Development and application of BOUT++ for large scale turbulence simulation

J. Leddy, B.Dudson, P.Hill, B.Shanahan, N.Walkden

PPPL Theory seminar
28 February 2017

Outline

Numerical developments

New coordinate system
Flux-coordinate independent method

A new plasma model (Hermes)

2-fluid cold ion model in divergence form
Including neutral interactions

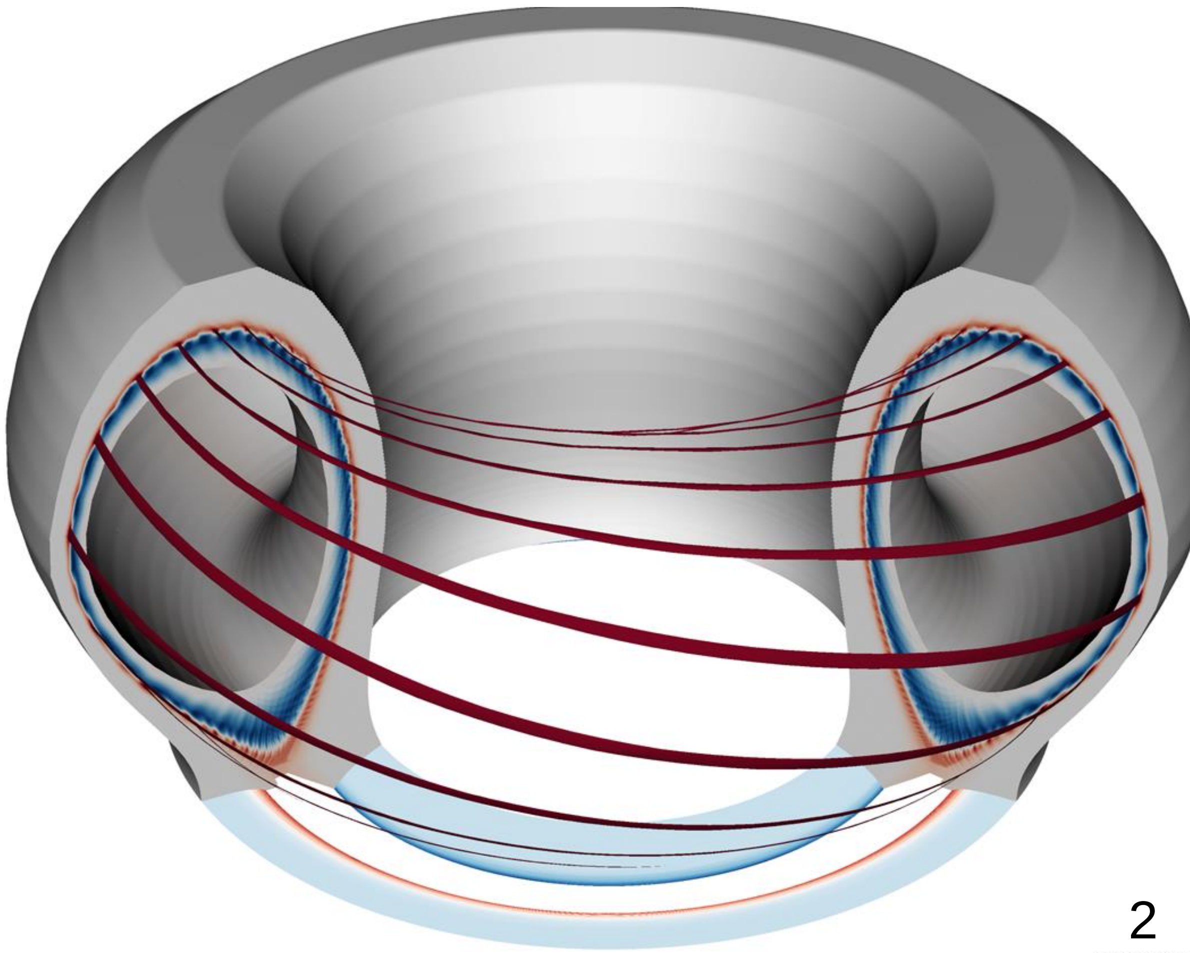
Turbulence and Neutral Simulations

Linear device
MAST-U
DIII-D

What is BOUT++

- Framework for solving systems of PDE's
- Flexible numerical methods and geometries
  - Pvode, PETSc, grids from EFIT
- Easy to implement physics models
  - ddt(Ni) = -Div(Ni * Vi)
- Designed with tokamaks in mind
  - Axisymmetry
  - Parallelization
- Open source at:

https://github.com/boutproject/BOUT-dev



Standard field-aligned coordinates

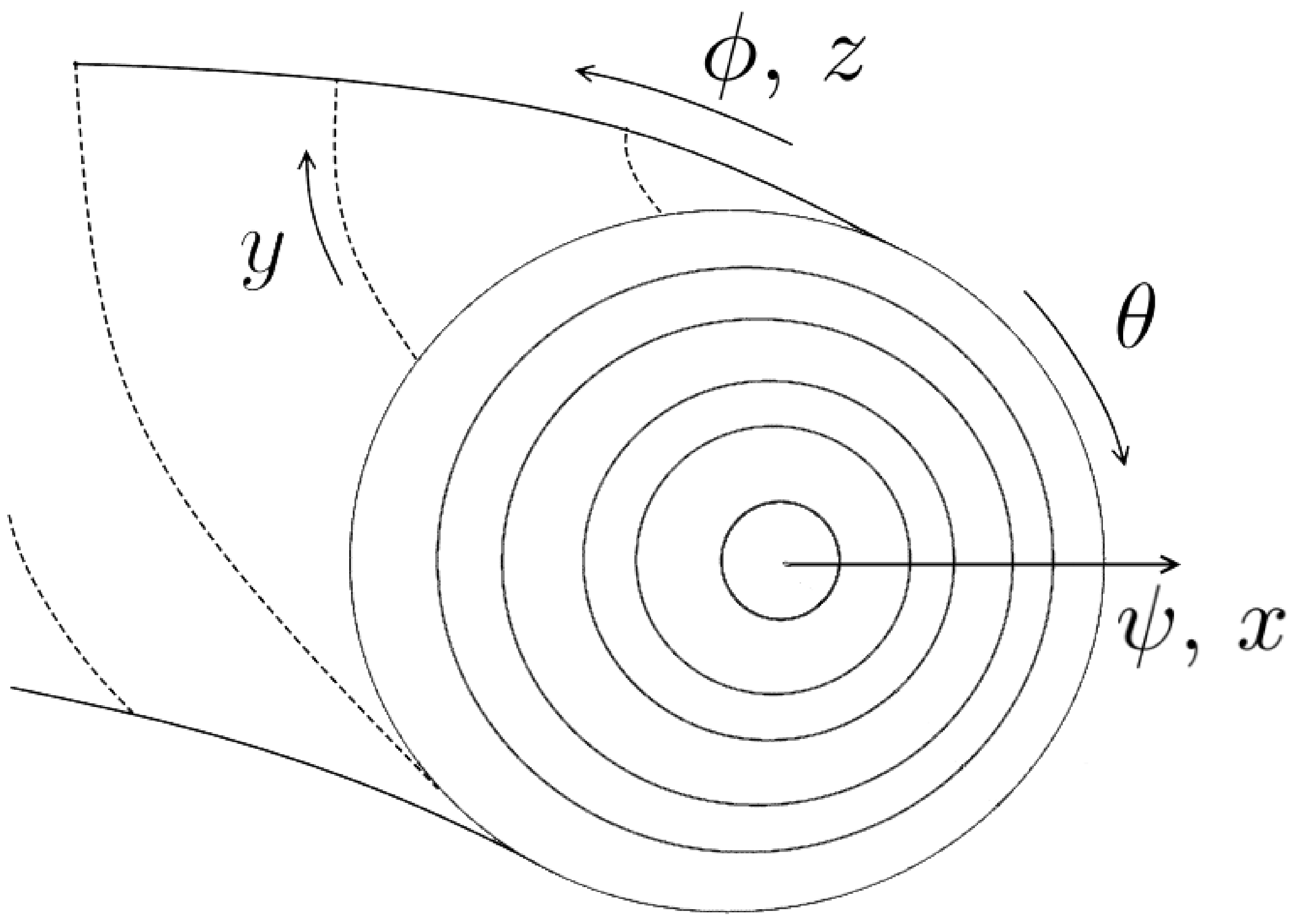
- Coordinate system should be **field-aligned**:
- Ease of parallel operations
- Perturbations tend to have low $k_{||}$

Coordinates:

$$x = \psi$$

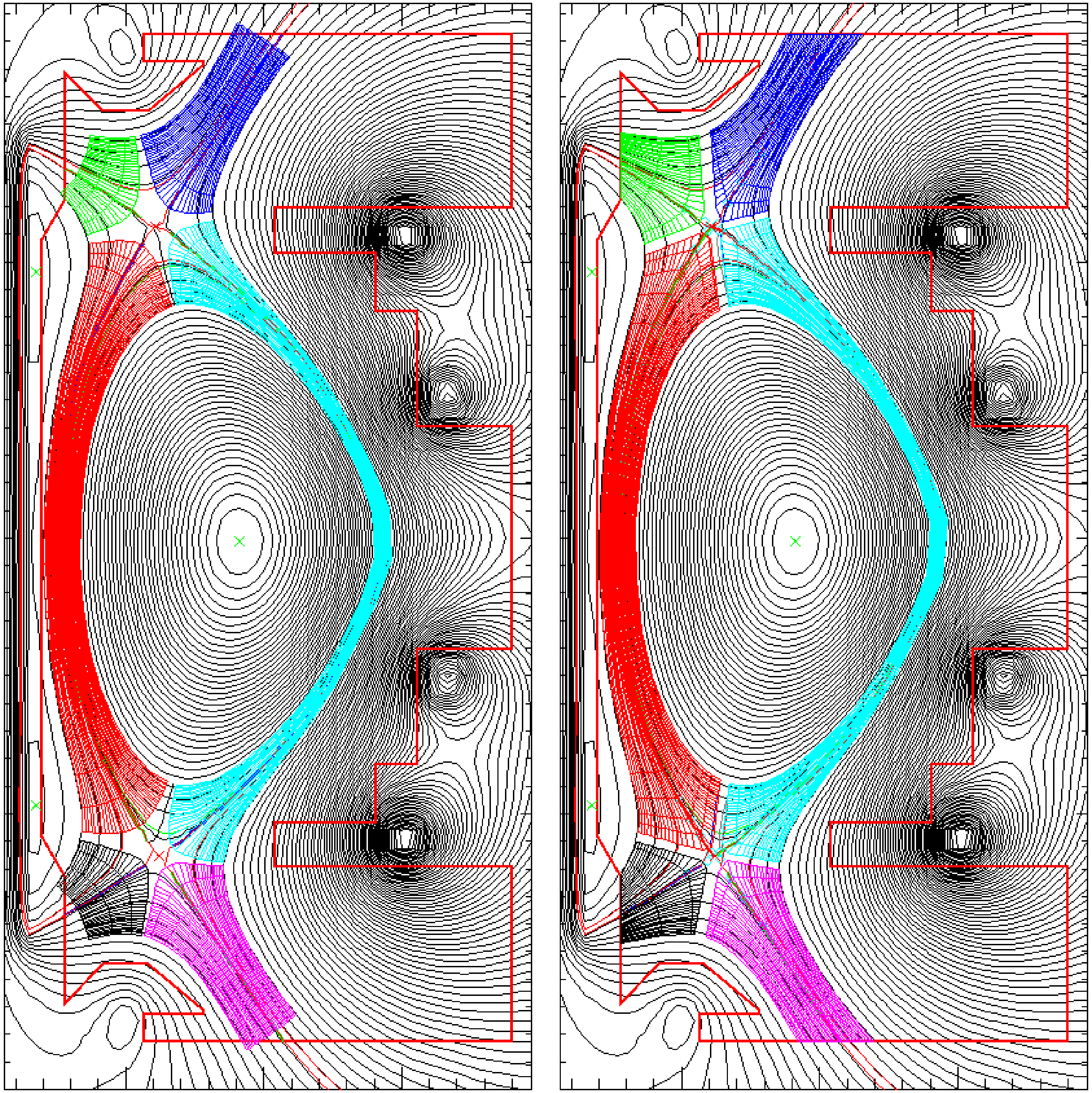
$$y = \theta$$

$$z = \phi - \int_{\theta_0}^{\theta} \nu \, d\theta$$



Why new coordinates?

