Statistical analysis of turbulent transport for flux driven toroidal plasmas – work in progress

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Outline

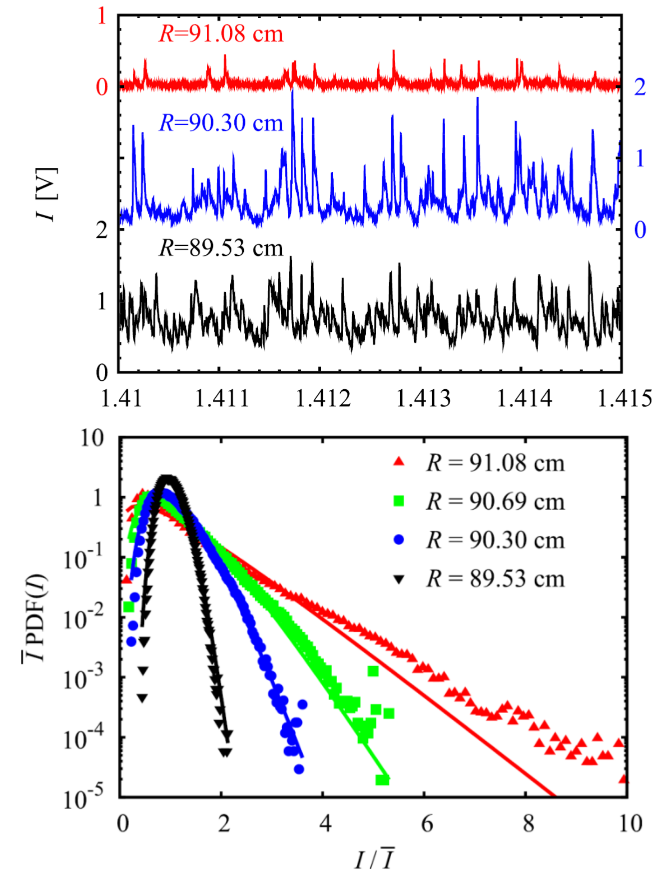
- Goals and motivation
- Introduction to intermittency and Probability Density Functions
- Theory overview – Instantons and coherent structures
- Results and recent developments: Comparison Local - Global.
- Summary

Goals and motivation

- Goal: Investigate the statistical properties of intermittent transport in turbulent plasmas.
- The PDF tails are found to be qualitatively and quantitatively different from Gaussian distributions.
- Motivation: There is theoretical and experimental evidence that for understanding transport (involving many scales and amplitudes) a probabilistic description is needed.
- We will compare data from numerical simulations and with predicted PDFs.
- In particular, we are interested in using AutoRegressive Integrated Moving Average (ARIMA) to separate noise and oscillatory trends present in the numerical data. The noise data can be modeled theoretically. Previously, only treatment of the full signal has been done.

Probability distribution function

- PDF tail – rare events, but large amplitude (e.g. large heat load on the wall.) – In this case blobs.
- Near the center, the PDF is often close to Gaussian but reveals a significant deviation from Gaussianity at the tails (intermittency - the events contributing to the tails are strongly non-linear.).
- Rather than a transport coefficient, a flux PDF is required in order to substantially characterize the transport process.



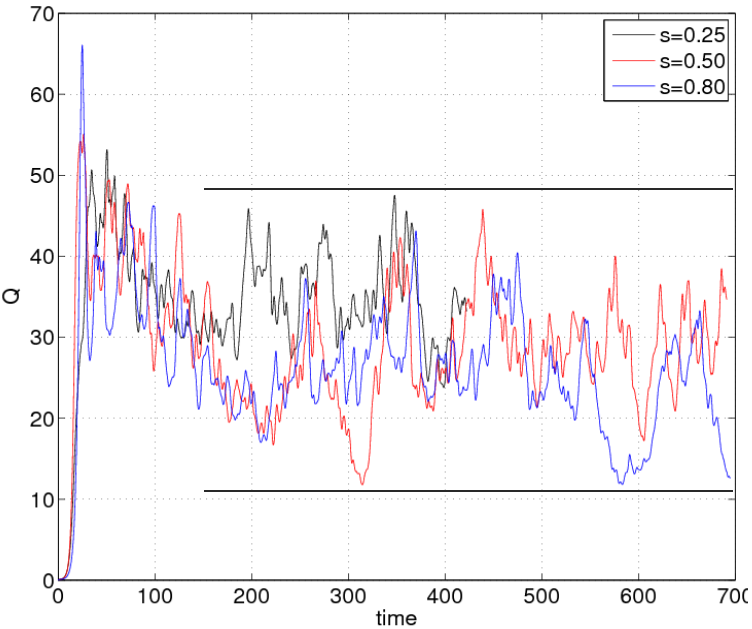
Garcia PoP 2013

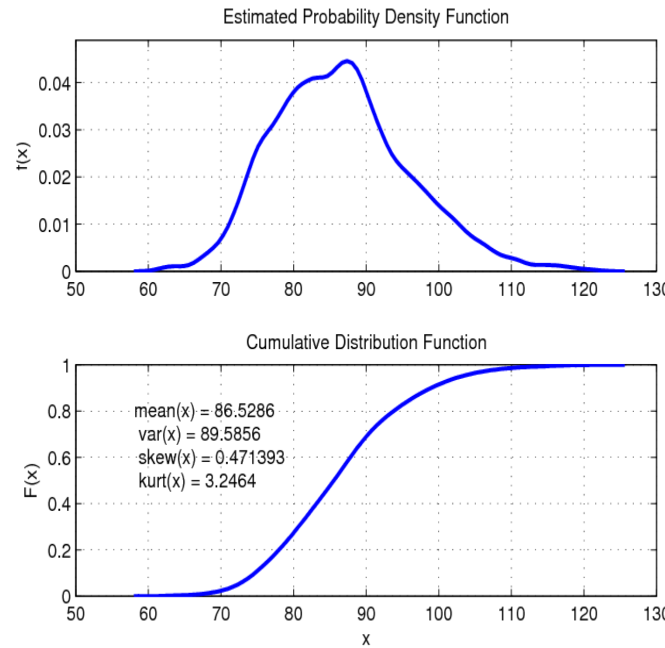
$$P(Z) \propto \exp(-\xi\langle Z\rangle)$$

Garcia PRL 2012

Falcovich PRE 2011

Statistical analysis of time traces





Divide Q in sufficiently small regions and produce a histogram. (Kurtosis(Gaussian) = 3!)

$$skewness = \frac{m_3}{m_2^{3/2}} = \frac{\sum (x_i - x)^3 / n}{(\sum (x_i - x)^2 / n)^{3/2}}$$

Time traces exhibit bursts (rare events with high Amplitude – Intermittent character.)

