Nonlinear quantum electrodynamics in strong laser fields:
From basic concepts to electron-positron photoproduction

Princeton Plasma Physics Laboratory (PPPL)


Sebastian Meuren

Karen Z. Hatsagortsyan, Christoph H. Keitel, Antonino Di Piazza

MPI for Nuclear Physics, Heidelberg
First part: introduction to SFQED

- Why should we study Strong-Field QED?
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
- Lasers as a tool to study the critical field
  - Nonlinear Compton scattering
  - Nonlinear Breit-Wheeler pair production
- From a single vertex to a QED cascade
  - QED-PIC approach
  - Formation region and hierarchy of scales
- Radiative corrections
  - Quantum dressing: exact wave functions
  - Fully nonperturbative regime of QED

More details can be found, e.g., in:
W. Dittrich, H. Gies, Probing the Quantum Vacuum (Springer, 2000)
E.S. Fradkin, D.M. Gitman, S.M. Shvartsman, QED with Unstable Vacuum (Springer, 1991)
V. I. Ritus, J. Sov. Laser Res. 6, 497–617 (1985)
Motivation: Why do we want to test nonlinear QED?

QED: electrons, positrons and photons

\[ \mathcal{L}_{\text{QED}} = \bar{\psi} \left( i \gamma^0 \partial - eA - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \]

- Here, \( \epsilon_0, \hbar \) and \( c \) are set to unity (sometimes restored for clarity)
- The characteristic scales of atomic physics and QED are determined by the electron mass \( (m) \) and charge \( (e < 0) \)

QED

- \( \mathcal{E} = mc^2 \sim 10^6 \text{ eV} \)
- \( \lambda_C = \hbar c/(mc^2) \sim 10^{-13} \text{ m} \)
- \( E_{\text{cr}} = (mc^2)^2/(|e|\hbar c) \sim 10^{16} \text{ V/cm} \)

Atomic physics

- \( \mathcal{E}_H = (Z\alpha)^2 \mathcal{E}/2 \sim Z^2 \times 10 \text{ eV} \)
- \( a_B = \lambda_C/(Z\alpha) \sim Z^{-1} \times 10^{-10} \text{ m} \)
- \( E_{\text{eff}} = (Z\alpha)^3 E_{\text{cr}} \sim Z^3 \times 10^{10} \text{ V/cm} \)

\( \alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137 \): fine-structure constant, \( Z \): atomic number

Conceptual changes

- Energy \( \mathcal{E} \): nonrelativistic vs. relativistic description
- Length \( \lambda_C \): classical vs. quantum field theory
- Field \( E_{\text{cr}} \): vacuum vs. nonlinear QED
Sauter-Schwinger vacuum instability

- A pure electric field \( E \geq E_{cr} \) is unstable, it decays spontaneously.
  
  First observation: Sauter (1931), First modern calculation: Schwinger (1951)

Vacuum fluctuations

Instead of being empty, the vacuum is filled with quantum fluctuations

Heuristic tunneling picture

“Tilted” energy levels \( \rightarrow \) tunneling

Probability: \( \sim \exp \left( -\pi \frac{E_{cr}}{E} \right) \)

- Heuristic derivation of the critical field \( E_{cr} = 1.3 \times 10^{16} \) V/cm:
  - Spatial extend of the fluctuations (Heisenberg): \( \sim \lambda_C = \hbar/(mc) \)
  - Energy gap between virtual and real (Einstein): \( \sim mc^2 \)
  - Work by the field (Lorentz force): \( \sim E |e| \lambda_C \rightarrow E_{cr} = mc^2/(|e| \lambda_C) \)

- In vacuum \( I_{cr} = 4.6 \times 10^{29} \) W/cm\(^2\) is not achievable in the near future:

