transport to access ∇n
- Eventually the mode loses free energy drive: "starved" by larger Fp





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We now examine simulation results, with these concepts in mind

- We start with a tokamak case with high α and negative shear and analyze it in detail
  - Experiments with such parameters frequently have strong TBs
- The relative simplicity of the tokamak geometry allows us to more easily demonstrate important ITG/TEM properties and metrics relevant to TBs
- We show that many experimental TBs are in this regime (both ITBs and H-mode pedestals)- both tokamaks and stellarators
- We apply the concepts and metrics we developed to stellarators after developing them for the computationally easier case of tokamaks
- We then consider analytic theory results to further understand the physics

We use simulations to compare transport in both high α and core-like magnetic geometries for TB-like gradients

- Analyze a Transport Barrier Base Case modeled on ITB (JET, JT-60U) but also similar to pedestals (**α = 3, $\hat{s}$=-1**)

- Steep gradients R/L_{press} = 40 (like steep ITB or top of pedestal)
  - This is about 4-8 times steeper than typical core R/L_{press}

$1/L_{press}$ = (1/p) dp/dr

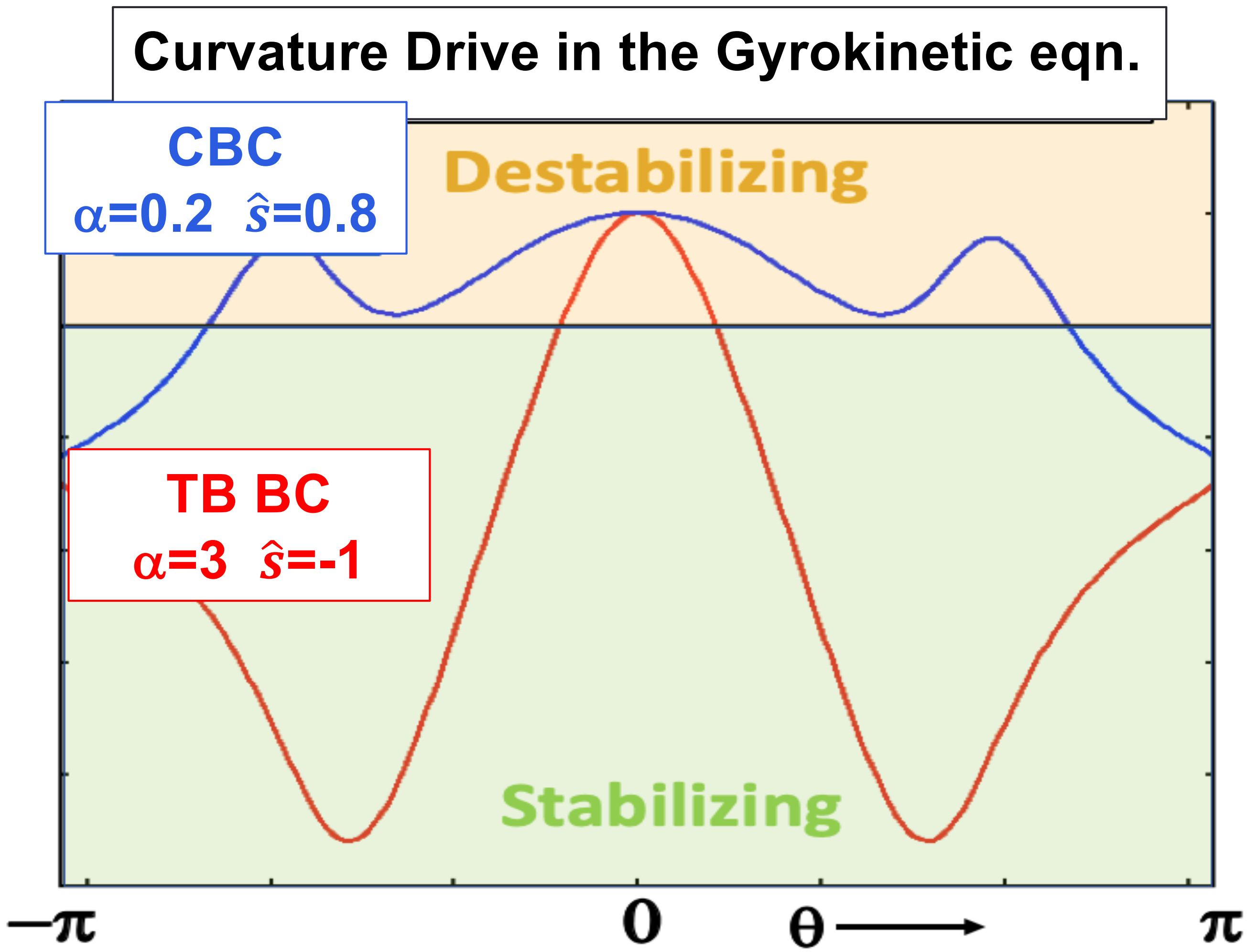
For a typical core-like case w/o velocity shear, such a steep TB gradient will gives heat loss from ITG/TEM ~ 2 orders of magnitude too high

A reduction of ~ 2 orders of mag is needed for a viable TB

- Use GENE
  - Electrostatic ITG/TEM
  - Local linear and nonlinear (i.e., ρ^* -> 0 limit), γ_{ExB} = 0

The TB BC captures the main qualitative feature of high α, negative $\hat{s}$ geometries relative to typical core-like geometries

- Compare the curvature driving term in the gyrokinetic equation (usual local ballooning limit)
- Typical core-like geometry: Cyclone Base Case (w shaping)
- **TB BC: small region of destabilizing curvature, much larger region of highly stabilizing curvature**



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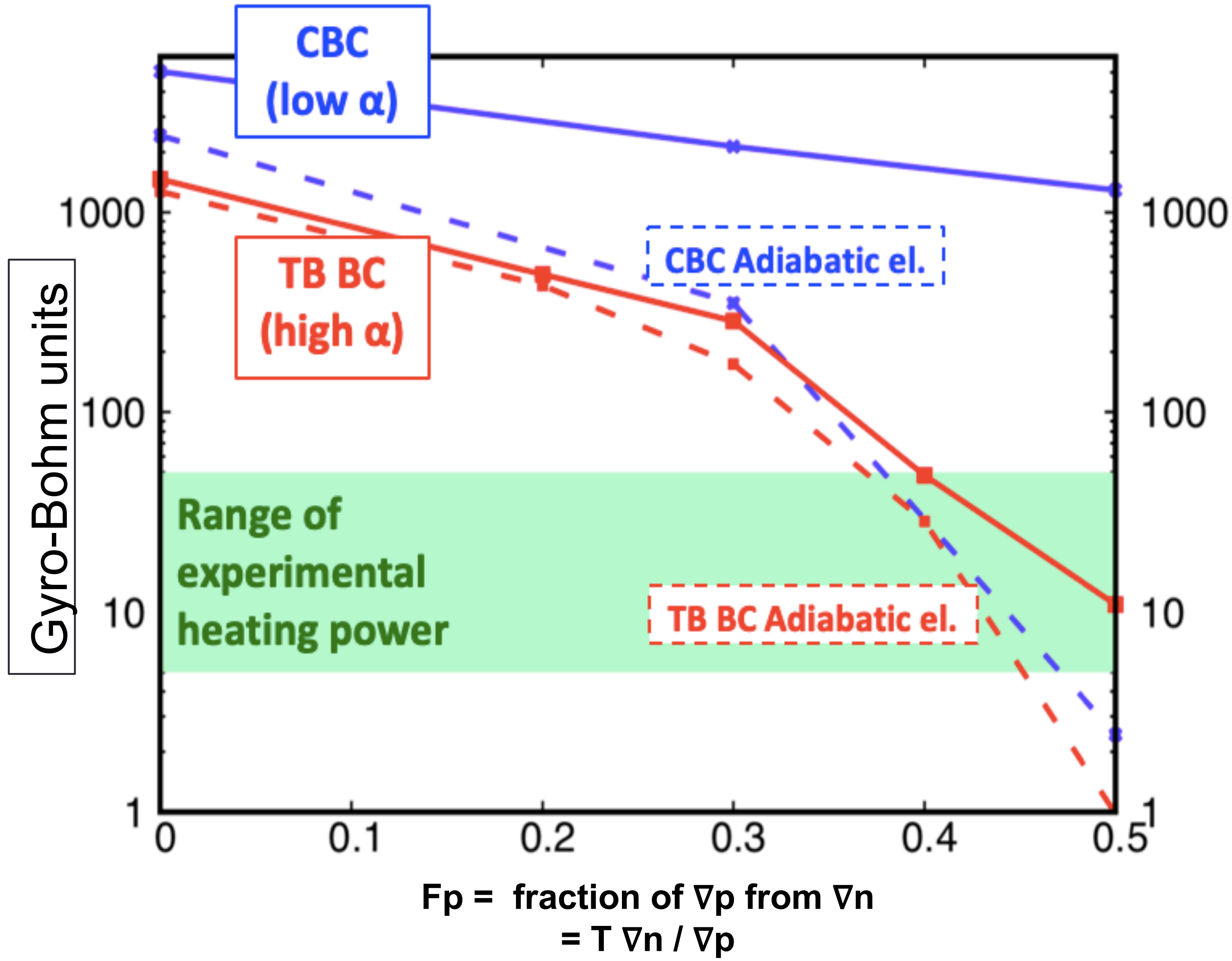
Nonlinear Heat Flux results

CBC: heat flux **always far too high**

TB BC: Two order of magnitude drop for F_p ~ 0.4- 0.5

With adiabatic electrons instead of full electron dynamics:

BOTH cases behave very similar to the TB BC



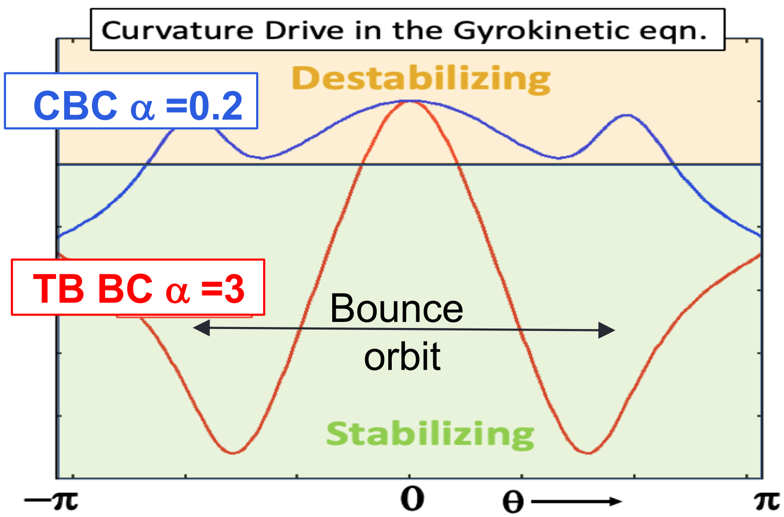
What is the physics of this ?

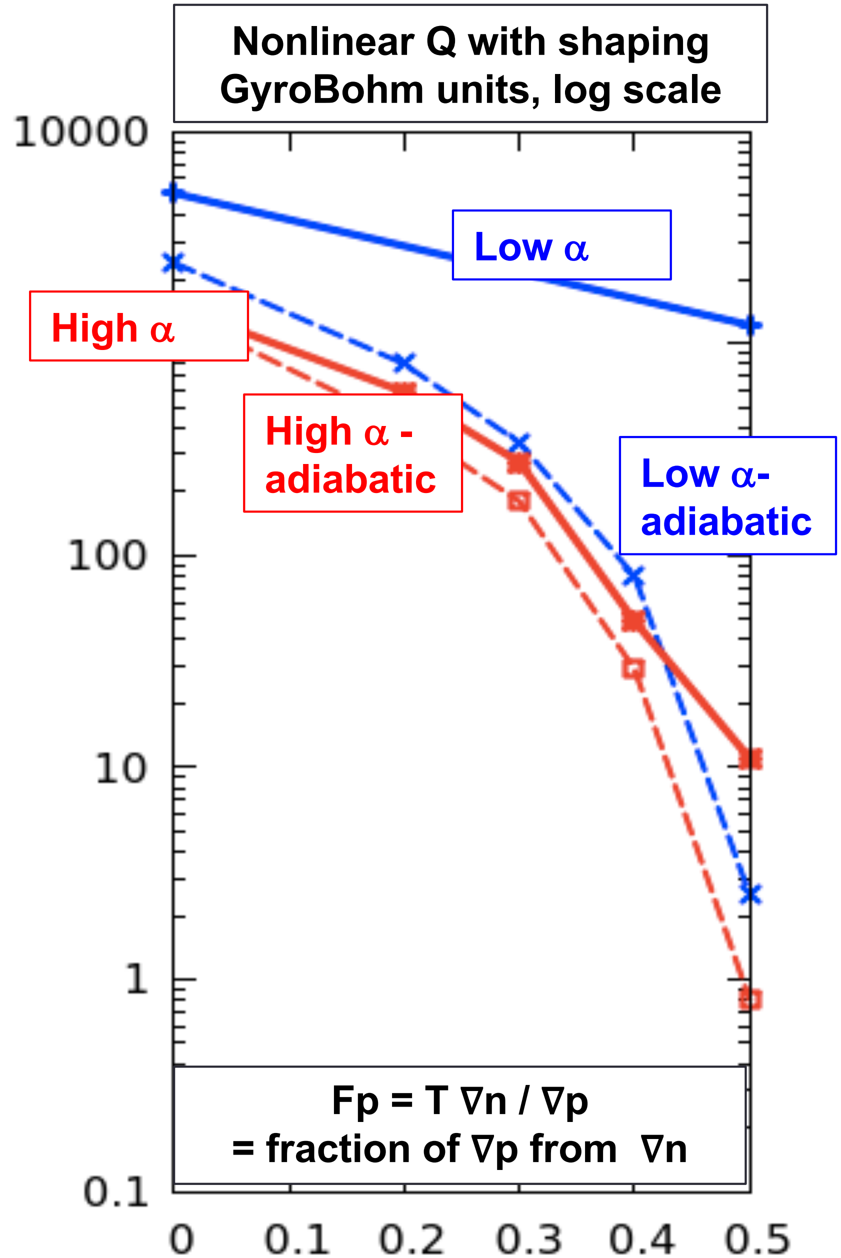
- Run with adiabatic electrons:
- **The high α case changes relatively little**

Evidently high α *suppresses non-adiabatic electron effects*

- **The low α case now behaves extremely similarly to high α**

The degree of adiabaticity is what distinguishes low and high α with full electrons !





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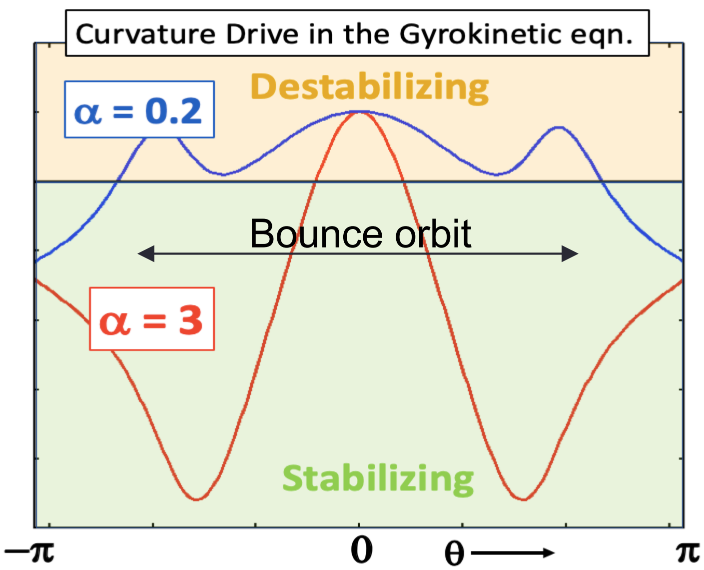
Physics explanation (after lengthy investigation):

- As α increases, unstable eigenfunction becomes narrower in θ, AVOIDING the stabilizing curvature region
- Narrow eigenfunction couples poorly to trapped electrons, whose typical bounce orbits are MUCH larger



- F_p = amount of density gradient in ∇p
- As F_p increases, the free energy available to drive instability transitions from all ∇T to a mix of ∇n and ∇T
- Instability cannot feed on density gradients because:
  - poor coupling of eigenfunction to trapped electrons =>
  - Electrons react essentially adiabatically =>
  - no particle transport to access ∇n
- Eventually the mode loses free energy drive: "starved" by larger Fp

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Previous interpretations of such results

- Previous work has shown that, for high α and/or negative $\hat{s}$, the bounce average electron curvature is quite stabilizing for most trapped electrons
- Results such as the ones described have been interpreted in terms of stabilization of trapped electron modes by this bounce avg. curvature
- It has not been recognized that, simultaneously,
  - The eigenfunction adjusts to the curvature to stay in the bad curvature region
  - When this is accounted for by defining “eigenfunction averaged” curvatures, the ion and electron curvatures stay destabilizing even at high α
  - The coupling to trapped electrons is strongly decreased, as determined by a quantitative metric
  - It is the latter which explains the simulation results
- We use simulations to examine these behaviors

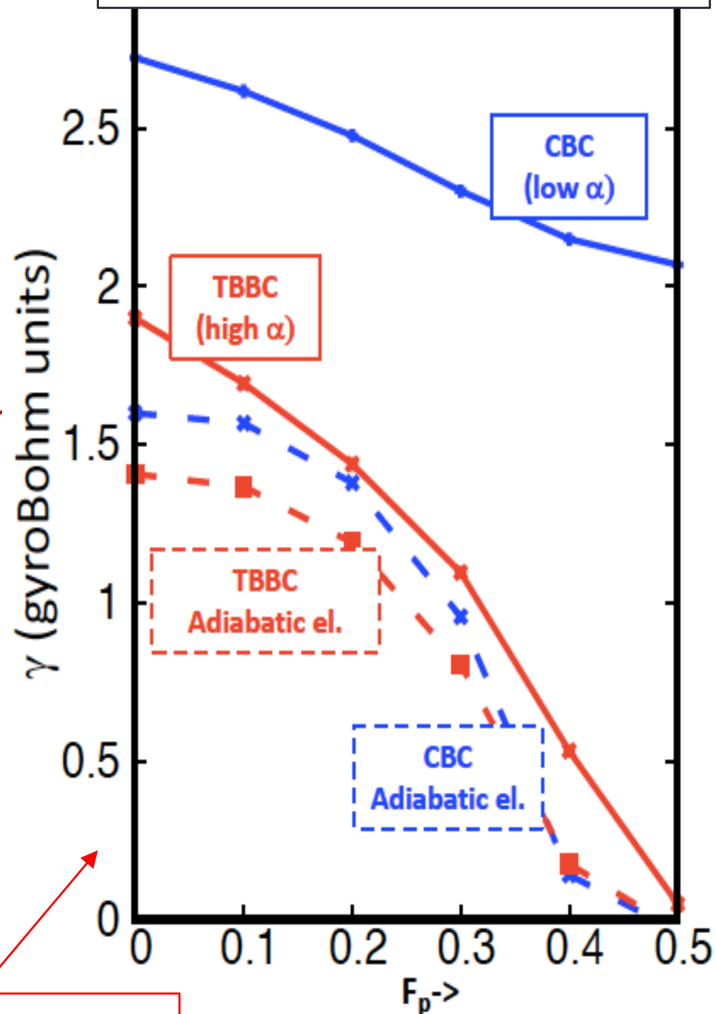
Testing for such adaptive behaviors in simulations

- Compare the same run with adiabatic e⁻
  - Adiabatic e⁻ runs have NO trapped e⁻ effects
- High alpha case:
  - If γ changes little, trapped e⁻ were decoupled
  - If γ drops, trapped electrons were destabilizing:

    Eigenfunction adapted to the curvature to find a way to use them to assist in growth, even if slightly
  - If the γ increases, the e⁻ were a stabilizing effect

    Eigen-function strongly sampled the good curv. regions- poor adaptation
- Also: compare to low alpha case
  - Is low α with adiabatic e⁻ similar to the high α case?
  - Adaptive mode concentrates in bad region for either low or high α – with adiabatic constraint they're similar
  - Low α has strong coupling to trapped e⁻ : much more unstable with full e⁻