- Still desire field-aligned system
- But poloidal projection of *x* and *y* are constrained to be orthogonal
- With new coordinate system we can:
  - Match divertor geometry
  - Approach X-point more closely and evenly



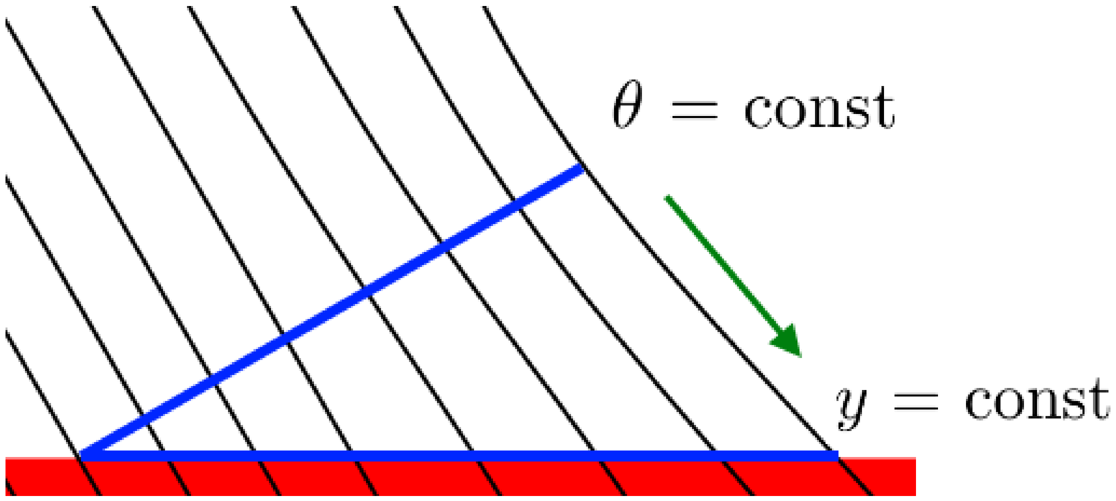
J Leddy *et al* (2017) *Computer Physics Communications*

Flexible field-aligned coordinates

$$x = \psi$$

$$y = \theta \boxed{- \int_{\psi_0}^{\psi} \eta \, d\psi}$$

$$z = \phi - \int_{y_0}^{y} \nu \left(1 + \int_{\psi_0}^{\psi} \eta \, d\psi\right) \, dy$$

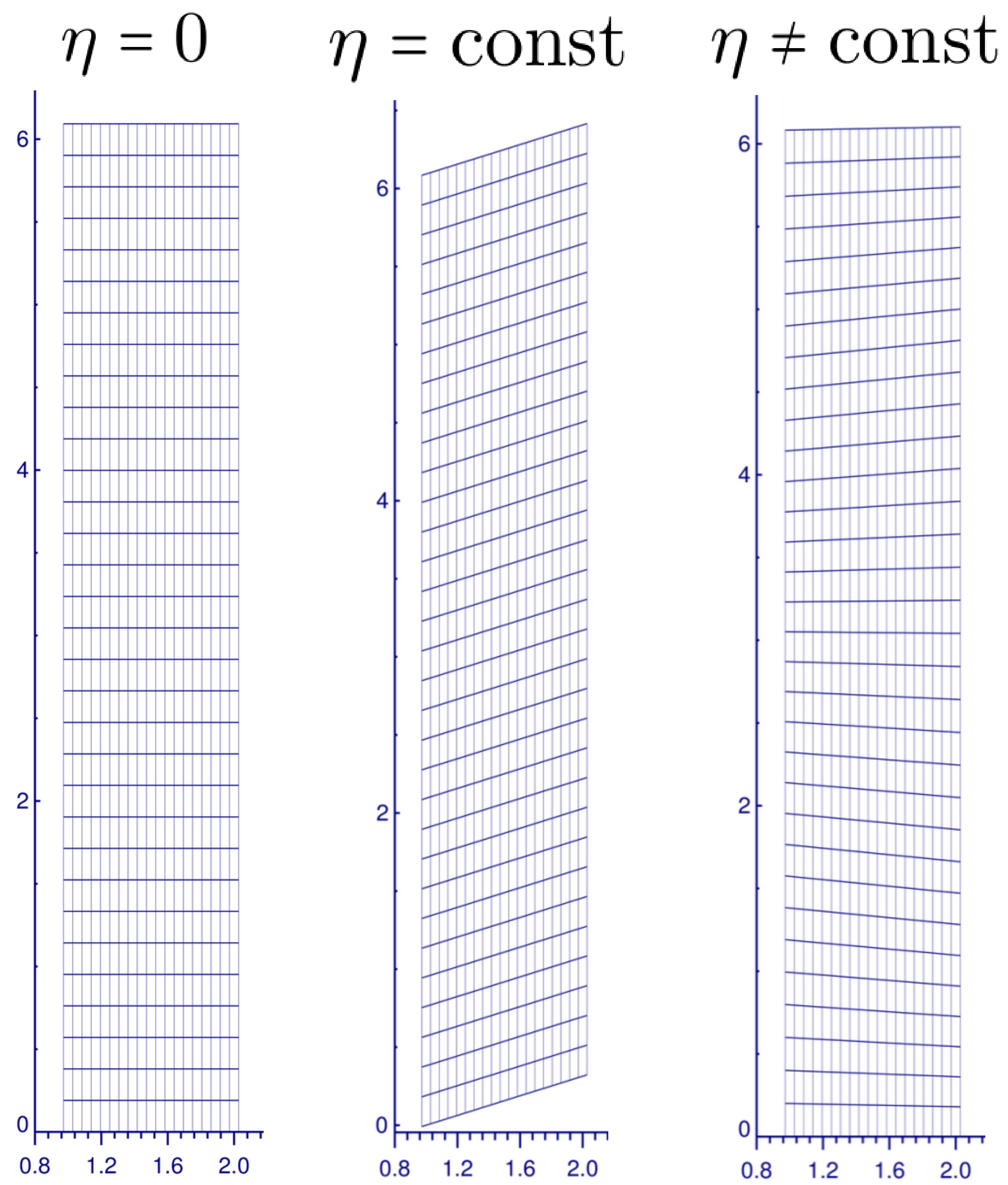


Can now calculate metric tensors for spatial operators

J Leddy *et al* (2017) *Computer Physics Communications*

Numerical accuracy

- Tested via the method of manufactured solutions[1]
- Nine combination of orthogonalities tested
- Implementation in BOUT++ is 2nd order accurate

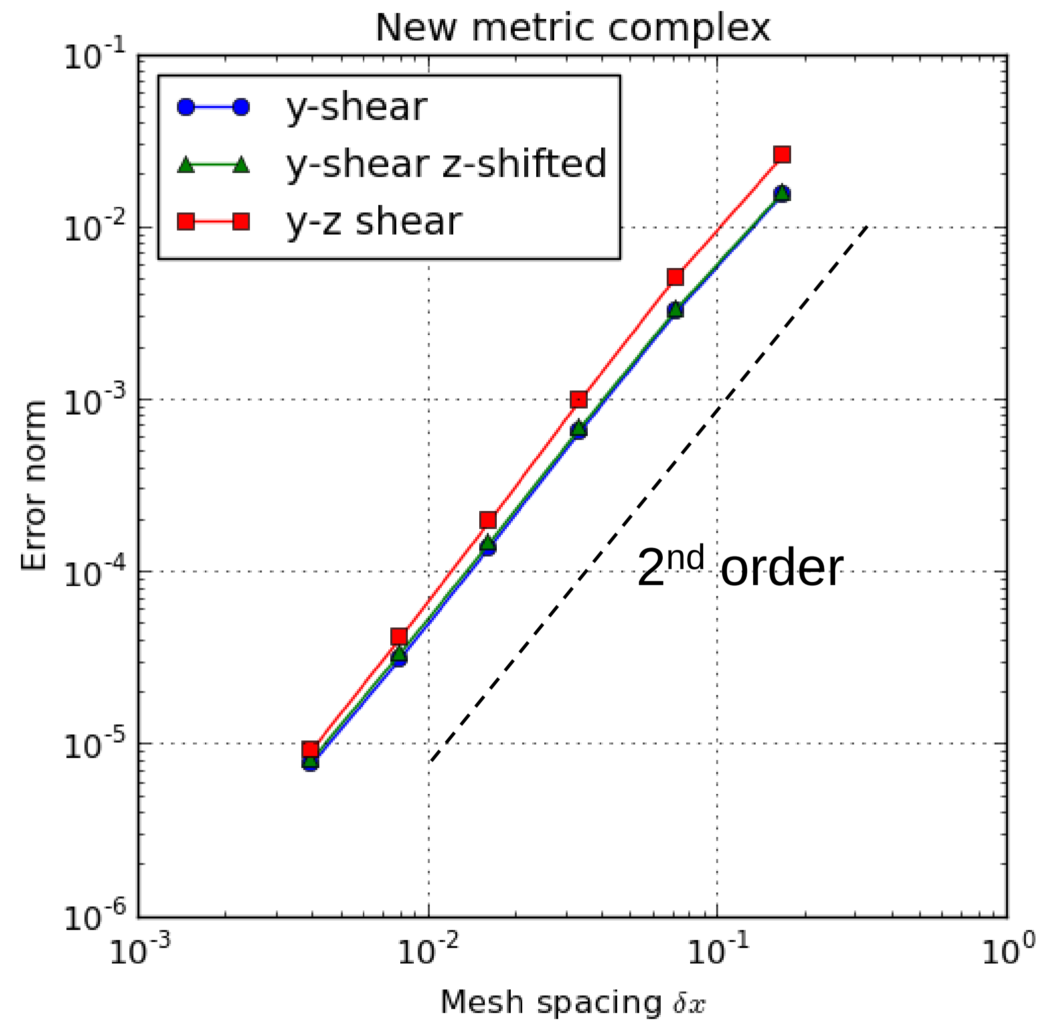


[1] Salari and Knupp, (2000) Tech. Report **SAND2000-1444**

J Leddy *et al* (2017) *Computer Physics Communications*

Numerical accuracy

- Tested via the method of manufactured solutions[1]
- Nine combination of orthogonalities tested
- Implementation in BOUT++ is 2nd order accurate



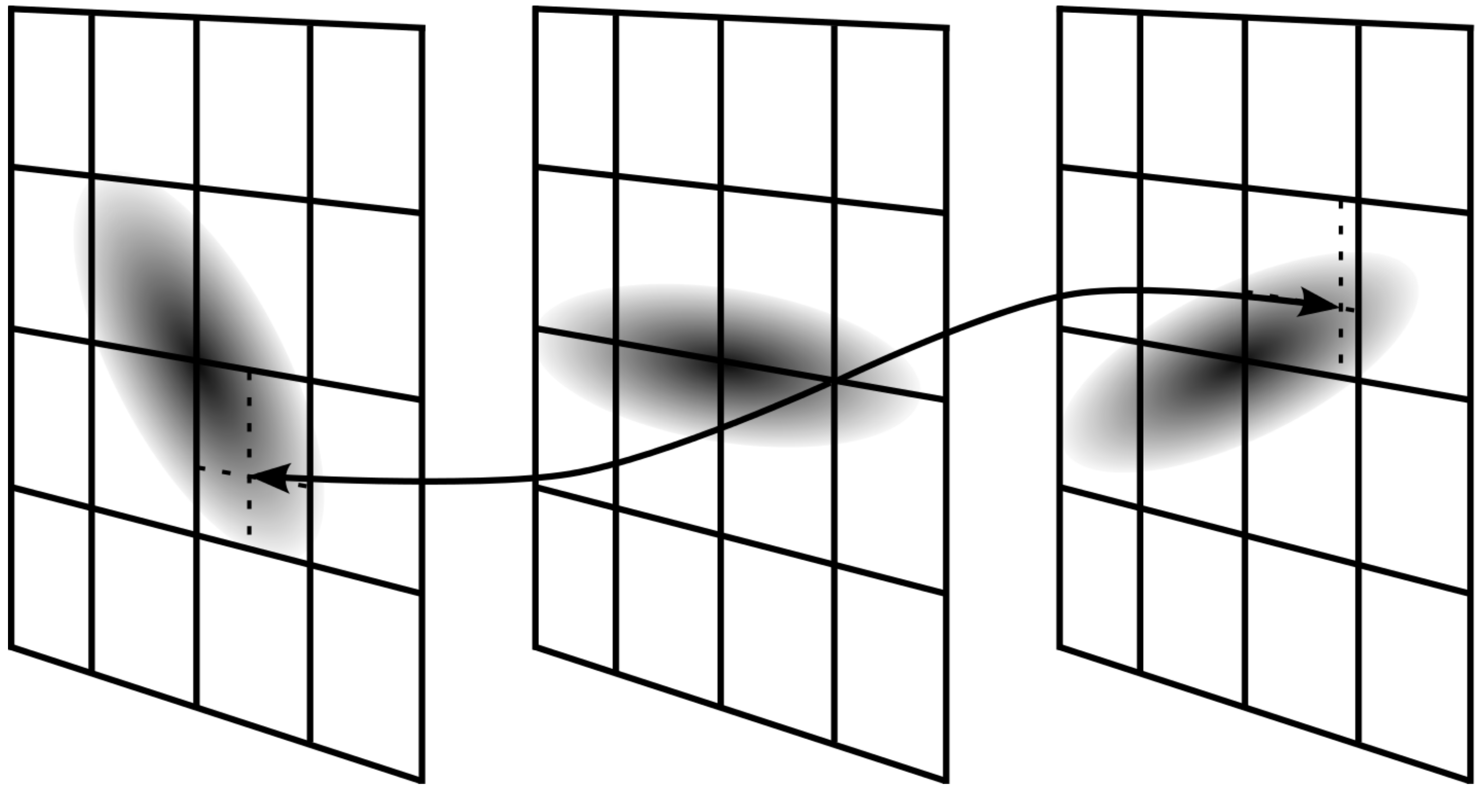
	Orthogonal ($\eta = 0$)	Poloidal pitch (η = const)	Poloidal shear ($\eta \neq$ const)
No pitch ($\nu = 0$)	2.00	2.14	2.00
Constant pitch (ν = const)	2.02	2.04	2.02
Shear ($\nu \neq$ const)	2.14	2.14	2.13

