M3D-K simulations of Linear & Nonlinear Fishbone Dynamics in STs and Tokamaks

Feng Wang, G. Y. Fu

PPPL

SciDAC Center for Nonlinear Simulation of Energetic Particles in Burning Plasmas (CSEP)

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Outline

1. Background of fishbone.
2. M3D-K model.
3. Fishbone linear instability and nonlinear dynamics with toroidal rotation in NSTX.
4. Simulations of passing particles driven Fishbone mode in HL-2A.
Fishbone in PDX

- With near perpendicular NBI, a strong MHD activity induces a significant loss of fast ions (20%~40%). The characteristic signals on the Mirnov coils are similar to the bones of a fish.

- The oscillation frequency is ~10kHz, and has a burst structure. The dominated mode number is m/n=1/1.

- L. Chen (1984 PRL) showed the fishbone events can be excited by energetic trapped particles through the resonance between mode and particles’ toroidal precession drift frequency.

K. McGuire (1983 PRL)
Fishbone activities in Tokamaks and STs

• PBX: **NBI tangential** (W. Heidbrink, PRL, 1986).

• DIII-D: **NBI between perpendicular and tangential**, (W. Heidbrink, Nucl. Fusion, 1990).

• JET: NBI or **ICRF heating** (Isotropic).


• NSTX: almost tangential but still has some part of trapped particles, Spherical Torus, (E. Fredrickson, Nucl. Fusion, 2003).

• HL-2A: tangential NBI.
Fast ion-driven Fishbone instability predicated by theory

Fishbone driven by trapped particles:
- Trapped particles driven through resonance with energetic particles toroidal precession drift frequency (L. Chen PRL 1984):
  \[ \omega \approx \omega_{dh} \]
- Trapped particles driven fishbone with mode frequency close to thermal ion diamagnetic frequency (B. Coppi, PRL, 1986):
  \[ \omega = \omega_{*i} \]

Fishbone driven by passing particles:
- Passing particles-driven fishbone with resonance condition (R. Betti PRL 1993) (low frequency branch):
  \[ \omega = \omega_{*i} \approx \omega_\phi - \omega_\theta \]
- Fishbone driven by passing particles with resonance condition (S.J. Wang, PRL 2001) (high frequency branch):
  \[ \omega = \omega_\phi \]
Motivation

• Fishbone is one of the most important energetic particle driven mode (EPM) in tokamak plasmas. It has global mode structure, can interact with thermal plasma strongly: affect equilibrium, stability and transport. It is a key element for understanding and controlling burning plasmas.

• Despite lots of previous work, there are still experimental observations that are not well understood, especially concerning fishbone nonlinear dynamics.

• M3D-K simulations of beam-driven modes in NSTX are carried out for code validation and physics understanding.

• Alpha particle driven fishbone instability is possible in ITER.
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M3D-K is a global nonlinear kinetic/MHD hybrid simulation code for toroidal plasmas


\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

\[
\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v} - \nabla \cdot \mathbf{P}_h
\]

\[
\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \left( \kappa \cdot \nabla \frac{p}{\rho} \right)
\]

\[
\mathbf{P}_h = P_{\perp} \mathbf{I} + (P_{\parallel} - P_{\perp}) \mathbf{b} \mathbf{b}
\]

\[
P_{\parallel}(x) = \int M v_{\parallel}^2 \delta(x - x - \rho_0) F(x, v_{\parallel}, \mu) d^3 x dv_{\parallel} d\mu d\theta
\]

\[
P_{\perp}(x) = \int \frac{1}{2} M v_{\parallel}^2 \delta(x - x - \rho_0) F(x, v_{\parallel}, \mu) d^3 x dv_{\parallel} d\mu d\theta
\]

\[
F = F(x, v_{\parallel}, \mu) = \sum_i \delta(X - X_i) \delta(v_{\parallel} - v_{\parallel,i}) \delta(\mu - \mu_i)
\]

\[
\frac{d\mathbf{X}}{dt} = \frac{1}{B^{**}} v_{\parallel} \left[ v_{\parallel} \left( \mathbf{B}^{**} - \mathbf{b}_0 \times \left( \langle \mathbf{E} \rangle - \frac{1}{e} \mu \nabla (B_0 \langle \delta B \rangle) \right) \right) \right]
\]

\[
\frac{d\mathbf{v}_{\parallel}}{dt} = \frac{e}{B^{**}} \mathbf{B}^{**} \cdot \left( \langle \mathbf{E} \rangle - \frac{1}{e} \mu \nabla (B_0 - \langle \delta B \rangle) \right)
\]

\[
\dot{\mu} = 0
\]

- The energetic particle stress tensor, \( P_h \), is calculated using drift kinetic or gyro-kinetic equation via PIC.
- Mode structures are evolved self-consistently including non-perturbative effects of energetic particles.
- Include plasma rotation.