Growth rate γ for a representative $k_y\rho_i$ ($\approx \frac{1}{3}$)



The combination of results such as these => that adiabaticity constraint is the key to stabilization, not curvature structure

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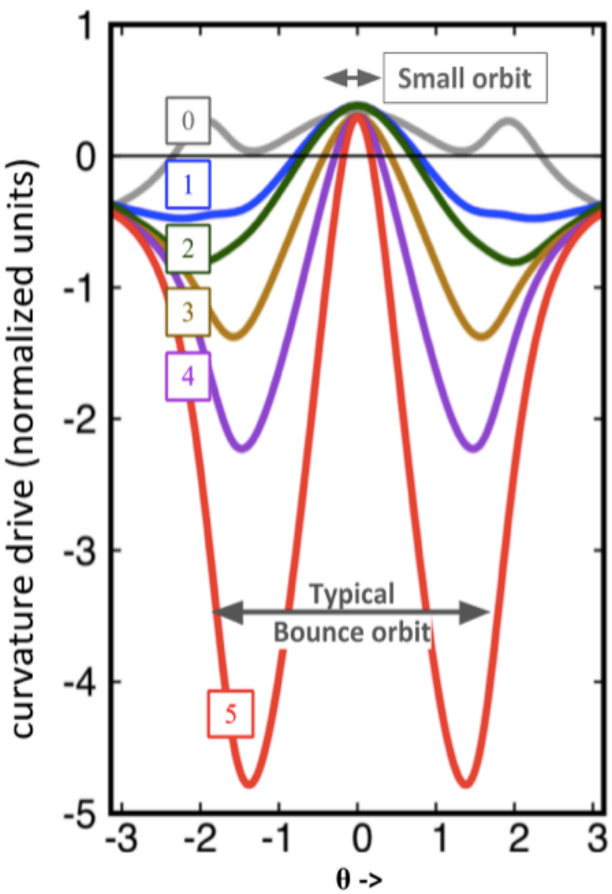
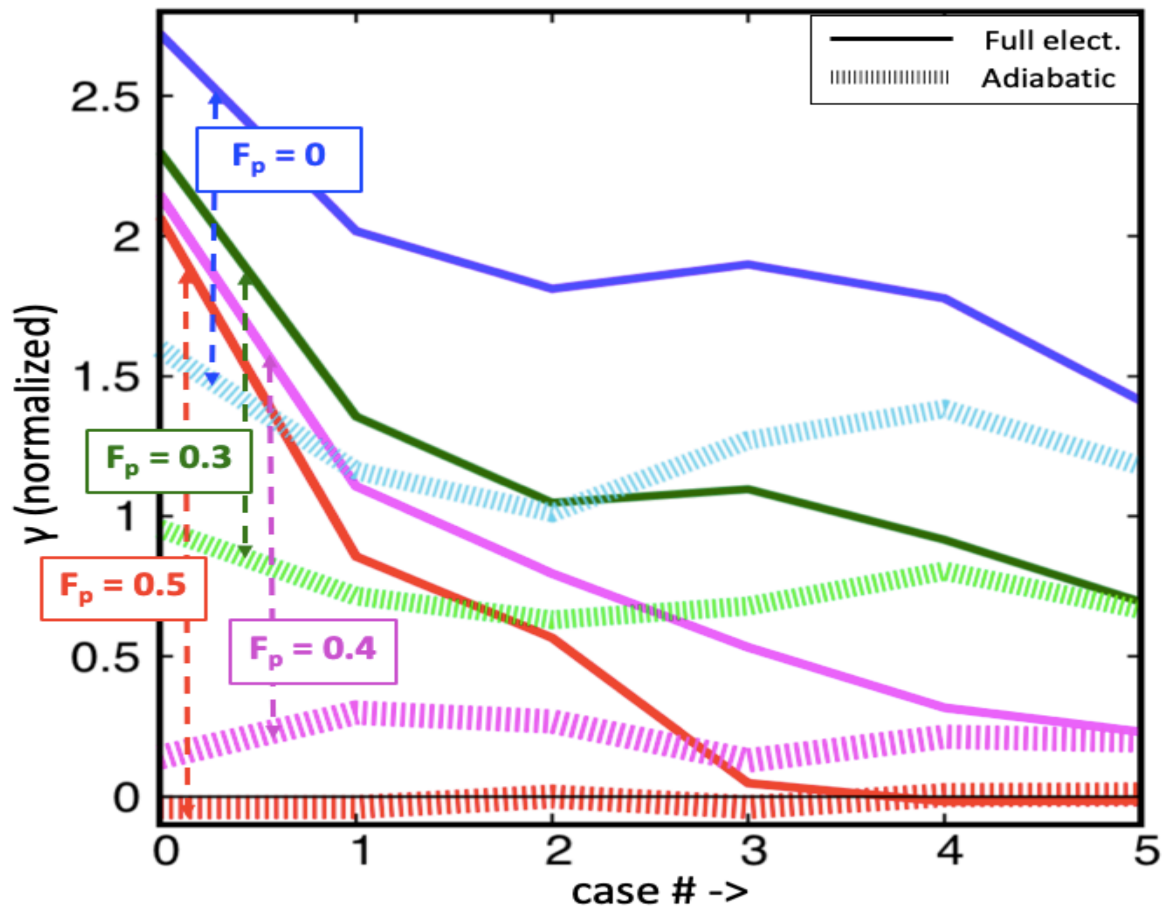
To demonstrate adaptivity, we simulate the following sequence of equilibria going from $\alpha = 0.2$ to $\alpha = 27$





- Going from case 0 to case 5, $\gamma_{full\ e^-}$ approaches $\gamma_{adiabatic\ e^-}$
- $\gamma_{adiabatic\ e^-}$ is remarkably constant for each F_p, despite HUGE curv. variations
- $\gamma_{full\ e^-} > \gamma_{adiabatic\ e^-}$

What kind of fluctuations are present in these simulations?

- Analytic theory: take asymptotic limit of electrostatic gyrokinetic equation in the asymptotic limit of steep ∇p (like ∇p in TBs

$$\gamma^2 = \frac{<\omega_{curv\, i}>\omega_{*pi}\,(\frac{T_e}{T_i}) + <f_{Trapeff}><\omega_{curv\, e}>\omega_{*pe}}{(1 - <f_{Trap\, eff}>)} - \frac{1}{4}\omega_{*ne}^2$$

- Dispersion relation is qualitatively similar to ideal MHD ballooning modes
- Comparison with MHD,
  - Instability is driven by press. gradients of electrons and ions × eigenfunction avg. of curvature
  - The contribution from electrons is in terms of bounce averaged quantities
  - As with ideal MHD, the dispersion relation can be written in terms of quantities averaged over the eigenfunction (all quantities in < >)
- Though the asymptotic limit is not well satisfied for typical TB parameters, the < > quantities are extremely informative as to the character of the instabilities

Eigenfunction averages have a clear interpretation, which we can use to diagnose the behavior of simulations

Eigenfunction average of the curvature drift

$$< \omega_{curv\ i} > = \frac{\int dl\ |\phi|^2 \omega_{di}}{\int dl\ |\phi|^2}$$

Generalization of the eigenfunction average curvature drift to trapped e[-] case

$$< \omega_{curv\ e} > = \frac{\int dl\, d\Omega_v < \phi >^2_{bounce} < \omega_{de} >_{bounce}}{\int dl\, d\Omega_v\ < \phi >^2_{bounce}}$$

The average over trapped particle effects requires an integration over the (solid) angle $d\Omega_v$ in velocity space

$$< f_{Trap\ eff} > = \frac{\int dl\, d\Omega_v < \phi >^2_{bounce}}{\int dl\, d\Omega_v\, \phi^2}$$

Incorporates eigenmode structure into the usual definition of trapped particle fraction

Equals usual trapped fraction when $< \phi >^2_{bounce} = \phi^2$

$\rightarrow 0$ when ϕ^2 has spatial structure highly disparate from trapped orbits

This mathematically quantifies the degree of de-coupling of trapped e[-] to the eigenfunction

Note: we define $< >_{bounce} = 0$ for untrapped region of velocity angle

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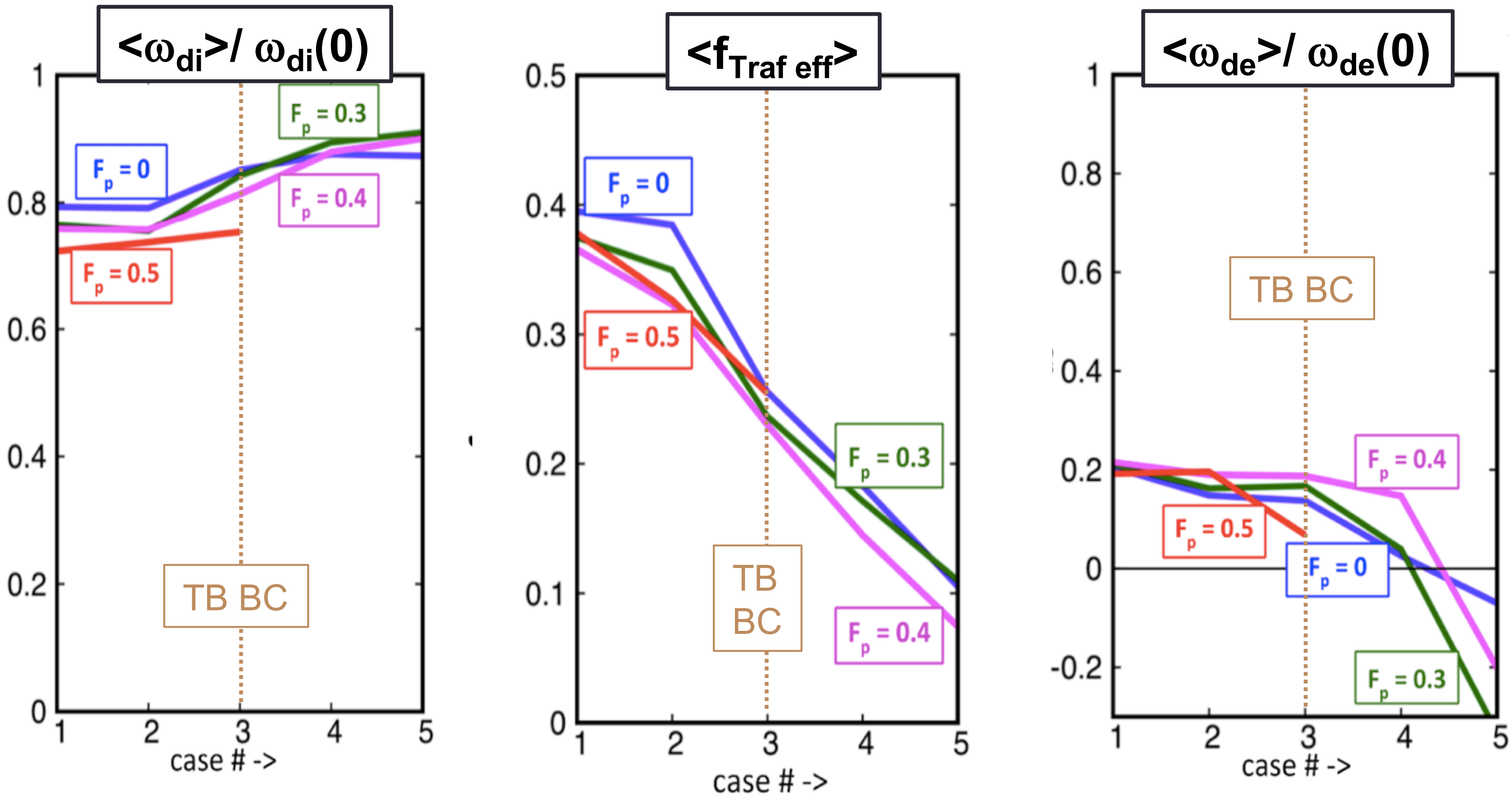
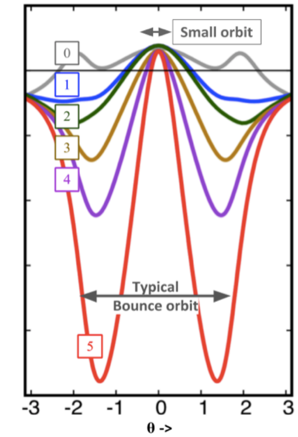
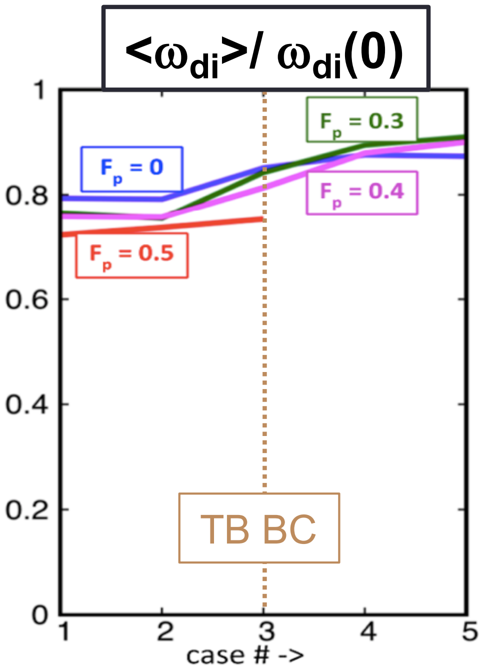
These quantities greatly clarify the physics

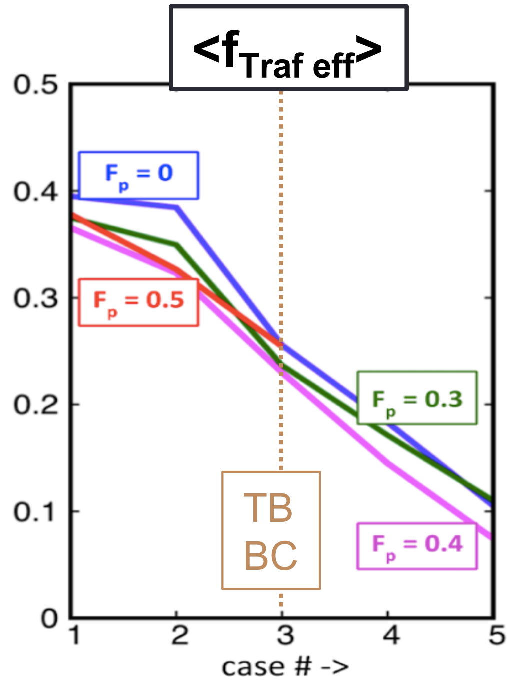


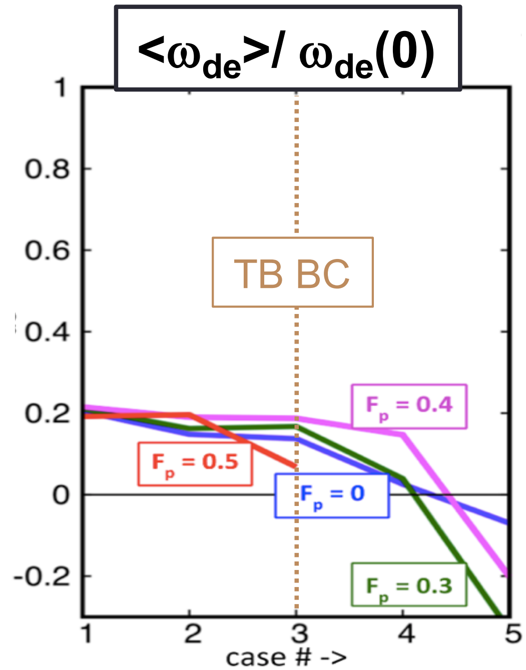


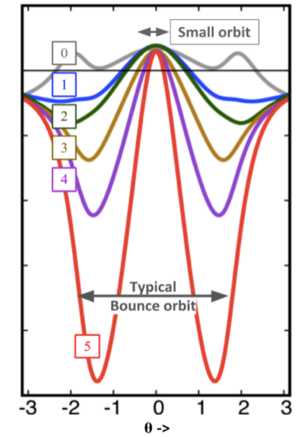
- Despite the huge variation in curvature, the eigenfunctions adapt to keep $<\omega_{di}>$ nearly equal to the peak value $\omega_{di}(0)$ (at outboard midplane)
- The $<f_{Trap\ eff}>$ monotonically decreases => weaker coupling to trapped electrons
- The eigenfunction avg. bounce average curvature $<\omega_{de}>$ stays positive until the most extreme case- the eigenfunctions adapt to utilize trapped e^- as a destabilizing effect

These quantities greatly clarify the physics









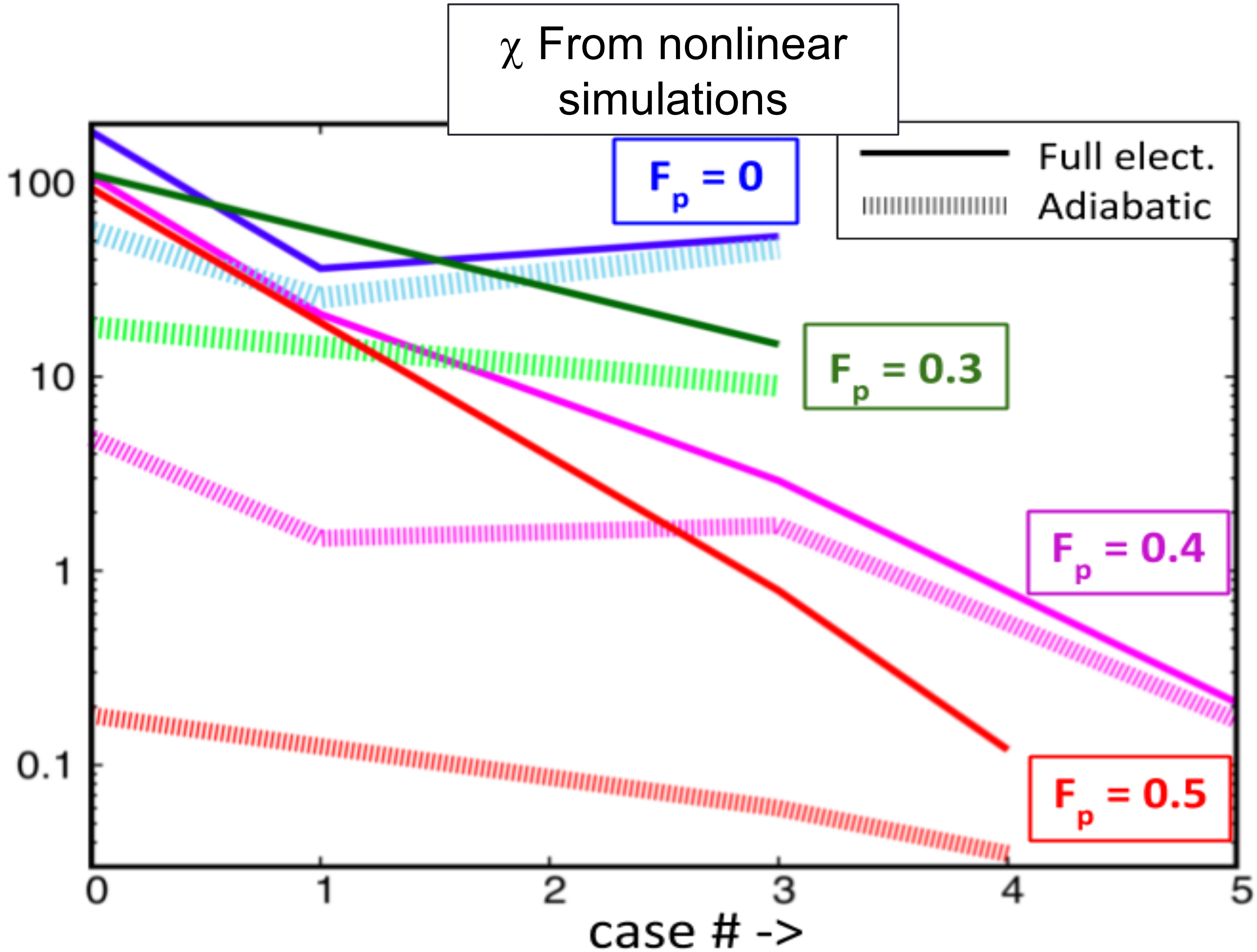
- There is no difference in the curvatures $\langle\omega_{di}\rangle/\ \omega_{di}(0)$ and $\langle\omega_{de}\rangle/\ \omega_{de}(0)$ for high F_p (~ 0.4) for cases 1 – 4