<table>
<thead>
<tr>
<th></th>
<th>( \sim \hbar \omega )</th>
<th>Future facilities</th>
<th>( I ) (intensity)</th>
<th>current</th>
</tr>
</thead>
<tbody>
<tr>
<td>optical</td>
<td>1 eV</td>
<td>CLF, ELI, XCELS,...</td>
<td>( 10^{24-25} ) W/cm(^2)</td>
<td>( 10^{22} ) W/cm(^2)</td>
</tr>
<tr>
<td>x-ray</td>
<td>10 keV</td>
<td>LCLS-II, XFEL,...</td>
<td>( 10^{27} ) W/cm(^2) (goal)</td>
<td>( 10^{18} ) W/cm(^2)</td>
</tr>
</tbody>
</table>
Light-Light interaction in the local limit

- In vacuum (i.e. without real charges and currents) the Lagrangian density for the electromagnetic field is given by $\mathcal{L} = (E^2 - B^2)/2$. Accordingly, the field equations are linear (superposition principle):
  \[ \nabla E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla B = 0, \quad \nabla \times B = \frac{\partial E}{\partial t} \]

- In quantum field theory photons couple via virtual electric charges. Effectively, we obtain nonlinear terms in the Lagrangian:

### Euler-Heisenberg Lagrangian density (1936)

\[
\mathcal{L} = \frac{1}{2} (E^2 - B^2) + \frac{2\alpha}{45 E_{cr}^2} \left[ (E^2 - B^2)^2 + 7 (EB)^2 \right] + \ldots
\]

![Diagram showing leading-order contribution to the EH-Lagrangian (local limit)]

This description is applicable if:
- The wave length is much larger than the Compton wavelength
- The field strength is much smaller than the critical field
How to reach the critical field with existing technology

- The laser intensity $I$ is not a Lorentz scalar ($I' \sim \gamma^2 I$, $\gamma = \epsilon/m$)

- Critical intensity $I_{cr} = 4.6 \times 10^{29} \text{ W/cm}^2$ is obtainable in the boosted frame if $\gamma \sim 10^3 - 10^4$ even if $I \lesssim 10^{22} \text{ W/cm}^2$ (optical Petawatt system)

**Electron-Laser interactions**

Electrons with an energy $\epsilon \gtrsim \text{GeV}$ are obtainable via laser-wakefield acceleration

**Light-by-light scattering**

Photons with an energy $\hbar \omega \gamma \gtrsim \text{GeV}$ are obtainable via Compton backscattering

- For very strong fields the simultaneous interaction with several laser photons becomes important – describable using “dressed” states:
How to reach the critical field with existing technology

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<th>$q^\mu$</th>
<th>$p^\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon = 4 \text{ GeV (}\gamma \sim 10^4\text{)}$ W. P. Leemans, et al., PRL 113, 245002 (2014)</td>
<td>$\hbar \omega \gamma = 2.9 \text{ GeV}$ N. Muramatsu, et al., NIMA 737, 184–194 (2014)</td>
</tr>
</tbody>
</table>
Dressed states and the classical intensity parameter

- Dressed states are solutions of the interacting Dirac equation:

\[(i\partial - m) \psi_p = 0, \quad \psi_p = \quad \quad (i\partial - eA - m) \psi_p = 0, \quad \psi_p = \quad \quad\]

\[\quad = + \quad + \quad + \quad + \quad \cdots\]

The dressed propagator/external line includes an arbitrary number of interactions with the classical background field.

- A single interaction with the background scales as \(\sim \xi (\xi = a_0)\)

\[\xi \sim \frac{|e| \sqrt{\langle -A^2 \rangle}}{mc} \sim \frac{|e| E}{mc \omega},\]

Intensity parameter

\[\quad = \frac{p + m}{p^2 - m^2} \sim \frac{1}{m},\]

Free propagator

\[\quad = -ieA \sim |e| \sqrt{-A^2}\]

Coupling vertex

\((E, \omega): \text{field strength and angular frequency of the laser field, respectively})