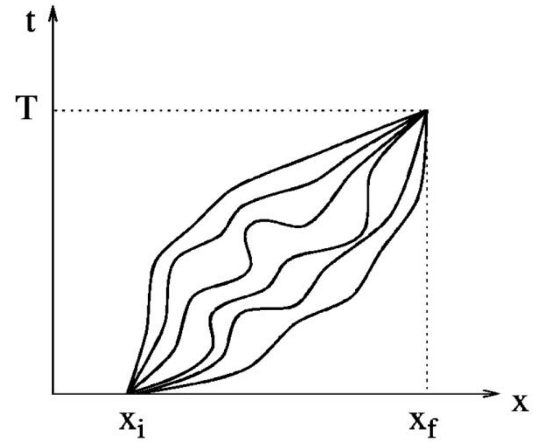
$$kurtosis = \frac{m_4}{m_2^2} = \frac{\sum (x_i - x)^4 / n}{(\sum (x_i - x)^2 / n)^2}$$

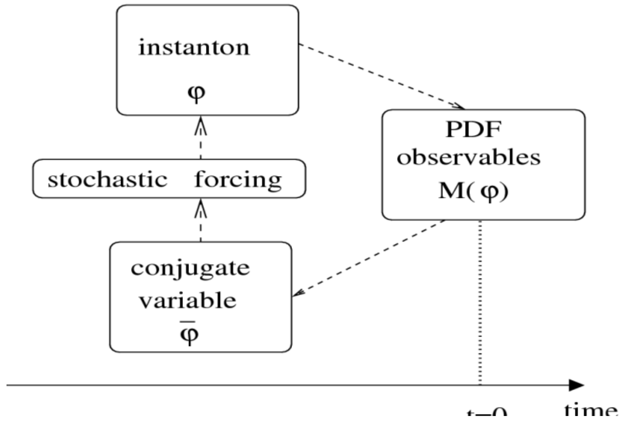
Anderson and Xanthopolous Phys. Plasmas 2010

Instanton method

Kim and Anderson PoP (2008)

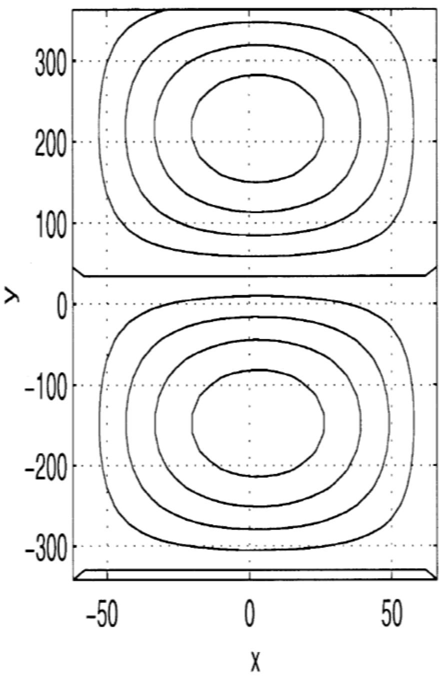
- The instanton method is a non-perturbative way of calculating the Probability Distribution Function tails.
- The PDF tail is viewed as the transition amplitude from a state with no fluid motion to a final state governed by the coherent structure.
- The creation of the coherent structure is associated with the bursty event.
- The optimum path is found by using the saddle-point method.

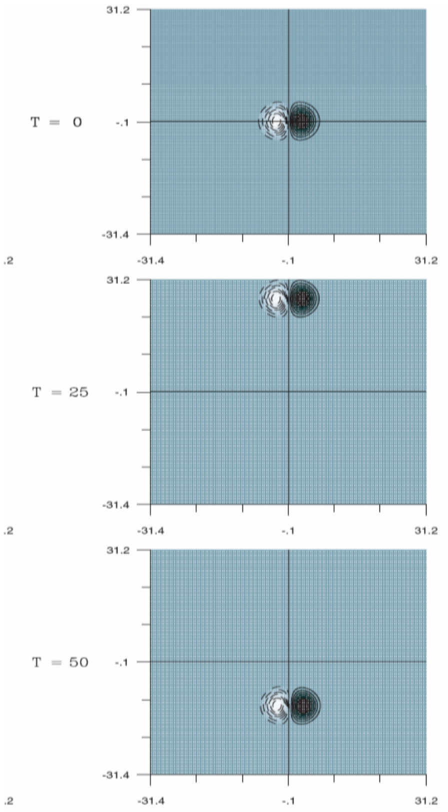




Coherent structures

- Coherent structures are major players in transport dynamics through the formation of avalanche-like events with large amplitude.
- There are several examples of coherent structures (c.f. modon or bipolar vortex soliton) to the non-linear governing equations.
- Strong theoretical evidence that a probabilistic formulation is needed to characterize the problem.





Left:Dastgeer IEEE TPS 2003, Right: Waelbroeck et al PPCF 46 1331 (2004)

Previous work

Modon and Coherent structures

- Modon solution in Rossby wave turbulence Larichev et al Dokl. Akad. Nauk SSSR 231, 1077 (1976).
- ITG modon solution Shukla et al Phys. Lett. A 136, 59 (1989) and Hong et al Phys. Fluids B 3, 615 (1991).
- ITG Modon stability, coherent structures and invariants Waelbroeck et al PPCF 46, 1331 (2004).

PDF tails using path-integral formulation

- Burgers turbulence (bi-fractal turbulence with ramps and shocks) Gurarie et al Phys. Rev. E 54, 4908 (1996).
- Hasegawa-Mima turbulence for local Reynolds stress Kim et al Phys. Rev. Lett. 88 225002 (2002) and PoP 9, 71 (2002).
- ITG mode turbulence for local Reynolds stress Kim et al, NF 43, 961 (2003).

PDF tail of a general moment using the instanton method

The PDF tails of moment (m) and with the order of the highest non-linear interaction term (n) Kim & Anderson PoP 2008.

$$\frac{\partial \phi}{\partial t} + K\phi^n = f \qquad P(Z) \propto \exp(-\xi Z^s) \qquad s = \frac{n+1}{m}$$

Examples:

1. Linear system with PDF tails of first moment (n,ϕ) – Gaussian s=2.
2. Linear system with PDF tails of flux (n*v) – s=1
3. Hasegawa–Mima system with PDF tails of momentum flux – s=3/2 (Kim et al 2002, confirmed in experiments at CSDX by Yan et al 2007)
4. Burgers turbulence with velocity differences – s=3 (Cheklov 1995, Gurarie 1996, Balkovsky 1997,)

The PDF tails can be calculated provided that the integral mean value over the considered coherent structure is non-zero. A coherent structure For the HM system is the modon. Kim and Anderson 2008

Physics model for the ions with adiabatic electrons

Ion continuity and energy equation

$$\frac{\partial\tilde{\phi}}{\partial t}-\left(\frac{\partial}{\partial t}-\alpha_i\frac{\partial}{\partial y}\right)\nabla_\perp^2\tilde{\phi}+\frac{\partial\tilde{\phi}}{\partial y}-\varepsilon_n g_i\frac{\partial}{\partial y}(\tilde{\phi}+\tau(\tilde{\phi}+\tilde{T}_i))=\left[\tilde{\phi},\nabla_\perp^2\tilde{\phi}\right]$$

$$+\tau\left[\tilde{\phi},\nabla_\perp^2(\tilde{\phi}+T_i)\right]$$

$$\frac{\partial\tilde{T}_i}{\partial t}-\frac{5}{3}\varepsilon_n g_i\frac{\partial\tilde{T}_i}{\partial y}+\left(\eta_i-\frac{2}{3}\right)\frac{\partial\tilde{\phi}}{\partial y}-\frac{2}{3}\frac{\partial\tilde{\phi}}{\partial t}=-\left[\tilde{\phi},T_i\right]$$