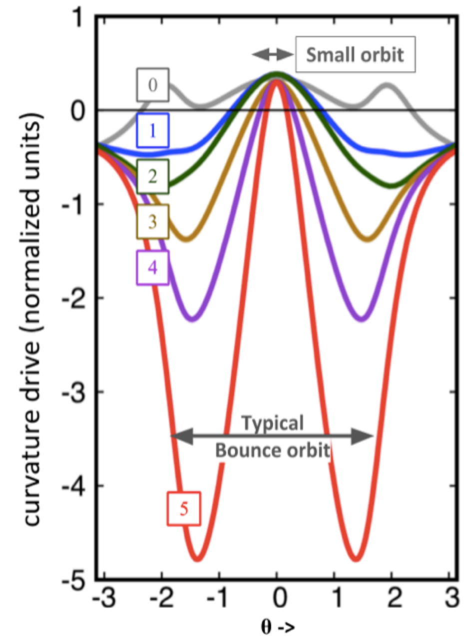
Only $\langle f_{Traf\ eff}\rangle$ correlates with the stabilization with F_p

- The stabilization is due to small values of $\langle f_{Traf\ eff}\rangle$, the coupling to trapped electrons, not a curvature stabilization effect- **so the modes approach the adiabatic limit**

Case #	Stabilization with F_p
1	None
2	Weak
3	Strong
4	Stronger

Nonlinear simulations behave the same as linear- heat flux approaches the value for adiabatic electrons





The full electron values approach the adiabatic electron values for higher case number

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A simple formula based on linear runs explains the large majority of the variation of the nonlinear χ

- $\chi_{lin\ est} = 5\ \theta_{width}\ D_{mixing}$

"5" is chosen to give a good match to nonlinear χ

- D_{mixing} is the usual definition $\gamma / \langle k_\perp^2 \rangle$, where $< k_\perp{}^2 > = \dfrac{\int dl\ |\phi|^2\ k_\perp{}^2}{\int dl\ |\phi|^2}$

- θ_{width} is a quantitative measure of the width of the eigenfunction
  - $\theta_{width} = \dfrac{\int d\vartheta\ |\phi|^2}{|\phi|^2_{max}}$
  - where $|\phi|^2_{max}$ is the maximum value of $|\phi|^2$

- Narrow eigenfunctions have a commensurately small value of θ_{width}

- One expects turbulent heat flux is less for fluctuations with a narrow θ_{width} , since they produce transport over a smaller region in θ

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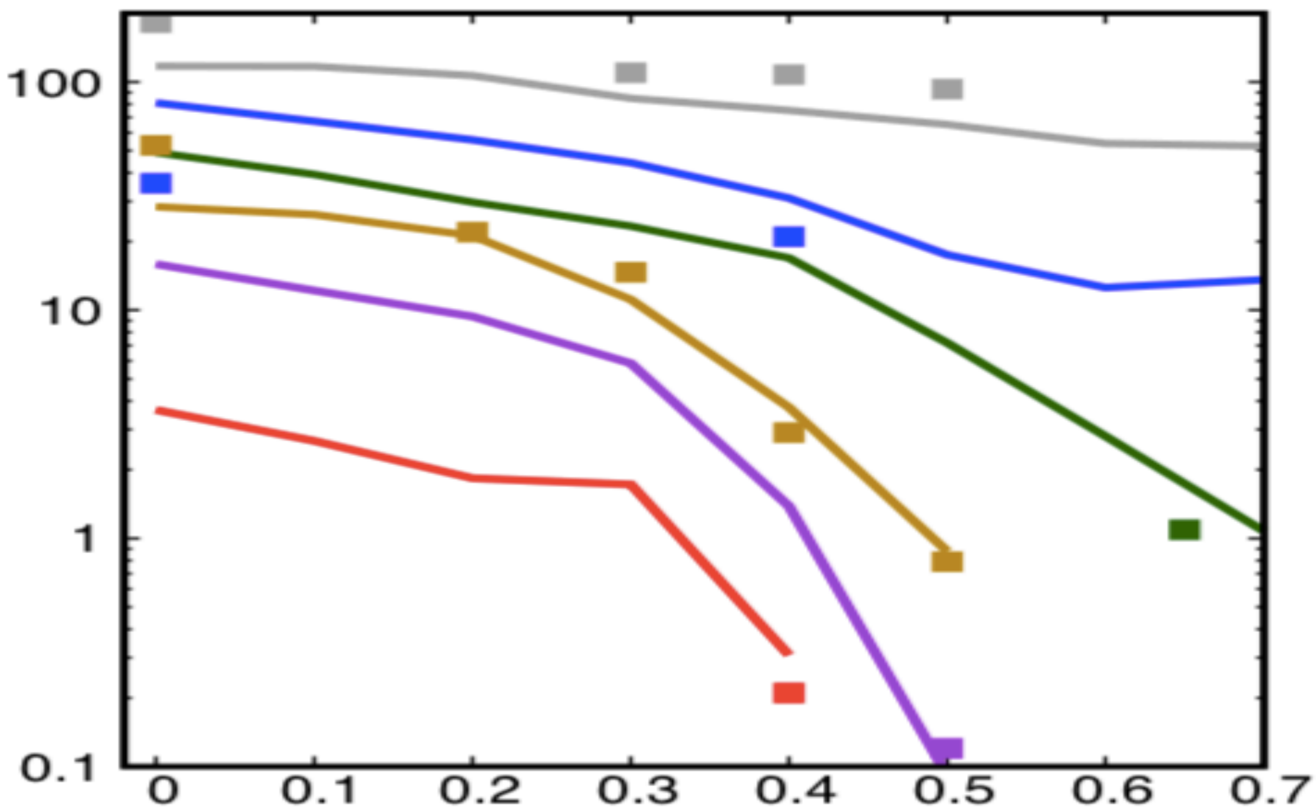
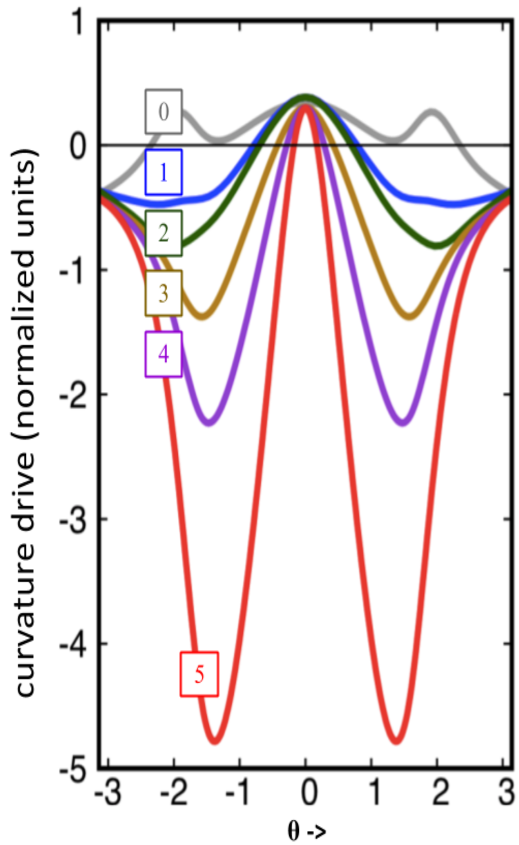
The formula $\chi_{lin\ est}$ explains the large majority of the 3 orders of magnitude variation of nonlinear χ





- NOTE: variation in nonlinear χ by factor > 1000
- Maximum errors of $\chi_{lin\ est}$ ~ a factor of 2.5
- About 2 orders of magnitude of variation in $\chi_{lin\ est}$ is from D_{mixing}, about one order from θ_{width}

There are important inferences to be drawn from this

- The ITG/TEM behavior that leads to a TB is mainly due to the electrons becoming nearly adiabatic
- When the electrons are adiabatic, the behavior is not strongly dependent on the geometry
- The huge reduction in χ and γ and comes from stabilization of the ITG mode with adiabatic electrons by density gradients (well known)
- Stellarators may have different geometrical route to achieve decoupling of the eigenfunction from the trapped orbits than tokamaks, but this is irrelevant-

ONCE THE e^- ARE ADIABATIC, IT DOESN"T MATTER HOW THEY GOT THAT WAY – THEY HAVE REACHED A WELL DEFINED LIMIT

Plus, ITG MODES WITH ADIABATIC e^- ARE INSENSITVE TO GEOMETRY

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Other corollaries

- The eigenfunction is highly adaptive so it concentrates in the bad curvature region
- It only “cares” about the geometry in this region- it ignores the rest
- Hence, even very different geometries that are locally similar should have similar fluctuation dynamics
  - E.g. tokamaks and stellarators
- (Further, we will see that the eigenmode behavior is mainly determined by a few low-order moments of the eigenfunction)

Another corollary

- We have emphasized geometry as a route to adiabatic behavior
- However, there are other parameters than can do this as well
- In particular, Coulomb collisional diffusion can strongly damp a trapped electron response, as is known
- This has a similar effect to high α, and in fact, works synergistically with it

Collisions can strongly affect the electrons to give TBs

- Trapped electrons exist with only a limited range of angles in velocity space $\Delta\theta_{trap} \sim \sqrt{\varepsilon}$
- In neoclassical theory, the effective collision frequency for scattering outside the trapped region is $\nu_{e\,eff} \sim \nu_e/\Delta\theta_{trap}^2 \sim \nu_e/\varepsilon$
- If $\nu_{e\,eff}$ is much higher than the mode frequency, the trapped particle response is strongly damped by collisions: $\nu_{e\,eff} > \omega$
- This damping can be strong even deeply into the banana regime, since ω << the electron bounce frequency
- Thus, we would expect that strong enough collisions can cause the modes to be in the adiabatic limit

Simulations show this trend

- High ν_e acts just like higher α:

  approaches the adiabatic electron limit

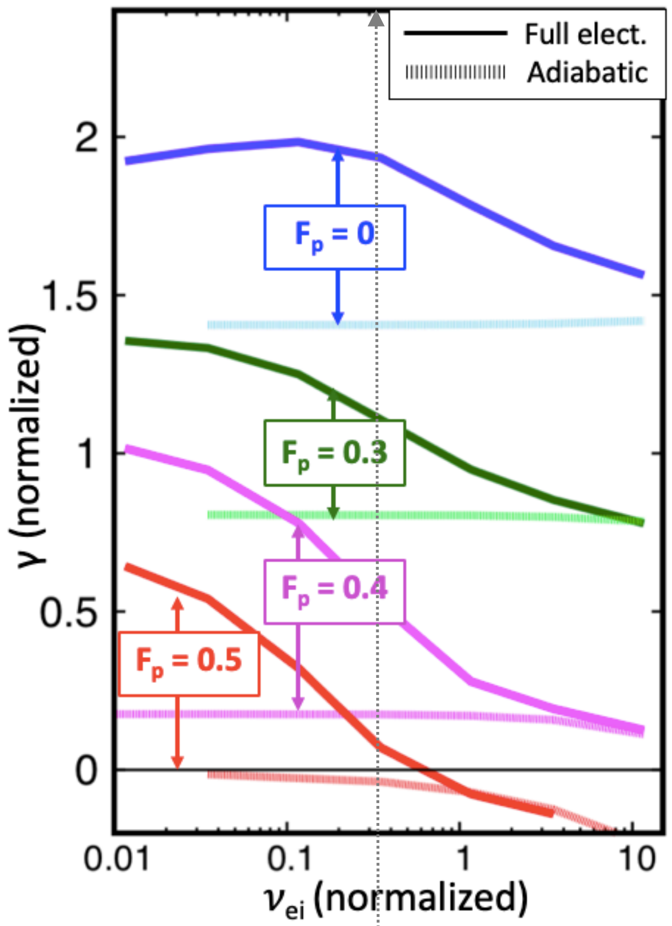
- $\gamma_{full\ e^-}$ is $always\ higher\ than$ $\gamma_{adiabatic\ e^-}$, consistent with the point of view of an adaptive mode:

  If there is a possibility of a trapped response,

  the mode will adjust to use it to grow faster

- The observed trend is surprising from a conventional point of view:
  - Sufficient collisions should introduce destabilizing dissipative trapped electron effects
  - Also, within the picture of trapped electrons as being stabilizing due good bounce avg. curvature, collisions should reduce this stabilizing effect
  - In both cases above, there should be some range over which increasing ν_e is destabilizing
  - The simulation trends are unlike this

γ vs ν_e for the TB BC



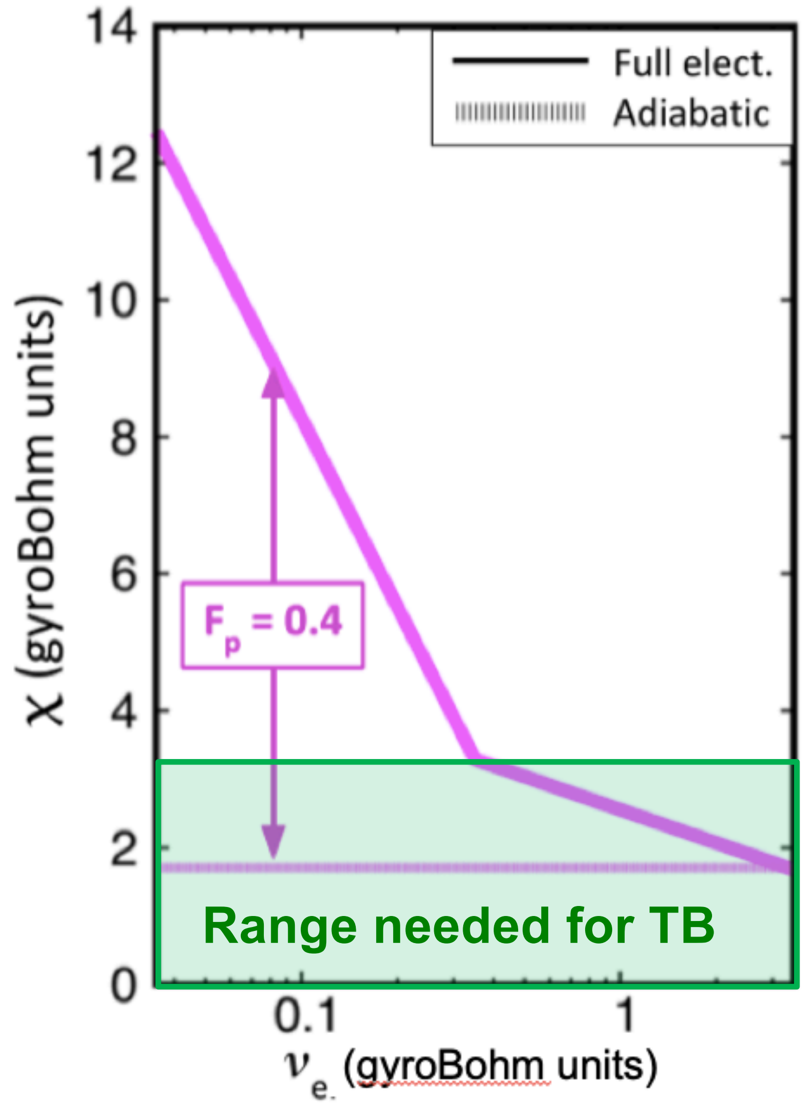
Nominal value of ν_e for TB BC: chosen to be roughly in the midrange of expt. values

This collisional effect is seen in nonlinear simulations

- Nonlinear χ approaches the adiabatic value at high ν_e

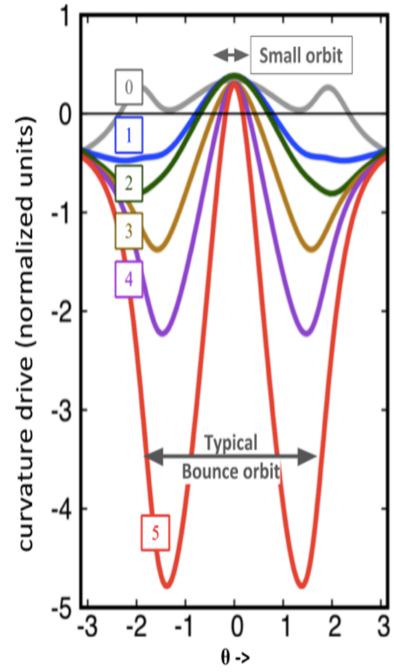
- Collisional effects can be the difference between attaining a good TB or not

Nonlinear $\chi_i + \chi_e$ vs ν_e
for the TB BC and $F_p = 0.4$



There is a synergism between geometry (e.g. high α) and collisions

- For an eigenmode that has a narrow width, only a fraction of the trapped electrons couple strongly to the mode
- These have an even smaller velocity angle range than trapped electrons as a whole $\Delta\theta_{interact} < \Delta\theta_{trap}$
- The effective collision frequency scattering out of the coupling range is therefore even higher $\nu_{interact} \sim \nu / \Delta\theta_{interact}^2 > \nu_{eff}$
- This effect is seen in the simulations….
  - And as we will see, this may explain trends seen in several experiments