<table>
<thead>
<tr>
<th>Perturbative regime (\xi \ll 1)</th>
<th>Each coupling suppressed by (\xi^2) (probability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonperturbative regime (\xi \gg 1)</td>
<td>(n)-photon absorption scales as (\xi^{2n})</td>
</tr>
<tr>
<td>Semiclassical regime (\xi \gg 1)</td>
<td>Dressing becomes important</td>
</tr>
<tr>
<td></td>
<td>([I \gtrsim 10^{18} \text{ W/cm}^2 \text{ for optical lasers } (\hbar \omega \sim 1 \text{ eV})])</td>
</tr>
<tr>
<td></td>
<td>Probability amplitude is highly oscillating, classical interpretation of stationary points</td>
</tr>
</tbody>
</table>
Pair production and the quantum nonlinearity parameter

**Sauter-Schwinger effect**

Spontaneous decay of the vacuum

- Sizable if $E \gtrsim E_{cr} = m^2 c^3 / (\hbar |e|)$ (at the QED critical field)
- Probability: $\sim \exp \left( - \pi E_{cr} / E \right)$ (for a pure electric field)

**Breit-Wheeler pair production**

Decay of an incoming photon

- Sizable if $\chi \gtrsim 1$ (critical field reached in the boosted frame)
- Probability: $\sim \exp \left[ -8 / (3 \chi) \right]$ (if $\chi \ll 1$ and $\xi \gg 1$)

Electron-positron photoproduction depends crucially on the quantum nonlinearity parameter

\[
\chi \sim \frac{|e| \hbar}{m^3 c^4} \sqrt{\langle q^\mu F^2_{\mu\nu} q^\nu \rangle} \sim (2\hbar \omega \gamma / mc^2)(E / E_{cr})
\]

[$\hbar \omega \gamma$: energy of the incoming photon; last relation assumes a head-on collision]

- The photon four-momentum is transferred at the vertex
- Pair is produced ultra relativistic, background field is boosted
From a single vertex to a QED cascade

**Photon emission**

\[ q^\mu \rightarrow p^\mu \]

In general an electron can radiate more than only once

**Pair production**

\[ p_1^\mu \rightarrow e^- \]

\[ e^+ \rightarrow \gamma \rightarrow q^\mu \]

The survival probability of a photon can become exponentially small

- The total probability \( P \sim \alpha \xi N \) for the fundamental processes can become very large \( [\alpha \approx 1/137, \ N: \text{number of laser cycles}] \)
- At a certain point processes with many vertices become important
- Starting from a single particle a cascade develops

**Trident pair production**

\[ p_1^\mu \rightarrow e^- \]

\[ p_2^\mu \rightarrow e^+ \]

\[ q^\mu \rightarrow p^\mu \]

Simplest cascade process

**QED cascade**

Exponential increase of particles
From a single vertex to a QED cascade

Photon emission

\[ q^\mu \rightarrow p^\mu \]

In general an electron can radiate more than only once

Pair production

\[ \gamma \rightarrow q^\mu, e^-, e^+ \]

The survival probability of a photon can become exponentially small

- The total probability \( P \sim \alpha \xi N \) for the fundamental processes can become very large \([\alpha \approx 1/137, N: \text{number of laser cycles}]\)
- At a certain point processes with many vertices become important
- Starting from a single particle a cascade develops

Trident pair production

QED cascade

Seminal SLAC E-144 experiment:

\[ \epsilon = 46.6 \text{ GeV} \ (\gamma \sim 10^5), \ h\omega = 2.4 \text{ eV}, \ I \sim 10^{18} \text{ W/cm}^2 \ (\xi \approx 1) \]


PIC approach to QED cascades

**S-matrix approach**
- Ab initio calculation, all effects included, arbitrarily precise
- Only asymptotic probabilities, no description of the dynamics
- Complicated if many vertices must be taken into account

**PIC-approach**
- Separates quantum processes from classical propagation
- Intuitive picture, complicated processes can be considered
- No reliable error estimates, question of applicability