[1] Salari and Knupp, (2000) Tech. Report **SAND2000-1444**

J Leddy *et al* (2017) *Computer Physics Communications*

FCI method

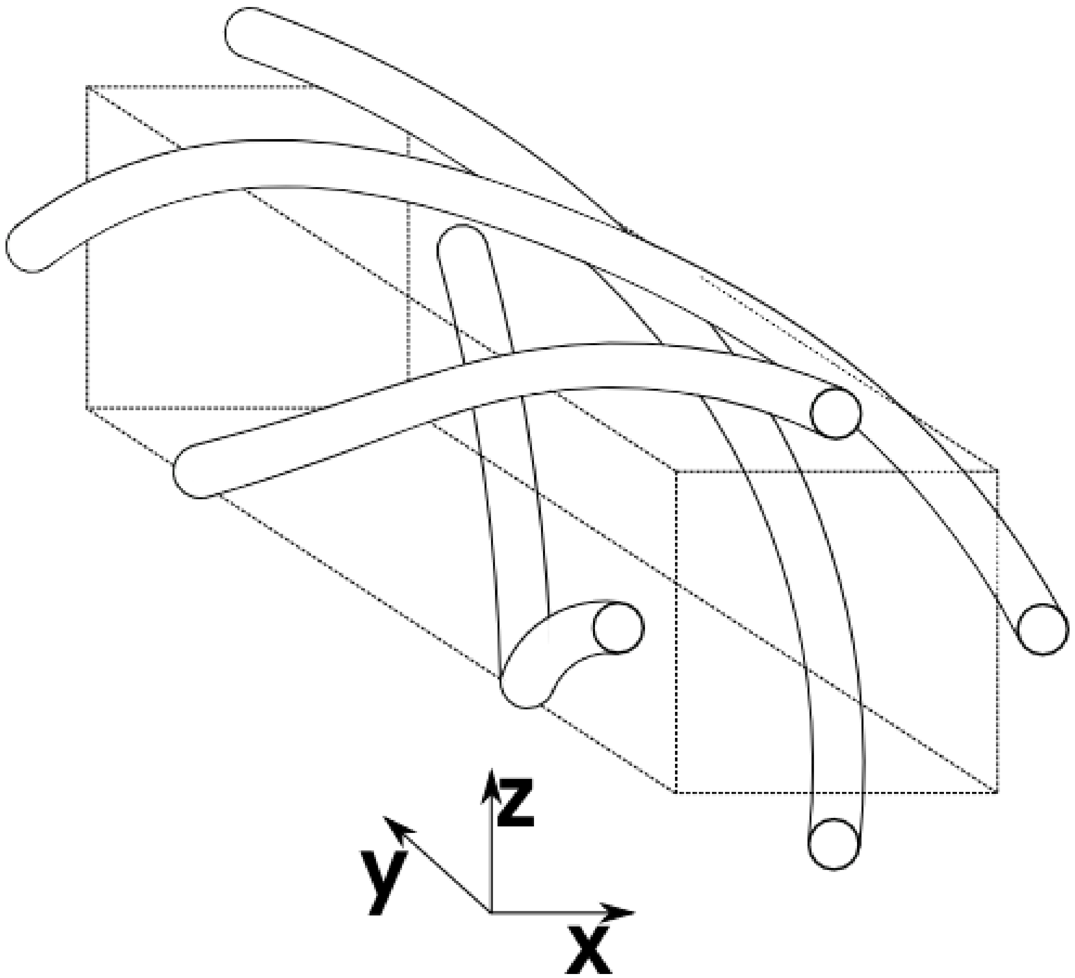
- In irregular and stochastic magnetic fields, having a flux coordinate independent (FCI) system can be preferable
- Cartesian planes – follow field lines and interpolate to perform parallel derivatives
- Benefits:
  - No assumption of flux surfaces
  - Parallel derivative entirely in parallel direction so no singularities in metric



Straight stellarator test

- As a test of the FCI Method, a straight stellarator was constructed
- Solved parallel diffusion equation to trace flux surfaces
- Inherent perpendicular diffusion reduced to tolerable levels ($<10^{-8}$) for ~1mm resolution

$$d_t\left(f\right) = \nabla^2_{\|} f$$

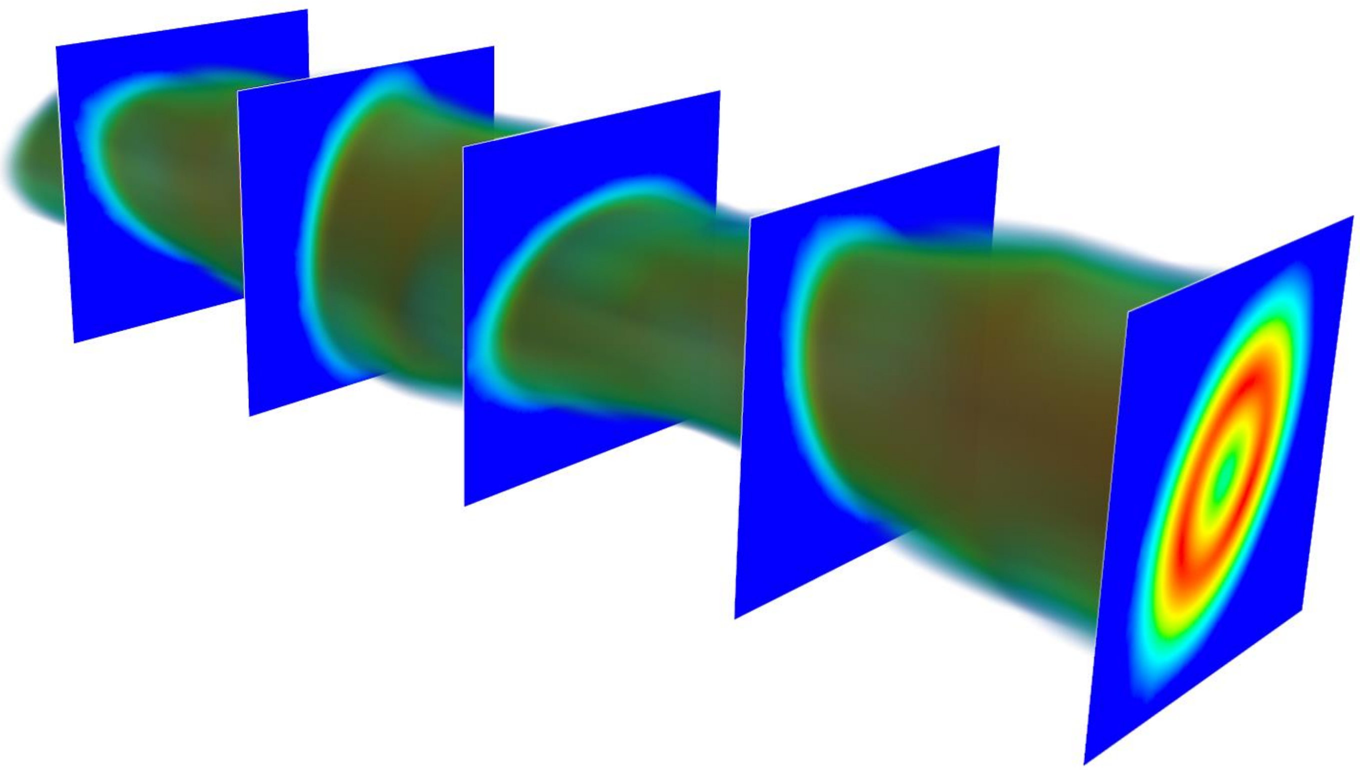


P Hill *et al* (2017) *Computer Physics Communications*
B Shanahan *et al* (2016) Accepted by *JP;CS*

Straight stellarator test

- As a test of the FCI Method, a straight stellarator was constructed
- Solved parallel diffusion equation to trace flux surfaces
- Inherent perpendicular diffusion reduced to tolerable levels ($<10^{-8}$) for ~1mm resolution

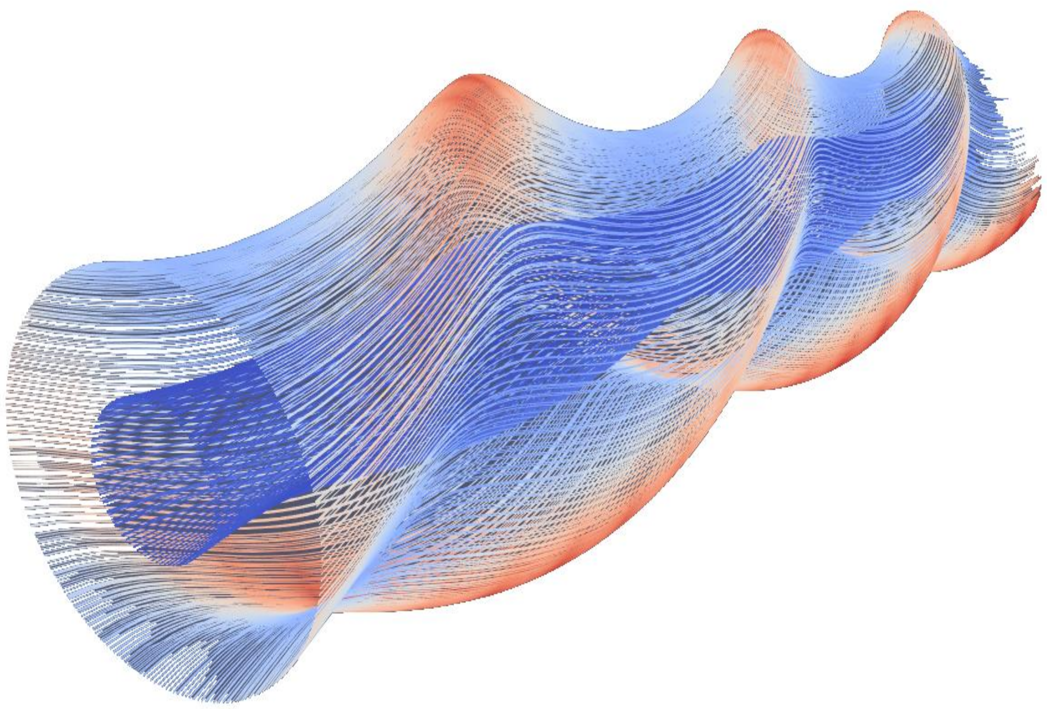
$$d_t\left(f\right) = \nabla^2_{\|} f$$

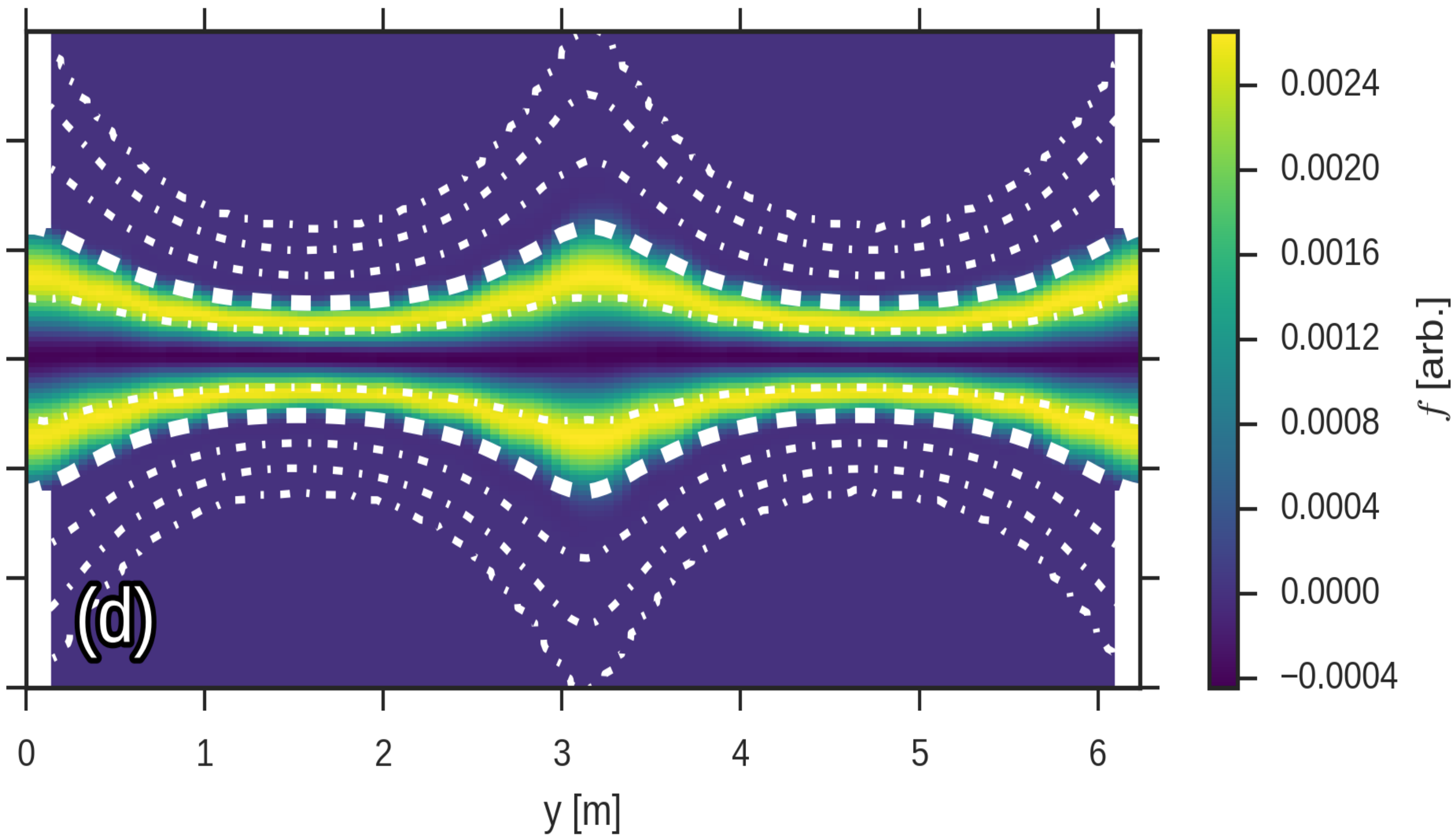


P Hill *et al* (2017) *Computer Physics Communications*
B Shanahan *et al* (2016) Accepted by *JP;CS*