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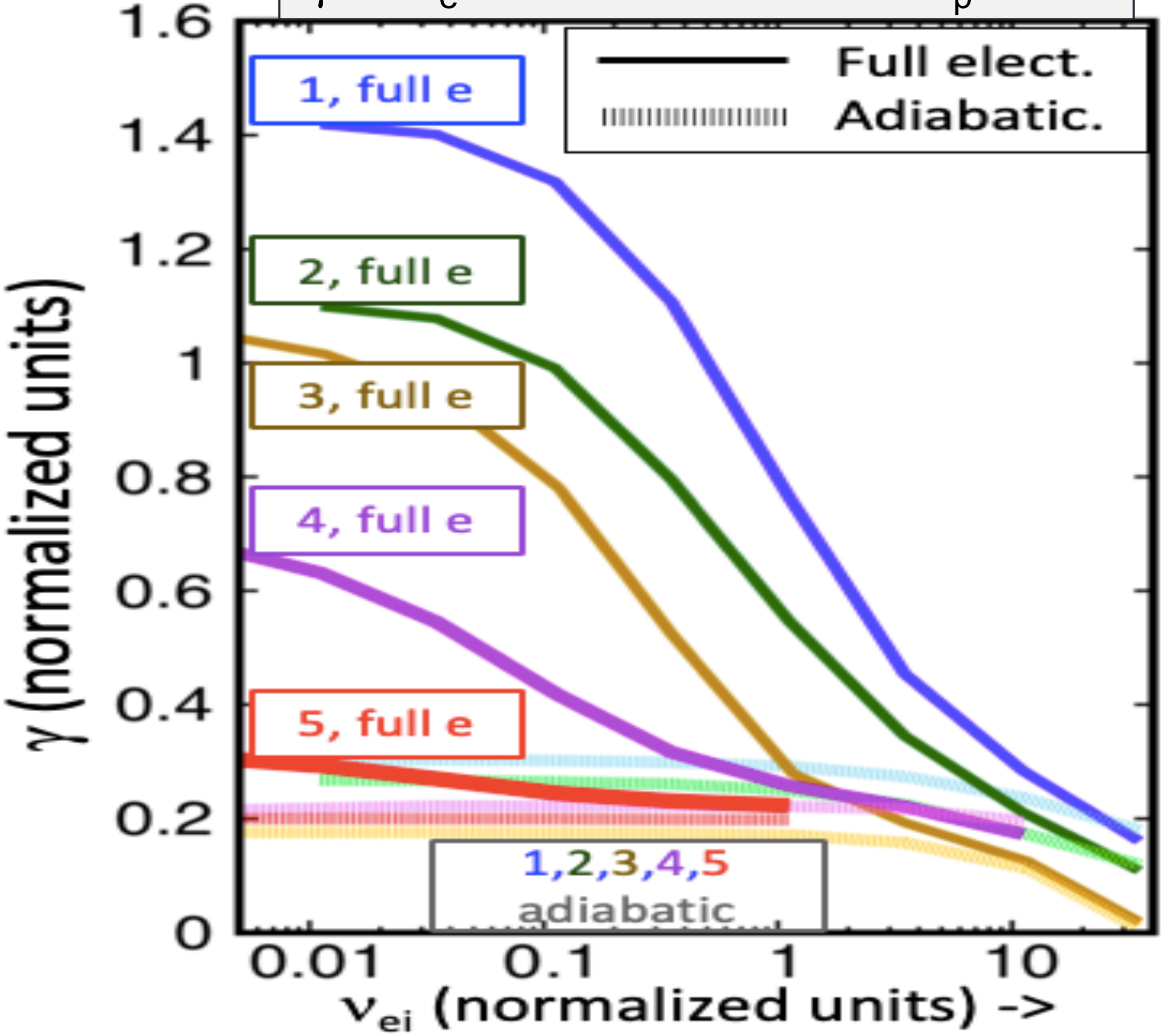
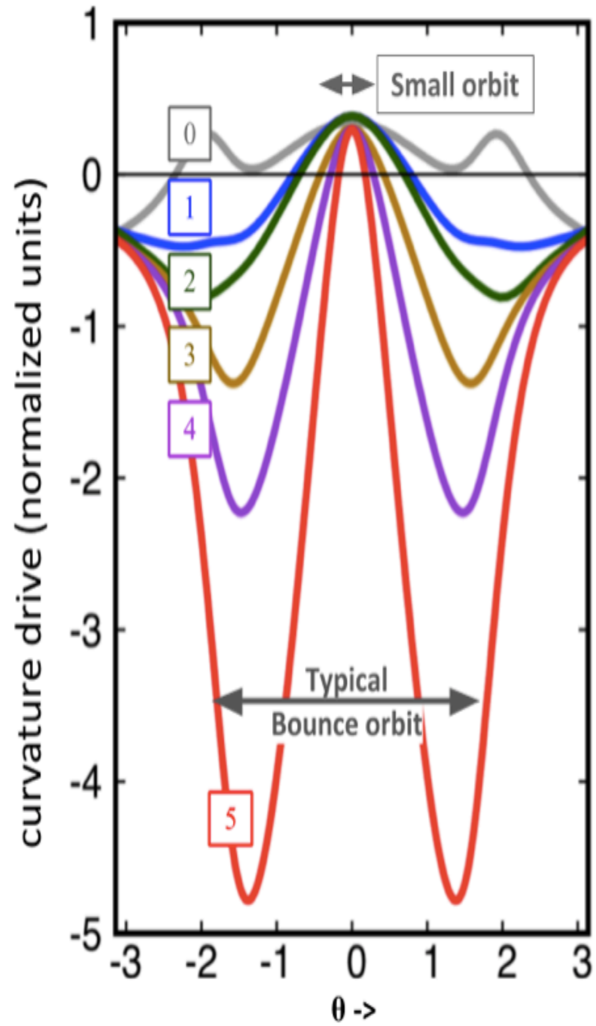
Cases with narrow eigenfunctions are stabilized at lower ν_e

- We attribute this to the effect described

γ vs ν_e for cases 1 – 5 and F_p = 0.4



Log scale



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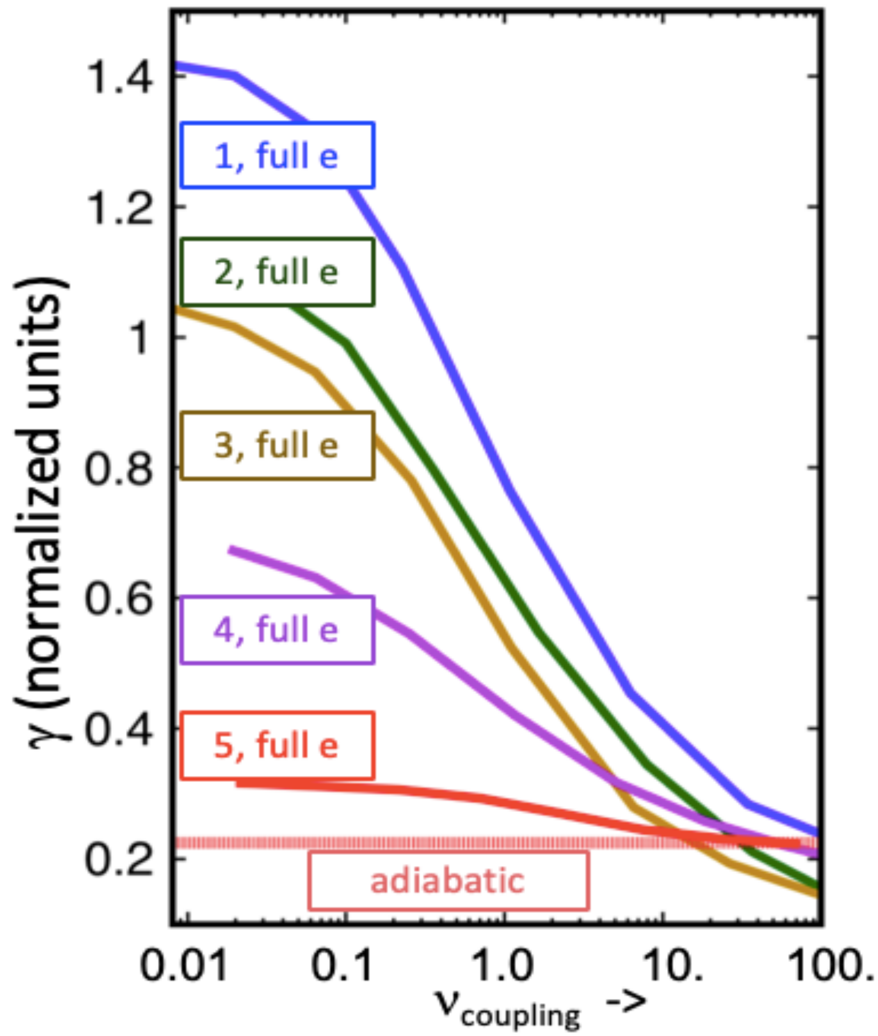
Quantifying the synergism between geometry (e.g. high α) and collisions

- One can estimate the width $\Delta\theta_{interact}$ from the value of <$f_{Trap\ eff}$>

- We obtain a parameter to estimate when collisions are become important when eigenfunctions are narrow:

- $$\nu_{coupling} = \frac{\nu_{ei}}{10\ |\omega|\ f_{Trap\ eff}{}^2} > 1$$

- This orders the simulations results fairly well: collisions become significant when $\nu_{coupling} \sim 1$



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Comparison with experimental Transport Barriers where velocity shear is expected to be weak

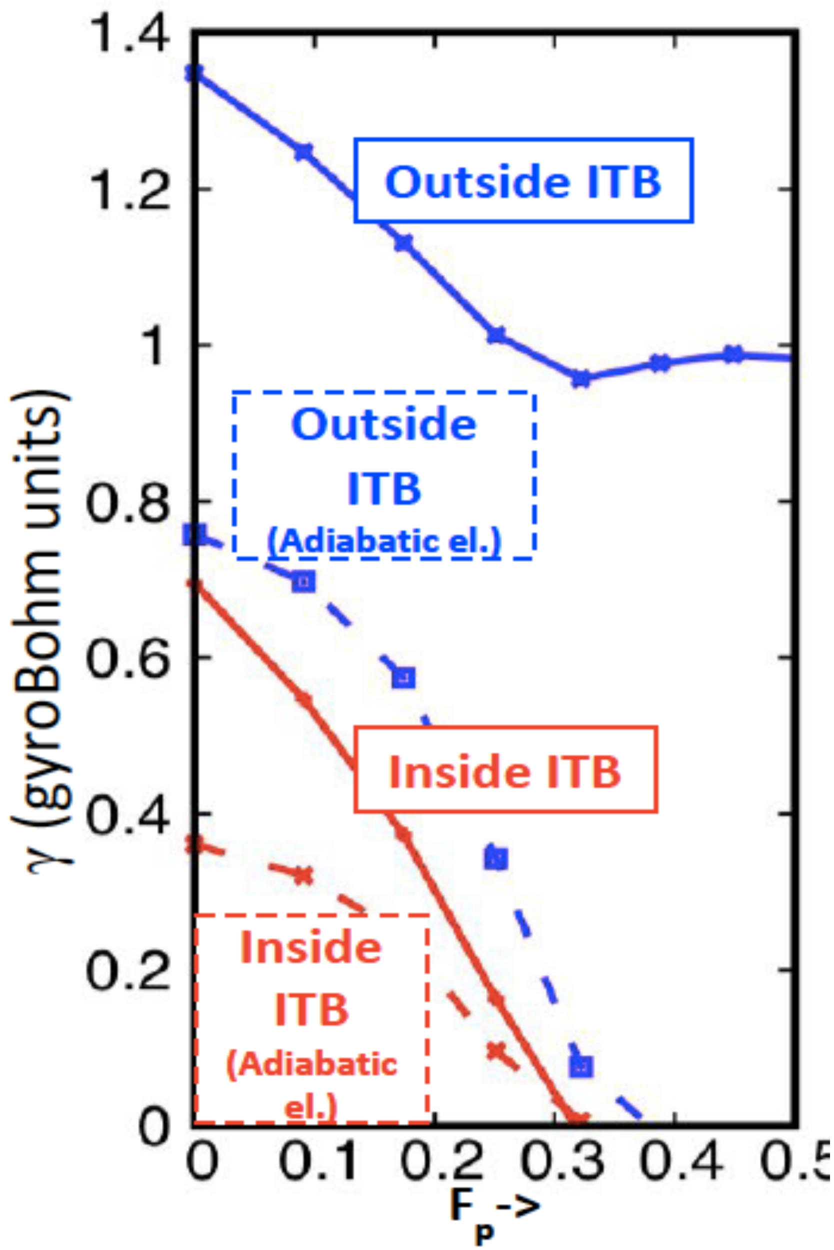
- We examine:
- JET ITB with pellet injection and reversed shear
- DIIID ITB with high $\beta_{poloidal}$ (=> high α)
- DIIID wide pedestal QH mode
- JET -ILW pedestal
- EAST ITB

JET ITBs after pellet injection

- Simulations include $T_i \neq T_e$, impurities, EM effects, etc.
- Compare behaviors for a radial positions inside and slightly outside ITB
  - Inside ITB: Strong stabilization with F_p
  - Outside ITB: weak stabilization
  - With adiabatic electrons, both similar to the ITB
  - Hence: the adiabatic constraint is the crucial ingredient for the ITB
    - Also, the trapped electrons were destabilizing

The comparison inside vs outside the ITB is just like TB BC vs CBC

JET # 57941[1]



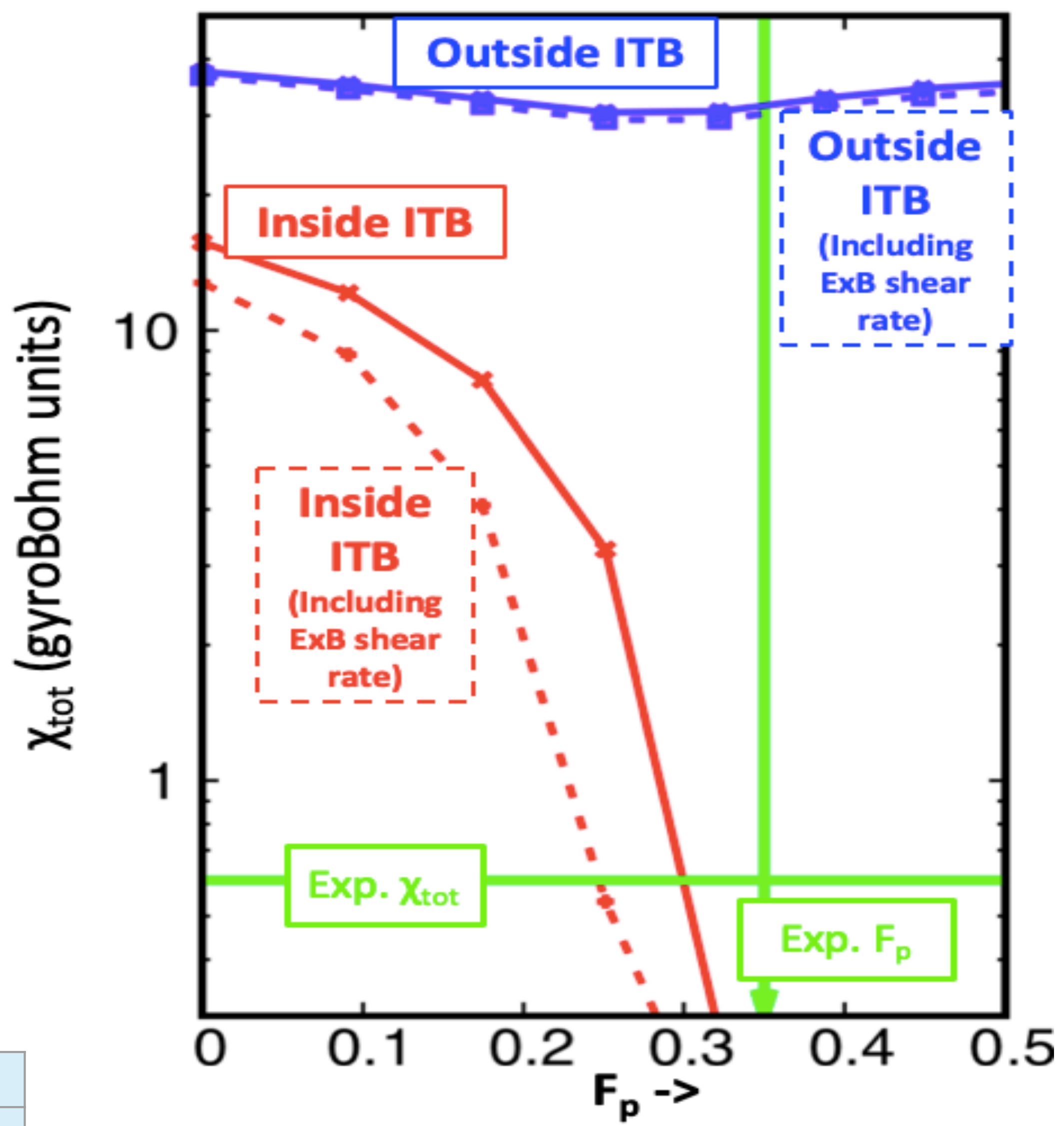
[1]D. Frigione,, L. Garzotti, C. D. Challis et. al. and JET EFDA contributors, Nucl. Fusion 47 (2007) 74–84

JET ITBs after pellet injection: χ

- Use the $\chi_{\text{lin est}}$
- The experimental F_p value is sufficient to give χ small enough to be consistent with power balance estimates
- Velocity shear is not a major factor, as experimentally inferred
- The magnetic geometry outside the ITB would not give an ITB (as observed)

parameter	JET ITB	outside
$\langle\omega_{di}\rangle / \omega_{di}(0)$	~ 0.81	~ 0.81 - 0.84
$\langle\omega_{de}\rangle / \omega_{de}(0)$	~ 0.17 - 0.22	~ 0.27 - 0.30
$\langle f_{\text{Trap eff}}\rangle$	~ 0.2	~ 0.36 - 0.39

JET # 57941[1]



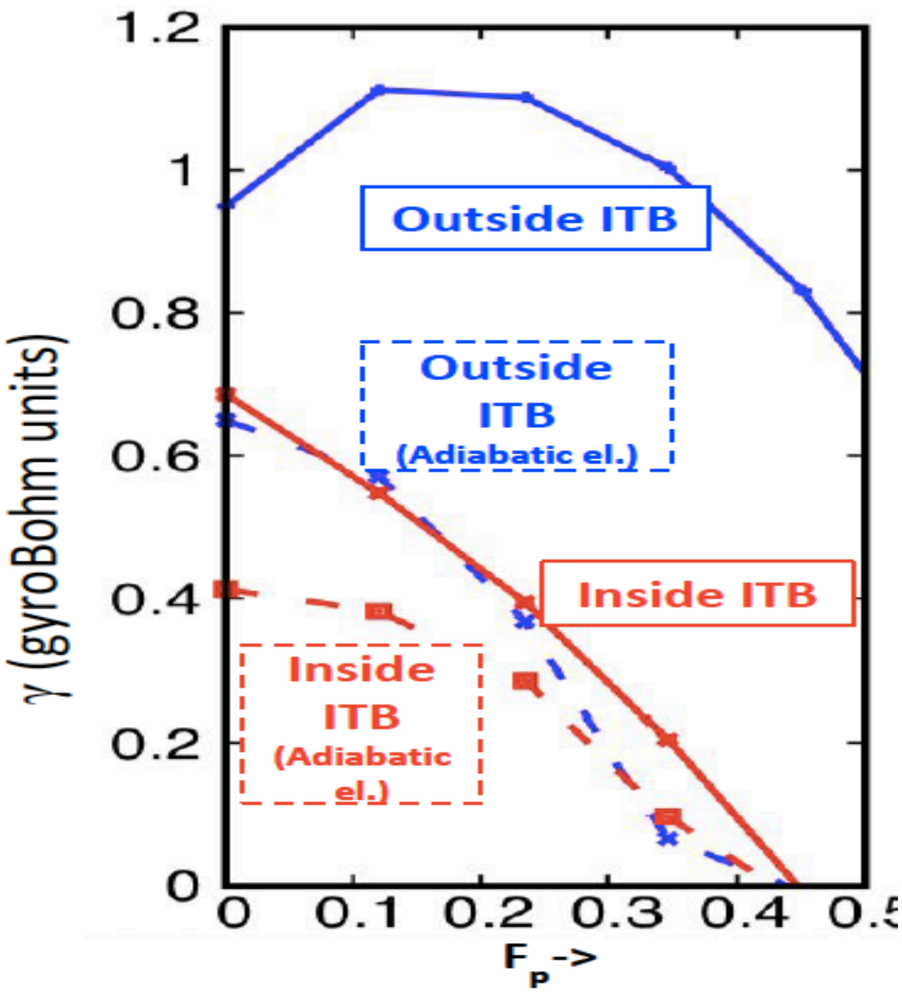
[1]D. Frigione,, L. Garzotti, C. D. Challis et. al. and JET EFDA contributors, Nucl. Fusion 47 (2007) 74–84

DIII-D shot # 154406 with high β_{pol} (= high α)

- From equilibrium reconstruction (EFIT, profiles, etc.)
- Behavior inside and slightly outside the ITB is the same as TB BC and CBC
- Inside the ITB, coupling to trapped electrons is weak, whereas it is strong outside
- With the adiabatic constraint imposed, both inside and outside are similar
- The adiabatic constraint is the key to strong stabilization

parameter	DIIID ITB	outside
$\langle\omega_{di}\rangle / \omega_{di}(0)$	~ 0.72 - 0.76	~ 0.94 - 1.35
$\langle\omega_{de}\rangle / \omega_{de}(0)$	~ 0.19 - 0.25	~ 0.80 - 0.83
$\langle f_{Trap\ eff}\rangle$	~ 0.23 - 0.26	~ 0.43 - 0.50

DIII-D # 154406[1,2,3]



(Mcclenaghan 2017[1], Ding 2017[2], Pan 2107[3],)