**PIC scheme**

**PIC approach to QED cascades**

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Many recent papers on QED cascades:

- Green and Harvey, CPC (2015)
- Bashmakov, Nerush, Kostyukov, Fedotov, and Narozhny, POP (2014)
- Mironov, Narozhny, and Fedotov, PLA (2014)
- Narozhny and Fedotov, EPJST (2014)
- Ridgers, Kirk, Duclos, Blackburn, Brady, Bennett, Arber, and Bell, JCP (2014)
- Tang, Bake, Wang, and Xie, PRA (2014)

...
**Hierarchy of scales**

**QED processes in strong laser fields**

**Important length scales for plane-wave laser fields**

1. $L = N\lambda_L$: total length of the laser pulse ($N$: number of cycles)
   *Characterizes the space-time volume which contains a strong field*

2. $\lambda_L$: laser wavelength (scale on which the field changes its sign)
   *Determines the highest possible classical energy transfer*

3. $\delta\lambda$: formation region of the basic single vertex QED processes
   *Its relation to $\lambda_L$ determines the qualitative properties of the QED processes*

4. $\lambda_C$: electron/positron Compton wavelength
   *Fundamental length scale of QED (quantum fluctuations become important)*
Intermezzo: classical dynamics in a plane-wave field

Equation of motion

- The electron four-momentum \( P^\mu \) is determined by the Lorentz force:

\[
\frac{dP^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} P_\nu \quad \Rightarrow \quad \frac{dP^\mu(\phi)}{d\phi} = \frac{e}{kP_0} F^{\mu\nu}(\phi) P_\nu(\phi),
\]

\( \tau \): proper time, \( \phi = kx \): laser phase, \( F^{\mu\nu} \): field tensor, \( kP_0 = kP(\phi) \) is conserved

- The position four-vector \( x^\mu \) is obtained by integrating:

\[
x^\mu(\phi) = x^\mu_0 + \int_{\phi_0}^{\phi} d\phi' \frac{P^\mu(\phi')}{kP_0}, \quad \frac{d\phi}{d\tau} = \frac{kP_0}{m}, \quad F^{\mu\nu}(\phi) = \sum_{i=1,2} f_i^{\mu\nu} \psi_i(\phi)
\]

Solution for a plane-wave field

- Result depends only on the integrated field tensor:

\[
P^\mu(\phi) = P^\mu_0 + \frac{e \tilde{\mathcal{S}}^{\mu\nu}(\phi, \phi_0) P_0\nu}{kP_0} + \frac{e^2 \tilde{\mathcal{S}}^{2\mu\nu}(\phi, \phi_0) P_0\nu}{2(kP_0)^2},
\]

\[
\tilde{\mathcal{S}}^{\mu\nu}(\phi, \phi_0) = \int_{\phi_0}^{\phi} d\phi' F^{\mu\nu}(\phi') = \sum_{i=1,2} f_i^{\mu\nu} [\psi_i(\phi) - \psi_i(\phi_0)].
\]
Plane-wave dynamics: qualitative properties

Classical electron four-momentum

\[ P^\mu(\phi) = P_0^\mu + \sum_{i=1,2} \left\{ \frac{e}{kP_0} f^{i\mu\nu} P_0^\nu \left[ \psi_i(\phi) - \psi_i(\phi_0) \right] + k^\mu \frac{m^2}{2kP_0} \xi_i^2 \left[ \psi_i(\phi) - \psi_i(\phi_0) \right]^2 \right\} \]

\( F^{\mu\nu}(\phi) = \sum_{i=1,2} f^{i\mu\nu} \psi'_i(\phi), \quad f^{i\mu\nu} = k^\mu a_i^\nu - k^\nu a_i^\mu, \quad \xi_i = \frac{|e|}{m} \sqrt{-a_i^2}, \quad \xi = \sqrt{\xi_1^2 + \xi_2^2} \)