Limiter boundary condition

- Recently implemented:
  - Grid generator which takes input from analytic functions, VMEC equilibria, etc.
  - Parallel boundary conditions/poloidal limiters





P Hill *et al* (2016) Submitted to Computer Physics Communications
B Shanahan *et al* (2016) Accepted by *JP;CS*

Outline

Numerical developments

- New coordinate system
- Flux-coordinate independent method

A new plasma model (Hermes)

- 2-fluid cold ion model in divergence form
- Including neutral interactions

Turbulence and Neutral Simulations

- Linear device
- MAST-U
- DIII-D

Multi-fluid codes

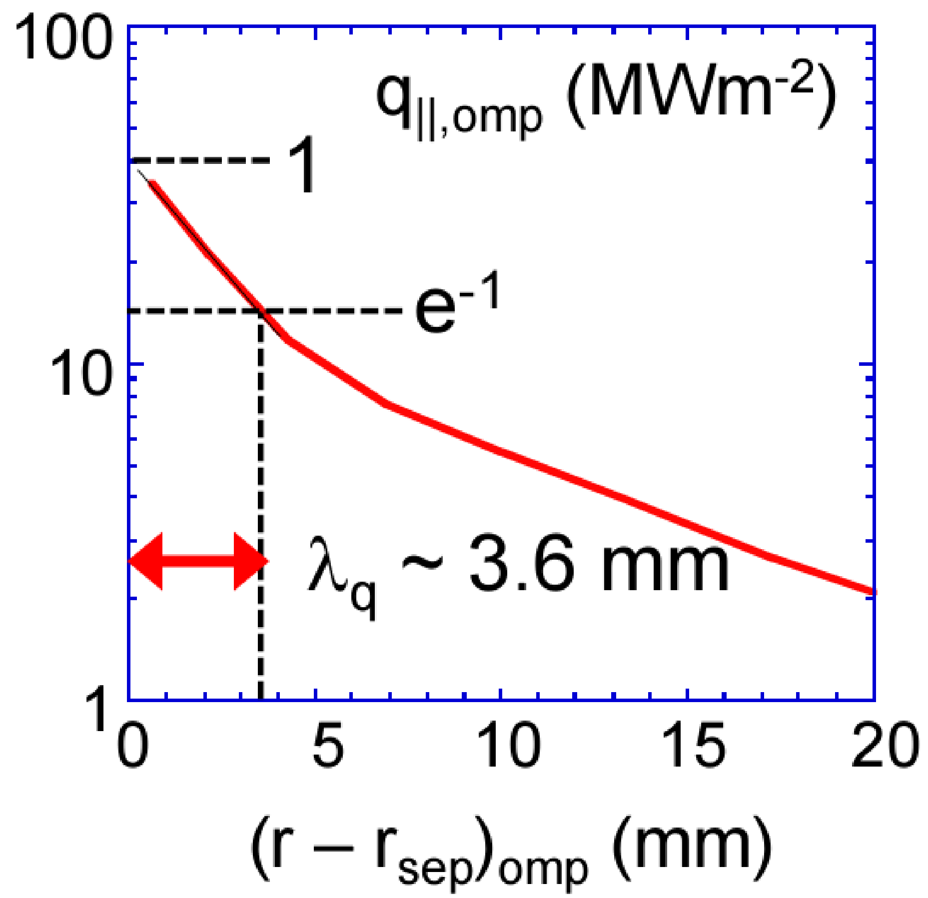
The workhorse of plasma boundary studies (e.g SOLPS, EDGE2D, UEDGE, SONIC, ...)

Include detailed physics of plasma-wall interaction

- Parallel transport of heat and particles
- Sheath physics
- Neutral gas recycling
- Impurities
- Divertor plates, baffles, ducts, slots, pumps,...

But

Simplified cross-field transport



$D_\perp$ = 0.3 m^{2}s^{-1}, $\chi_{\perp i,e}$ = 1.0 m^{2}s^{-1}

λ_q (omp) = 3 – 4 mm

R.Pitts, IAEA TM on Divertor Concepts (2015)

R Schneider et al. (2006) *Contributions to Plasma Physics*

S. Wiesen et al. (2015) *Journal of Nuclear Materials*

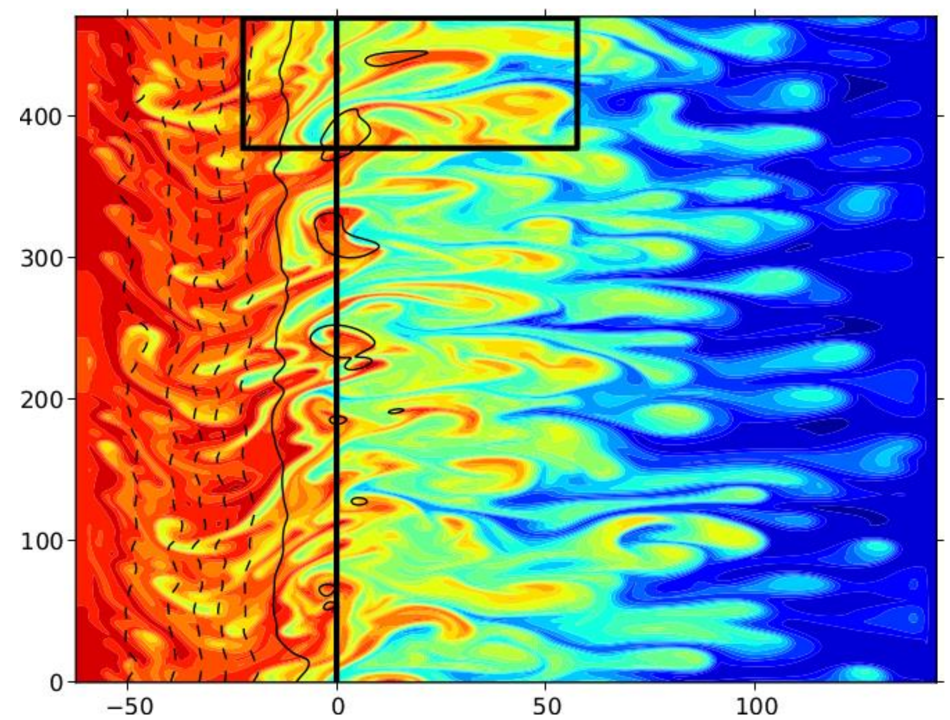
X. Bonnin et al. (2016) *Plasma Fusion Research*

Turbulence codes

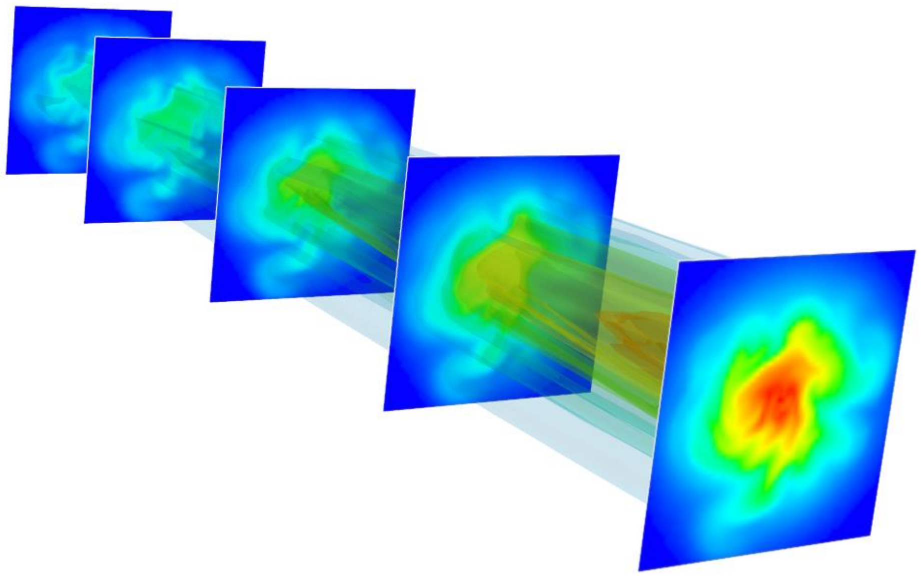
Calculating the turbulent transport requires solving for the time-varying plasma currents and electric fields

- Drift waves, ballooning/interchange instabilities, small-scale structure
- Computationally demanding, timesteps < ion cyclotron time
- Several codes under development (e.g. GBS, TOKAM-X, HESEL, BOUT++)

- Have not previously included detailed geometry, impurities, neutrals, ...



Dudson, IAEA TM on Plasma Instabilities (2014)



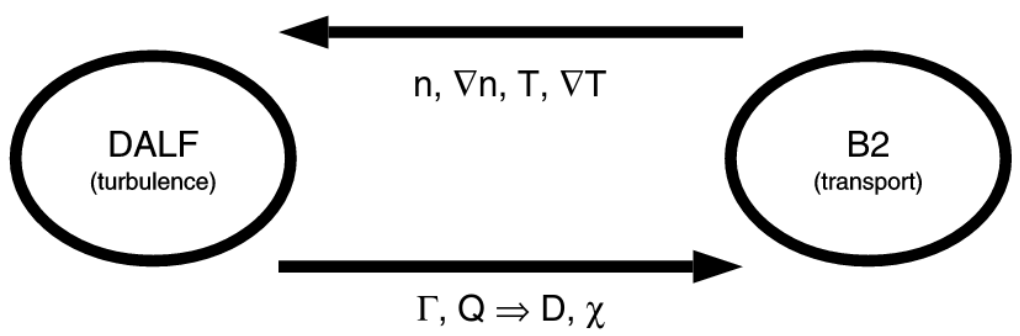
J.Leddy, PSI 2016

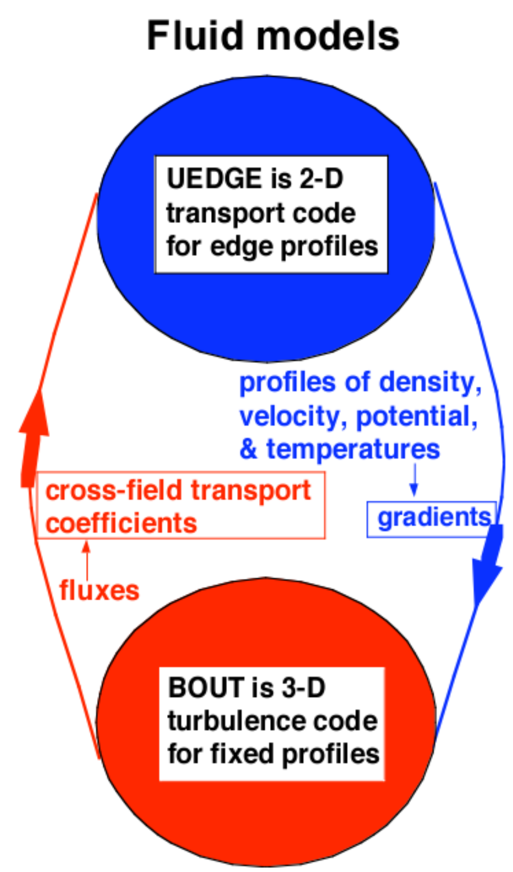
F D Halpern et al. (2016) *Journal of Computational Physics*

P Tamain et al. (2016) *Journal of Computational Physics*

Combining models

- Several attempts to combine transport models with turbulence codes
- Difficulties include
  - Consistency of underlying models
  - Separation of scales
  - Nonlinearity of atomic processes with density, temperature





Here the aim is to combine everything into one simulation, modelling “transport” and turbulence together

R Schneider et al. (2006) *Contrib. Plasma Phys.*
T.D.Rognlien, (2004) ECC meeting
Umansky M V and Rognlien T D (2005) *J. Nucl. Mater.*
F.Guzman et al. (2015) *PPCF*