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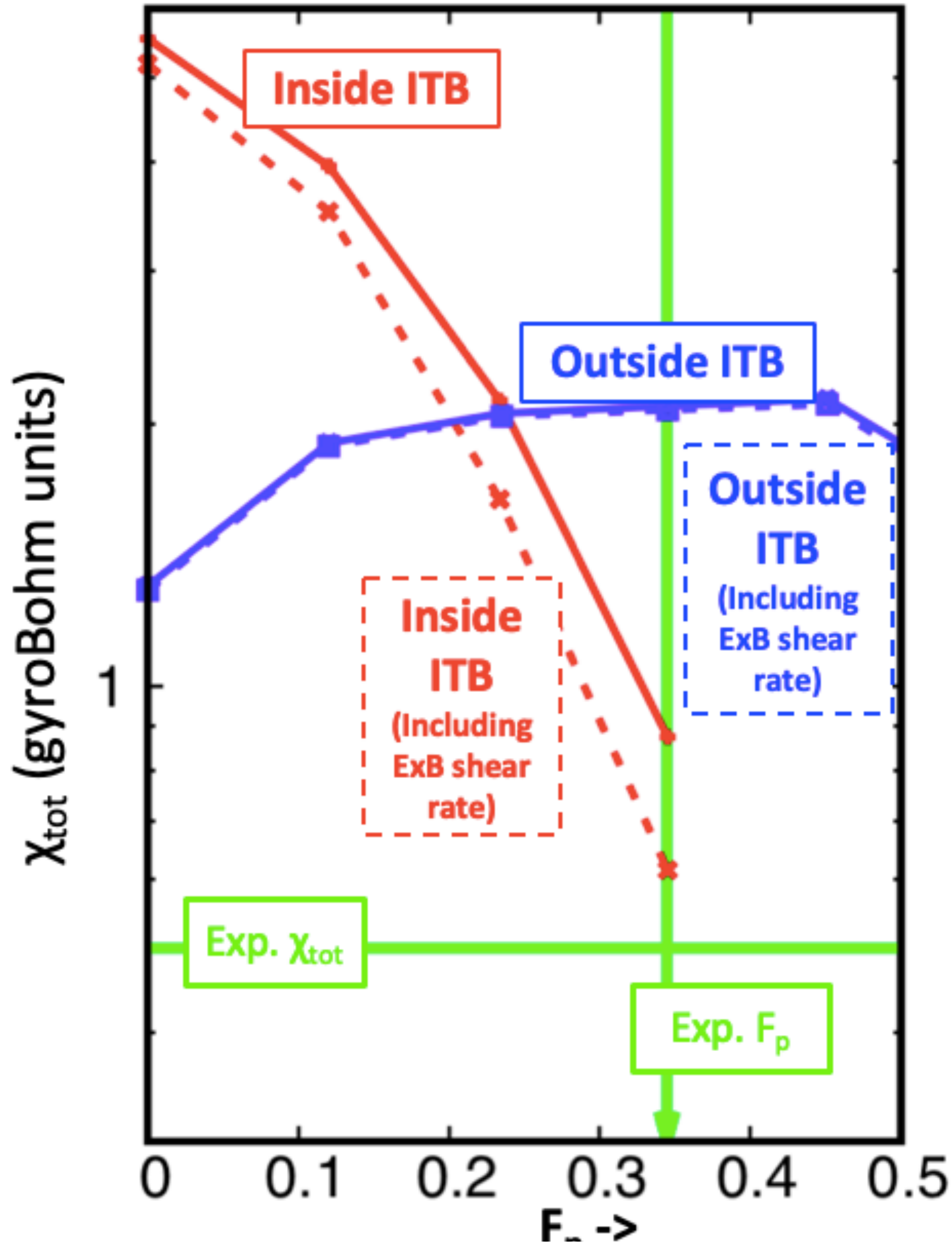
DIII-D shot # 154406 with high β_{pol} (= high α)

DIII-D # 154406[1,2,3]

- The estimated χ is only consistent with experiment because of strong stabilization from F_p
- Velocity shear is secondary (previously inferred[1,2,3])
- For the magnetic geometry outside, χ is too large for a TB



(Mcclenaghan 2017[1], Ding 2017[2], Pan 2107[3],)

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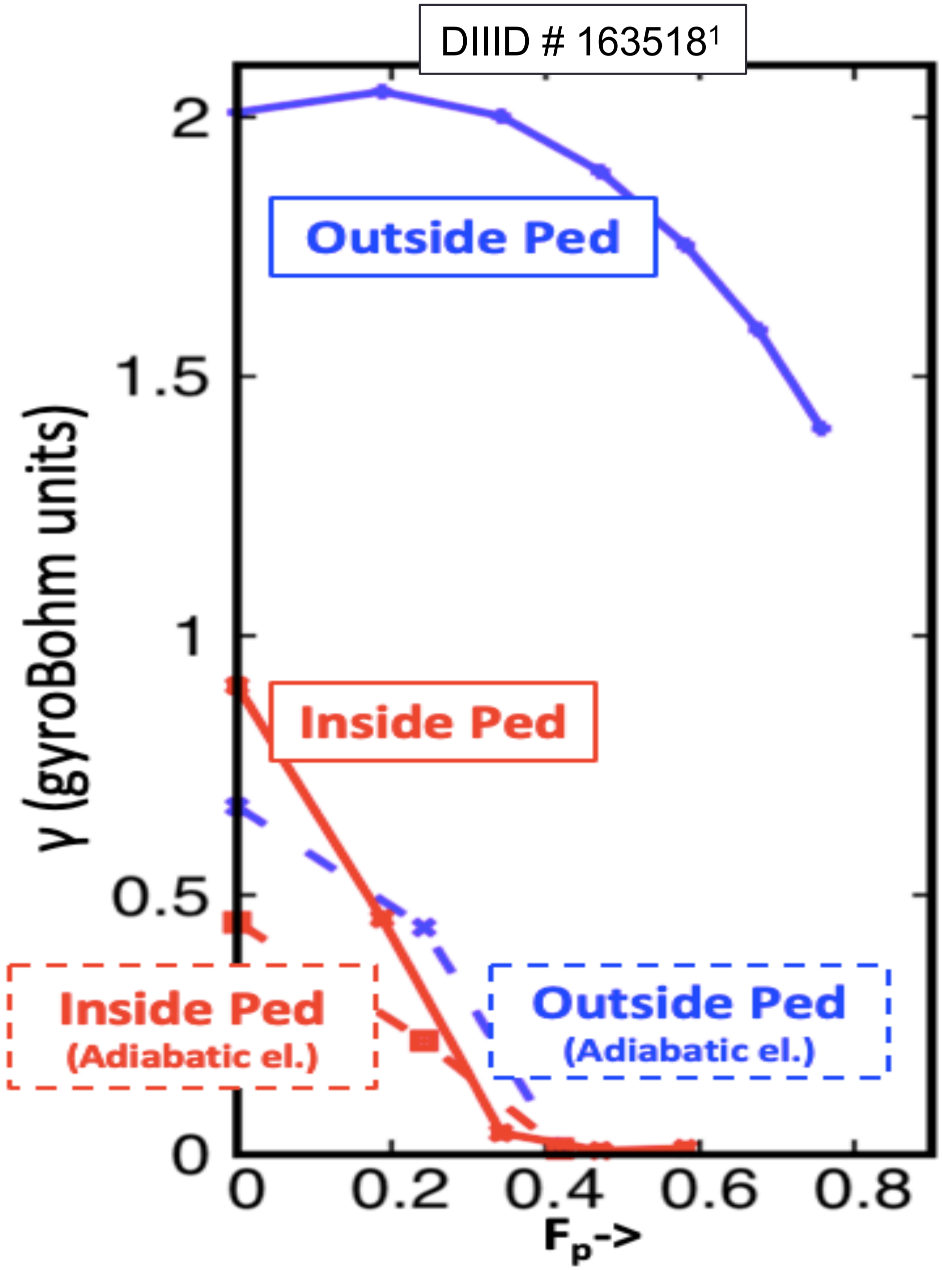
DIIID "wide pedestal" QH mode

- Velocity shear is considerably lower in these pedestals compared to standard H-modes
- n_e profile is much steeper than T_i, giving high F_{pi}
- Without density gradients (F_p =0), transport would be inconsistent with a TB
- The behavior of adiabatic runs, and r/a different from the TB, is the same as for TB BC and CBC

The adiabaticity constraint is the key



[1]X. Chen et. al., Nucl. Fusion 57 (2017) 022007

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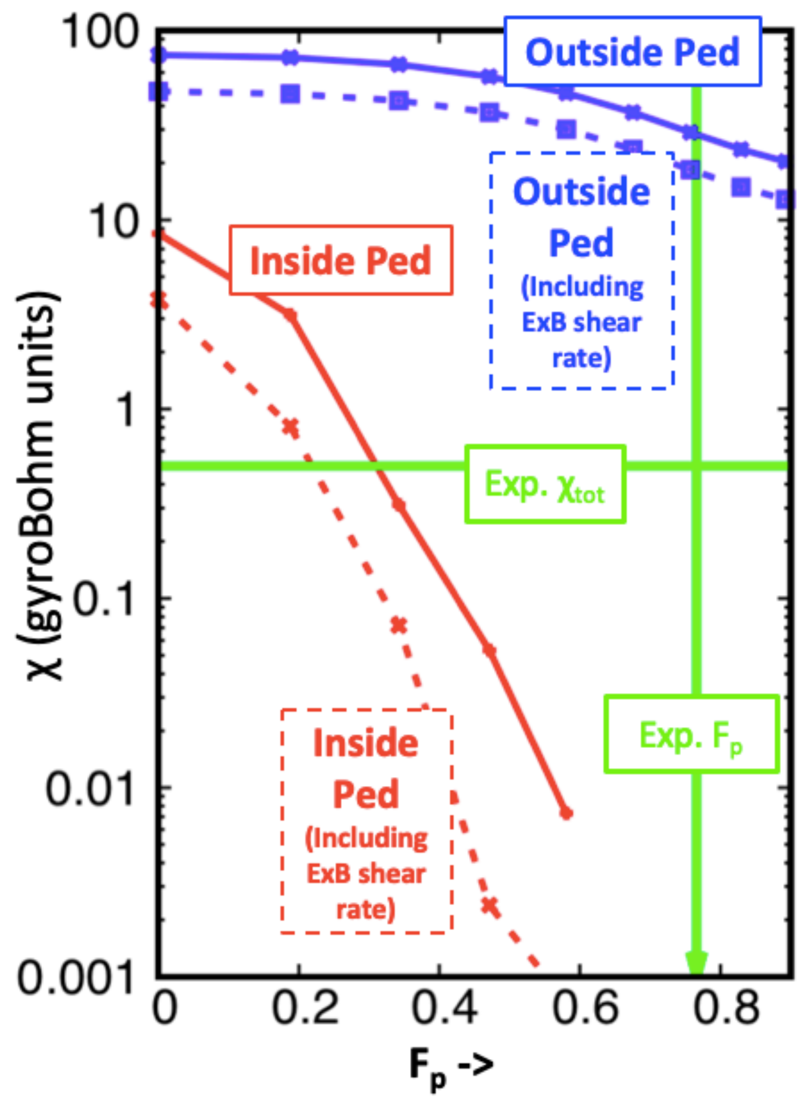
QH mode pedestal : χ

- **Use the $\chi_{\text{lin est}}$**
- χ is consistent with experiment only due to F_p
- In this case, F_p is much larger than is needed for stabilization
- The magnetic geometry outside the ITB would not give an ITB

parameter	DIIID QH	outside
$\langle\omega_{di}\rangle / \omega_{di}(0)$	~ 0.64 - 0.67	~ 1.
$\langle\omega_{de}\rangle / \omega_{de}(0)$	~ 0.18 - 0.24	~ 0.63
$\langle f_{\text{Trap eff}}\rangle$	~ 0.32 - 0.36	~ 0.53

DIIID # 163518[1]



[1]X. Chen et. al., Nucl. Fusion 57 (2017) 022007

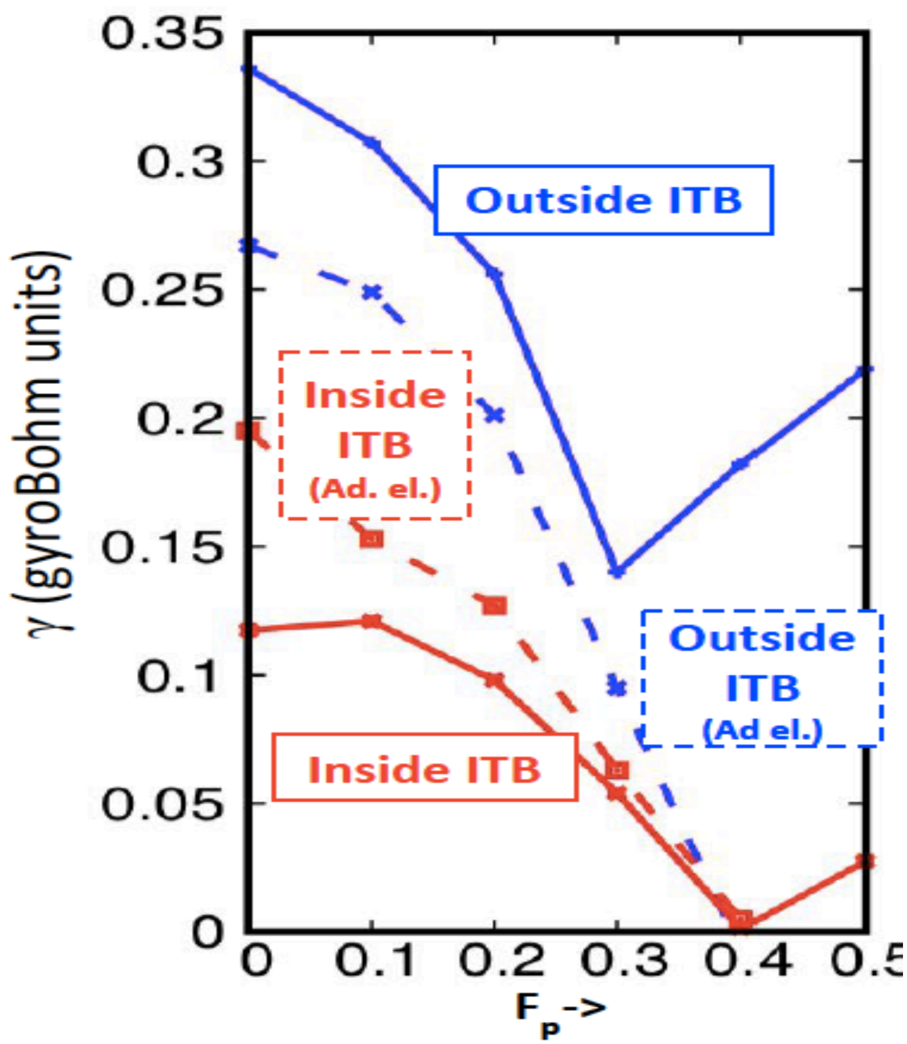
EAST ITB

- This ITB is close to the axis

- The behavior of adiabatic runs, and r/a different from the TB, is the same as for TB BC and CBC

The adiabaticity constraint is the key

EAST # 71320[1]



[1]X. Gao and the EAST team, Physics Letters A 382 (2018) 1242–1246

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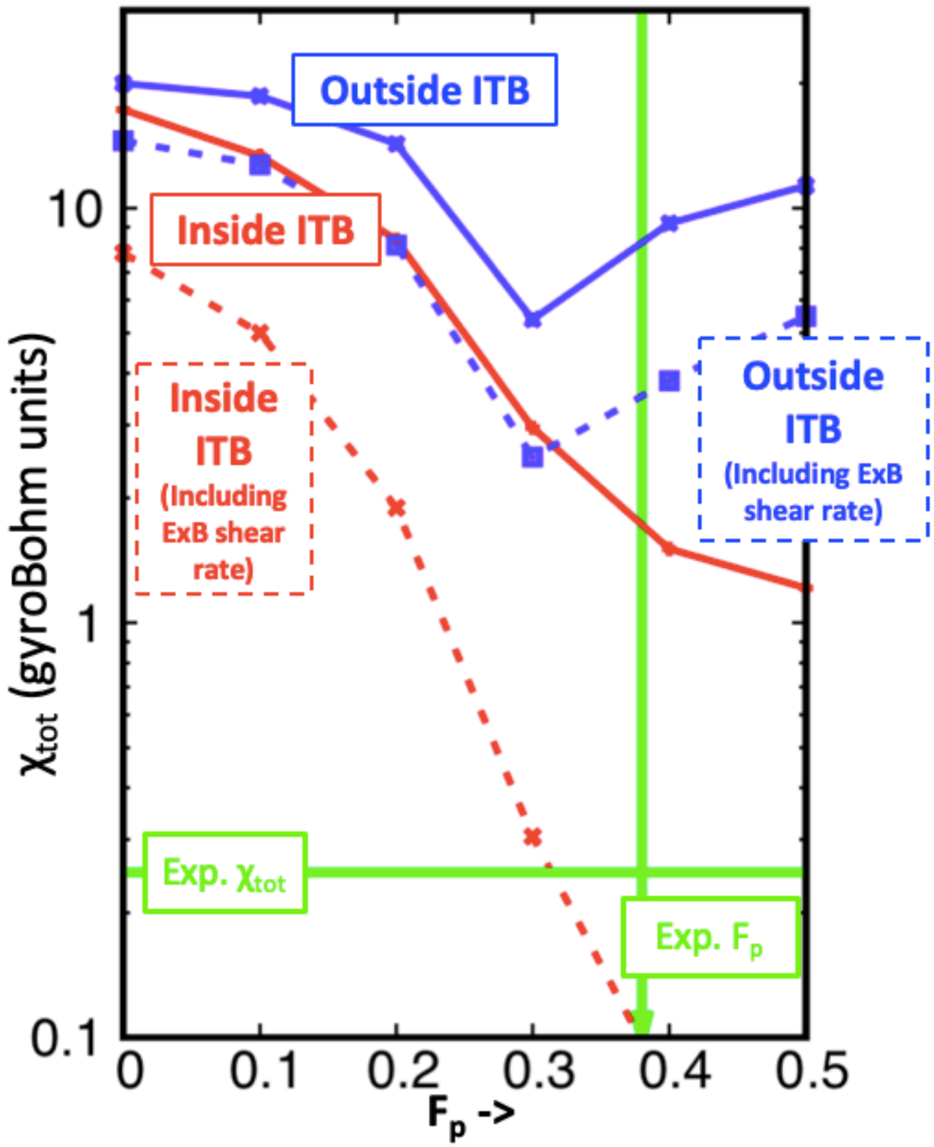
EAST ITB

- This ITB is close to the axis
- In this shot, both F_p and velocity shear are needed
- But without F_p stabilization, χ would be over an order of magnitude too large
- This ITB would not be possible at a larger r/a with more trapped particles
- Collisions are important for this ITB - $\nu_{coupling}$ > 10

EAST # 71320[1]