Normalization: \( |\psi_i(kx)|, |\psi'_i(kx)| \lesssim 1; \) no dc component: \( \psi_i(\pm\infty) = \psi'_i(\pm\infty) = 0 \)

- **Lawson-Woodward theorem:**
  - no net acceleration \([P^\mu(+\infty) = P^\mu(-\infty)]\)
  - Important exception: acceleration of particles created inside the field

- **Momentum and energy scales:**
  - Transverse momentum: linear term, \( \sim m\xi_i \)
  - Energy absorption: quadratic term (ponderomotive force)
  - Absorption inside \( \delta\phi \lesssim 1 \) around \( \phi_0 \): \( k^\mu [m^2/(2kP_0)][\xi_i \psi'_i(\phi_0) \delta\phi]^2 \)

- **Conservation of kP:**
  - \( kP \) is both classically and quantum mechanically conserved
  - Quantum nonlinearity parameter \([\chi_e = (kP_0/m^2)\xi] \) is conserved
  - Inside a plane-wave field a QED cascade stops at a certain point
A plane-wave field depends only on the laser phase $\phi = kx$.

Accordingly, four-momentum is conserved up to a multiple of $k^\mu$:

$$p_1^\mu + p_2^\mu = q^\mu + nk^\mu, \quad n \geq 2m^2/kq$$

(threshold: $p_1^\mu = p_2^\mu$)

Within the (small) formation region $\delta \phi$ the four-momentum $nk^\mu$ with $n \sim (\xi \delta \phi)^2(m^2/kq)$ can be absorbed (classically) from the laser field:

$$(\xi \delta \phi)^2(m^2/kq) \sim m^2/kq \quad \rightarrow \quad \delta \phi \sim 1/\xi$$

Two different regimes can be distinguished:

- **Multiphoton regime**

  $\xi \ll 1$: Large formation region, the process “feels” an oscillatory field

- **Tunneling & classical propagation**

  $\xi \gg 1$: Small formation region, the process happens instantaneously
Formation region: multiphoton vs. tunneling process

- A plane-wave field depends only on the laser phase $\phi = kx$
- Accordingly, four-momentum is conserved up to a multiple of $k^\mu$:
  $$p_1^\mu + p_2^\mu = q^\mu + nk^\mu, \quad n \geq 2m^2/kq \quad \text{(threshold: } p_1^\mu = p_2^\mu)$$
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  $$(\xi \delta \phi)^2 (m^2/kq) \sim m^2/kq \quad \rightarrow \quad \delta \phi \sim 1/\xi$$
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**Multiphoton regime**

- Nonlinear Breit-Wheeler process in the multiphoton regime:
  (important application: x-ray lasers)
- M. J. A. Jansen and C. Müller, PRA 88, 052125 (2013)
Comparison with ionization in atomic physics

- Pair production is similar to ionization in atomic physics.
- The Keldysh parameter distinguishes the two regimes:
  - AP: \( \gamma = \omega \sqrt{2mI_p}/(|e|E) \), SFQED: \( 1/\xi = \omega mc/(|e|E) \)  \( (I_p = 2mc^2) \)
  - \([\omega, E]: \) laser angular frequency/field strength, \( I_p: \) atomic ionization potential
Single vertex processes revisited

Meaning of laser-dressed Feynman diagrams

- Emission of exactly one photon
- Decay into a single $e^+ e^−$-pair

- This interpretation is correct if the total probability remains small
- For large probabilities radiative corrections become important
- The $S$-matrix is unitary $\rightarrow$ optical theorem (cutting rules): 

- The imaginary part of loop diagrams ensures a unitary time evolution

SM and A. Di Piazza, PRL 107, 260401 (2011)
Total pair-creation probability: Numerical results

By combining available optical petawatt lasers with existing GeV gamma sources, the pair-production probability can become very large.

