Understanding mechanisms underlying ohmic breakdown in a tokamak by considering multi-dimensional plasma responses

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   • Field quality analyses of external EM structure

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   • Slow plasma formation & homogeneous plasma structure along B

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**Introduction**

**What is the ohmic breakdown?**

- Ohmic breakdown is a first operation phase in tokamak to produce initial plasmas
- Plasma density increases exponentially due to the electron avalanche
- Limited toroidal electric field ($E_{tor} \leq 0.3 \text{ V/m}$) in ITER due to operation limit
Distinct characteristics of Ohmic breakdown

- **Open magnetic field lines**
  - External magnetic fields are dominant
  - Plasma current is negligible

- **Low pitch angle**
  - $B_{RZ}/B_{tor} \sim 10^{-3}$
  - Long connection lengths (> 1000 m)

- **Toroidal electric fields**
  - $E_{tor} \leq 1 \text{ V/m}$
  - Strongest toroidal electric fields during tokamak operation

Underlying physical mechanism of the breakdown have been obscured over 50 years
Introduction

Time-varying complex electromagnetic fields

- (time-varying) CS currents + PF currents + eddy currents

Deep understanding of Ohmic breakdown physics is essential to design robust and optimized breakdown scenarios.
Experimental observations of ohmic breakdown phenomena are very limited.
1. Electrons move following the magnetic field lines (parallel transport dominance)

\[ v_{e,\parallel} = -\mu_e E \quad v_{e,\parallel} \gg v_{e,\perp} \]

2. Electron density increases exponentially (Townsend avalanche)\(^2\)

\[ n(x) = n_0 \exp(\alpha x_{\parallel}) \]

\[ \alpha = A p \exp \left(-Bp/E_{\text{ext,\parallel}}\right) \]

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**Previous works**

Adopting simplest theory: Townsend avalanche theory

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Previous works

Field quality analyses of external EM structure

Based on the Townsend avalanche theory...

- Electron avalanche physics are determined by external fields only
- Detail avalanche physics are out of interest
- Evaluations of complex external electromagnetic fields are important

Field quality analysis

<table>
<thead>
<tr>
<th>0D effective parameters</th>
<th>Empirical condition</th>
<th>2D field-line-integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{eff}} \approx 0.25 a_{\text{eff}} B_T / B_{\rho} )</td>
<td>( E_T B_T / B_{\perp} &gt; 1000 \text{ V/m} )</td>
<td>( L = \int_{\vec{B}} dl \quad V = \int_{\vec{B}} \vec{E} \cdot d\vec{l} )</td>
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Limitations

- Static field analysis at a specific time \( \Rightarrow \) No dynamic plasma evolution (ne, Te)
- **Self-electric fields** are ignored \( \Rightarrow \) No plasma response

Motivation

Existence of strong self-electric fields in tokamak

- The existence of strong self-electric fields (~kV/m) in RZ plane was measured in CASTOR tokamak 30 years ago [3]
  
  M. Valovic, NF (1987)

- CASTOR tokamak ($R_0 = 0.4$ m, $a = 85$ mm)

- Convection loss by ExB drifts 2-3 orders of magnitude larger than Bohm diffusion

Roles of self-electric fields have been still shrouded in mystery due to complex EM topology

Motivation
Mysterious experimental results

Townsend avalanche simulation

- Calculated Hα image
- Observed Hα image

Visible camera from Experiment

1. Fast avalanche
2. Localized & asymmetric structure
   (Exponential density profile along $\vec{B}$)

≠

1. Much slower avalanche
2. Elongated & symmetric structure
   (Homogeneous density along $\vec{B}$)
 Townsend theory cannot explain the avalanche phenomena in tokamak

Townsend avalanche (Snow avalanche)

Actual avalanche in tokamak

Snowball = Electrons
Gravity = Toroidal E fields
Slope = B field lines

✓ Exponential profile

\[ n(x_{\text{slope}}) = n_0 \exp(\alpha x_{\text{slope}}) \]

✓ Homogeneous profile

\[ n(x_{\text{slope}}) \approx M \times n_0 \]

Mysterious experimental result
Homogeneous plasma structure along magnetic field line

⇒ Townsend theory cannot explain the avalanche phenomena in tokamak
A new type of breakdown theory is required for the strongly magnetized system by considering...