Anderson et al PoP 9, 4500 (2002) and Anderson and Xanthopoulos 2010

$$\varepsilon_n=2L_n/R,\quad \alpha_i=\tau(1+\eta_i),\ \eta_\mathrm{i}=L_n/L_{Ti},\quad \tau=T_i/T_e$$

$$\tilde{\phi}=(L_n/\rho_s)e\delta\phi/T_e,\ \tilde{T}_i=(L_n/\rho_s)\delta T_i/T_0$$

The model for calculating the PDF tails

The PDF for global Reynolds stress can be defined as:

$$P(R) = \langle \delta(\langle v_x v_y \rangle - R) \rangle = \int d\lambda e^{i\lambda R} \langle e^{-i\lambda v_x v_y} \rangle = \int d\lambda e^{i\lambda R} I_\lambda$$

For details: Zinn-Justin, QFT and Critical Phenomena

The integrand can be re-written as a path-integral:

$$I_\lambda = \int D\phi D\bar{\phi} e^{-S_\lambda}$$

Here, the effective action can be written:

$$S_\lambda = -i \int d^2 x dt \left(\frac{\partial \phi}{\partial t} - \left(\frac{\partial}{\partial t} - \alpha_i \frac{\partial}{\partial y} \right) \nabla^2 \phi + (1 - \varepsilon_n g_i \beta) \frac{\partial \phi}{\partial y} - \beta [\phi, \nabla^2 \phi] \right)$$

$$+ \frac{1}{2} \int d^2 x d^2 x' \bar{\phi}(x) \kappa(x - x') \bar{\phi}(x')$$

$$+ i\lambda \int d^2 x dt \left(-\frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \right) \delta(t)$$

The forcing κ is a Gaussian with a delta-correlation in time and the ion temperatur effects are included in the factor β (using a linear relation ship between ϕ and T_i).

Instanton (saddle-point) solutions

- The path-integral can be solved in the large λ limit using the saddle-point method.
- Assuming that the modon is the coherent structure that is mostly contributing the intermittent state.

$$\phi(x,y,t) = \psi(x,y-Ut)F(t)$$
$$\psi(x,y-Ut) = c_1 J_1(kr)(\cos\theta + \varepsilon\sin\theta) + c_2 K_1(kr)\cos\theta + \alpha r\cos\theta$$

$$\frac{\delta S_\lambda}{\delta F} = 0$$
$$\frac{\delta S_\lambda}{\delta \overline{F}} = 0$$

The function F is associated with the instanton time dependency. F=0 at the initial point (a long time ago) and F $\neq$ 0 at t=0 (now). The action is expressed in the modon solution and the variations of F and the conjugate variables to F are computed.

The analytically predicted PDF tails of heat flux from the two-fluid model

Using the instanton method (on the two fluid model for ϕ and T_i) we only predict the right tail whereas here we assume that the PDF is symmetric.

$$P(Q) = \frac{N}{2b} \exp\{-\frac{1}{b}(\frac{Q-\mu}{Q_0})^{3/2}\}$$

The same exponent has been found in experiments at CSDX by Yan et al. (2007)

This gives the probability of having a normalized heat flux Q.

$$b = b_0\left(\frac{R}{L_n} + 2\langle g_i\rangle\beta - U - \langle k_\perp^2\rangle\left(U + \frac{R}{L_{Ti}}\right)\right)$$

$$\langle f\rangle = \frac{\int \phi\, f\, \phi d\theta}{\int \phi^2 d\theta} \quad \text{with } \phi(\theta) \text{ taken from GENE}$$

$$\beta = 2 + \frac{2}{3}\frac{R/L_n - U}{U + 10/3\,\tau\langle g_i\rangle}$$

$$\langle g_i\rangle = A + Bs \qquad \langle k_\perp^2\rangle = k_y^2(1 + Cs^2) \qquad s = \frac{r}{q}\frac{dq}{dr}$$

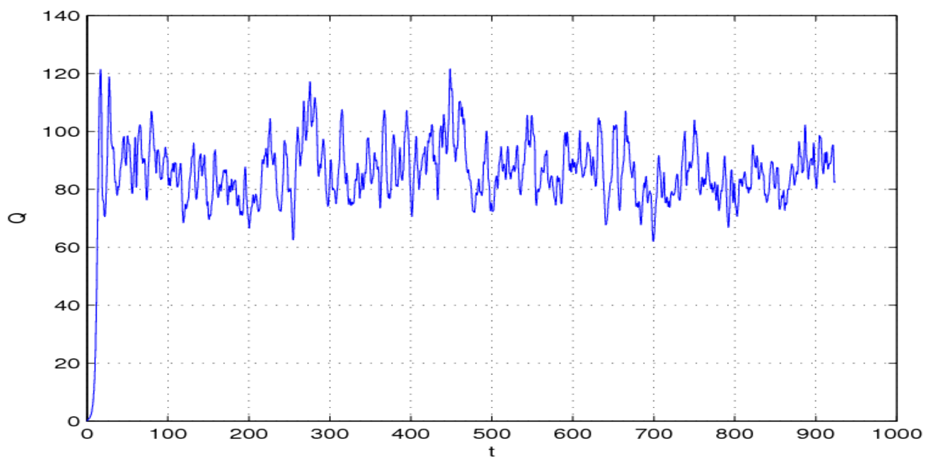
R/L_n and R/L_{Ti} normalized gradients in density and temperature, U is the speed of the modon soliton solution and s is the magnetic shear.

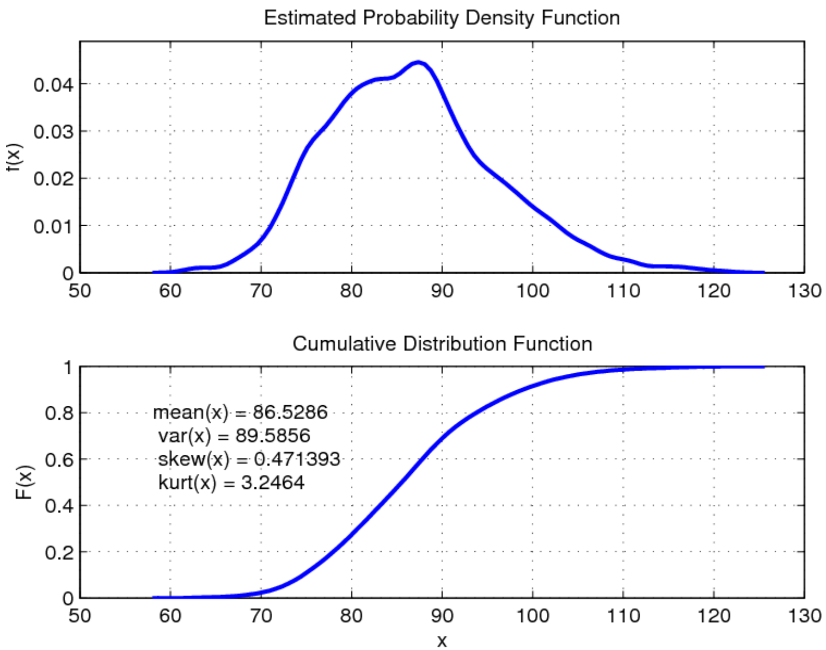
U=1 is assumed hereafter. Anderson PoP 2008 and Anderson PoP 2010

