Relativistic Electrons and Magnetic Reconnection in Tokamaks

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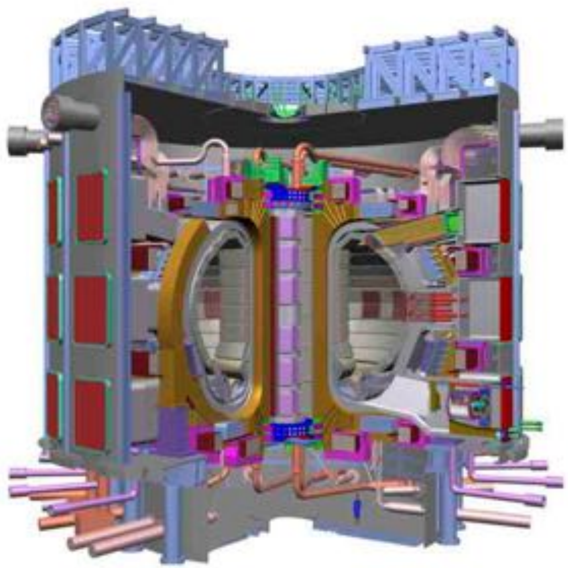
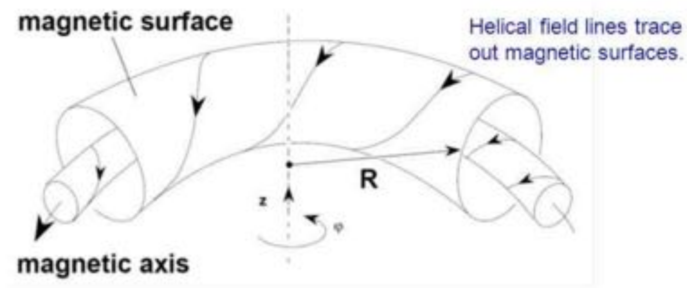
ITER is the most expensive science device ever built. Designed to develop magnetic fusion energy.

The machine could be damaged if the plasma current is transferred to relativistic electron carriers.

Fast breaking of magnetic topology (reconnection) has an important role in the current transfer.



Tokamaks and ITER

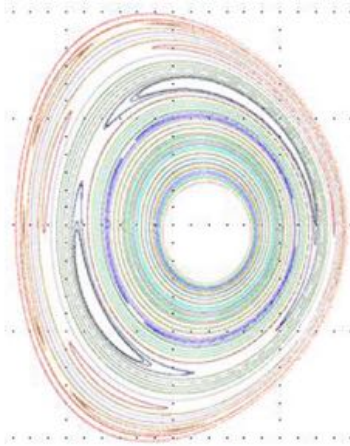


A large current, 15 MA in ITER, is required for plasma confinement.

If plasma is suddenly cooled, electric field acceleration of electrons can exceed the Coulomb drag and energetic electrons runaway.

The number of runaway electrons can exponentiate. A single Coulomb collision can transfer enough energy to a cold electron for it to run away.

Causes of Sudden Plasma Cooling



1. Magnetic islands can grow in the plasma interior until thermal confinement is lost.

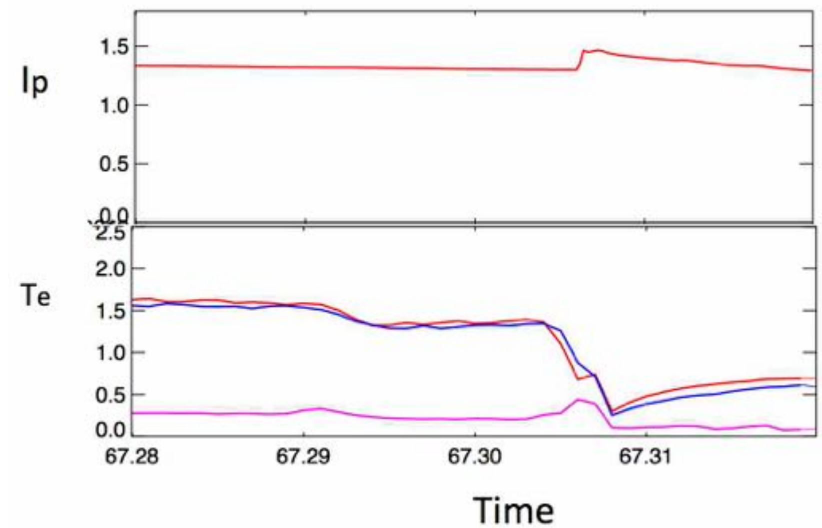
2. To keep a tokamak plasma centered in the chamber, an externally produced vertical magnetic field is required, which depends on the plasma state.