G.Y. Fu et al, PHYSICS OF PLASMAS 13, 052517 (2006)
Self-consistent simulations of fishbone by M3D-K

• G.Y. Fu POP 2006 trapped particle driven fishbone. Trapped particles driven fishbone in tokamaks.

• F. Wang POP 2013, fishbone simulation in NSTX. Fishbone in spherical tori with weak reversed $q$ profile and $q_{\text{min}}$ above 1.

• W. Shen POP 2015, fishbone-like mode simulation in DIII-D. Fishbone-like, lower frequency mode with between perpendicular and tangential NBI.
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Beam-driven fishbones are routinely observed in NSTX.
Equilibrium profiles and parameters

NSTX #124379 at t=0.635s

$B_0=0.44T$, $R=0.86m$, $a=0.60m$

$n_e(r=0)=9.3 \times 10^{13} \text{ cm}^{-3}$

$\beta_{\text{tot}}(r=0)=30\%$

Analytic fast ion distribution (slowing down) with center pitch

$$\lambda \equiv \frac{v_{\parallel}}{v} \simeq 0.6$$
Previous work:

Linear stability and nonlinear dynamics of the fishbone mode in spherical tokamaks

• We consider NSTX plasmas with a weakly reversed q profile and $q_{\text{min}}$ close but above unity. For such q profile, fishbone and non-resonant kink mode (NRK) have been observed in NSTX and MAST.

• M3D-K simulation results show that both NRK and fishbone can be unstable in such profile. A fishbone instability preferentially excited at higher $q_{\text{min}}$, which consistent with the observed appearance of the fishbone before the “long-lived mode” in MAST and NSTX experiments.

• Nonlinear simulations show that an m/n=2/1 magnetic island is found to be driven by the fishbone instability, which could provide a trigger for the NTM.

New results in this work

• Effects of toroidal rotation on linear stability.

• Particles nonlinear phase space dynamics, frequency chirping.
Rotation effect is destabilizing for fishbone at higher $q_{\text{min}}$. 

\[ \beta_t = 0.395 \text{ without rotation} \]

\[ \beta_t = 0.395 \text{ with rotation} \]

\[ \frac{\beta_h}{\beta_t} = 0.35 \]
The mode structure is different at low $q_{\text{min}}$ and high $q_{\text{min}}$. At high $q_{\text{min}}$, the m/n=2/1 component becomes more important, and the mode has strong ballooning feature.
Passing and trapped linear resonant particles in phase space

The main resonance for passing particles is:

$$\omega = \omega_\phi + \omega_\theta$$

$\omega_\theta$ and $\omega_\phi$ are poloidal and toroidal transit frequency.

For trapped particles, the main resonance condition is:

$$\omega = \omega_d$$

where $\omega_d$ is the toroidal precession drift frequency.

$q_{\text{min}} = 1.321, \beta_h/\beta_t = 0.2$ and $\mu = [7.2, 7.3]$ ($E$, $P_\phi$ and $\mu$ are in code units)
Both passing and trapped particles have contribution to drive the mode.
Mode structure broaden at low field side nonlinearly.
The key difference between passing and trapped particles: trapped particles drift frequency decreases as $P_\Phi$ increases, while passing particle resonant frequency decreases.

Unperturbed trapped and passing resonance particles and near resonance particles:
Nonlinear dynamic of trapped particles with initial frequency close to the linear mode frequency: almost all of those particles stay in resonance as frequency chirps down.
Nonlinear dynamic of trapped particles with initial frequency (70%) less than the linear mode frequency: most of those particles become resonant particles.

\[ \sim 80\% \text{ particles become resonant particles} \]
Nonlinear dynamic of trapped particles with initial frequency larger (130%) than the linear mode frequency: some of those particles become resonant particles.

\[ \sim 30\% \text{ particles turn into resonance} \]
Nonlinear dynamic of passing particles with initial frequency close to the linear mode frequency: some of those particles stay in resonance, and they may also break from resonance.

~30% particles keeping in resonance
Trapped particles keeping resonance more easily than passing particles, most particles with initial frequency lower than mode linear frequency become resonant non-linearly.

\[ \kappa \equiv \frac{\text{Resonant particle number}}{\text{Total particle number}} \]
Particles with initial frequency lower than mode linear frequency can contribute more energy than linear resonant particles in nonlinear phase.