![Graph showing numerical results]

- **Dashed lines:** without wave-function decay
  - \( \xi = 10, 20, 50, 100, 200 \) \((N = 5)\)
  - \( F^{\mu\nu}(x) = f^{\mu\nu} \sin^2[\phi/(2N)] \sin(\phi) \), \( \phi = kx \)

- **Solid:** \( F^{\mu\nu}(x) = f^{\mu\nu} \sin^2[\phi/(2N)] \sin(\phi) \)

- **Dashed:** \( F^{\mu\nu}(x) = f^{\mu\nu} \sin^4[\phi/(2N)] \sin(\phi) \)
  - \( \xi = 100, N = 5, \omega = 1.55 \text{ eV} \)

**Problem:** For certain values of \( \chi, \xi \) the evaluation of the leading-order Feynman diagram violates unitarity

**Solution:** The back-reaction of the decay on the photon wave function must be taken into account

Radiative corrections: exact wave functions

**Photon**

\[ = + + + \cdots \]

- a) Exact photon wave function (includes radiative corrections)

\[ = + + + + \cdots \]

- b) Polarization operator (all one-particle irreducible diagrams)

**Electron**

\[ = + + + \cdots \]

- c) Exact electron wave function (includes radiative corrections)

\[ = + + + + \cdots \]

- d) Mass operator (all one-particle irreducible diagrams)

Exact wave functions obey the Schwinger-Dyson equations, e.g.,

\[ - \partial^2 \Phi_{q}^{\text{in}\mu}(x) = \int \! d^4 y \ P^{\mu\nu}(x, y) \Phi_{q\nu}^{\text{in}}(y), \quad \Phi_{q}^{\text{in}\mu}(x) : \text{incoming photon} \]
Furry-picture approach to strong-field QED:

- Strong background fields ($\xi \gtrsim 1$) are included exactly (dressed states)
- The radiation field (non-occupied modes) is treated perturbatively
  $\rightarrow$ QED becomes a nonperturbative theory (like QCD?) for $\alpha \chi^{2/3} \gtrsim 1$

Full breakdown of perturbation theory

\[
\begin{align*}
\text{Mass operator: perturbation theory with respect to the radiation field}
\end{align*}
\]

Different regimes for strong background fields ($\xi \gg 1$):

1. $\chi \ll 1$: classical regime
   Quantum effects are very small, pair production is exponentially suppressed
2. $\chi \gtrsim 1$, $\alpha \chi^{2/3} \ll 1$: quantum regime
   Recoil and pair production are important, but the radiation field is a perturbation
3. $\alpha \chi^{2/3} \gtrsim 1$: fully nonperturbative regime
   Perturbative treatment of the radiation field breaks down

V. I. Ritus, J. Sov. Laser Res. 6, 497–617 (1985)
Second part: nonlinear Breit-Wheeler process

- Semiclassical description
  - Classical interpretation of the stationary points
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation

- Numerical results
  - Momentum distribution of the created pairs
  - Importance of interference effects

More details can be found in:

Nonlinear Breit-Wheeler process

Leading-order Feynman diagram

- Photon: four-momentum $q^\mu \ (q^2 = 0)$
- Electron: four-momentum $p_1^\mu \ (p^2 = m^2)$
- Positron: four-momentum $p_2^\mu \ (p'^2 = m^2)$

(we do not introduce dressed momenta!)

Semiclassical approximation

- We assume a strong plane-wave laser pulse ($\xi \gg 1$)
- The $S$-matrix is solvable analytically (to leading order)
- Stationary-phase analysis: main contribution to the process at $\phi = \phi_k$
- We propagate the final momenta back in time $p_{1,2}^\mu \longrightarrow p_{1,2}^\mu(\phi)$

$$p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu$$

- At the stationary phases $\phi_k \ n(\phi) > 0$ is minimal
- Process happens where the pair becomes real as easy as possible!