   If control is lost plasma, the plasma current must be quenched faster than the drift into the walls ~150ms, about 10^4 times faster than the natural decay time. Requires strong plasma cooling.

Natural Thermal Quenches

de Vries et al, NF 56, 026007 (2016).

Magnetic fluctuations are observed to increase for ~0.5s, then the temperature drops and the plasma current spikes on a ~1ms time scale.



The time scale and the magnitude of the current spike given by the speed and extent of a magnetic-surface-destroying reconnection.

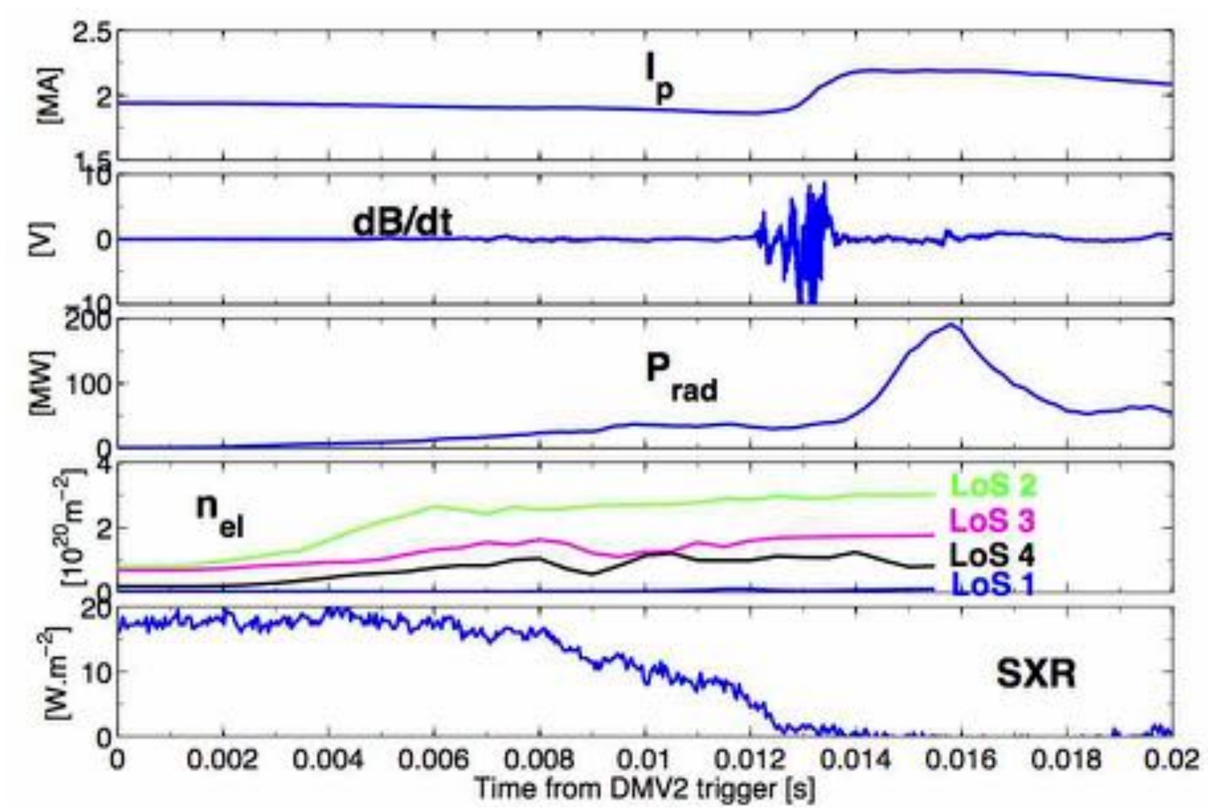
If reconnection complete, prevents electrons from running away. If not, catalyzes runaway.