[1]X. Gao and the EAST team, Physics Letters A 382 (2018) 1242–1246

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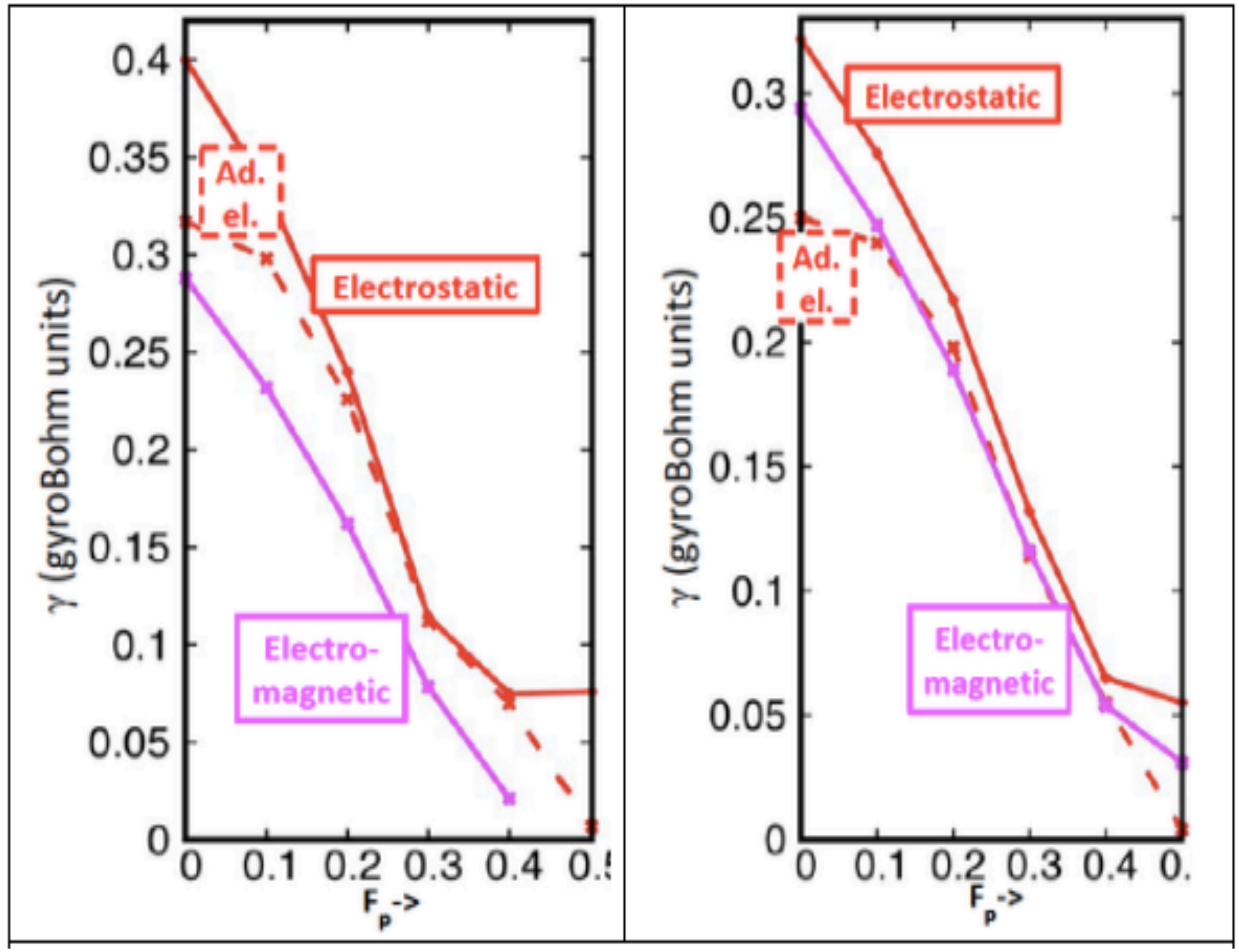
Wendelstein 7X is similar

- High density gradients in W7X are observed to give ITBs
- The geometry of W7X places the bad curvature region at a different location along the field line than the position where most trapped particles are
  - Most trapped electrons are in the good curvature region, and have bounce average good curvature
  - However, this also leads to low coupling of eigenmodes to trapped electrons because of the position mis-match

Optimized

Un- optimized

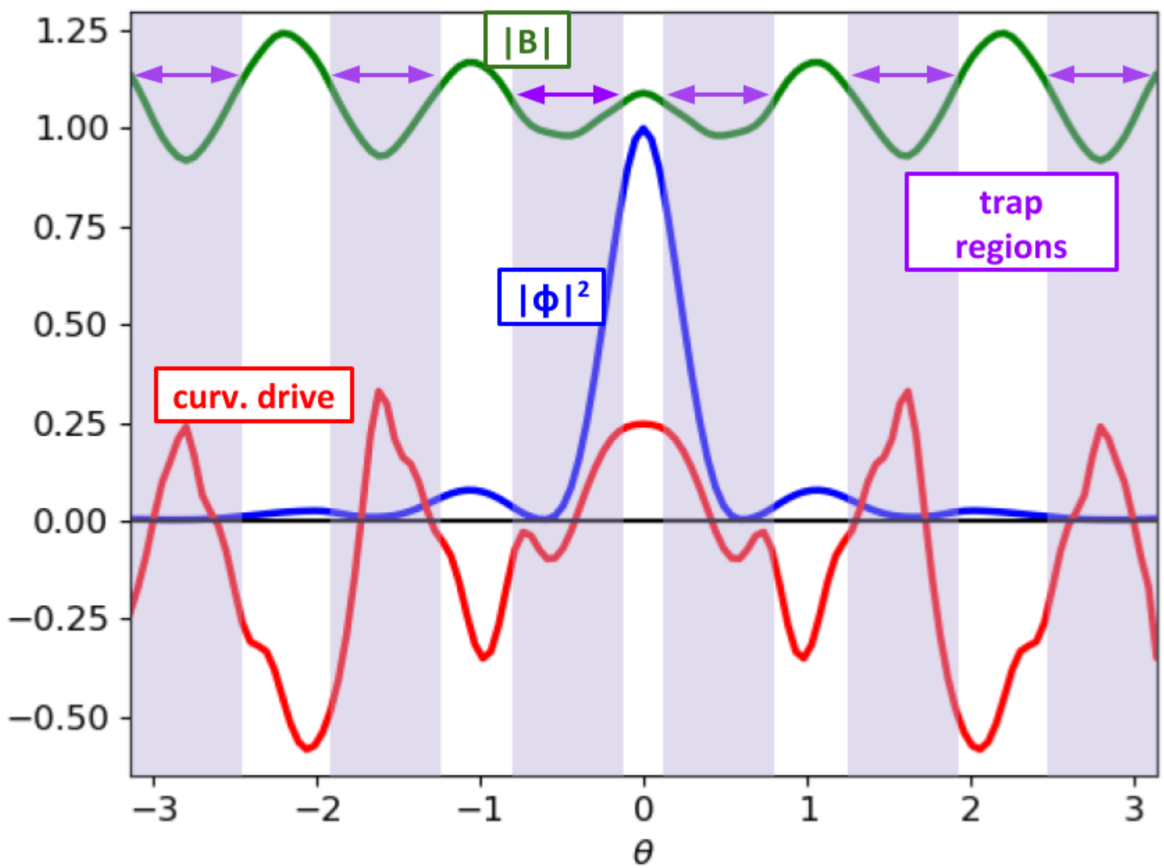


Bozhenkov 2019

Wendelstein 7X is similar

- Bad curvature region at a different location along the field line than the position where most trapped particles are
  - Most trapped electrons are in the good curvature region, and have bounce average good curvature
  - However, this also leads to low coupling of eigenmodes to trapped electrons because of the position mis-match
  - The electrons which have the best coupling to the eigenfunction have bounce points in the bad curvature region

The upshot: eigenfunction average quantities are very similar to high α tokamak



Wendelstein 7X is similar

- We evaluate the quantities **$\langle\omega_{di}\rangle/\ \omega_{di}(0)$, $\langle\omega_{de}\rangle/\ \omega_{de}(0)$, and** $\langle f_{Trap\ eff}\rangle$ for a range of a/L similar to experiments

- In the initial range of stabilization $F_p < 0.4 - 0.5$, values and behaviors are similar to high a tokamak cases 3 – 4 which showed strong stabilization with F_p

parameter	W7X	High α tokamak
$\langle\omega_{di}\rangle/\ \omega_{di}(0)$	~ 0.75	~ 0.8 - 0.9
$\langle\omega_{de}\rangle/\ \omega_{de}(0)$	~ +0.1	~ 0.2
$\langle f_{Trap\ eff}\rangle$	~ 0.15 -0.25	~ 0.2

- In particular:
    - The eigenfunction averaged bounce average curvature is slightly destabilizing
    - The value of $\langle f_{Trap\ eff}\rangle$ is similar to high a tokamaks
        - Recall this was apparently the most significant parameter for tokamaks

Collisions can be very important for Wendelstein 7X

- Estimate $\nu_{e\,eff} \sim \nu_e/\varepsilon$ where ε is estimated from the depth of the magnetic wells
- Then ν_e/ε can easily be an order of magnitude greater than the mode frequencies for high density cases
- $\nu_{couling}$ can also exceed 1 by about an order of magnitude
- Hence the trapped electron response would be strongly damped by collisions
- For high density cases on W7X
- This fact alone rules out bounce averaged curvature (or any other trapped e^- effect) as an explanation for the experimental observations
- The results are only consistent with an interpretation based on the ***lack*** of a trapped e^- response- i.e., a nearly adiabatic e^- response

NCSX shows can this effect over a very large range – the outer half – even at β = 0 (!) (for a negative shear case)

- Effect is essentially the same as the tokamak cases: decoupling from the trapped electrons
- More detailed analysis (below) substantiates this is the same as high a tokamaks

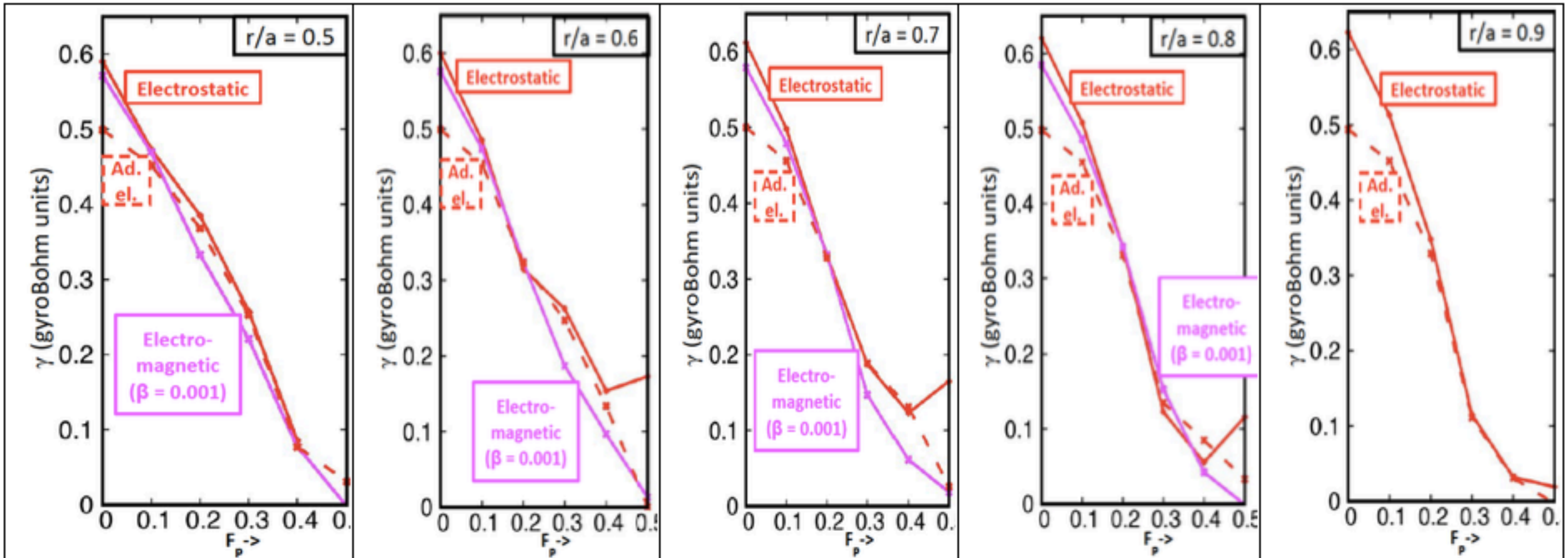
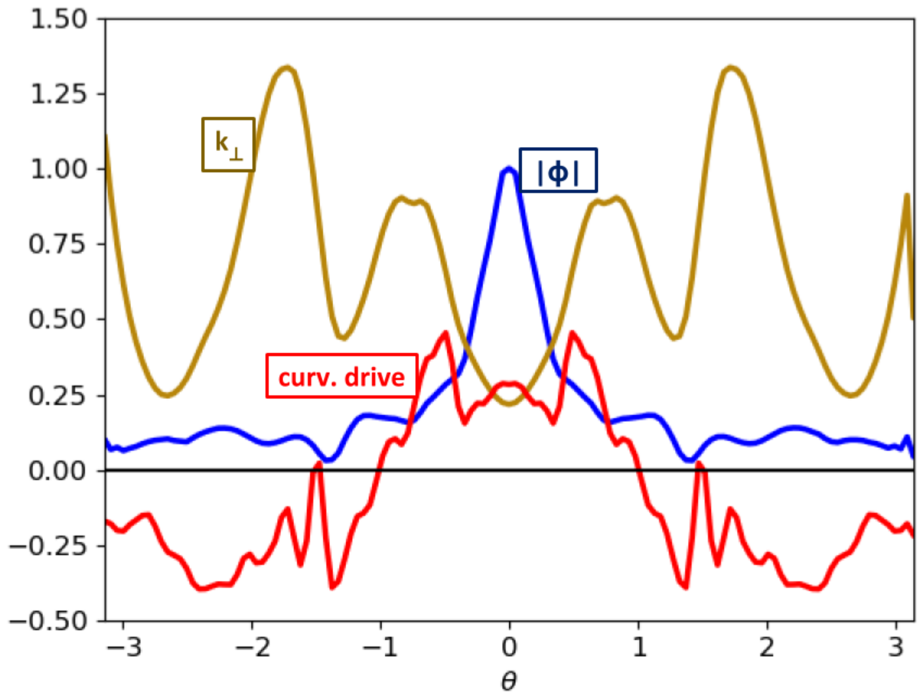


Fig 7. NCSX equilibria with β=0. Strong stabilization with F_p is found over a much wider range of minor radii than in tokamaks, from r/a = 0.5 outward. The effect is akin to the electrons behaving adiabatically. This is evidently not an effect of high α.

Eigenfunction on NCSX (r/a = 0.8)



- The eigenfunction is localized in θ by the $k_\perp \rho_i$ becoming ~ 1, not, apparently, by the curvature
- This is a consequence of high local magnetic shear
- It leads to low values of the $\langle f_{Trap\ eff} \rangle$, just as for high α tokakmak

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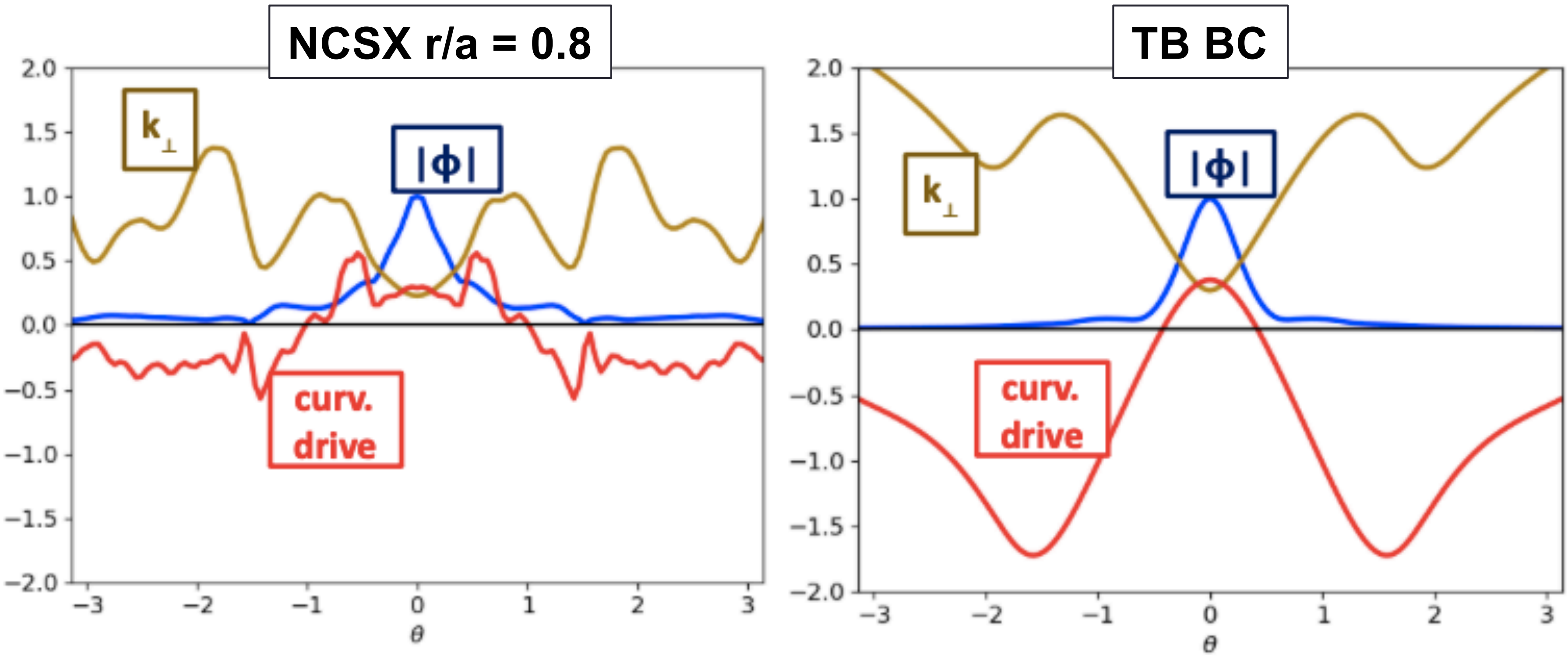
Eigenfunction average parameters

- The values $<f_{Trap\ eff}>$, is even lower for NCSX than in tokamak experiments

parameter	r/a = 0.7	r/a = 0.8	r/a = 0.9
$<\omega_{di}>/ \omega_{di}(0)$	~ 0.81 - 0.85	~ 0.85 - 0.88	~ 0.81 - 0.87
$<\omega_{de}>/ \omega_{de}(0)$	~ 0.4	~ 0.4	~ 0.34 - 0.37
$<f_{Trap\ eff}>$	~ 0.10 - 0.14	~ 0.13 - 0.17	~ 0.18 - 0.23

- However, the $<\omega_{curv\ e}>$ is higher (destabilizing) for NCSX
- The reason for this is clear by comparing NCSX and the TB BC….

Compare eigenfunctions and curvatures for NCSX (β = 0, $\hat{s} = -0.4$) and TB BC (α = 3, $\hat{s} = -1$)

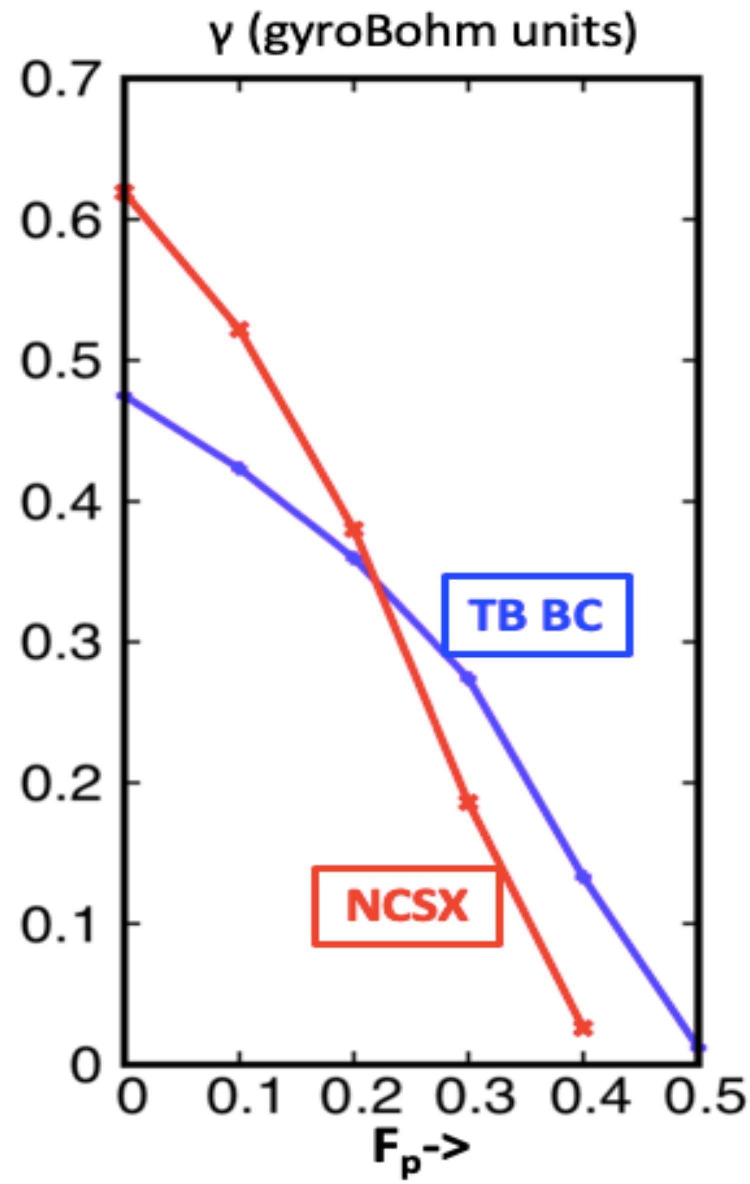


- The curvature in TB BC is much more deeply negative over a much larger range
- This explains the larger value of $\langle \omega_{\text{curv e}} \rangle$
- But this was not found to be the most important parameter for stabilization in tokamaks. How do the growth rate rates of these cases compare?