Initial mode frequency  Mode frequency at the end
The wave particle trapping is in adiabatic regime ($\alpha << 1$)

$$\alpha \equiv \frac{d\omega}{\omega_b^2 dt}$$
The distribution function becomes flat around the resonant region. Flatting region increases as the mode frequency chirping down.

Flatting region increases with resonant particles keep moving out. The trapped particle induce redistribution has a structure like a hole, while the resonant island in phase space is wide, comparable with the hole shift distance.
The distribution function become flattened around the resonant region, and as the mode frequency chirping down, trapped particles transport from the core to the edge.
The redistribution induced by a large growth rate case: \((\gamma/\omega=0.15)\) vs. the low growth rate case: \((\gamma/\omega=0.037)\)
Conclusions

• Rotation effect is destabilizing for fishbone at higher and lower $q_{\text{min}}$.
• Linearly, passing particles are important to drive fishbone mode.
• The fishbone nonlinear chirping is mainly due to the trapped resonant particles moving outward radially while keeping resonance with the mode.
• Due to the frequency profile in space, passing particles are difficult to keep in resonance nonlinearly.
• Nonlinearly, as the mode frequency chirping down, linearly non-resonant particles could turn into resonance. This additional factor plays an important role to sustain the mode nonlinearly.
• The phase space island is large in $P_\Phi$, and induces a significant flattening region in the distribution function. As a result, the hole-clump structure is not clear.
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Fishbone modes in HL-2A

Liming Yu, SWIP
Fishbone is excited just before sawtooth crash in HL-2A

Discharge 22485, t=600~650
f(fishbone)=25kHz
in plasma frame, the initial frequency of fishbone: f(fishbone)~17kHz
HL-2A Discharge #22485, t=600~650 parameters

- $B_t = 1.34T$
- $N_e = 1.31 \times 10^{13} \text{ cm}^{-3}$ (Deuterium plasma)
- $T_i = 1750 \text{ eV}$
- $T_e = 810 \text{ eV}$
- $\beta = 0.78\%$
- $R_0 = 1.65$
- $V_a = 5.7070 \times 10^6 \text{ m/s}$
- $f_0 = 549 \text{ KHz}$
- $T(\text{se})$ slowing down time $\sim 50 \text{ ms}$
- Global energy confinement time $\sim 50 \text{ ms}$
- NBI injection energy 40KeV
- Pitch angle parameter $\sim 0.28$
Passing particle driven Fishbone mode in HL-2A

With the NBI injection pitch angle $v_\perp^2/v^2 \approx 0.28$, fast ions are mainly passing particles, which may correspond to passing particles driven fishbone in either low frequency branch: (Betti, 1993 PRL), or high frequency branch (S.J. Wang, PRL 2001):

- **Low frequency branch**, the mode frequency is given by:

  \[ \omega = \omega_{*i} \simeq 5kH \omega \ll \omega_{fishbone} \simeq 17kH \]

- **High frequency branch**

  \[ \omega \simeq \omega_{\phi,0} \simeq 170kH \gg \omega_{fishbone} \simeq 17kH \]

- A new low frequency branch:

  \[ \omega_{fishbone} \simeq \omega_{\phi} - \omega_\theta \simeq (1 - \frac{1}{q})\omega_{\phi} \simeq 17kH \]
M3D-K simulation results

$\omega \sim 0.147 \sim 80\text{kHz} \gg 17\text{kHz}$ in experiential.
Resonant particles in M3D-K results

- The general resonance condition for wave particles

\[ \omega = \omega_\phi + P \omega_\theta, \quad P = -1, 0, 1 \]

- Only \( P=0 \) can be satisfied the resonance condition. The resonance particles’ energy \( \sim 7.5 \text{KEV} \).

- It is corresponding to the high frequency branch fishbone.
Fishbone stability dependences on fast ion orbit width and velocity

\[ \gamma \propto \rho_h \]

\[ \omega \sim const \]

\[ \omega \propto v_h \]
Summary

• The fishbone in HL-2A may refer to a new branch of passing particles driven fishbone.

• The fishbone in M3D-K is corresponding to high frequency branch fishbone, and numerical results show that fast ions’ orbit width and injection velocity has significant effects on the mode instability.

• Fishbone instability is sensitive to the q profile and fast ion distribution function, to better understand the fishbone in HL-2A, we need some more on the q profile and fast ion distribution function effects on fishbone.