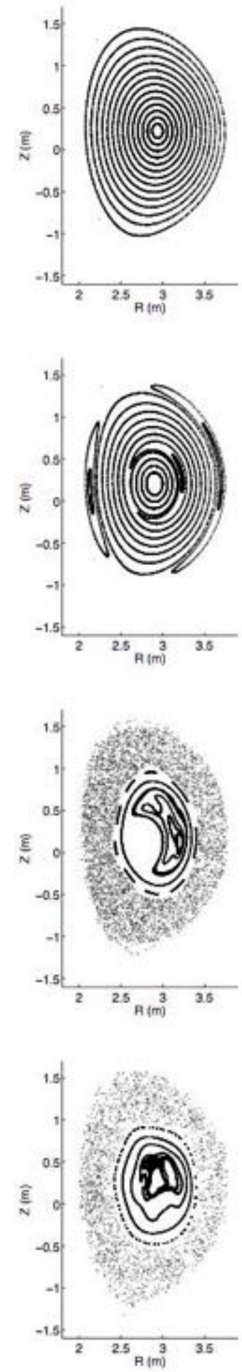
Thermal Quench from Gas Injection

From Nardon et al, EPS and PFCF (2016)

Gas is injected into JET, ~14ms later on a 1ms time scale, temperature collapses and current spikes.

Simulations at t=0, 4.1, 5.7, and 6.2ms.





Basic Points

When magnetic field lines intercept walls, electron loss is too fast for electrons to runaway to relativistic energies.

Rapid change in poloidal flux during a reconnection can accelerate electrons along magnetic field lines that do not intercept walls.

Relativistic electrons from large stochastic regions can be dumped on walls in a pulse along a narrow flux tube.

Plan of Analysis

Properties of magnetic flux and helicity

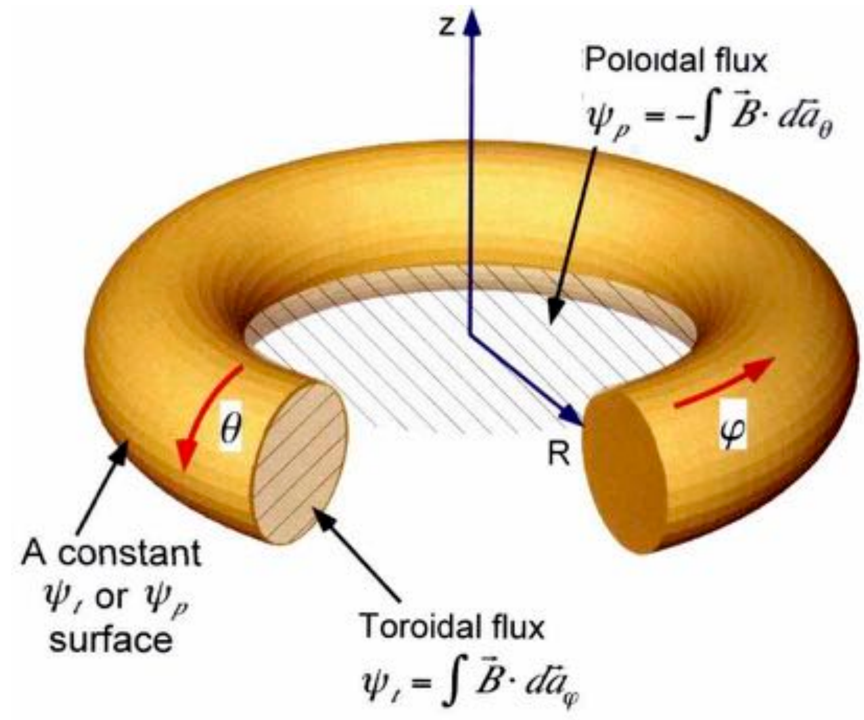
Fast magnetic reconnection

Electron acceleration, exponentiation, and number

Poloidal and Toroidal Flux

Any vector can be represented as
$\vec{A} = \frac{\psi_t \vec{\nabla}\theta}{2\pi} - \frac{\psi_p \vec{\nabla}\varphi}{2\pi} + \vec{\nabla} g$, so

$$2\pi\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}\theta + \vec{\nabla}\varphi \times \vec{\nabla}\psi_p.$$



Field lines $\quad \frac{d\psi_t}{d\varphi} = \frac{\vec{B}\cdot\vec{\nabla}\psi_t}{\vec{B}\cdot\vec{\nabla}\varphi} = -\frac{\partial\psi_p}{\partial\theta}$ and $\frac{d\theta}{d\varphi} = \frac{\vec{B}\cdot\vec{\nabla}\theta}{\vec{B}\cdot\vec{\nabla}\varphi} = \frac{\partial\psi_p}{\partial\psi_t}$.