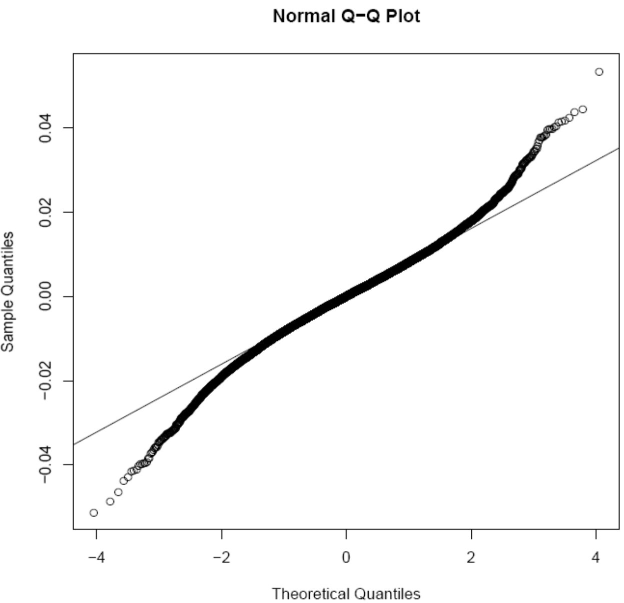
The nonlinear Gyro-Kinetic code GENE is used for the local simulations

- Eulerian solver for the Vlasov-Maxwell equations on $(x,y,z,v_{||},\mu)$ grid (initial value or eigenvalue mode).
- Includes multi-species (fully gyro-kinetic), collision operators, electromagnetic effects.
- Excellent scaling up to at least 32K processors on BlueGene/L.
- Flux-tube domain for stellarators and global for tokamak using CBC parameters.

Statistical analysis of time traces of radial heat flux







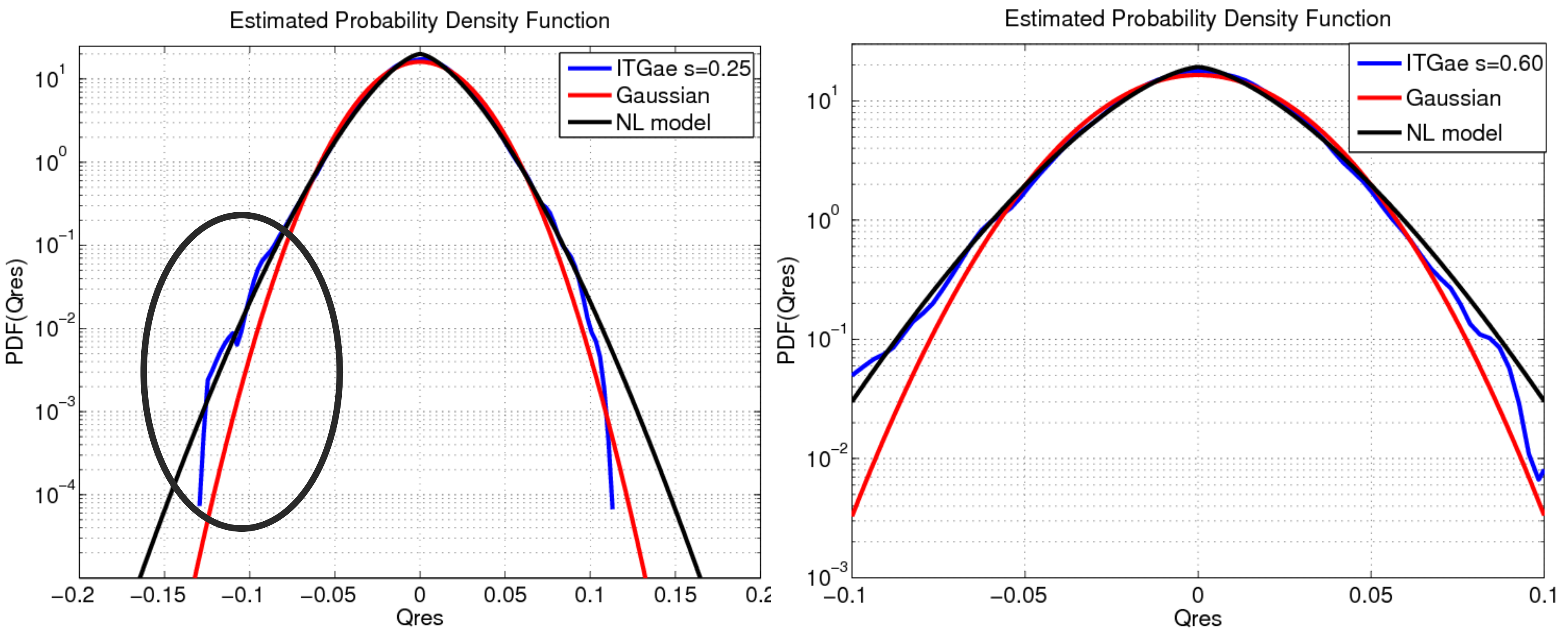
1. Key element: Remove autocorrelations from time traces.

2. Achieved using Box-Jenkins modeling via Matlab.

$$Q(t+1) = \sum_{n=0}^{\infty} a_n Q(t-n) + Q_{res}$$

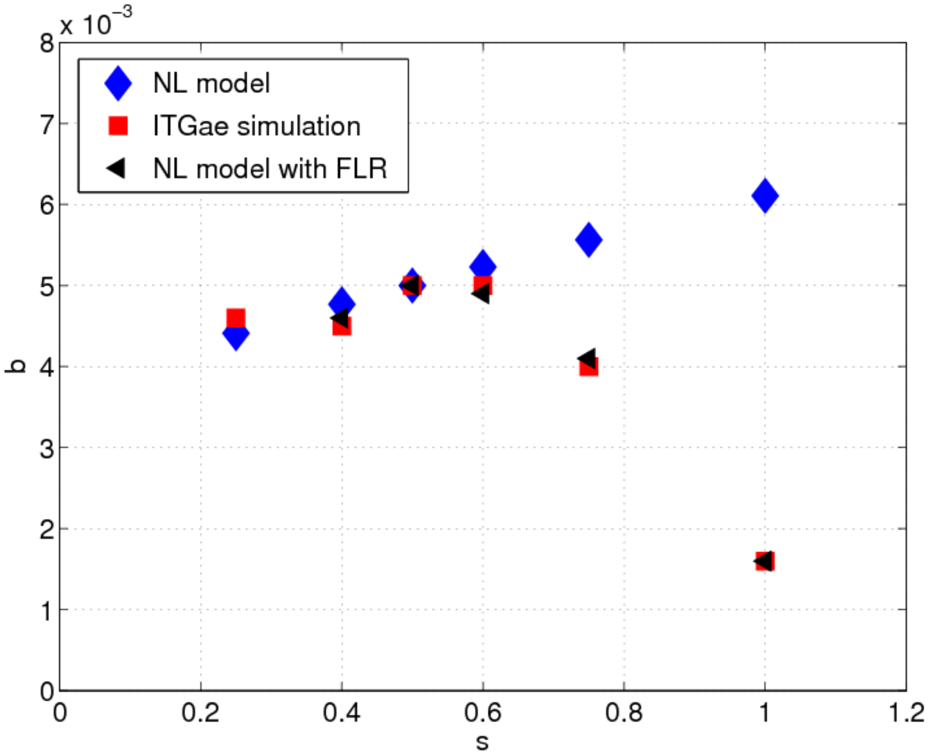
3. Residuals systematically manifest long non-Gaussian tails.