NCSX has lower growth rates at higher F_p

- The lower $\langle f_{\text{Trap eff}} \rangle$ of NCSX was key- it leads to better stabilization at high F_p even though the curvature of the TB BC appeared more favorable

- As in the tokamak cases, $\langle f_{\text{Trap eff}} \rangle$ was the most important parameter



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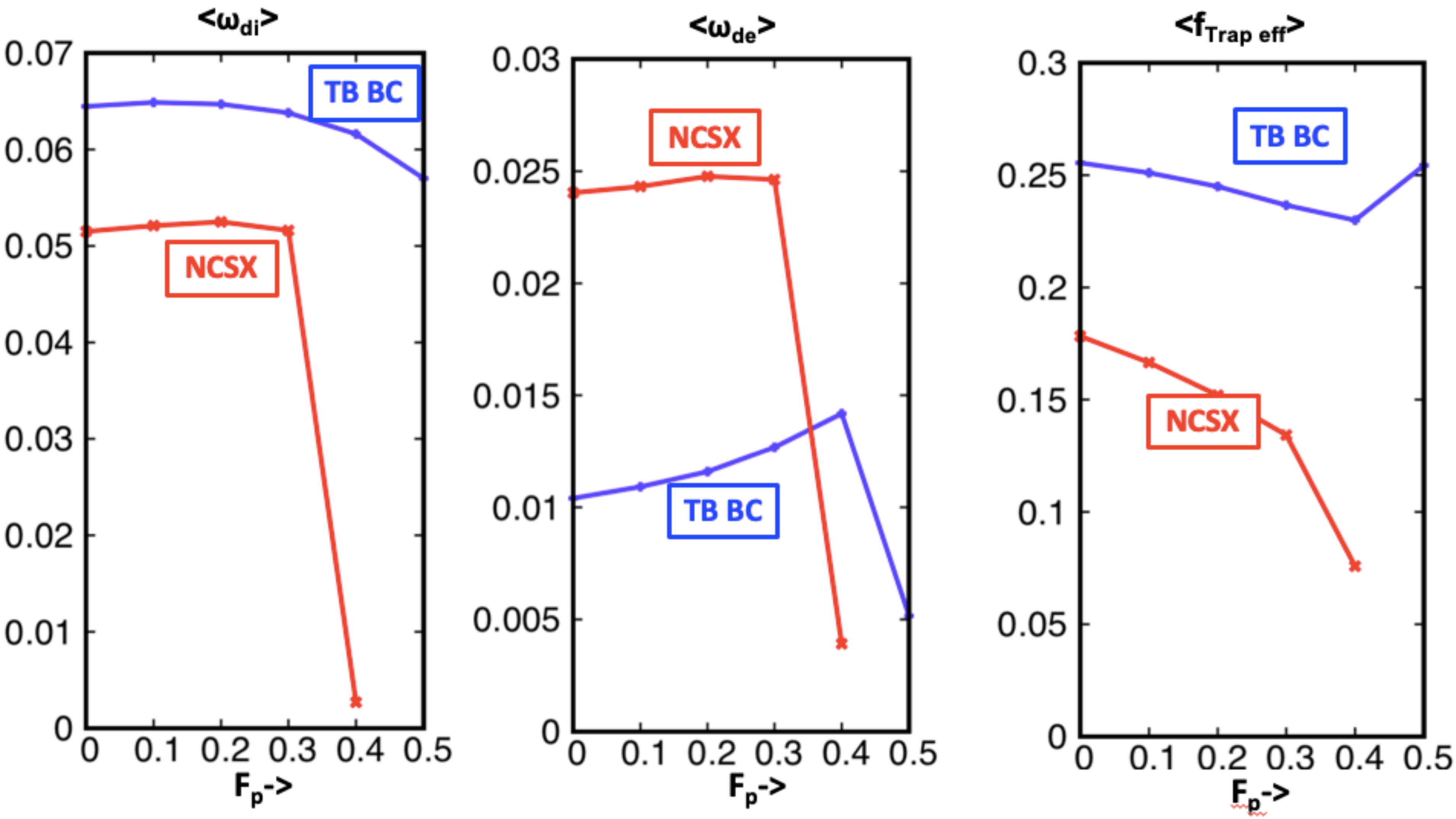
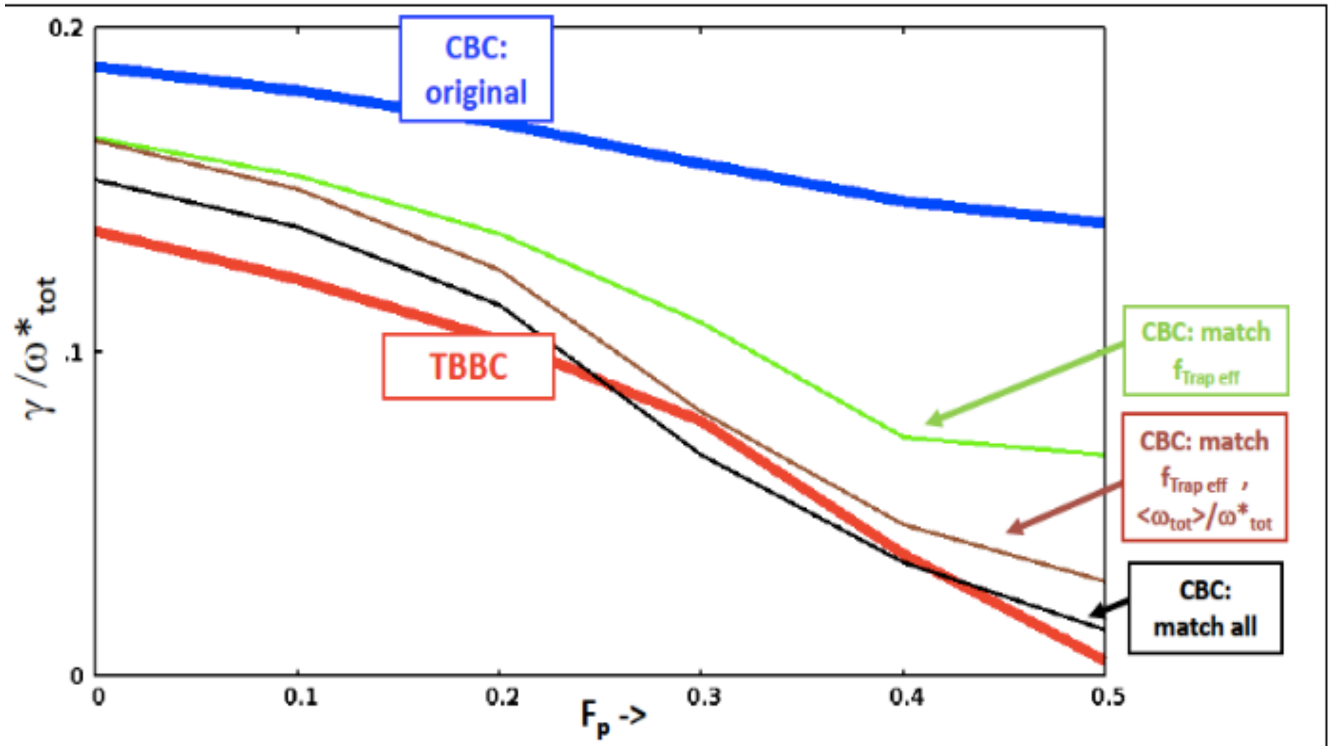
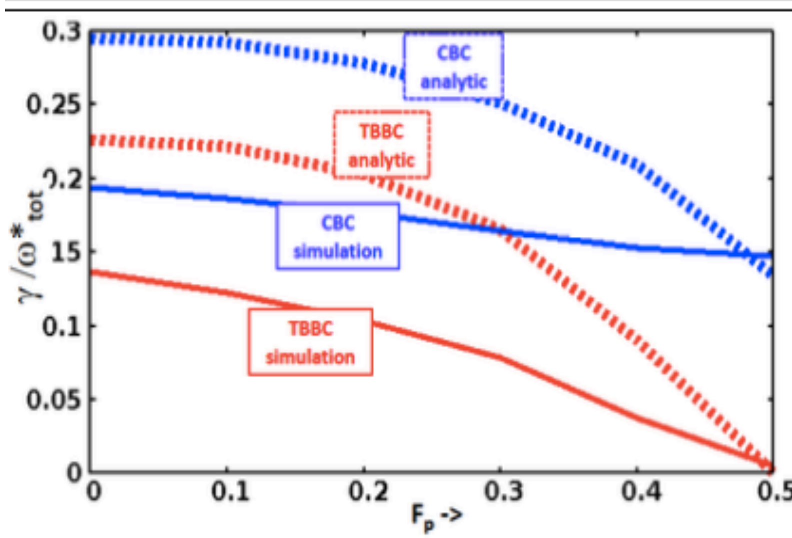
NCSX has lower growth rates at higher F_p

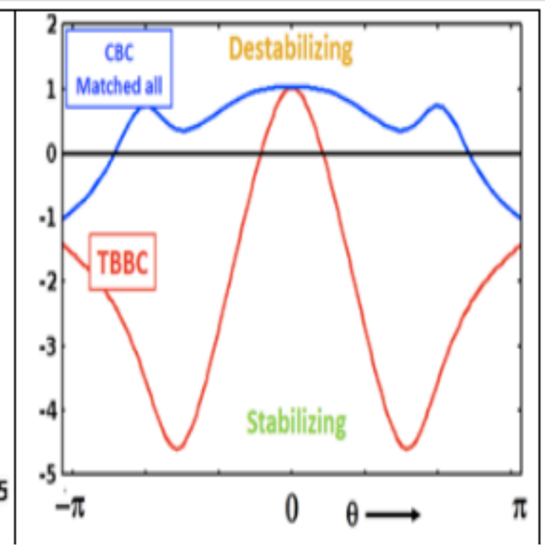


The metrics are quantitave “control parameters” – as in the theory of self-organized states !

- Keeping the curvature the same, we change the control paramters of the CBC to the values of the TBBC
  - ➢ Change ε to match $f_{\mathrm{Trap\ eff}}$
  - ➢ Change R/L to match $\omega_{d\ \mathrm{tot}}/\omega^*_{\mathrm{tot}}$
  - ➢ Change q to match $k_{||}$
- **When done: γ matches DESPITE HUGELY DIFFERENT CURVATURES !**
- **The metrics are much more accurate as control parameters than in the DR!**







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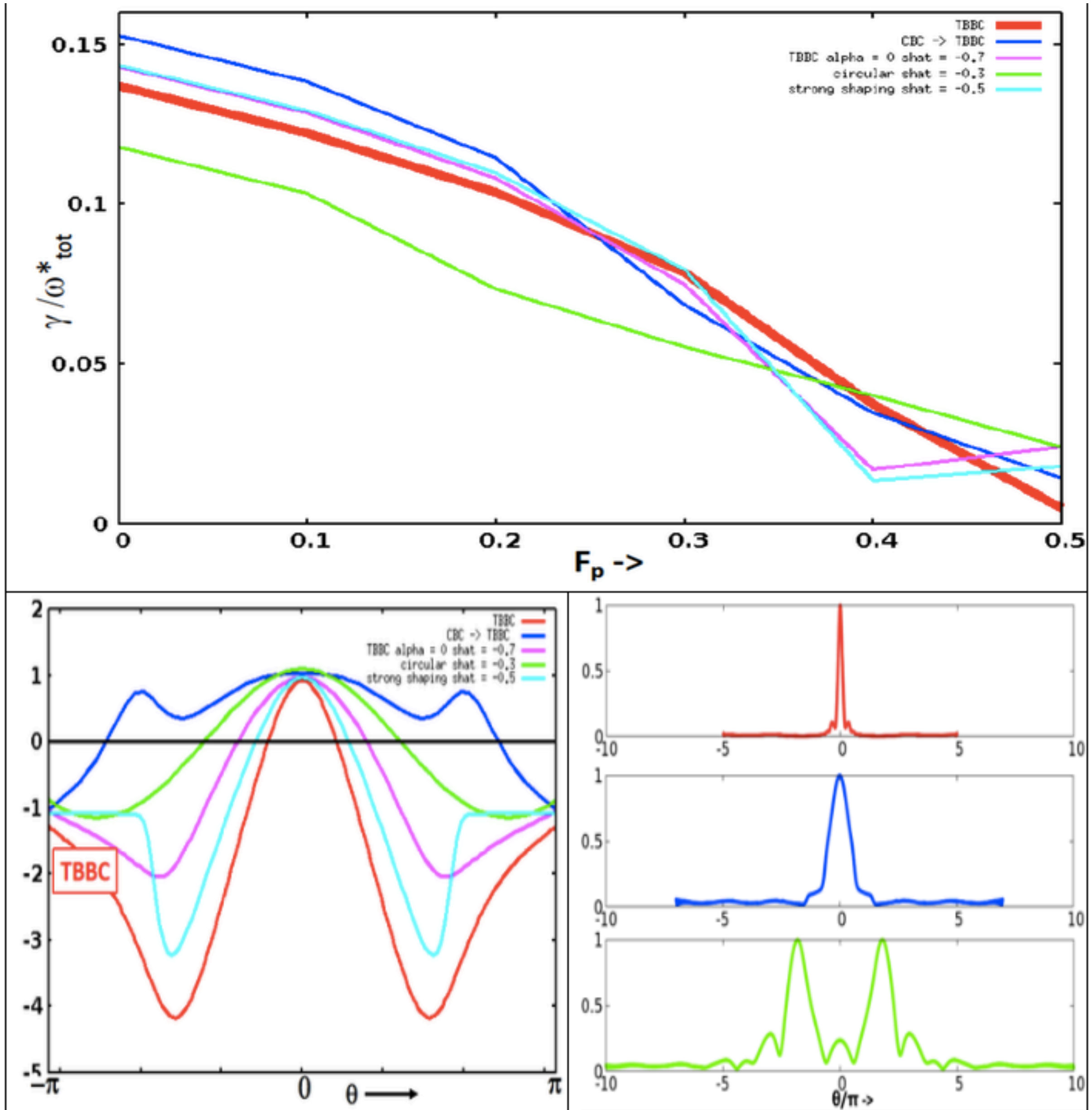
Similar matching is possible in diverse geometries

- We do the same for various different curvatures with essentially the same result
  - ➢ Curvatures hugely vary
  - ➢ Eigenfunction shapes hugely vary

- **The metrics are much more accurate as control parameters!**

- **Corrollary:**

- **The same physics operates in hugely different geometries, if they have the same control parameters!**



A Simplified Kinetic Model (SKiM) gives much improved accuracy

- Start with simplest 0D model for a uniform plasma: constant curvature drift ω_D, $k_{||}$, $k_\perp$, etc
- Include trapped particles as a 0D trapped species
- Well known dispersion relation results; it gives semi-quantitative agreement with GENE when the control parameters are used

$$\frac{1}{T_i} - \frac{1}{T_i}\int d^3v\,\hat{F}_0 J_0^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\frac{\omega-\omega_{*,i}}{\omega - k_\parallel v_{\parallel,i} - \omega_{D,i}}$$

$$+\frac{1}{T_e} - \frac{f_{trap,eff}}{T_e}\int d^3v\,\hat{F}_0 \frac{\omega-\omega_{*,e}}{\omega - \langle\omega_{D,e}\rangle_{bounce,eff} + i\frac{\nu_e(v)}{\epsilon}} = 0$$

F_M = Maxwellian

J_0 = Bessel Function (gyroavg.)

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Similar matching is possible in diverse geometries

- We do the same for various different curvatures with essentially the same result
  - ➢ Curvatures hugely vary
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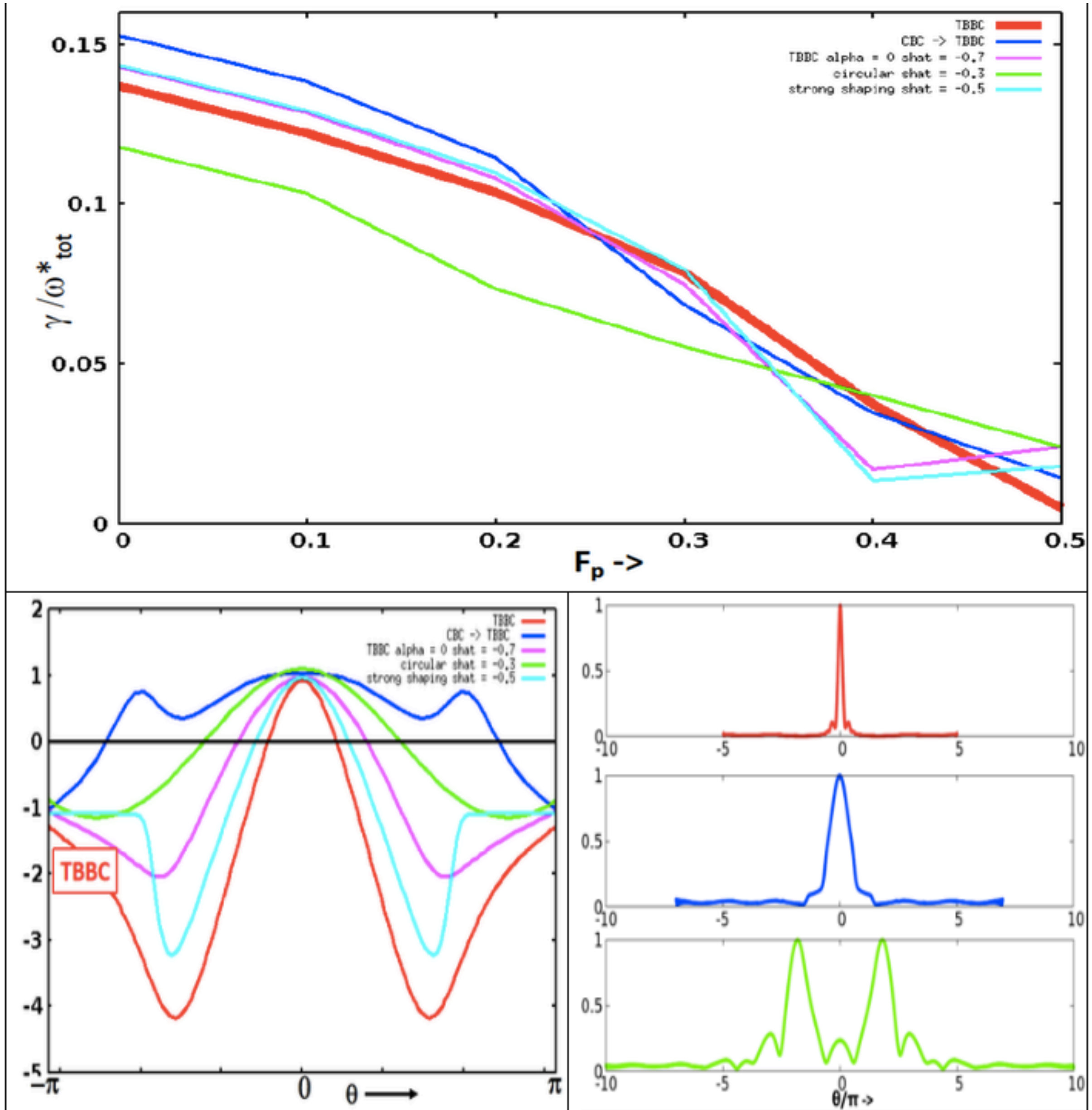
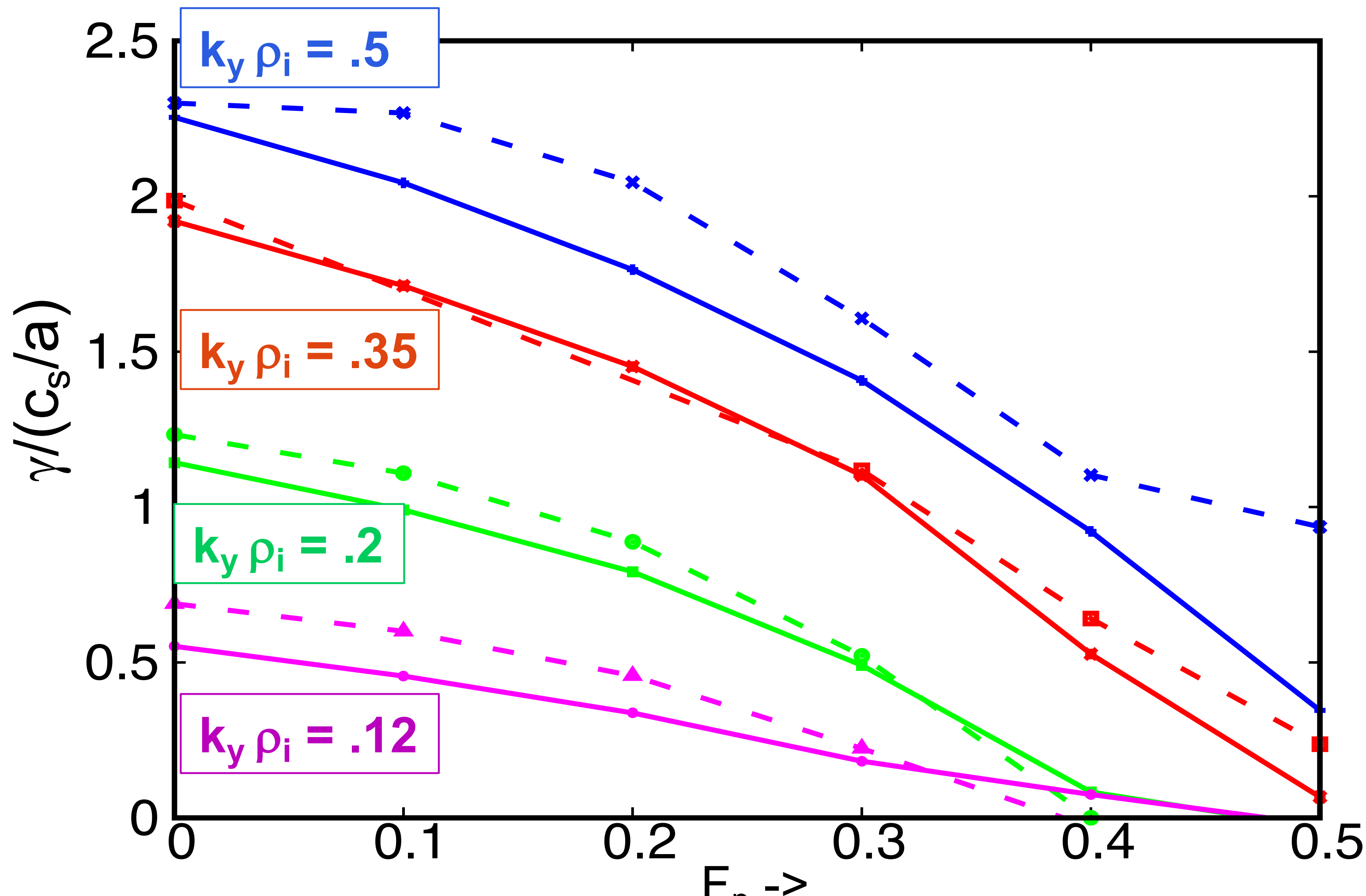