- The nature of gyromotion and guiding center motion
- The roles of self-electric fields regarding strong magnetic fields
Toroidally symmetric plasma model

Roles of self-electric fields

\[ E = -\frac{\partial A}{\partial t} - \nabla V = E_{\text{ext}} + E_{\text{self}} \]

- **n ≪ n_c** (low density)
  - \( |E_{\text{self},\parallel}| \ll |E_{\text{ext},\parallel}| \) \Rightarrow Self-electric fields are negligible

- **n ≳ n_c** (high density)
  - \( |E_{\text{self},\parallel}| \sim |E_{\text{ext},\parallel}| \) \Rightarrow Total electric fields is reduced \( (E_{\text{tot},\parallel} = E_{\text{ext},\parallel} + E_{\text{self},\parallel} \rightarrow 0) \)
  - \( |E_{\text{self},\perp}| \gg |E_{\text{ext},\perp}| \) \Rightarrow \( \vec{E} \times \vec{B} \) drift motions

**Self-electric fields** play significant roles in the ohmic breakdown regarding parallel and perpendicular dynamics
Toroidally symmetric plasma model
Roles of self-electric fields

Parallel dynamics

Slow avalanche
Reduced heating
Reduced parallel transport

\[ n_{c,\parallel} \equiv \left( \frac{e_0}{kT_e} \right) \cot^2(\theta_B)(E_{\text{ext}})^2 \left( \frac{1}{\gamma} \right)^2 \]

(\(10^{12} \sim 10^{14} \text{ m}^{-3}\))

Fast avalanche
Dominant parallel transport

Perpendicular dynamics

Dominant perp. transport via ExB drift

\[ n_{c,\perp} \equiv \left( \frac{e_0B^2}{m_e} \right) \tan^2(\theta_B) \left( \frac{1}{\gamma} \right)^2 \]

(\(10^{12} \sim 10^{14} \text{ m}^{-3}\))

Negligible perp. transport

New regime due to plasma responses

Townsend avalanche

Plasma Density
Toroidally symmetric plasma model

Parallel dynamics

- Separation force \( (E^\phi_{\text{ext}}) \) vs. Attracting force \( (E^{RZ}_{\text{self}}) \)

\[
E^\phi_{\text{ext}} = E^\phi_{\text{ext}} \cos(\theta_B)
\]

\[
E^{RZ}_{\text{self}} = E^{RZ}_{\text{self}} \sin(\theta_B)
\]

- Equilibrium state in parallel direction

\[
E^\phi_{\text{ext}} = -E^{RZ}_{\text{self}} \quad \Rightarrow \quad |E^{RZ}_{\text{self}}| = |E^\phi_{\text{ext}}| \cot(\theta_B) \gg |E^\phi_{\text{ext}}|
\]

\((1000 \text{ V/m})\) \((1 \text{ V/m})\)

- Cancellation of external electric fields

\[
E^\parallel_{\text{tot}} = E^\phi_{\text{ext}} + E^{RZ}_{\text{self}} \approx 0
\]

- Parallel transport
- Heating power
- Electron temperature
- Avalanche growth rate

Key mechanism for mysterious slow plasma formation

Parallel critical density

\[
n_{c,\parallel} \equiv \left( \frac{e_0}{kT_e} \right) \cot^2(\theta_B) E^2_{\text{ext}} \left( \frac{1}{V} \right)^2
\]
**Toroidally symmetric plasma model**

**Perpendicular dynamics**

- **Mean ExB across $B_{RZ}$**
  - Induced by spatial-temporal average $\overline{E_{\text{self}}^{RZ}}$
  - Determine overall plasma flow and position
  - Dominant convective loss term
  - Perpendicular confinement time of the plasma
    \( \tau_\perp = a/v_{E\times B} \)