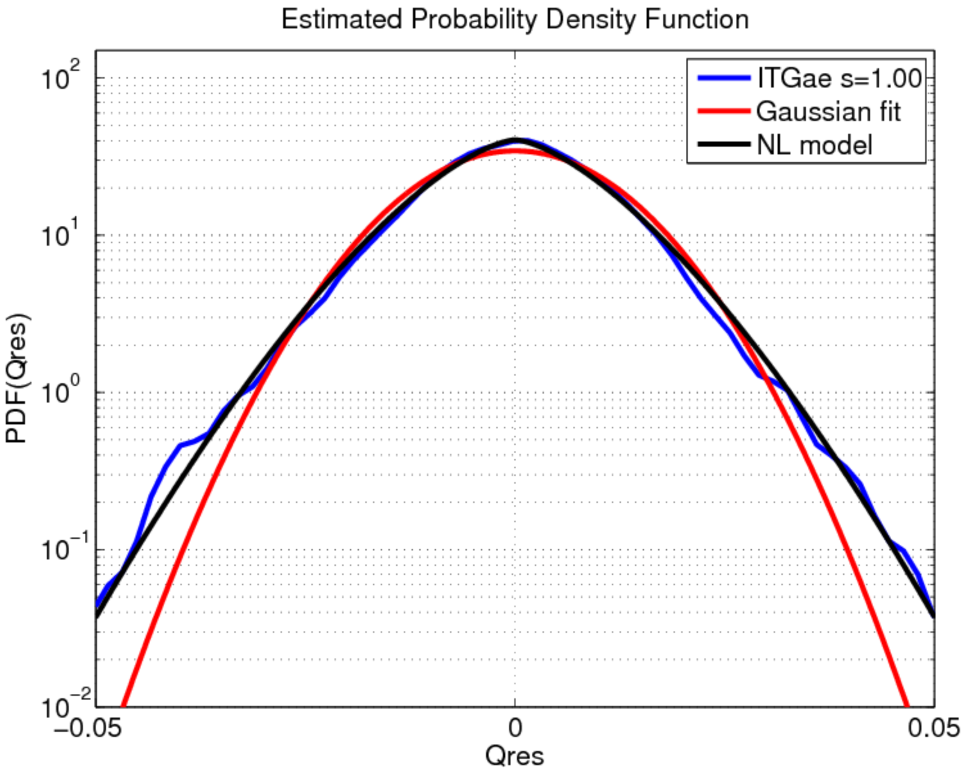
Result 1: PDFs at different magnetic shears



The analytically predicted PDFs are in good agreement with the numerically estimated PDFs for several orders of magnitude. In particular the NL model fits are considerably better than Gaussian fits (using the variance and mean from the distributions). $R/L_{Ti} = 9$, $R/L_n = 2$, $\tau=1$, $k = 0.5$.

Result 2: The PDF shear scaling





We fix the constant $b_0/Q^{1.5}_0$ at s=0.50. The GENE FLR is obtained by using the eigenfunctions from the simulations and estimate ky ≈0.45.

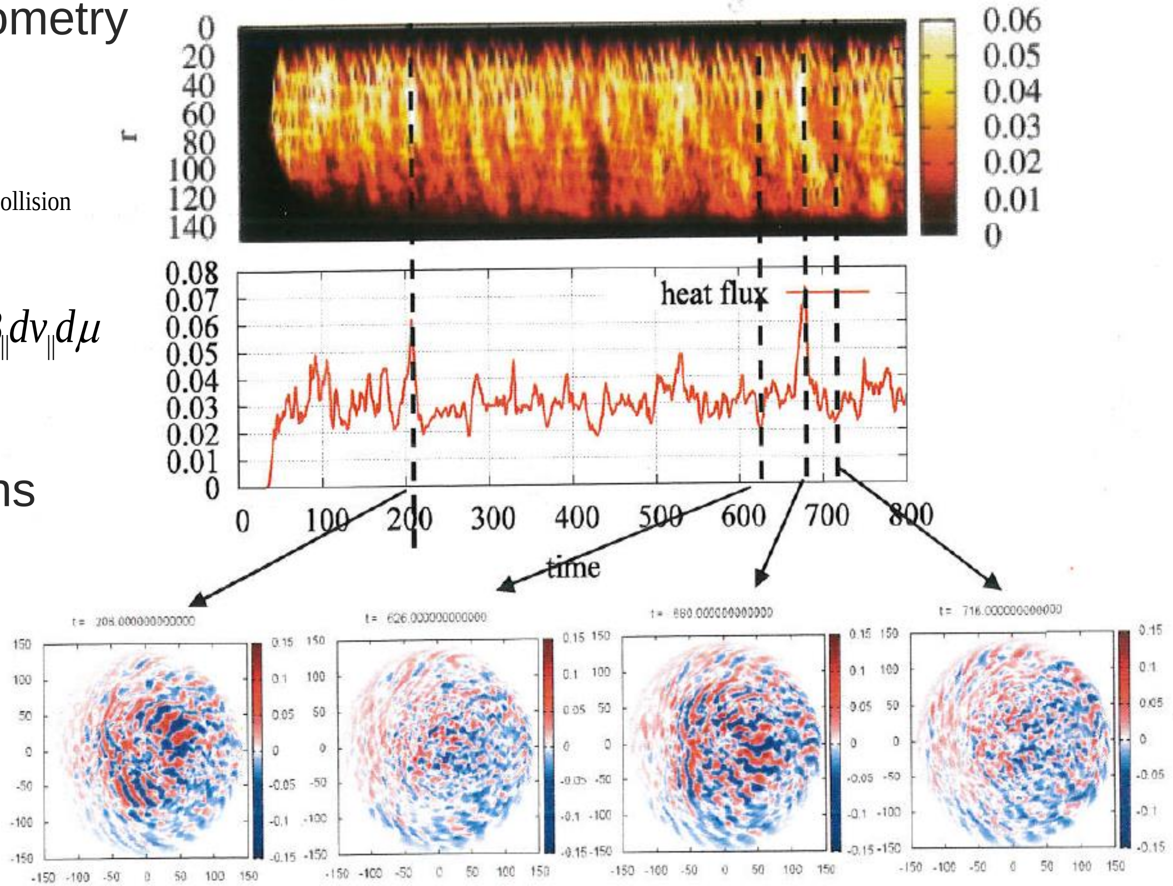
Global flux driven simulations - GKNET

Gyro-Kinetic based Numerical Experiment of Tokamak (GKNET)