When $\psi_p(\psi_t, \theta, \varphi, t)$ is independent of time, evolution ideal.

Position vector $\vec{x}(\psi_t, \theta, \varphi, t)$ changes in an ideal evolution.

Conservation of Poloidal Flux

Faraday's law, $\vec{\nabla}\times\frac{\partial\vec{A}}{\partial t} = -\vec{\nabla}\times\vec{E}$, transformed into canonical $(\psi_t, \theta, \varphi, t)$ coordinates, implies

$$\vec{E} + \vec{u}_c\times\vec{B} = \left(\frac{\partial\psi_p}{\partial t}\right)_c \vec{\nabla}\left(\frac{\varphi}{2\pi}\right) - \vec{\nabla}\Phi, \quad \text{where } \vec{u}_c \equiv \left(\frac{\partial\vec{x}}{\partial t}\right)_c.$$

Rate of change of poloidal flux outside a region defined by the toroidal magnetic flux is given by a loop voltage

$$\boldsymbol{\bar{V}_\ell = \frac{\partial}{\partial t}\oint \psi_p \frac{d\theta d\varphi}{(2\pi)^2}} = \oint\left(\vec{E} + \vec{u}_c\times\vec{B}\right)\cdot\frac{\partial\vec{x}}{\partial\varphi}\frac{d\theta d\varphi}{2\pi}$$

$$= \frac{\partial}{\partial\psi_t}\left(\int \vec{E}\cdot\vec{B}d^3x + \oint \Phi\vec{B}\cdot d\vec{a}_\psi\right).$$

Common misconception is that the poloidal flux in a region enclosed by a given toroidal flux is conserved. Clearly false at magnetic axis.

Magnetic Helicity Conservation

Magnetic helicity density $\vec{A}\cdot\vec{B} = \left(\frac{\partial\psi_p\psi_t}{\partial\psi_t} - 2\psi_p\right)\frac{(\vec{\nabla}\psi_p\times\vec{\nabla}\theta)\cdot\vec{\nabla}\varphi}{(2\pi)^2}$

Helicity content $\boldsymbol{K_c = -2\int \bar{\psi}_p d\psi_t}$, where $\boldsymbol{\bar{\psi}_p \equiv \oint \psi_p \frac{d\theta d\varphi}{(2\pi)^2}}$.

Note: $\int \vec{A}\cdot\vec{B}\, d^3x = K_c + \bar{\psi}_p\psi_t]_{bndry}$.

$$\boldsymbol{\frac{dK_c}{dt} = -2\int \bar{V}_\ell\, d\psi_t.}$$

$\int \bar{V}_\ell\, d\psi_t = \mathcal{R}I_P$, where $I_p \equiv \int \vec{j}\cdot\vec{\nabla}\frac{\varphi}{2\pi}d^3x$ net plasma current,

$\mathcal{R} = \frac{\langle 2\pi R\eta B\rangle_{j/B}}{\Psi_t}$ resistance, and $\Psi_t \equiv \int \vec{B}\cdot\vec{\nabla}\frac{\varphi}{2\pi}d^3x$ toroidal flux.

Magnetic Topology Conservation

Magnetic-field-line topology determined by $\psi_p(\psi_t, \theta, \varphi)$ not $\bar{\psi}_p$.

Unless ψ_p depends on both θ and φ magnetic surfaces exist;

$$\vec{B} \cdot \vec{\nabla}\psi_t \propto \frac{\partial \psi_p}{\partial \theta} \text{ and } \vec{B} \cdot \vec{\nabla}\psi_p \propto \frac{\partial \psi_p}{\partial \varphi}.$$

Breaking magnetic field line connections is exponentially easy in 3D, much more difficult in 2D.