The Hermes model

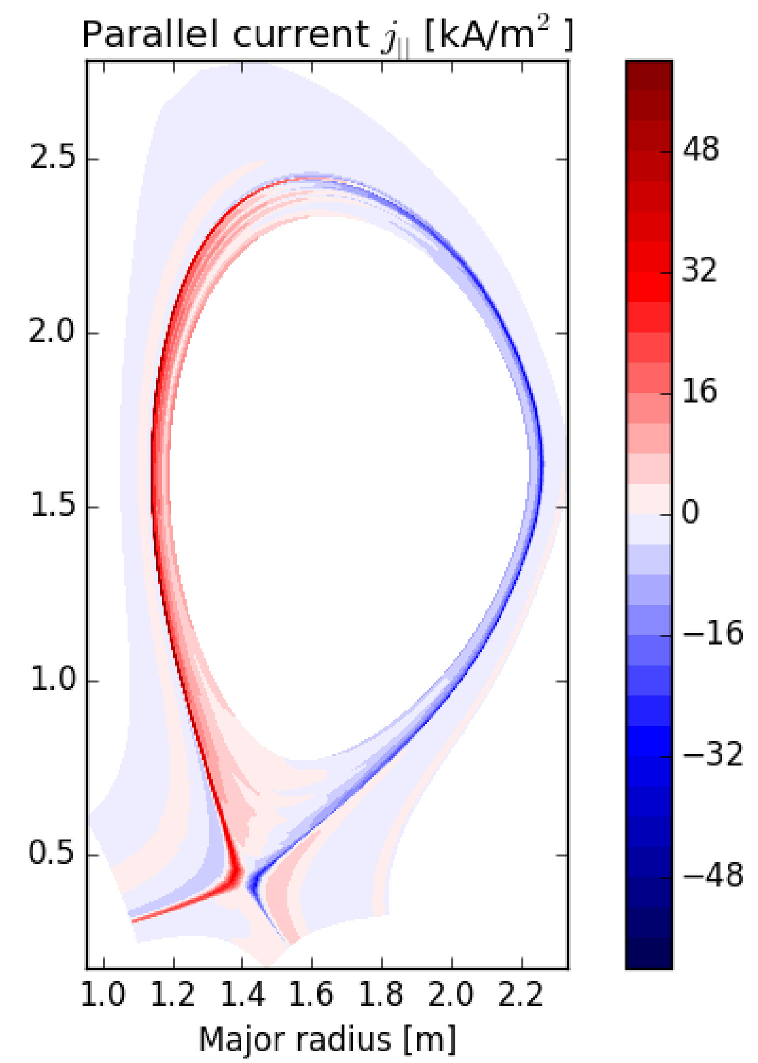
Based on **BOUT++**

https://github.com/boutproject/hermes

Current status

- Cold ion drift-fluid model
- Fluid neutrals: Diffusive, full Navier-Stokes, and hybrid models
- New differential operators for particle and energy conservation
- New electric field solver for n=0 mode

Flux-driven edge fluid simulations in X-point geometry



Under development

- Hot ion model
- EIRENE coupling for kinetic neutrals
- Pre-conditioners for faster simulation

Dudson and Leddy (2017) Submitted to *PPCF*

Model equations (1/2)

Evolving (electron) density n, electron pressure p

$$\frac{\partial n_e}{\partial t} = -\nabla \cdot \left[n_e \left(\mathbf{V}_{E\times B} + \mathbf{V}_{mag} + \mathbf{b}v_{||e}\right)\right] + \nabla \cdot \left(D_\perp \nabla_\perp n_e\right) + S_n$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} = -\nabla \cdot \left(\frac{3}{2}p_e \mathbf{V}_{E\times B} + \frac{5}{2}p_e\mathbf{b}v_{||e} + p_e\frac{5}{2}\mathbf{V}_{mag}\right) - p_e\nabla\cdot\mathbf{V}_{E\times B} + v_{||e}\partial_{||}p_e + \nabla_{||}\left(\kappa_{e||}\partial_{||}T_e\right) + 0.71\nabla_{||}\left(T_e j_{||}\right) - 0.71 j_{||}\partial_{||}T_e + \frac{\nu}{n}j_{||}^2 + \nabla\cdot\left(D_\perp T_e \nabla_\perp n_e\right) + \nabla\cdot\left(\chi_\perp n_e \nabla_\perp T_e\right) + S_p$$

With ExB and magnetic drifts given by:

$$\mathbf{V}_{E\times B} = \frac{\mathbf{b}\times\nabla\phi}{B} \qquad \mathbf{V}_{mag} = -T_e\nabla\times\frac{\mathbf{b}}{B}$$

Dudson and Leddy (2017) Submitted to *PPCF*

Model equations (2/2)

Flows and currents are evolved through the vorticity, ion parallel momentum, and vector potential

$$\frac{\partial \omega}{\partial t} = -\nabla\cdot\left(\omega \mathbf{V}_{E\times B}\right) + \nabla_{||} j_{||} - \nabla\cdot\left(n\mathbf{V}_{mag}\right) + \nabla\cdot\left(\mu_\perp \nabla_\perp \omega\right)$$

Boussinesq approximation

$$\omega = \nabla\cdot\left(\frac{n_0}{B^2}\nabla_\perp\phi\right)$$

$$\frac{\partial}{\partial t}\left(n_e v_{||i}\right) = -\nabla\cdot\left[n_e v_{||i}\left(\mathbf{V}_{E\times B} + \mathbf{b}v_{||i}\right)\right] - \partial_{||}p_e + \nabla\cdot\left(D_\perp v_{||i}\nabla_\perp n\right) - F$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\beta_e\psi - \frac{m_e}{m_i}\frac{j_{||}}{n_e}\right] = \nu\frac{j_{||}}{n_e} + \partial_{||}\phi - \frac{1}{n_e}\partial_{||}p_e - 0.71\partial_{||}T_e + \frac{m_e}{m_i}\left(\mathbf{V}_{E\times B} + \mathbf{b}v_{||i}\right)\cdot\nabla\frac{j_{||}}{n_e}$$

Finite electron mass, electromagnetic

Dudson and Leddy (2017) Submitted to *PPCF*

Conservation properties

- Movement of particles and thermal energy done using finite volumes (fluxes through cell faces), so particles conserved to high precision

Conserved energy

$$E = \int dv \left[\frac{m_i n_0}{2B^2} \left|\nabla_\perp \phi\right|^2 + \frac{1}{2} m_i n V_{||i}^2 + \frac{3}{2} p_e + \frac{1}{4}\beta_e \left|\nabla_\perp \psi\right|^2 + \frac{m_e}{m_i}\frac{1}{2}\frac{j_{||}^2}{n} \right]$$

B.Scott (2003) *Physics of Plasmas*

Boundary conditions

Interaction with plasma sheath a complex problem. Here relatively simple boundary conditions are used (multiple options in code for boundary conditions)

Ion velocity goes to the sound speed

$$v_{||i} \geq c_s \qquad c_s = \sqrt{eT_e/m_i}$$

Conducting wall

$$j_{||} = en_e \left[v_{||i} - \frac{c_s}{\sqrt{4\pi}} \exp\left(-\left\{\phi/T_e\right\}\right) \right]$$

Sheath heat flux transmission

$$q = v_{||i}\left(\frac{1}{2}m_i n_e v_{||i}^2 + \frac{5}{2}p_e\right) - \kappa_{||e}\partial_{||}T_e = \gamma_s n_e T_e c_s$$

with $\gamma_s = 6.5$

P C Stangeby. (2000) The Plasma Boundary of Magnetic Fusion Devices

J Loizu, et al. (2012) *Physics of Plasmas*

M U Siddiqui et al. (2016) *Physics of Plasmas*

New solver for electric potential

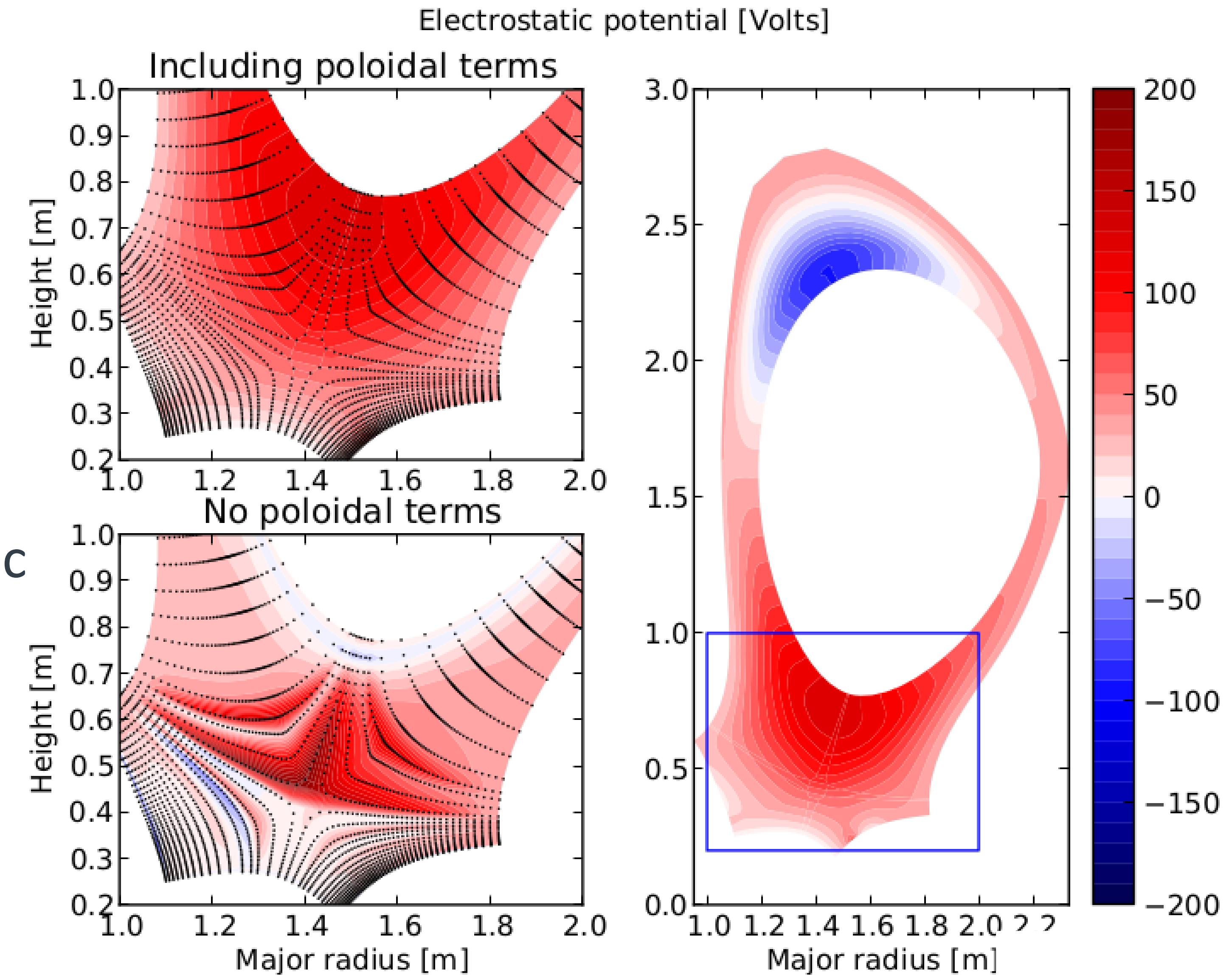
To calculate electrostatic potential we invert the vorticity:

$$\nabla \cdot \left(\frac{m_i n}{B^2}\nabla_\perp \phi\right) = \frac{1}{J}\frac{\partial}{\partial u^i}\left(J\frac{m_i n}{B^2} g^{ij}\left(\nabla_\perp \phi\right)_j\right)$$

For low-n modes the poloidal terms become important

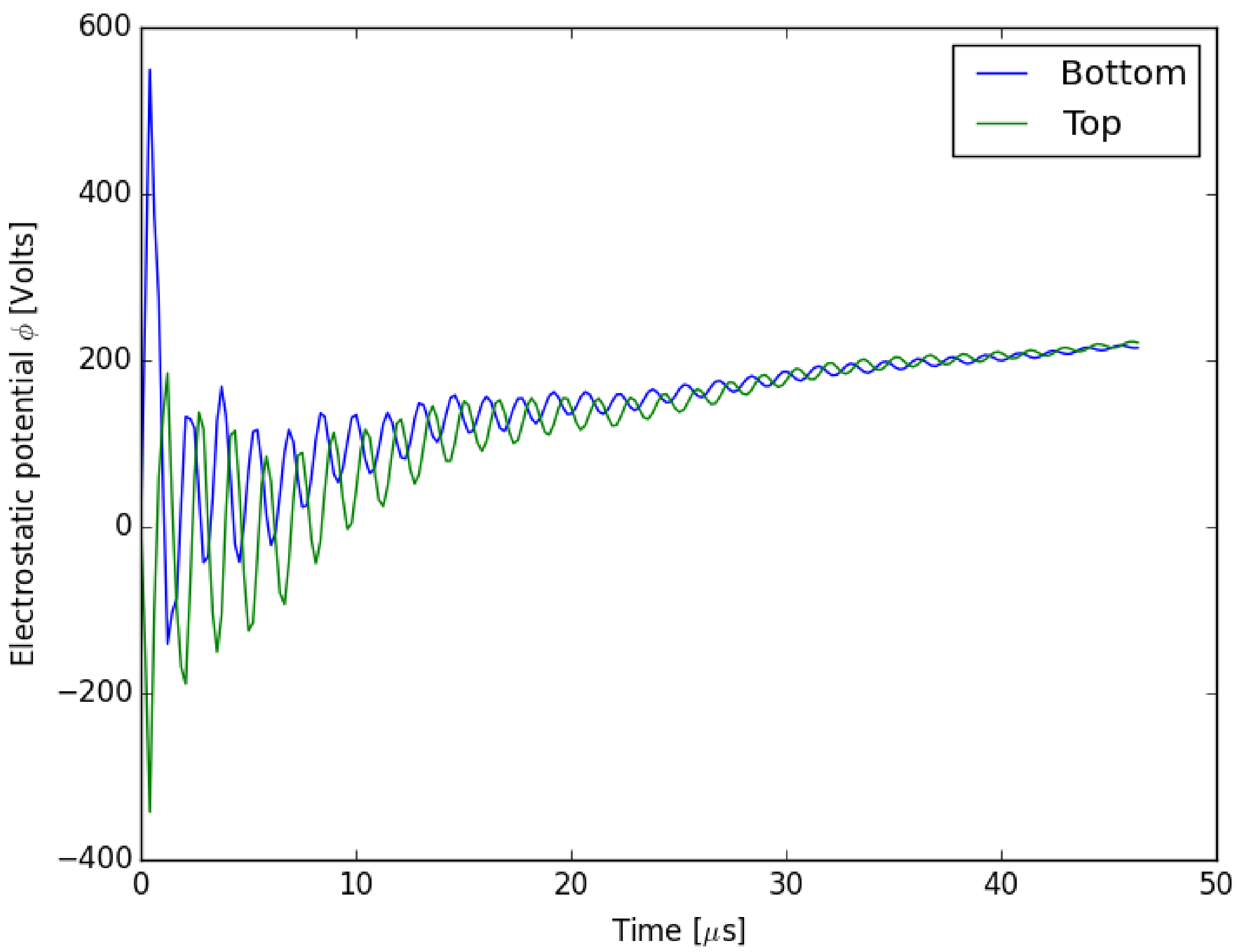
Around the X-point unphysical oscillations occur if poloidal terms are neglected