Basic system
Flux driven full-f global toroidal geometry with external source and sink

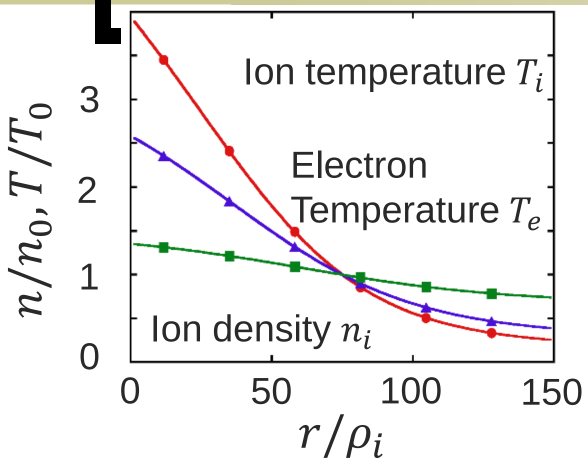
$$\frac{\partial f}{\partial t}+\{\vec{R},H\}\frac{\partial f}{\partial \vec{R}}+\{v_\parallel,H\}\frac{\partial f}{\partial v_\parallel}=S_{\mathrm{source}}+S_{\mathrm{sink}}+C_{\mathrm{collision}}$$

$$\Phi-\left\langle\langle\Phi\rangle_\alpha\right\rangle+\frac{1}{T_{e0}(r)}\left(\Phi-\langle\Phi\rangle_f\right)=\frac{1}{n_{i0}}\iint\langle f\rangle_\alpha B_\parallel dv_\parallel d\mu$$

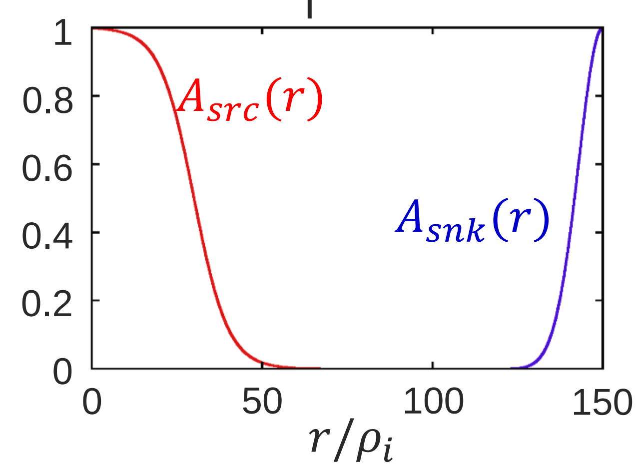


Electrostatic with adiabatic electrons
Full-order FLR
Conservative linear collision op.
Numerical method is 4th order Morinishi scheme for the Vlasov and 4th order RK scheme in time.
Parallelization is 5D (R-Z-φ-v-μ)

Parameters



Parameter	Value
a_0/ρ_i	150
a_0/R_0	0.36
$(R_0/L_n)_{r=a_0/2}$	2.22
$(R_0/L_{Ti})_{r=a_0/2}$	10.0
$(R_0/L_{Te})_{r=a_0/2}$	6.92
ν_*	0.28
P_{in}	16 [MW]
$\tau_{snk}^{-1}R_0/v_{ti}$	0.25