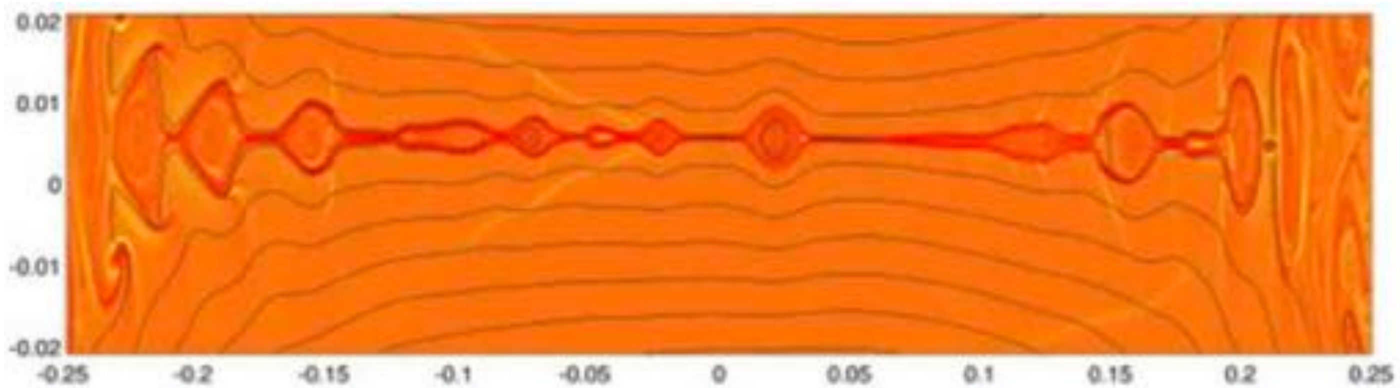
In 3D, a large current density is not necessary, and the electron inertial scale, c/ω_{pe}, can break magnetic field line connections.

Alfvénic Changes in Magnetic Field Line Topology

1. **Plasmoid reconnection**, as many plasmoids (islands) form as necessary to allow Alfvénic reconnection.

Loureiro et al, Phys. Plasmas 14, 100703 (2007).

Loureiro and Uzdensky, Plasma Phys. Control. Fusion 58, 014021 (2016).



Bhattacharjee et al, PoP **16**, 112102 (2009)

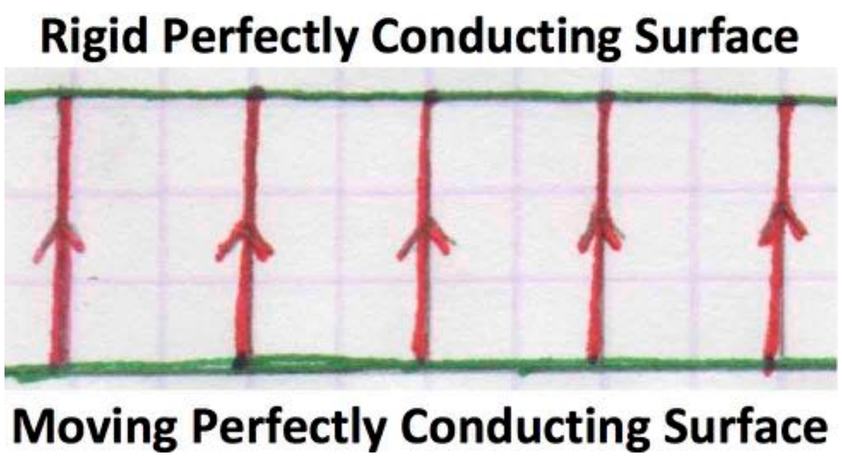
Model is essentially 2D or depends on double periodicity of a torus.