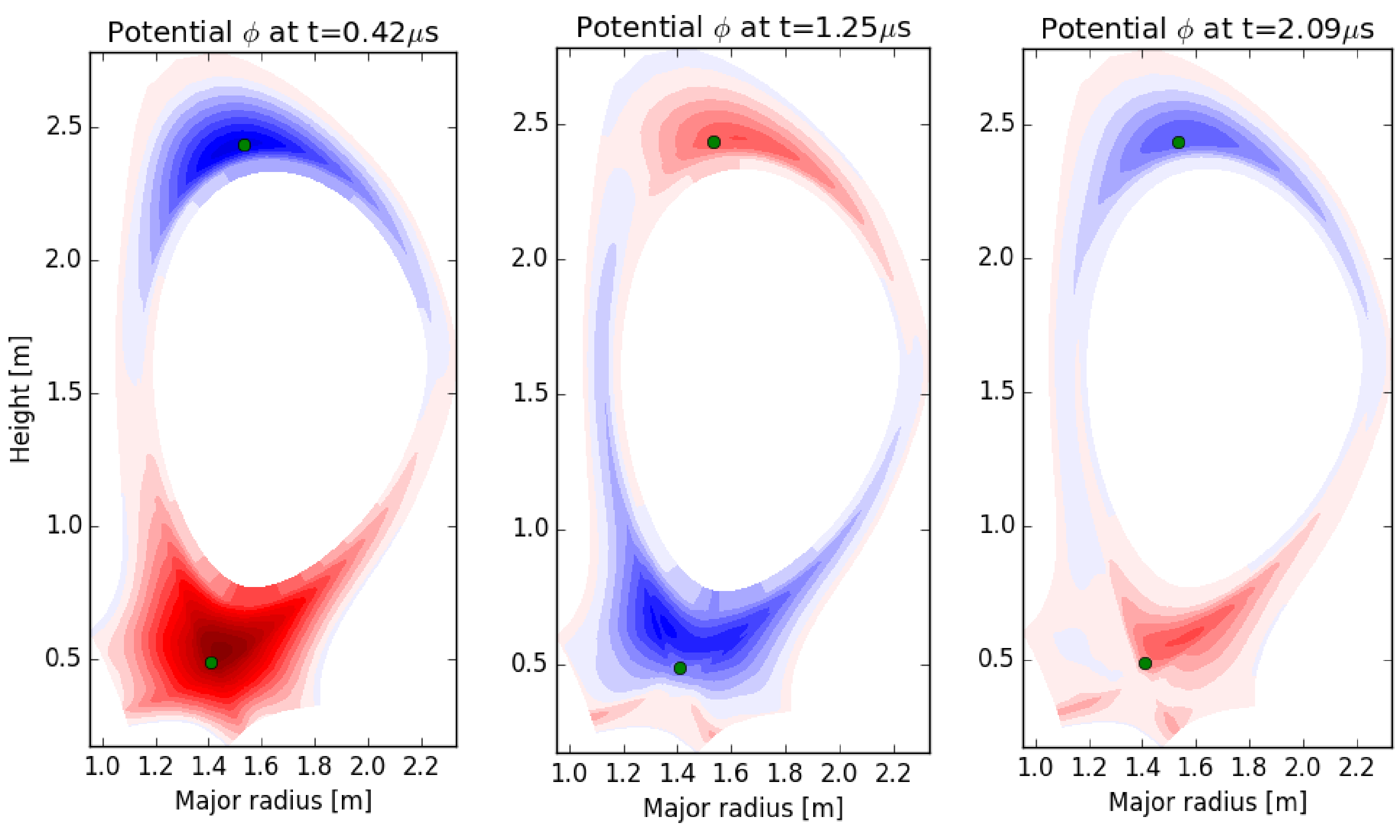
→ New solver implemented using PETSc for axisymmetric (n=0) component



Successfully evolve n=0 potential

- Initial Alfvénic oscillations f~500 kHz damp on ~20 μs timescale

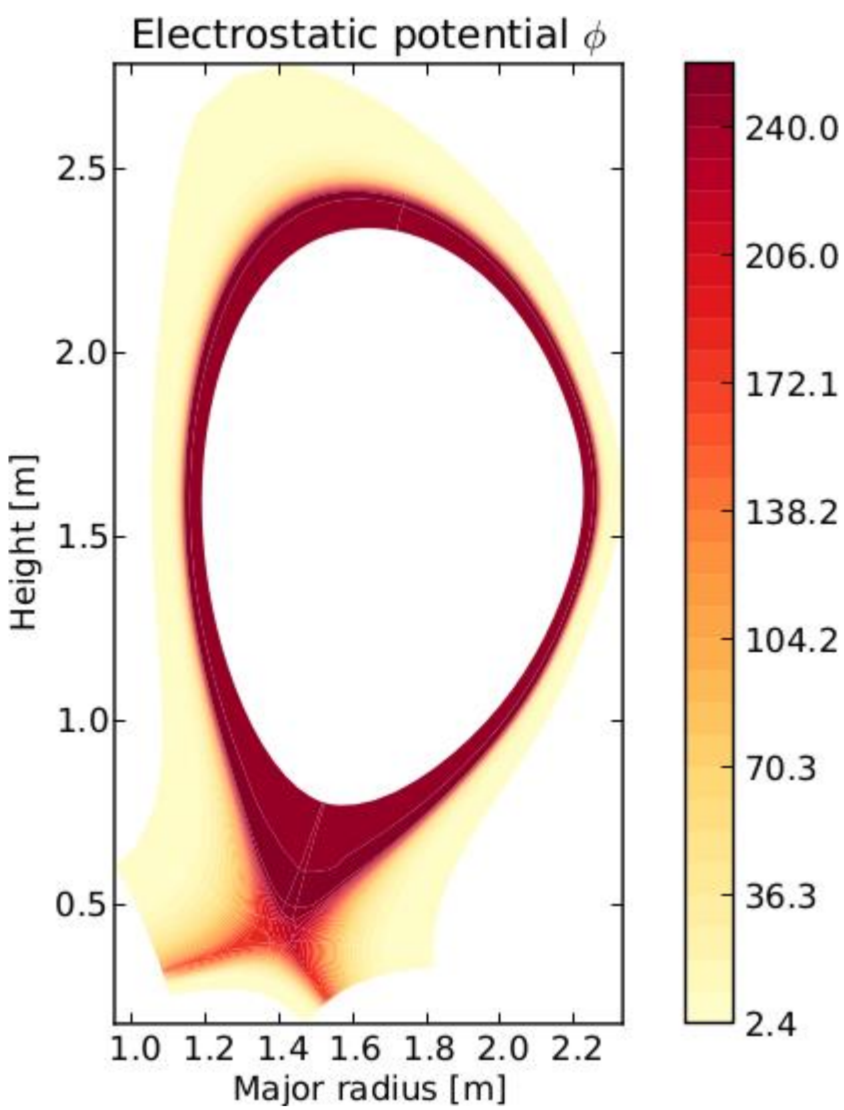


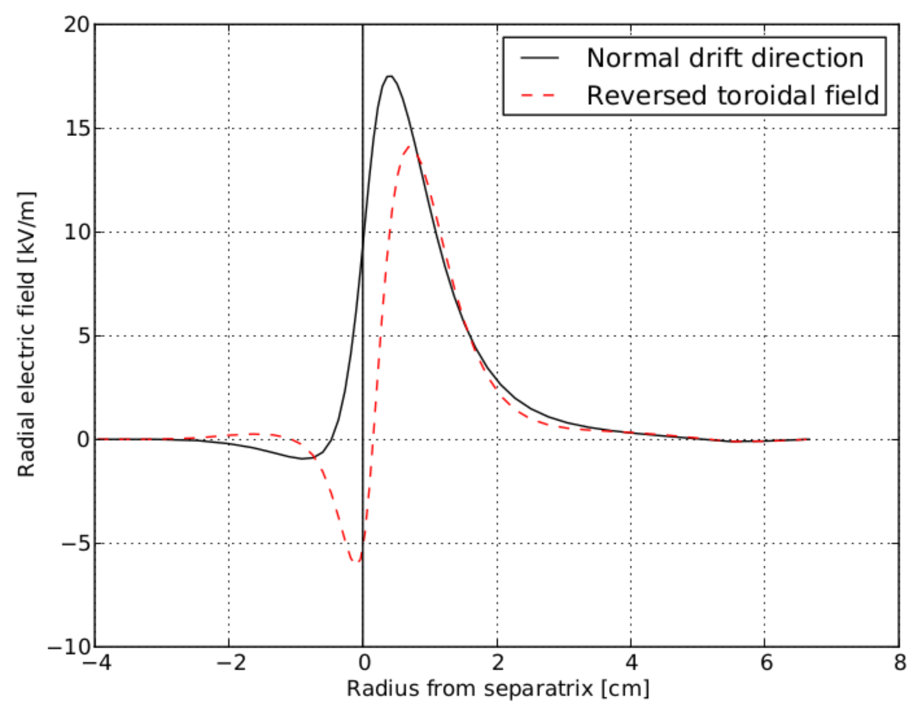


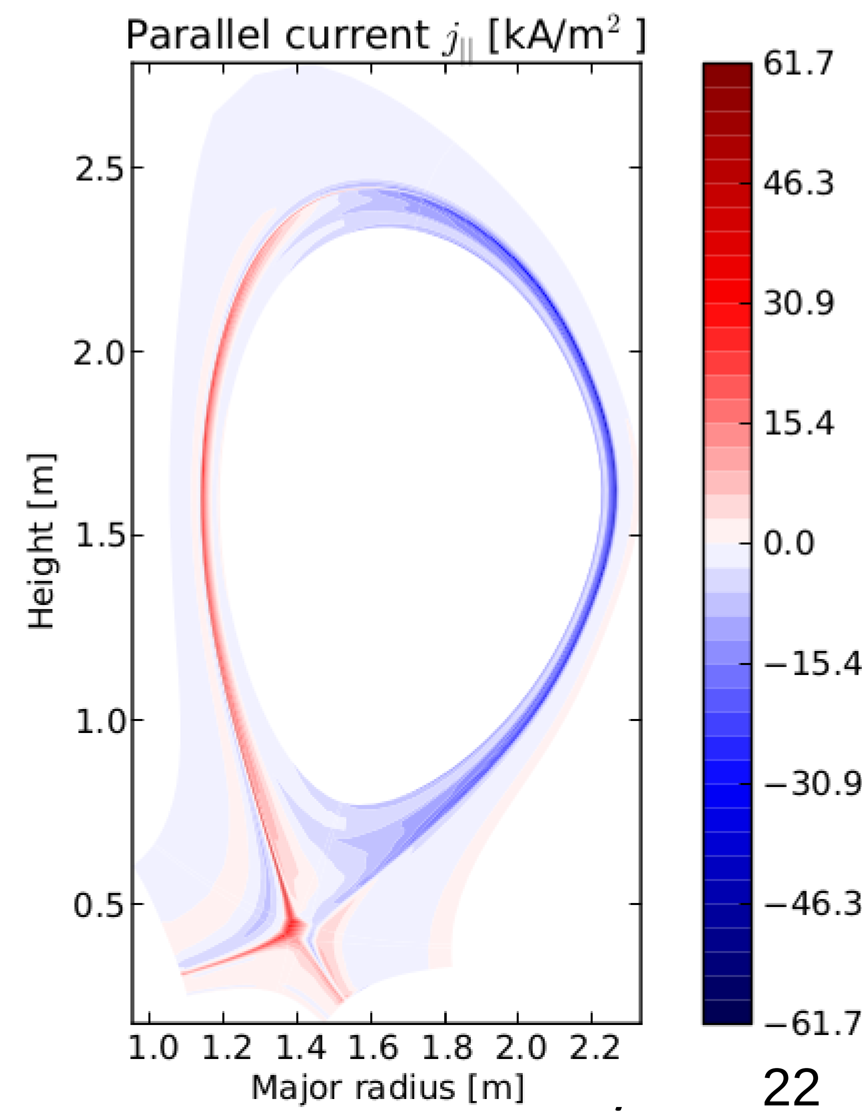
→ First time this has been possible with BOUT / BOUT++ in X-point geometry

Radial electric field

- Quasi-steady state has large radial electric field in SOL, driven by sheath and parallel electron force balance
- Reversing toroidal field modifies E_r near separatrix
- Poloidal rotation sensitive to subtle effects, missing e.g. ion pressure







Neutral gas model (1/2)

Neutral gas is modelled as a fluid

$$\frac{\partial n_n}{\partial t} = -\nabla \cdot [\mathbf{V_n} n_n] + S$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p_n\right) = -\nabla \cdot \mathbf{q}_n + \mathbf{V}_n \cdot \nabla p_n + E$$

$$\mathbf{q}_n = \frac{5}{2}p_n \mathbf{V}_n - \kappa_n \nabla T_n$$

Where S and E represent transfer of particles and energy between plasma and neutrals.