### Classical vs. quantum absorption

#### Global conservation law
\[ p_1^\mu + p_2^\mu = q^\mu + nk^\mu \]

#### Local conservation law
\[ p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu \]

#### Classical absorption
\[ n_{cl}k^\mu = p_1^\mu + p_2^\mu - [p_1^\mu(\phi_k) + p_2^\mu(\phi_k)] \]
Propagation from the stationary point

#### Quantum absorption
\[ n_{q}k^\mu = p_1^\mu(\phi_k) + p_2^\mu(\phi_k) - q^\mu \]
Absorption during the creation

- Pair production at \( \phi \): \( n(\phi)k^\mu \) must be absorbed “non-classically”
  \[ \rightarrow n(\phi)k^\mu \] is a measure for the effective tunneling distance
- Stationary-phase condition obeyed at \( \phi = \phi_k \):
  \[ \rightarrow n(\phi_k) \]: minimum laser four-momentum needed to be on shell

### Implications for the QED-PIC community
- We obtain the scaling laws: \( n_{q} \sim \xi/\chi \) and \( n_{cl} \sim \xi^3/\chi \), respectively
- The energy transver from the laser to the particles is dominated by classical physics (taken into account self-consistently in a PIC code)
- The quantum absorption is not taken into account in a PIC code
  \[ \rightarrow \] We have a definite error estimate now!
Four-momenta in the canonical light-cone basis

Characteristic four-vectors of the problem

- The Breit-Wheeler process is characterized by the quantities:
  \[ q^\mu, \ k^\mu, \ f_{1}^{\mu \nu}, f_{2}^{\mu \nu} \]
  \[ \gamma \text{ photon} \]
  \[ \text{laser photons} \]
  \[ \text{laser polarizations} \]

- They allow us to construct a canonical light-cone basis:
  \[ k^\mu, \ \bar{k}^\mu = q^\mu / kq, \ e_1^\mu = \Lambda_1^\mu, \ e_2^\mu = \Lambda_2^\mu, \ (q^2 = 0, \ kq \neq 0, \ \Lambda_i^2 = -1) \]

Invariant momentum parameters

- We define the Lorentz-invariant momentum parameters \( R, t_1 \) and \( t_2 \):
  \[ p_1^\mu = (1/2 + R)q^\mu + s' k^\mu + t_1 m\Lambda_1^\mu + t_2 m\Lambda_2^\mu, \quad p_1^2 = m^2, \]
  \[ p_2^\mu = (1/2 - R)q^\mu - s k^\mu - t_1 m\Lambda_1^\mu - t_2 m\Lambda_2^\mu, \quad p_2^2 = m^2 \]

- From the on-shell conditions we obtain the relations \( n = s' - s \):
  \[ s = \frac{1}{(2R - 1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2), \quad s' = \frac{1}{(2R + 1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2) \]
To include quantum processes into a PIC code, the initial conditions for the classical propagation of the created particles must be known.

Approach so far:
- Ignore the transverse degree of freedom
- All particles move initially into the forward direction

From first principles:
- We need to provide initial values for $R$, $t_1$ and $t_2$
- Constant-crossed field rate: distribution for $R$ and $t_2$

Question: which initial value for $t_1$? Our answer: $t_1 = 0$

Both $R$ and $t_2$ are constants of motion for a plane-wave (constant-crossed) field.

The corresponding distributions are not changed by the classical propagation.

Parameters:

\[ \chi = 1, \; \xi = 10, \; N = 5, \; \phi_0 = \pi/2 \]
The extend of the spectrum is determined by classical physics
- \( t_1 \sim \xi \) (classical acceleration)
- \( t_2 \sim 1 \) (quantum distribution)

Parameters:
\[ N = 5, \ \phi_0 = \pi/2, \ \chi = 1, \ \xi_1 = \xi = 20, 10, 5 \ (a,b,c) \]

The spectrum exhibits a strong CEP-dependence [K. Krajewska et al, PRA 86, 052104 (2012)]
Can be completely understood from classical electrodynamics