Source operator

$$S_{src} = A_{src}(r)\tau_{src}^{-1}[f_M(2\bar{T}) - f_M(\bar{T})]$$

✓ Constant power input near magnetic axis

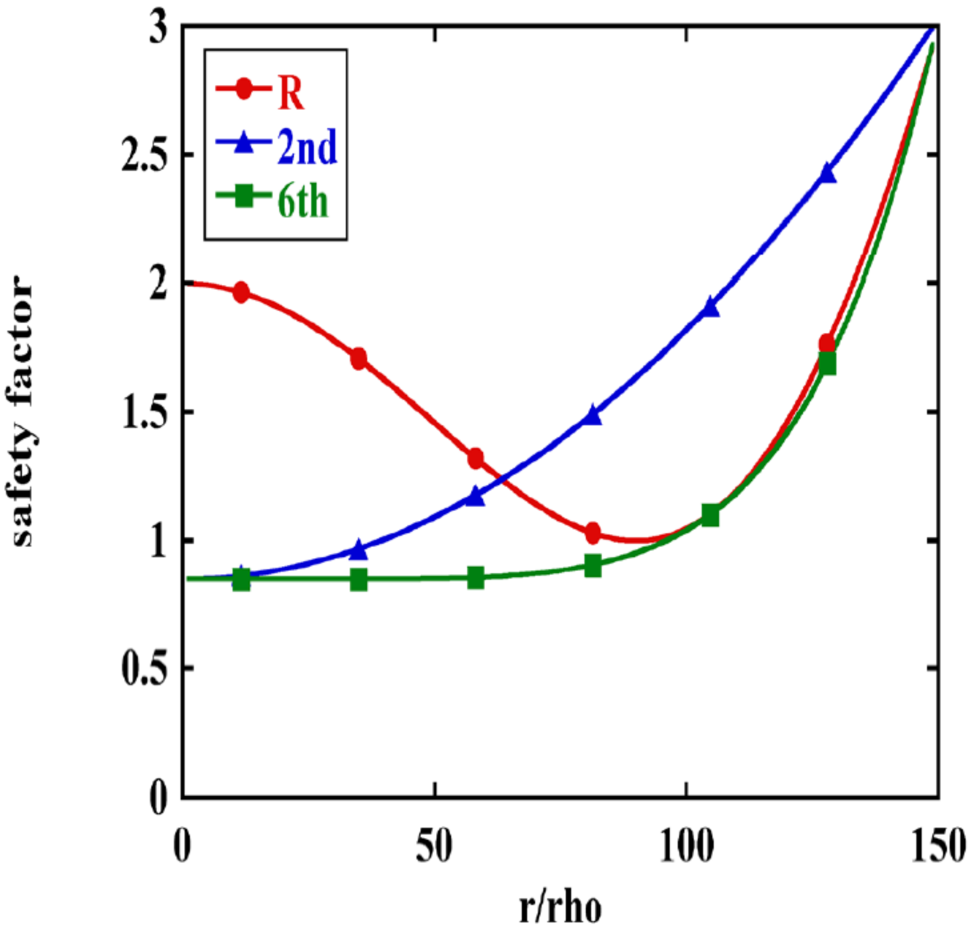
Sink operator

$$S_{snk} = A_{snk}(r)\tau_{snk}^{-1}[f(t) - f(t=0)]$$

✓ Krook-type operator to f in boundary region

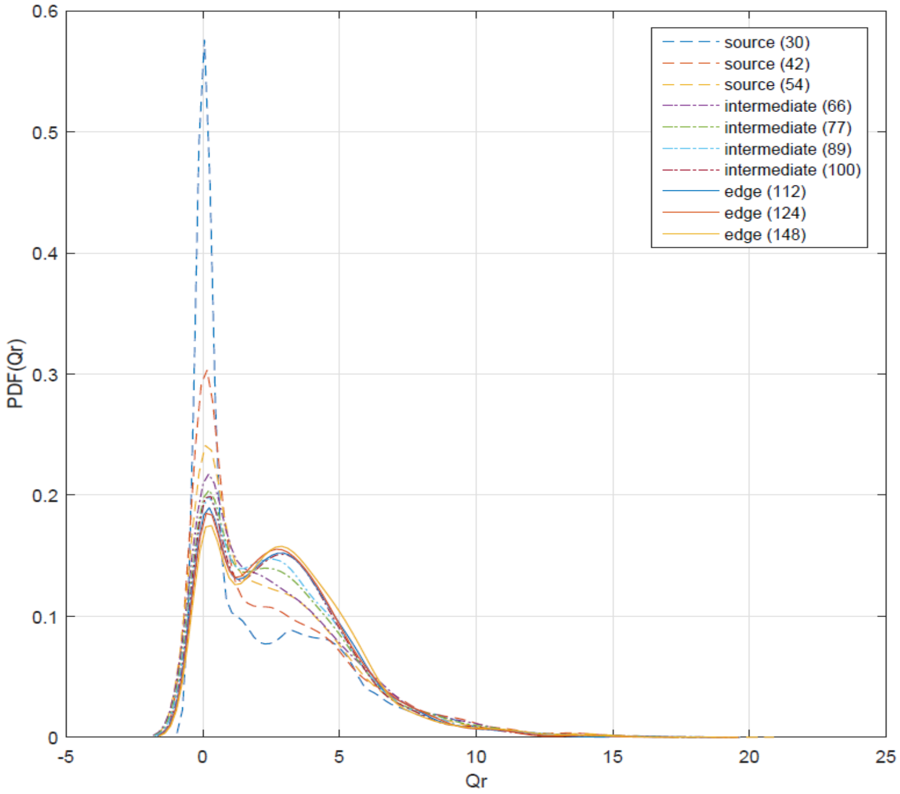
[Y. Idomura, *et. al*., Nucl. Fusion, **49**, 065029 (2009)]

Shear scan



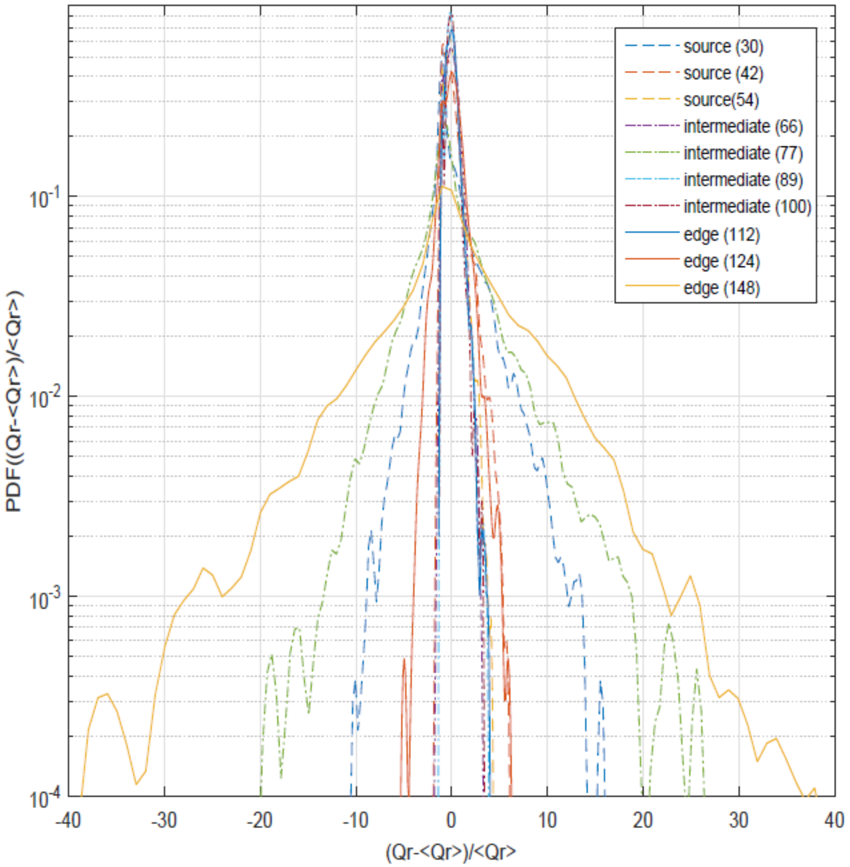
- Time traces with 4000 entries at 128 radial positions are analyzed.
- Flux surface average heat flux.
- Only results corresponding to the blue line will be discussed here.

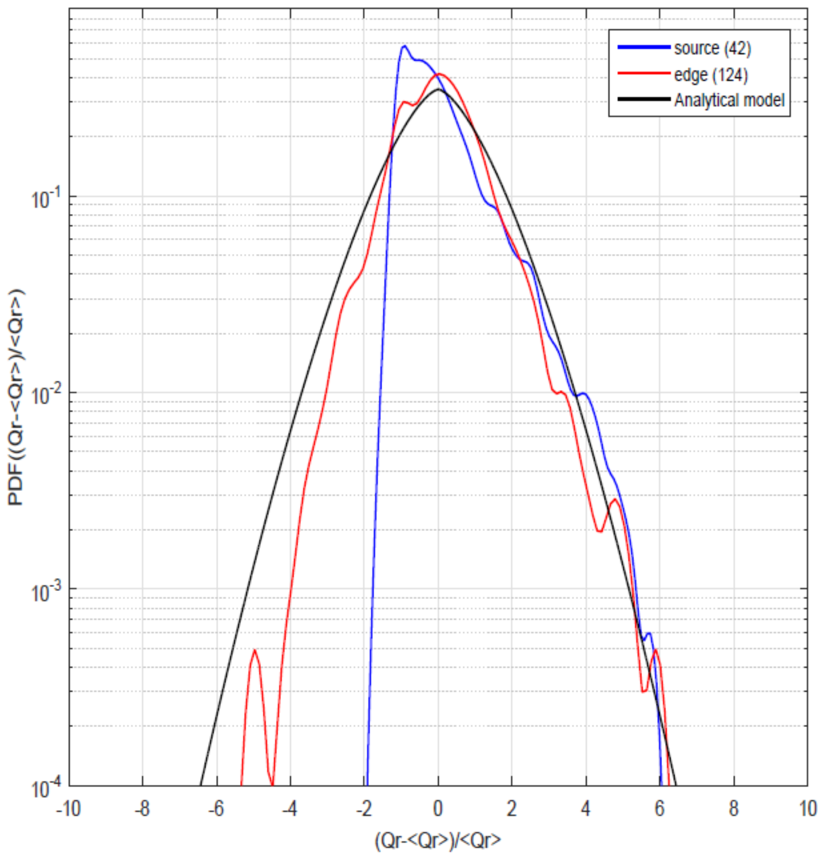
Result 1: Statistics of heat flux (2nd Order)