- **Turbulent ExB mixing along $B_{RZ}$**
  - ExB vortices at plasma edges are turbulent due to negligible viscosity
  - Plasma rapidly diffuses along $B_{RZ}$ by turbulent mixing
  - Homogeneous plasma density along $B_{RZ}$

**Dominant transport mechanism** in the RZ plane

when \( n > n_{\text{crit,}\perp} \equiv \left( \frac{\epsilon_0 B^2}{m_e} \right) \tan^2 (\theta_B) \left( \frac{1}{\gamma} \right)^2 \)
Toroidally symmetric plasma model

**Dominant ExB transport in the RZ plane**

✓ ExB perpendicular transport overwhelms parallel transports in the RZ plane due to very low pitch angle ($\sin \theta_B \sim 10^{-3}$)

$$v_e^{\perp} \approx v_i^{\perp} \approx v_{E \times B} \approx v_e^{\parallel} \sin \theta_B \gg v_i^{\parallel} \sin \theta_B$$

Perpendicular critical density

$$n > n_{c,\perp} \equiv \left( \frac{\epsilon_e B^2}{m_e} \right) \tan^2 \theta_B \left( \frac{1}{\gamma} \right)^2$$

1000 m/s 1000 m/s 1 m/s
**Toroidally symmetric plasma model**

**Perpendicular dynamics**

\[(a)\] \[B_{\Phi}, E_{\text{ext}}\]

\[B_{\text{RZ}}\]

\[\nabla \cdot j_{\parallel} < 0\]

\[\nabla \cdot j_{\parallel} > 0\]

\[\nabla \cdot j_{\parallel} < 0\]

\[\nabla \cdot j_{\parallel} > 0\]

**Charge accumulations via non-uniform parallel currents**

**Turbulent ExB vortices formation corresponding to charge densities**

**Density homogenization along \(B_{\text{RZ}}\) & Cross transports by mean ExB**

\[(b)\] \[B_{\text{RZ}}\]

\[(c)\] \[B_{\text{RZ}}\]

continous mixing by turbuelnt ExB

mean ExB
Toroidally symmetric plasma model

ExB mixing avalanche mechanism

Parallel dynamics

\[ n > n_{c,\parallel} \equiv \left( \frac{\varepsilon_0}{kT_e} \right) \cot^2(\theta_B) \left( \frac{E_{\text{ext},\parallel}}{1} \right)^2 \]

1. Cancellation of \( E_{\text{ext},\parallel} \)

\[ \Rightarrow \text{Slow avalanche growth} \]

(Mystery 1 solved)

Perpendicular dynamics

\[ n > n_{c,\perp} \equiv \left( \frac{\varepsilon_0 B^2}{m_e} \right) \tan^2(\theta_B) \left( \frac{1}{V} \right)^2 \]

2. Mean ExB across \( B_{\text{RZ}} \)

\[ \Rightarrow \text{Determine plasma position} \]

\[ \Rightarrow \text{Dominant plasma loss term} \]

3. Turbulent ExB mixing along \( B_{\text{RZ}} \)

\[ \Rightarrow \text{Homogeneous plasma density along} \ B_{\text{RZ}} \]

(Mystery 2 solved)
Particle simulation (BREAK)

Development of particle simulation code

- To study the ohmic breakdown physics under a realistic complicated situation considering the self-electric fields and kinetic effects consistently

- The ohmic breakdown phenomena span a broad range of spatio-temporal scales
  - $\Delta x \sim (10^{-6} - 1) \, \text{m}$, $\Delta t \sim (10^{-12} - 10^{-2}) \, \text{s}$