Parameters:
\[ N = 2, \ \chi = 1, \ \phi_0 = 0/\pi, \ \xi = 5/10 \]

Laser pulse shape: \( \psi'_1(\phi) = \sin^2[\phi/(2N)] \sin(\phi + \phi_0), \psi'_2(\phi) = 0 \) (linear polarization)
Semiclassical approximation

Total/Differential probability

\[ W(q, \epsilon) = \frac{m^2}{(kq)^2} \sum_{\text{spin}} \int_{-1/2}^{+1/2} dR \int_{-\infty}^{+\infty} dt_1 dt_2 \frac{w}{8} \frac{1}{(2\pi)^3} |\mathcal{M}(p_1, p_2; q)|^2 \]

\[ w = 4/(1 - 4R^2) \]

- Reduced S-matrix element: \( i\mathcal{M}(p_1, p_2; q) = \epsilon_{\mu} \bar{u}_{p_1} G^\mu(p_1, q, -p_2) v_{p_2} \)

\[ G^\rho = (-ie) \left\{ \gamma_{\mu} \left[ G_0 g^{\mu\rho} + \sum_{j=1,2} \left( G_1 \mathcal{S}_{j,1} f_{j}^{\mu\rho} + G_2 \mathcal{S}_{j,2} f_{j}^{2\mu\rho} \right) \right] + i\gamma_{\mu}\gamma^5 \sum_{j=1,2} G_3 \mathcal{S}_{j,1} f_{j}^{*\mu\rho} \right\}, \]

- The nontrivial information is contained in the master integrals

\[ \mathcal{S}_0 = \int_{-\infty}^{+\infty} d\phi \ e^{i\tilde{S}_\Gamma(t_1, t_2; \phi)}, \quad \mathcal{S}_{j,l} = \int_{-\infty}^{+\infty} d\phi \ [\psi_j(\phi)]' e^{i\tilde{S}_\Gamma(t_1, t_2; \phi)}. \]

- For strong fields \((\xi \gg 1)\): stationary-phase approximation

\[ \mathcal{S}_0 \approx \frac{kq}{m^2 w} \left[ \frac{w/2}{|\chi(\phi_k)|} \right]^{2/3} 2\pi \text{Ai}(\rho) e^{i\tilde{S}_\Gamma(\phi_k)}, \quad \rho = \left\{ \frac{w}{[2 |\chi(\phi_k)|]} \right\}^{2/3} (1 + t_2^2) \]

- The Airy functions typical for processes inside constant-crossed fields are obtained but also a phase factor (interference effects!)
Stationary-phase approximation possible for $\xi \gg 1$

Location of the stationary points: classical equation of motion

Probability amplitude: pair-creation inside a constant-crossed field

However: interference between different formation regions important

$\chi = 1, \xi = 5, N = 5$ cycle, e.g., $\omega_\gamma = 17$ GeV and $10^{20}$ W/cm$^2$

Stationary-phase approximation possible for $\xi \ll 1$

Local constant-crossed field approximation

Probability amplitude: pair-creation inside a constant-crossed field

However: interference between different formation regions important

$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17$ GeV and $10^{20}$ W/cm$^2$

Applied on the probability level, the local constant-crossed field approximation cannot reproduce the substructure!

This was observed for Compton scattering in:

Harvey, Ilderton, King, PRA 91 013822 (2015)
Summary: main topics of the talk

- Why should we study Strong-Field QED?
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
- Lasers as a tool to study the critical field
  - Nonlinear Compton scattering
  - Nonlinear Breit-Wheeler pair production
- From a single vertex to a QED cascade
  - QED-PIC approach
  - Formation region and hierarchy of scales
- Radiative corrections
  - Quantum dressing: exact wave functions
  - Fully nonperturbative regime of QED
- Nonlinear Breit-Wheeler process
  - Semiclassical description
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
  - Momentum distribution of the created pairs
  - Importance of interference effects

Thank you for your attention and your questions!