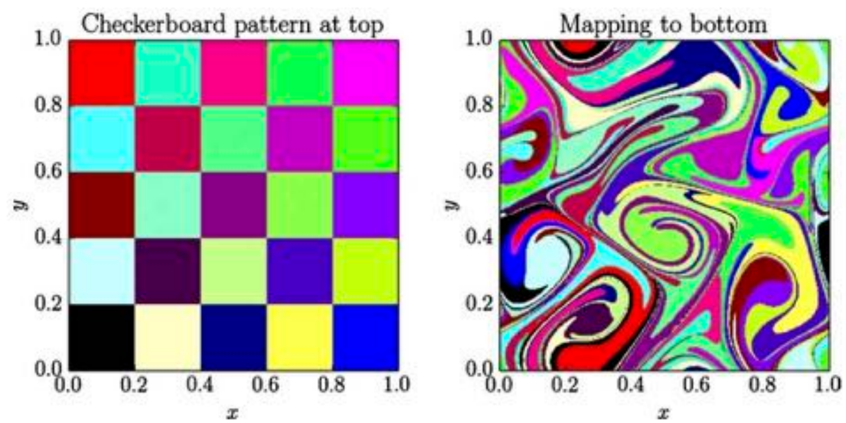
2. **Exponentially increasing separation** between neighboring magnetic field lines $\frac{d\vec{\delta}}{d\ell} = \vec{\delta} \bullet \vec{\nabla}\vec{B}$, so $|\vec{\delta}| \propto e^{\sigma(\ell)}$.

See Boozer, PoP 21, 072907 (2014).

Model requires 3D, but magnetic field connections broken with exponential, $e^{\sigma(\ell)}$, ease. *Solves problem of reconnection trigger*.

An ideal evolution $\vec{E} + \vec{u}\times\vec{B} = -\vec{\nabla}\Phi$ naturally gives a temporal increase in exponentiation. Electron inertia can break connections when $\sigma \approx \ln\left(\frac{a\omega_{pe}}{c}\right) \sim 8.5$ in ITER; $\vec{j}$ need not be large.





Huang et al, ApJ **793**, 106 (2014)

When Two Field Lines Connect

1. **Differing** $\boldsymbol{\frac{j_{||}}{B}}$**,** but $\vec{B} \bullet \frac{\vec{\nabla} j_{||}}{B} = \vec{B} \bullet \vec{\nabla} \times \frac{\vec{f}}{B^2}$, where $\vec{f} = \vec{j} \times \vec{B}$ is force exerted on the plasma.

   Inertial force $\vec{f} = \rho \partial \vec{v} / \partial t$ gives shear Alfvén wave

$$\boldsymbol{\frac{\partial^2}{\partial t^2} \frac{j_{||}}{B} = V_A^2 \frac{\partial^2}{\partial \ell^2} \frac{j_{||}}{B}}$$

   Current spike comes from $j_{||}/B$ flattening across plasma, time scale implies ~100 toroidal transits.

2. **Differing poloidal fluxes** $\boldsymbol{\psi_p = -\oint \vec{B} \bullet da_{hole}}$**.**

   Determined in post-current spike plasma by K_c conservation.

Electron Acceleration

For collisionless electrons

$$\frac{d(\gamma m v)}{dt} = -e\frac{\vec{E}\cdot\vec{B}}{B} = -e\left(\frac{\partial \psi_p}{\partial t}\right)_c \frac{\vec{B}\cdot\vec{\nabla}\left(\frac{\varphi}{2\pi}\right)}{B} \approx -\frac{e}{2\pi R}\left(\frac{\partial \psi_p}{\partial t}\right)_c$$

Characteristic flux change required to reach a relativistic energy

$$\boldsymbol{\psi_{pa} \equiv 2\pi R\frac{mc}{e}} \approx 0.0664\text{V•s} \text{ in ITER.}$$

Poloidal flux between plasma center and edge is $\Psi_p = 80$V•s.

$$\frac{\psi_{pa}}{\Psi_p} \sim 10^{-3}.$$

Electron Runaway

Maximum Coulomb drag force background electrons can exert on an electron is eE_{ch}, Connor-Hastie, NF 15, 415 (1975).

$$E_{ch} = \frac{ne^3}{4\pi\varepsilon_0^2 mc^2}\ln(\Lambda)$$

In ITER this corresponds to a loop voltage

$\boldsymbol{V_{ch} \equiv 2\pi R E_{ch}} \approx 2.9 n_{20}$Volts, where n_{20}=n/(10^{20}/m^3).

Post-thermal quench, $\bar{V}_\ell$ ≈80V•s/(150ms) ≈500Volts.