- **BREAK (Breakdown Realistic Evolution Analysis in tokamak)** (YOO, CPC 2017)
  - Written in C/C++ language
  - 2D / 3D implicit electrostatic particle-in-cell simulation code
  - Direct implicit method with D1 damping scheme is adopted to calculate charged particle motion
  - 6 species ($e$, $H_2^+$, $H^+$, $H_3^+$, $H_{2\text{(fast)}}$, $H_{\text{(fast)}}$) are considered
  - 26 collision reactions in the energy range of (0.01 – 1000) eV and plasma-wall interactions are treated by the MCC (Monte Carlo Collision) scheme
  - Coulomb collision is calculated by Nanbu’s method
  - Self-electric fields produced by plasma space charge are calculated
  - Hybrid parallel computing method (MPI + OpenMP)
BREAK simulation demonstrated the significant roles of self-electric fields

- Electron density
- Vortex
- Potential
- Mean ExB

✓ BREAK simulation demonstrated the significant roles of self-electric fields
KSTAR reference breakdown scenario

- **Breakdown scenarios** are designed by considering *eddy currents* as a ring model and *ferromagnetic incoloy 908 material effect* as a non-linear model [6].

- Magnetic field configurations varies with time (0 - 60 ms)

- **Initial condition**
  \[
  n_{\text{gas}} = 4 \times 10^{17} \text{ m}^{-3} \\
  n_{e0} = n_{i0} = 10^6 \text{ m}^{-3} \\
  T_{e0} = T_{i0} = 0.03 \text{ eV}
  \]

- **2 different simulations**

[w/o self-electric fields] [with self-electric fields]

difference

Particle simulation (BREAK)
Main results of KSTAR simulation

- Monotonic exponential growth
- Parallel transport

✓ without $E_{\text{self}}$
- Reduction of growth rate
- Anomalous perpendicular transport

✓ with $E_{\text{self}}$
- Reduction of growth rate
- Anomalous perpendicular transport
Particle simulation (BREAK)
Main results of KSTAR simulation

✓ without $E_{\text{self}}$

5 ms  10 ms  15 ms  17 ms  19 ms  21 ms  30 ms  40 ms

✓ with $E_{\text{self}}$
Particle simulation (BREAK)
0D results of KSTAR simulation

- **Plasma $T_e$ & $n_e$**
  - Maintaining low $T_e$ (~ 10 eV)
  - Drastic decrease of $n_e$ growth rate

- **Parallel heating**
  - $E_{\text{self}}^\parallel$ cancels out $E_{\text{ext}}^\parallel (E_{\text{tot}}^\parallel \downarrow)$
  - Heating power is reduced

- **Transports**
  - $v_{e^\parallel}$ is reduced
  - Perpendicular transports by ExB
  - Ion transport is greatly enhanced

$$(v_{e^\perp} \approx v_{H_2^+\perp}) > v_{e^\parallel} \gg v_{H_2^+\parallel}$$
Particle simulation (BREAK)

Phase 1-2: Heat up and Townsend avalanche (0 – 15 ms)

- $\log_{10}(n_e)$
- $T_e$ (eV)
- $|E_\parallel|$ (V/m)
- Potential (V)

- Fast growth rate
- Localized & up-down asymmetric structure due to parallel electron transport
### Particle simulation (BREAK)

**Phase 3: Self-electric fields dominated (15 – 40 ms)**

| log_{10}(n_e) | T_e (eV) | | E_{||} (V/m) | Potential (V) |
|---------------|---------|---|---------------|---------------|

- **✓** Slower growth rate
- **✓** Elongated plasma structure due to ExB perpendicular transports
- **✓** Strongly inhomogeneous plasma density and temperature
Particle simulation (BREAK)
Comparison with experiments

- Densities are higher at downstream regions
- Temperatures are lower at downstream regions
- Discrepancy between Balmer-α line emissions and electron density is observed

- CCD camera image shows emission structure at the inboard side along $B_{RZ}$
- Synthetic diagnostic well agrees with experiments qualitatively
- The differences result from uncertainties of initial conditions, magnetic field structure and camera view information
Implications for scenario design strategy
Mean ExB (Perpendicular dynamics)