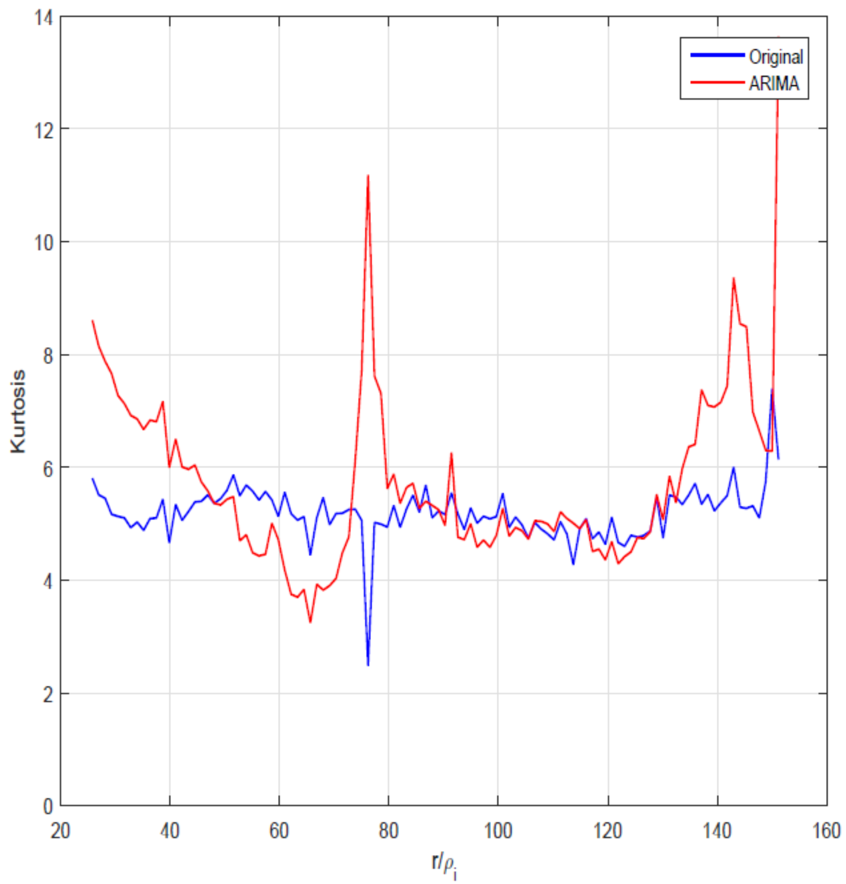
We have performed a statistical analysis of one base case where the parameters are similar to the cyclone base case parameters, i.e. $a/\rho_i = 150$, $a/R_0 = 0.36$, $(R_0/L_n)_{r=a_0/2} = 2.22$, $(R_0/L_{Ti})_{r=a_0/2} = 10.0$, $(R_0/L_{Te})_{r=a_0/2} = 6.92$, $\nu_\star = 0.28$, $P = 16MW$, $\tau_{snk}^{-1} R_0/v_{ti} = 0.25$.

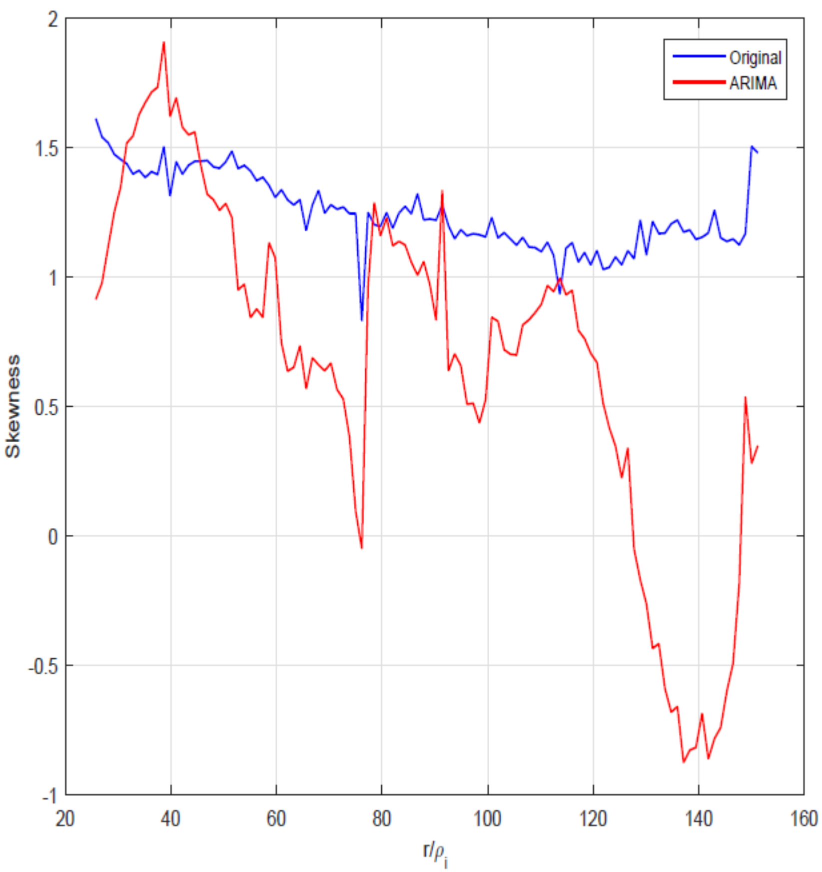
Result 2: ARIMA Modelled with fits





Result 3: Kurtosis and Skewness





NB

- Non-Gaussian distributions at most radial positions allowing for transport mediated by coherent structures such as blobs or streamers.
- Length of simulations is 800 (4000 time steps) although we have analyzed a three times longer time trace with similar results (shear scan the 6th order safety factor).
- The analytical model approximately reproduces the PDFs whereas there is a change as we go to the edge where the PDFs are Laplacian distributed.
- Positive skewness indicates excess transport outwards in radial direction.

Future work

- Statistical analysis of avalanches and SOC tokamaks and stellarators using GENE ~ Collaboration with Mavridis (Mavridis PoP 2014).

- Extending the work on statistical analysis of Hasegawa-Mima system to Hasegawa-Wakatani model. Collaboration with Hnat and Botha. ~ (Anderson PoP 2015)

- Would trapped particle effects change the PDFs? E.g. using kinetic electrons in the GENE runs. Is the same exponential form of the PDFs still valid?

Summary

- We have successfully used the ARIMA model to detrend data where noise and cyclic/oscillatory trends can be separated.
- The Box-Jenkins methodology is used to extract the noise characteristics from both experimental and numerical data.
- We have found good agreement between simulations and analytical estimated PDFs of heat flux and potentials in the core and edge respectively.
- The PDFs have been shown to have manifestly enhanced tails compared to Gaussian distributions.
- Note that most previous studies have focused on the full data including noise and oscillatory trends.

Thank You

References:

1. **J. Anderson** and Botha, Phys. Plasmas **22**, 052305 (2015).
2. **J. Anderson** et al, Phys. Plasmas **21**, 122306 (2014).
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