Electrons with kinetic energy K<<mc^2 cannot run away unless

$$K \geq \boldsymbol{K_r \equiv mc^2 V_{ch}/2\bar{V}_\ell} \approx 3\text{keV}.$$

Exponentiation in Electron Number

In a single Coulomb collision an energetic electron can scatter a cold electron above the runaway kinetic energy, K_r.

This collision rate has no $\ln(\Lambda)$ and is $\propto 1/K_r$, so the poloidal flux change required for an e-fold in the number of runaway electrons is

$\boldsymbol{\gamma_{ef}\psi_{pa}}$, where $\gamma_{ef} \approx 2\ln(\Lambda) \approx 25$, so $\Psi_p/\,\gamma_{ef}\psi_{pa} \approx 40$.

Without other drag forces energetic electron distribution function $\boldsymbol{f(p,t) = \frac{N_r(t)}{p_0 p^2} e^{-p/p_0}}$, where $\boldsymbol{p_0 \equiv \gamma_{ef} mc}$.

Pitch angle scattering and other effects can increase K_r and γ_{ef}.

Source of Seed Runaways

Primary source is Maxwellian tail. Fraction in tail is

$$\mathcal{F}_{tail} = \frac{2\sqrt{\epsilon}}{\sqrt{\pi}} e^{-\epsilon}, \text{ where } \epsilon \equiv mv^2/2T.$$

Sufficient tail electrons to carry the full plasma current survive Coulomb drag when they are accelerated within ~20ms.

A flux change $\sim\psi_{pa}$ transfers current to relativistic electrons. Probably seen on TFTR, Fredrickson et al, NF 55, 013006 (2015).

Runaway not possible on magnetic field lines that intercept walls in less than 500Volts/$\bar{V}_\ell \lesssim 100$ toroidal transits.

Electrons can be accelerated in flux tubes of non-intercepting lines.

Especially Dangerous Situation

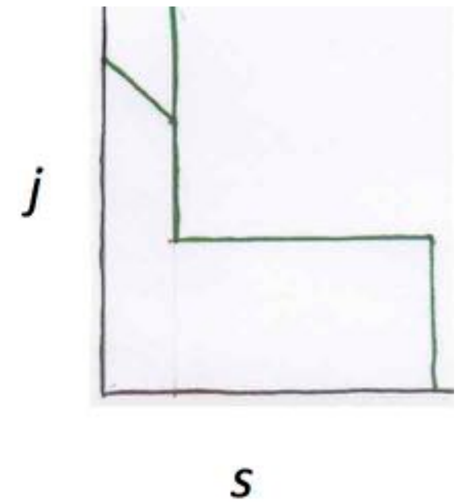
Boozer & Punjabi (Sherwood, 2016) related to turnstile theory

In simulations of experiments, magnetic surfaces in central region are not destroyed during the thermal quench.

If flux tubes defined by the surviving surfaces are not dissipated before at least some outer magnetic surfaces re-form, electrons in these tubes form seed electrons for runaway in the full stochastic region confined by the outer annulus of confining surfaces.

If the plasma drifts to the walls, the outer annulus is punctured and electrons in stochastic region dump on the walls in a short pulse $\approx \sqrt[3]{\tau_t \tau_{ev}^2}$ in a narrow flux tube $\psi_\ell/\psi_{st} \approx \sqrt[3]{\tau_t/\tau_{ev}}$, where $\tau_t = 2\pi R/c \approx 0.1\mu$s and $\tau_{ev} \approx 150$ms is evolution time. ψ_ℓ flux is escape tube and ψ_{st} toroidal flux in stochastic region.

Effect of Outer Surface Breakup



Let $s \equiv \frac{\psi_t}{\Psi_t}$.

Before breakup assume: Surface elongation κ_0.

The characteristic inductance $L_0 = \frac{2\kappa_0}{1+\kappa_0^2}\mu_0 R$.

The current $I(s) = s(2-s)I_p$ and $j(s) \propto dI/ds$.

$\iota(s) = \frac{1}{\Psi_t}\frac{d\psi_p}{ds} = \frac{L_0}{\Psi_t}\frac{I(s)}{s}$ and $\Psi_p = \frac{3}{4}L_0 I_p$, so $K_c = \frac{16}{9}\Psi_p\Psi_t$.