- Long mean free path of neutrals means Monte-Carlo treatment necessary in many cases
- Molecules not included. Can be important in high density regions
- Fluid model allows qualitative analysis and interpretation

Neutral gas model (2/2)

Model follows approach used in UEDGE

- Parallel to the magnetic field the neutral momentum equation is:

$$\frac{\partial}{\partial t}\left(m_i n_n V_{||n}\right) = -\nabla\cdot\left[m_i n_n V_{||n}\mathbf{b}V_{||n}\right] - \partial_{||}p_n + F$$

- Perpendicular to the magnetic field, neglect neutral inertia, and balance neutral pressure against friction:

$$\mathbf{F}_\perp \simeq -\nu\mathbf{V}_{n\perp}$$

$$\mathbf{V}_{n\perp} = -\frac{1}{\nu}\nabla_\perp p_n$$

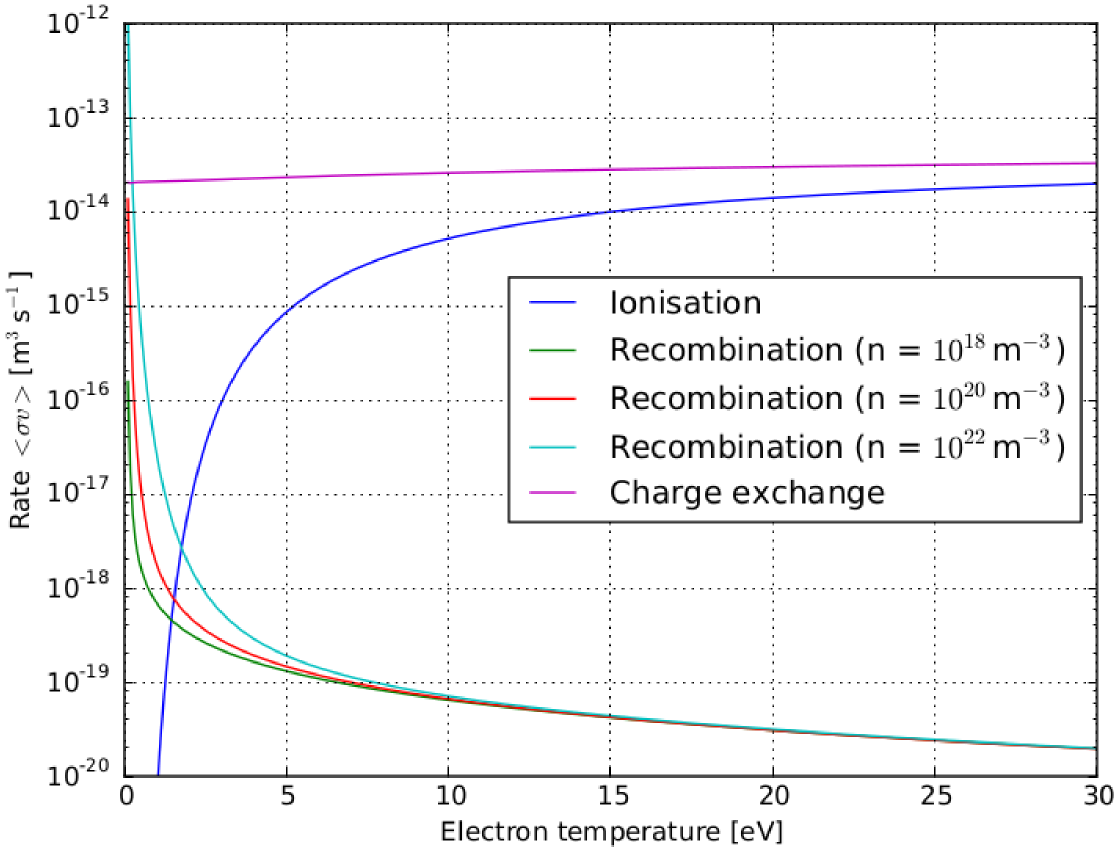
$$\nu = \nu_{cx} + \nu_{iz} + \nu_{nn}$$

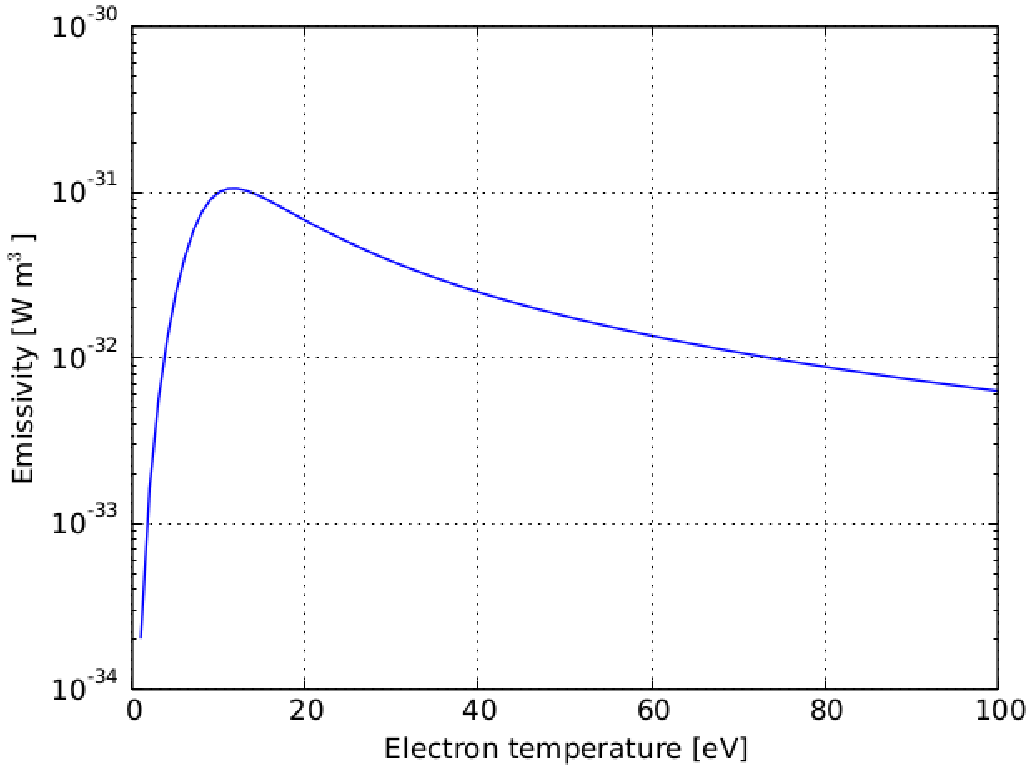
Collision rate = Charge exchange, ionisation, neutral-neutral

M.Umansky et al. (2003) *Journal of Nuclear Materials*
F.Wising et al. (1996) *Contrib. Plasma Phys.*

Atomic physics

- No molecular processes, only atoms evolved
- Simple semi-analytic fits used for atomic processes: Ionisation, recombination and charge exchange
- Provide source/sinks of particles, momentum and energy





- Carbon impurity included using fixed ion fraction (1% typically)
- Analytic radiation curve from Hutchinson thermal fronts paper

I.H.Hutchinson, (1994) *Nuclear Fusion*

Outline

Numerical developments

- New coordinate system
- Flux-coordinate independent method

A new plasma model (Hermes)

- 2-fluid cold ion model in divergence form
- Including neutral interactions

Turbulence and Neutral Simulations

- Linear device
- MAST-U
- DIII-D

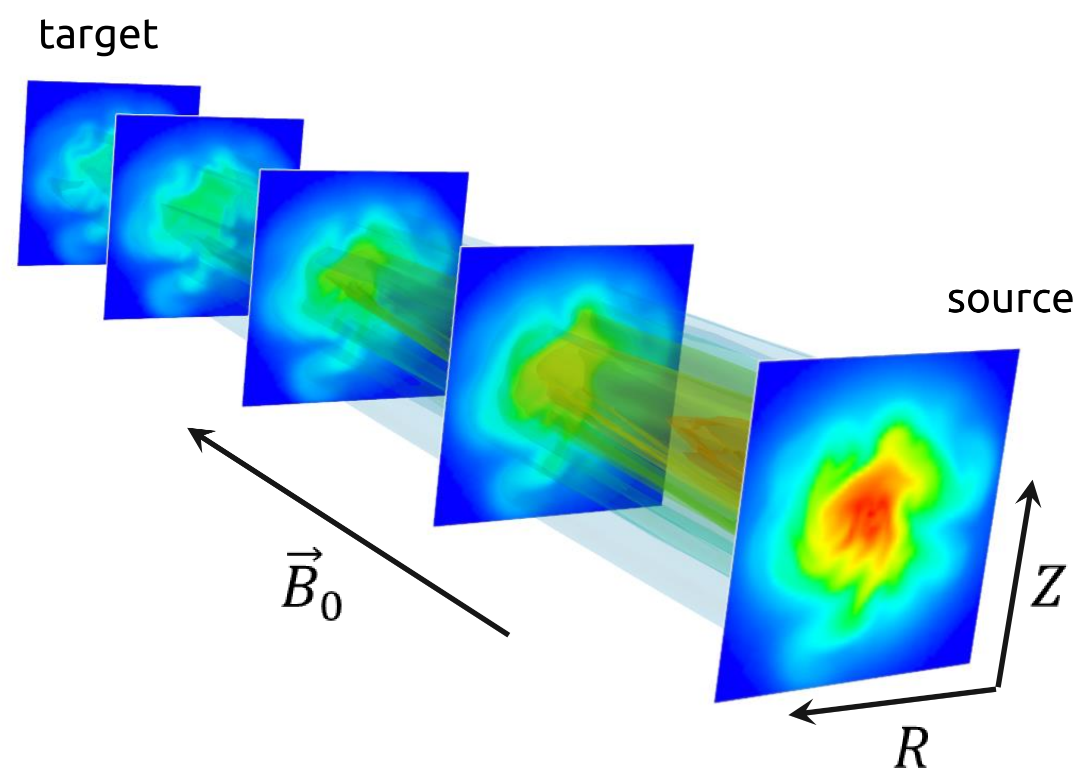
Combining turbulence + neutrals

Turbulence + neutrals, linear geometry

- Linear devices have simple geometries, making them a nice test-bed for plasma-neutral interaction

- We have simulated a small Magnum-PSI sized device with the following parameters:

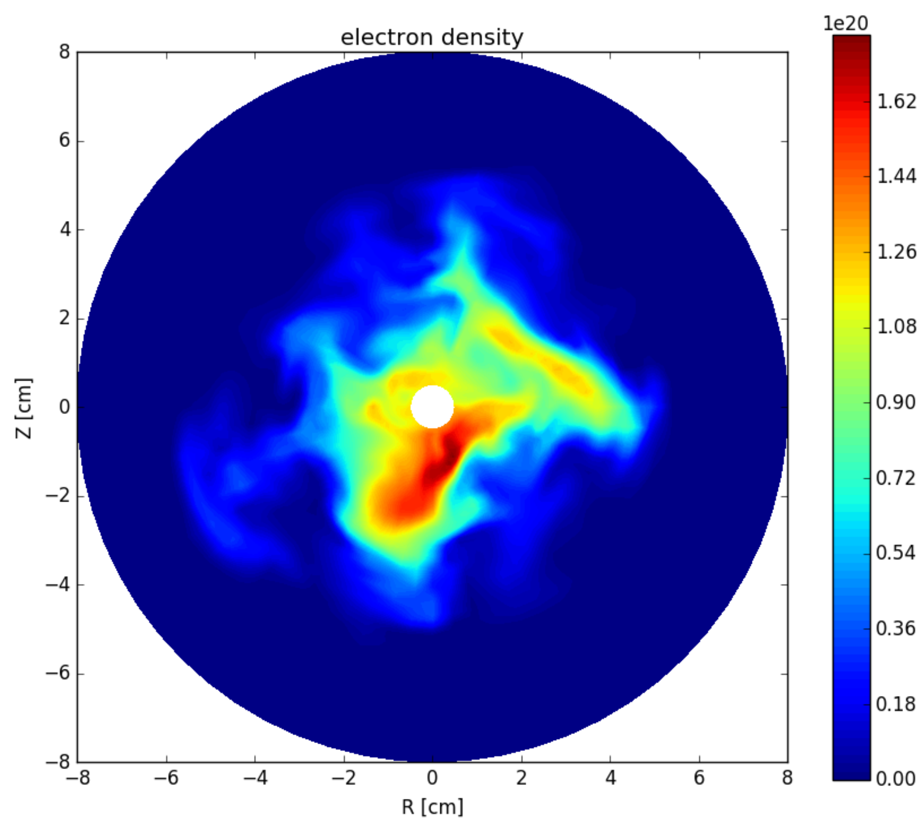
Magnetic field	0.15 T
Length	1.2 m
Radius	10 cm