- Mean ExB flow can have arbitrary direction depending on electromagnetic topology
- Easily predictable from external EM fields only

$$v_{E_{self} \times B_\phi} \parallel v_{E_{ext} \times B_{RZ}}$$

$$\frac{|v_{E_{self} \times B_\phi}|}{|v_{E_{ext} \times B_\phi}|} = \cot^2(\theta_B) = 10^5 \sim 10^7$$

1000 m/s

0.001 m/s
Implications for scenario design strategy

Comprehensive understanding of X-point topology

Critical plasma densities at X-point region

- $\theta_B \to 0$
- $n_{\text{crit},||} \propto \cot^2(\theta_B) \to \infty$
- $n_{\text{crit},\perp} \propto \tan^2(\theta_B) \to 0$

Parallel dynamics

- Higher heating at X-point region
  - $n \ll (n_{\text{crit},||} \approx \infty)$
- Lower heating at other regions
  - $n > n_{\text{crit},||}$

Perpendicular dynamics

- Two inflows + Two outflows by mean ExB
  - $n > n_{\text{crit},\perp}$

Topology analysis on external EM fields predicts overall plasma evolution

Plasma density would be higher at downstream region
Implications for scenario design strategy
Interpretation of KSTAR simulation results

- Plasma densities are higher at downstream regions
- Plasma temperatures are higher at X-point regions
- Balmer line emissions are observed at higher temperature regions
Implications for scenario design strategy

New guidelines for scenario design

**Topology analysis method** can predict overall spatial-temporal plasma evolution

- Main plasma position (High-density at downstream region)
- Inhomogeneous structure of plasma density and temperature
- New confinement time restricted by mean ExB convective loss \( \tau_{\perp} \sim a/v_{E\times B} \)

**Breakdown scenario can be optimized** by controlling X-point topology for

- Robust ohmic breakdown for low-E field operation
- Position control of initial closed surface
- Vertical stability control using PCS
- Effective ECH deposition (ex. Targeted to downstream region, not for X-point)
Implications for scenario design strategy
New guidelines for scenario design

\[ B_{RZ} \]

Forward \( B_{RZ} \)

\[ |B_{RZ}| @ t = 0.00 \text{ ms} \]

\( B_T \) ○

\( E_T \) ○

Reverse \( B_{RZ} \)

\[ |B_{RZ}| @ t = 0.00 \text{ ms} \]

\( B_T \) ○

\( E_T \) ○

→ Same magnitudes, but opposite direction of \( B_{RZ} \)
Implications for scenario design strategy

New guidelines for scenario design

✓ (Previous) Empirical condition

- Same prediction for 2 cases
- High plasma density at X-point region

✓ (New) Topology analysis method

- Different prediction for 2 cases
- High plasma density at downstream region
Implications for scenario design strategy
New guidelines for scenario design

Topology analysis method well predicted the plasma behaviors
Implications for scenario design strategy

New guidelines for scenario design

Topology analysis

Topology analysis method well predicted the plasma behaviors
Implications for scenario design strategy
New guidelines for scenario design

Artificial Double Null case 1

Artificial Double Null case 2

Plasma behaviors and positions could be predicted by topology analysis
Implications for scenario design strategy

New guidelines for scenario design

Artificial Double Null case 1

Artificial Double Null case 2

Topology analysis method well predicted the plasma behaviors

Plasma position could be controlled by designing of X-point topology
Conclusion

- Ohmic breakdown physics has not been revealed due to its complexity

- We propose **novel mechanisms of ohmic breakdown** by considering plasma response self-consistently which can explain mysterious experimental results

- **Crucial roles of self-electric fields** as underlying mechanism are **newly discovered**
  
  ✓ **Cancellation of external electric fields** (parallel)
    - Parallel transport ↓ Heating power ↓ Electron temperature ↓ Avalanche growth rate ↓
    - Responsible for slow plasma formation

  ✓ **Dominant new transport by ExB drifts** (perpendicular)
    - Dominant transport mechanism especially for heavier ions
    - Responsible for homogeneous plasma structure along magnetic field lines

- The new physical insights into the complex EM topology can provide **general guidelines for robust breakdown scenario strategy**