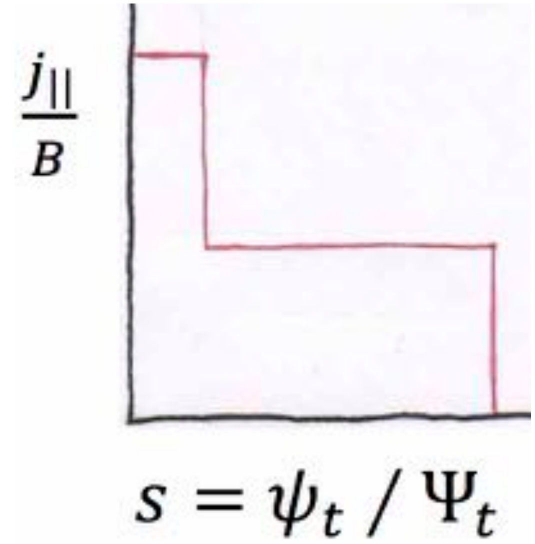
After breakup outside $\boldsymbol{s_r = 0.2}$:

$j_a(s < s_r) = j(s), j_a(s > s_r) = const.$ Fixed $K_c, \Psi_p, \psi_p(1)$ and Ψ_t.

Find $I_p^{(a)} = 1.14 I_p$; $I_{sing}^{(a)} = 0.3696 I_p$.

Relaxation of $I_{sing}^{(a)}$ can drive an electron runaway.

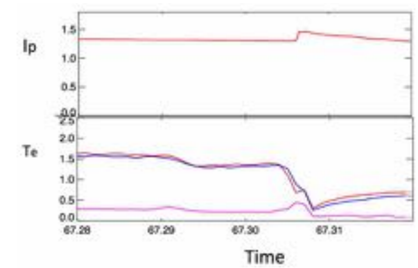
Two-Current-Density Model



The surface current at s_r relaxes resistively rapidly compared to the overall resistive dissipation.

Can use a two-current-density model to represent plasma state both before and after relaxation.

For JET pulse 83061 can use $\iota(0)/\iota(1)$ and $\ell_i(b)$ to fit equilibrium before thermal spike and $\ell_i(a)$, $I_p(a)/I_p(b)$, and $K_c(a)/K_c(b)$ to study final state.

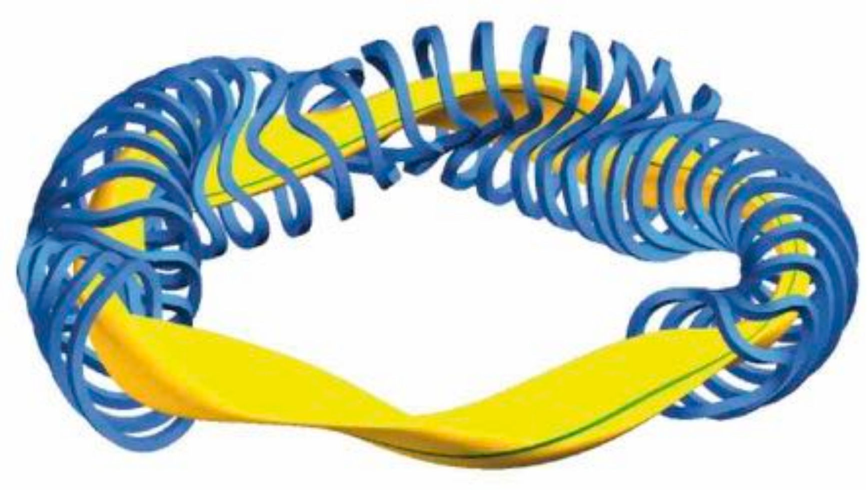


Find to obtain a solution need helicity to decay by ~13%, during relaxation, which is reasonable. Central change in poloidal flux is $\sim 0.14\Psi_p(b)$ could produce relativistic electrons.

Generality of Problem

Damage from relativistic electrons can be severe. Tokamaks must develop a method of avoiding or mitigating the damage.

Non-axisymmetric plasma confinement devices can avoid the problem by ensuring the plasma current is small.



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