Though not perfect, SKiM has surprising accuracy

- GENE (solid) vs SKiM (dashed) for JET-like ITB (high α)
  - Using GENE eigenfunctions to compute the geometrical quantities



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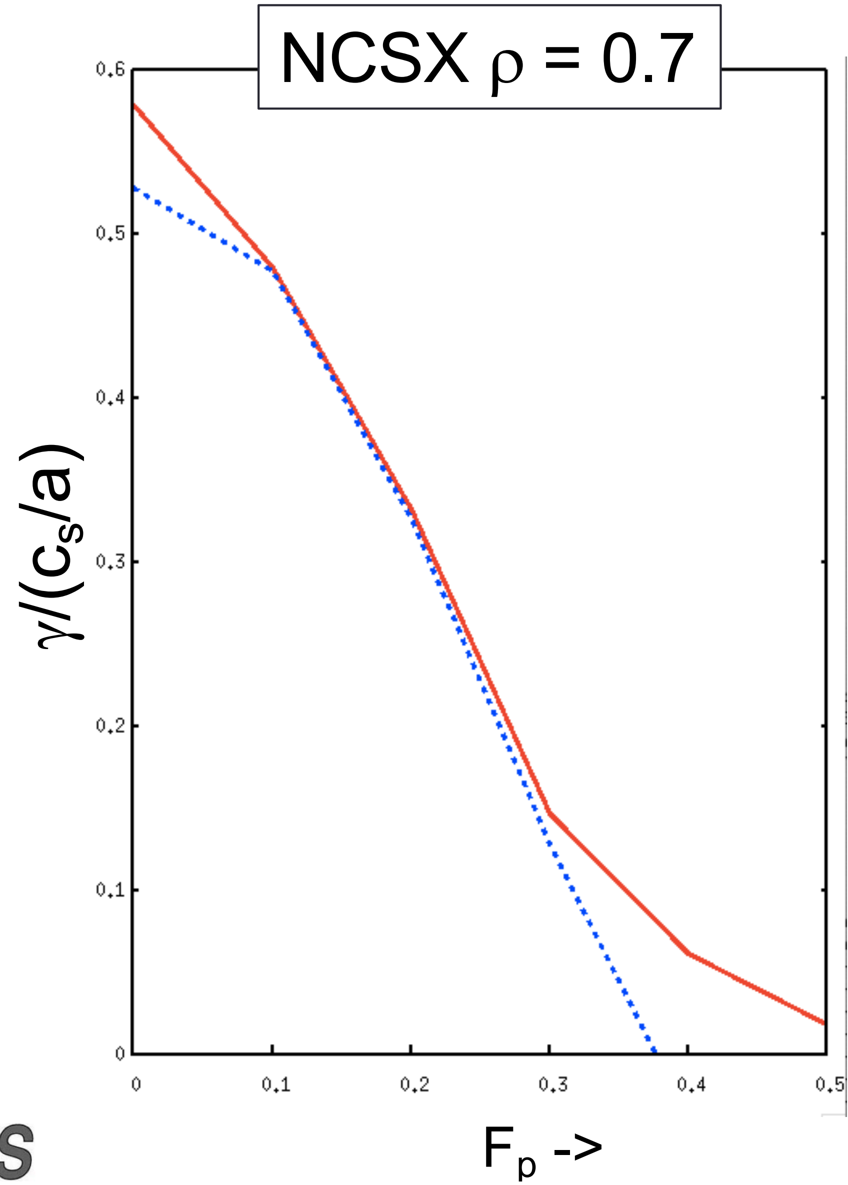
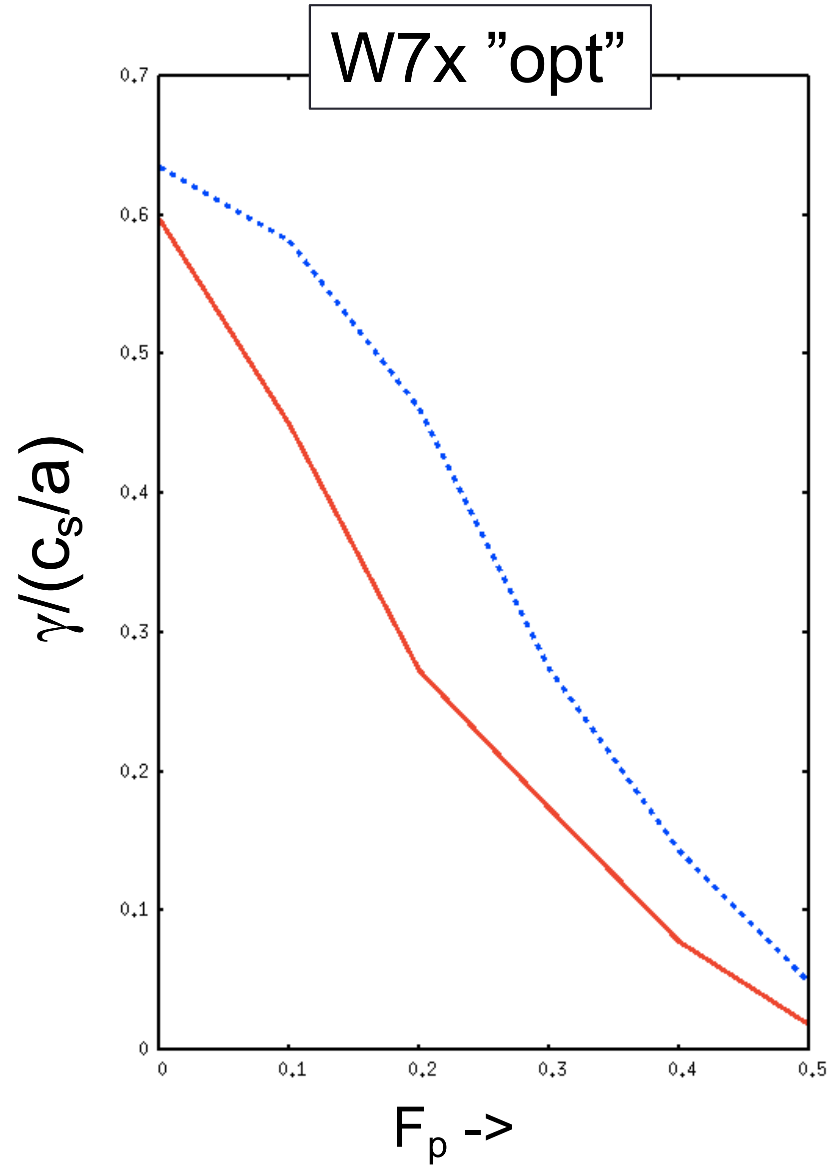
Stellarator NCSX and W7X cases:

GENE (solid) vs SKiM (dashed) for $k_y \rho_i$ near maximum D_{mixing}





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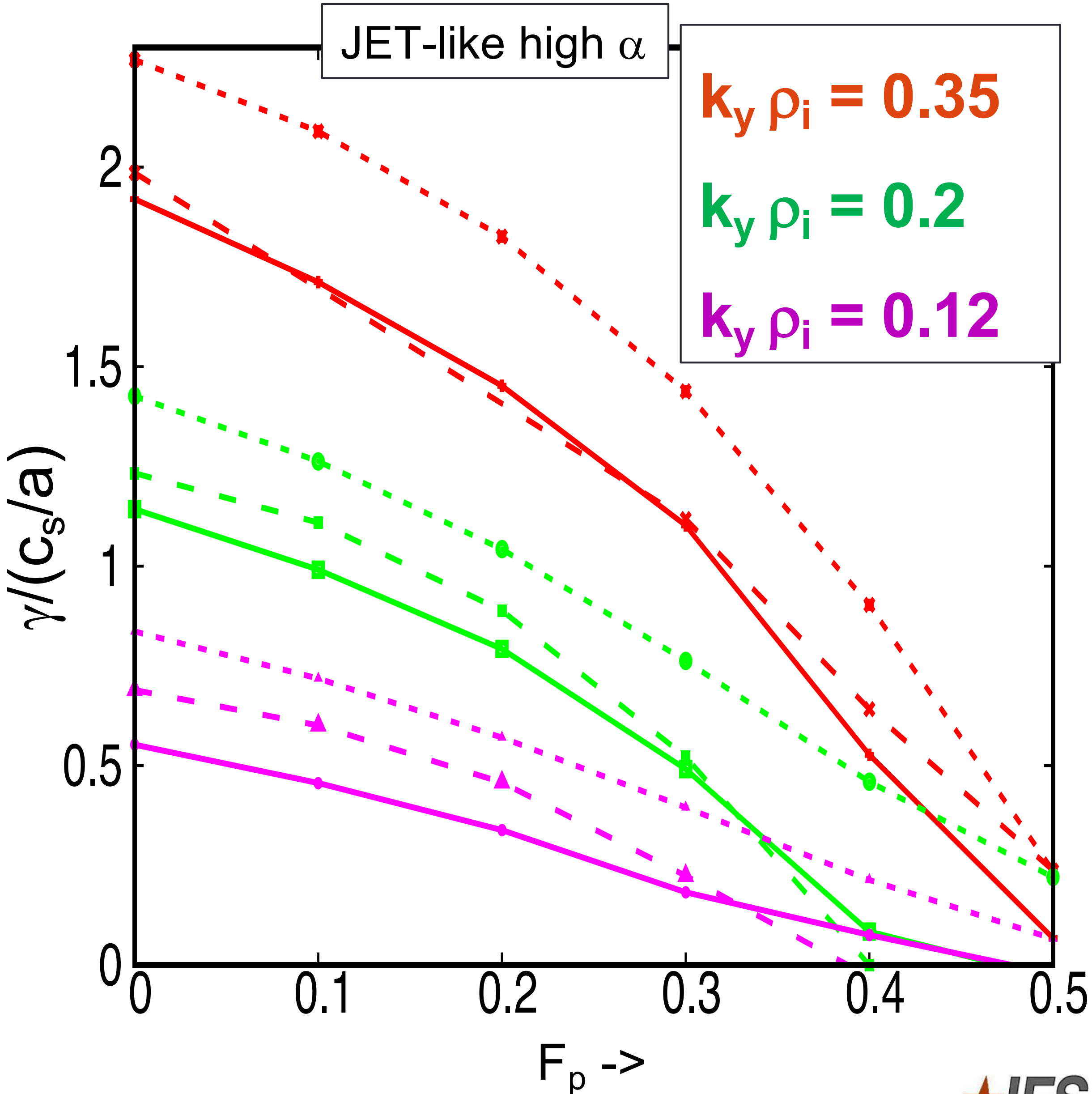
GENE roughly follows the maximum growth rate possible from SKiM (for assumed gaussian eigenfunction, varying width)

- Solid lines: GENE
- Dashed lines: SKiM from GENE eigenfunction
- Dotted lines: SKiM maximum γ from gaussian width scan

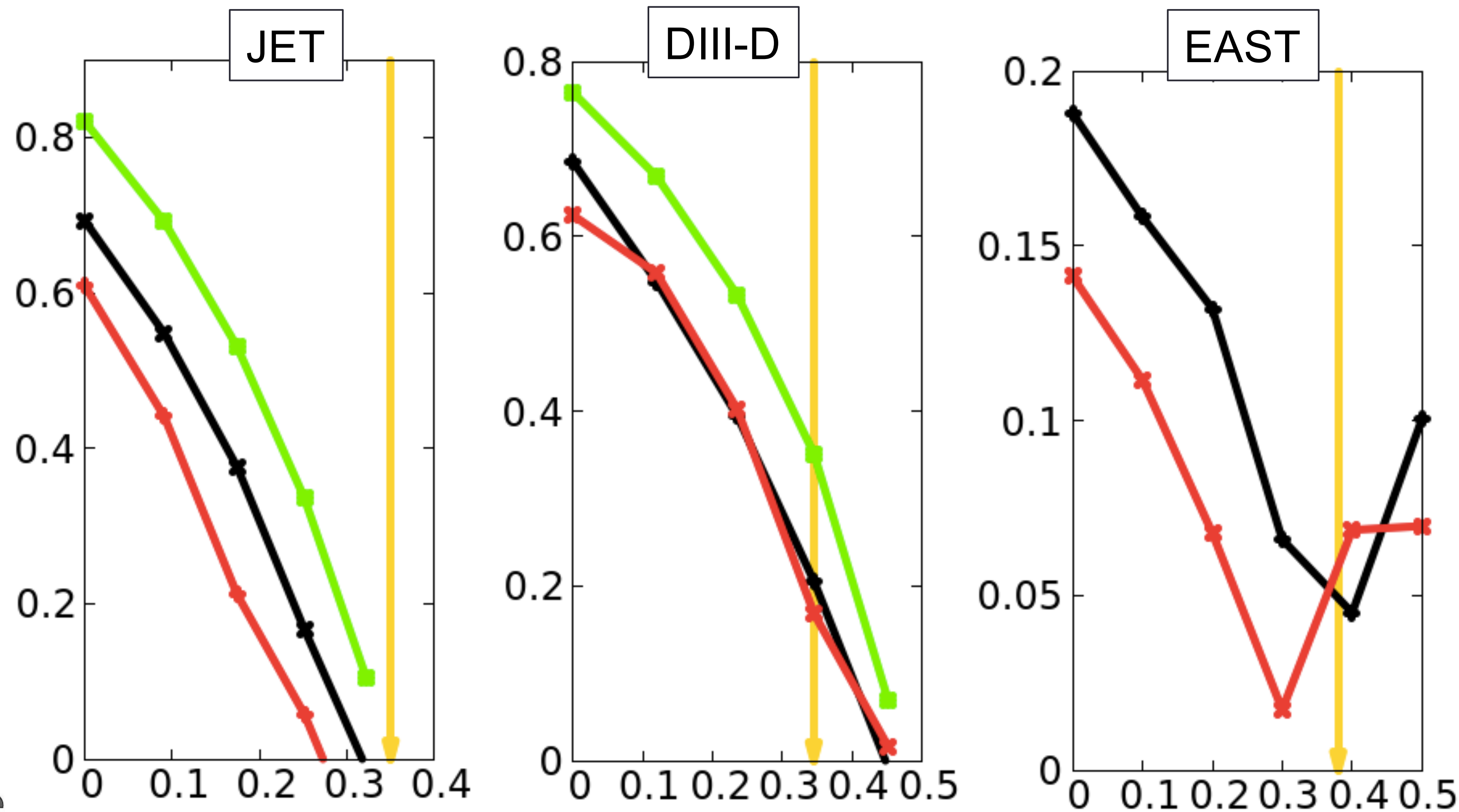
GENE roughly follows the maximum growth rate "possible"



Experimental cases:

Black = GENE   Red = SKIM with GENE eigenfucntions

Green = SKiM varying a Gaussian trial function width for maximum γ



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Putting these results in broader physical context

- Non-equilibrium thermodynamics: systems tend to maximize entropy production
  - Very active area of research in physics and beyond
- Linear eigenfunctions are “self-organized states” that attempt to maximize entropy production (γ)
- The closest fluid analogy to curvature driven ITG/TEM is fluid convection driven by simultaneous
  - Vertical temperature gradient across fluid
  - Gravity

These fluid systems are regarded as paradigmatic examples of self-organized states, and they appear to approximately maximize entropy production (heat flux)

Our results for ITG/TEM fit this general physical pattern

- Eigenfunctions adapt to maximize growth rates
  - They concentrate on the bad curvature region even when it is small
  - If there is no other thermodynamic constraint, their growth rate is not strongly decreased by geometry
  - This is the ITG/TEM behavior when F_p is small
- Basic thermodynamic and dynamical constraints are fundamental to the Max Entropy Production picture
- The powerful constraint here is the ability to tap free energy from the equilibrium gradients
  - Basic kinematics can make non-adiabatic electron response weak (small trapped electron response)
  - This constrains accessibility of the density gradient free energy

Our results for ITG/TEM fit the very general physical pattern

- Quite generally, if free energy input into a self-organized structure (eigenfunction) falls below a threshold, it does not organize (here becomes stable)
- If we neglect velocity shear, then:
- Because the eigenfunctions are quite adaptive, only very basic constraints (like the nearly adiabatic electron response) can allow steep gradients with low heat flux (low entropy production)
- The stability picture presented here, which appears to be grounded in very general thermodynamic concepts, might therefore be very robust

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General thermodynamic perspective

- In Transport Barriers, the T gradient ("thermodynamic force") is much higher, but the heat flux (the "conjugate flux") is not higher
- This is very unusual: for an *incredibly **VAST** variety* of physical systems within Non-Equilibrium Thermodynamics, increasing the thermodynamic force similarly increases the thermodynamic flux
- What is enabling the ex*tremely* unusual behavior in TBs, within the perspective of Non-Equilibrium Thermodynamics (NET)?