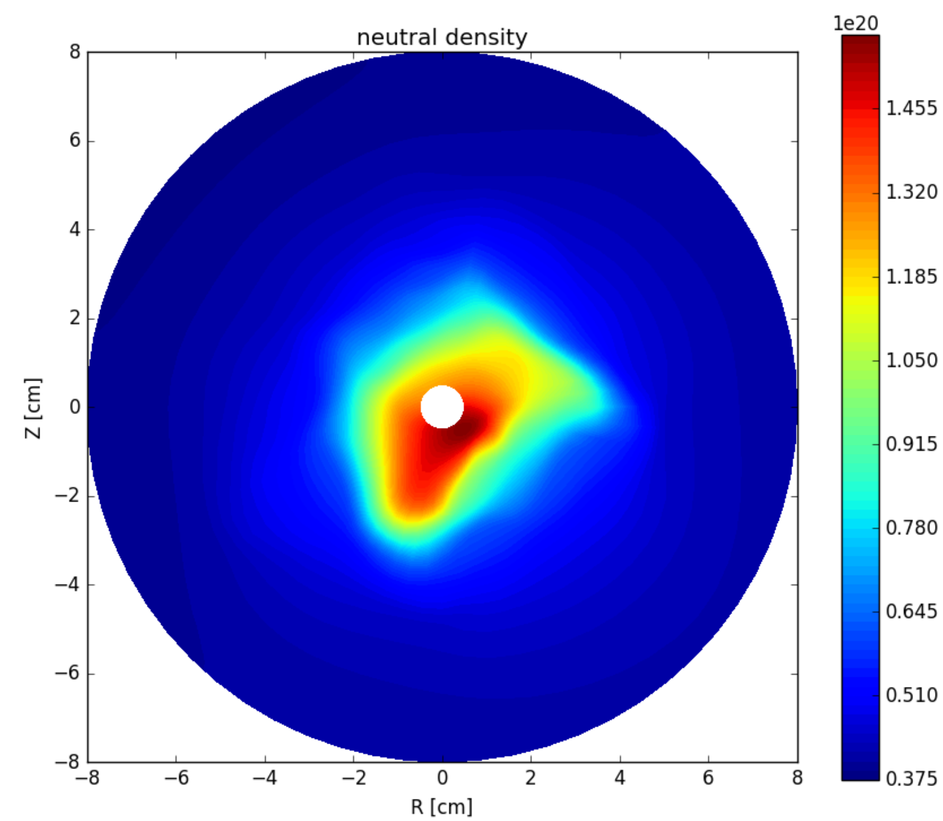
J.Leddy et al. (2016) In Press *Journal of Nuclear Materials & Energy*

Combining turbulence + neutrals

- Strong turbulence leads to significant modification of profiles (aided by insulating sheath boundary condition)
- Peak density off-axis at times
- Affects interaction with neutrals: only sources of neutrals are recycling at the target, and volume recombination



Electron density

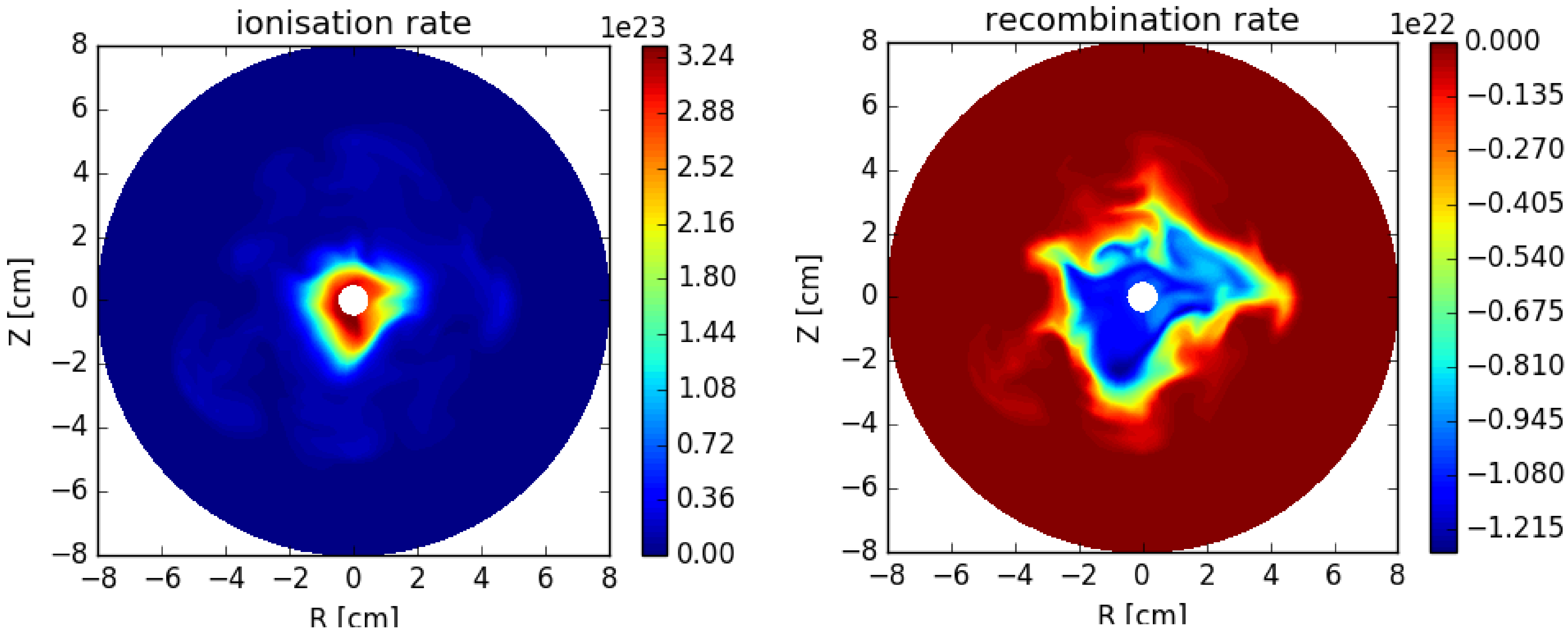


Neutral density

J.Leddy et al. (2016) In Press *Journal of Nuclear Materials & Energy*

Particle source/sinks

- Ionisation mainly occurs in highest density and temperature regions of the plasma (centre of eddies)
- Recombination is localised to the high density but low temperature regions (edge of the eddies)

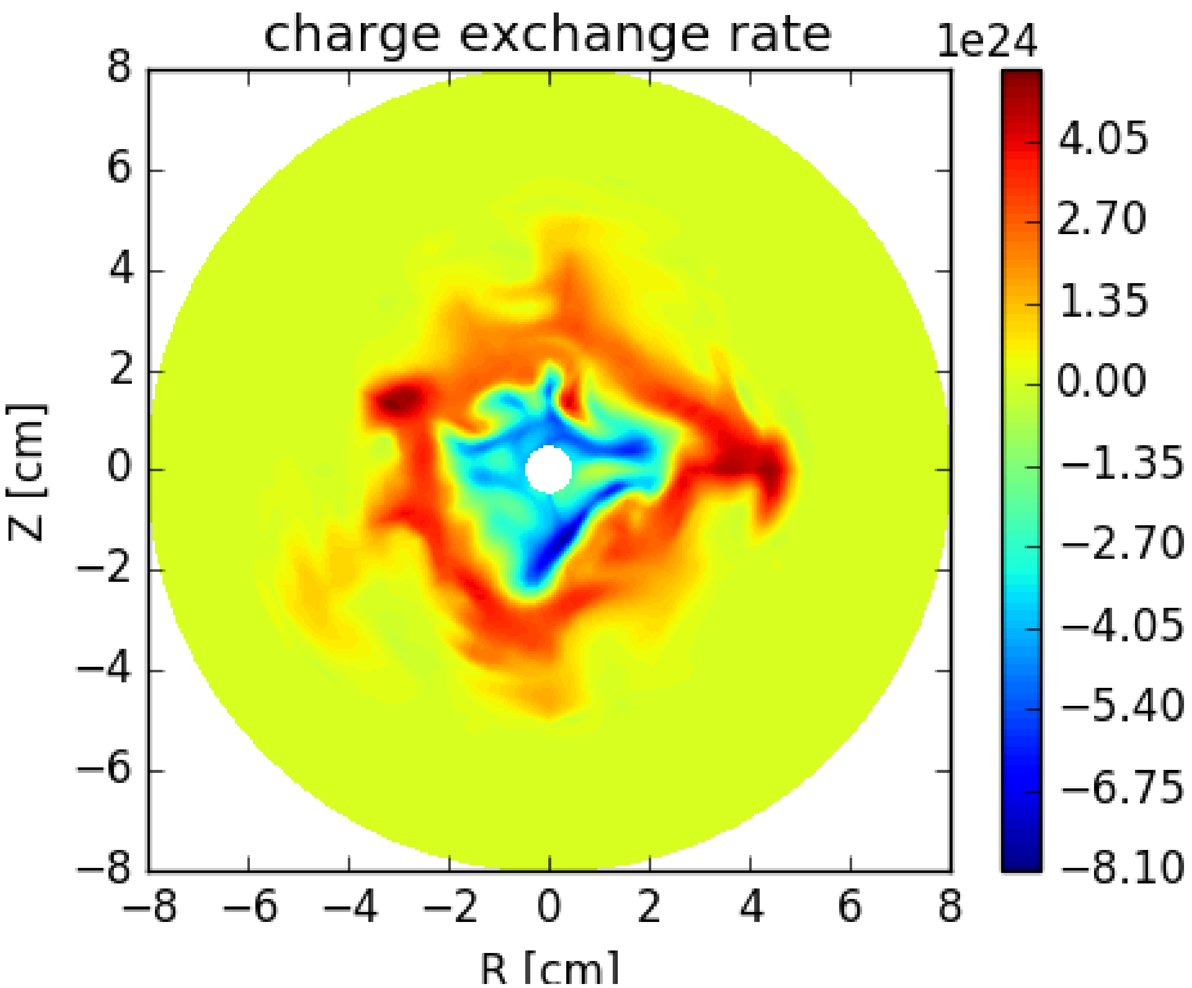


J.Leddy et al. (2016) In Press *Journal of Nuclear Materials & Energy*

Charge-exchange

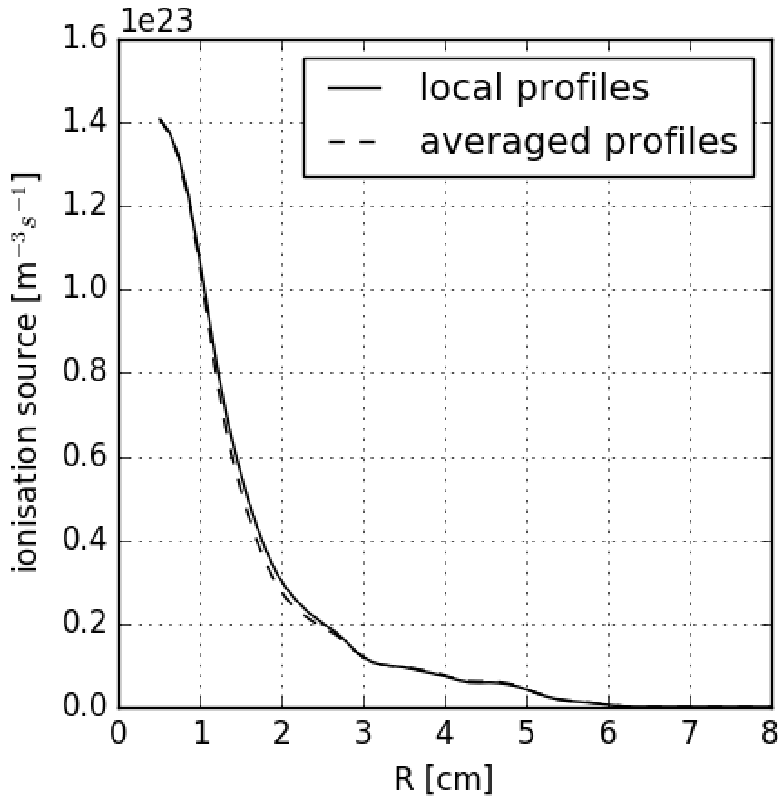
- Significant energy is only removed where the temperature difference is greatest ($T_e - T_n$)
- Energy removed from plasma in centre (hottest region)
- Energy transferred to plasma in the edge, where $T_n > T_e$

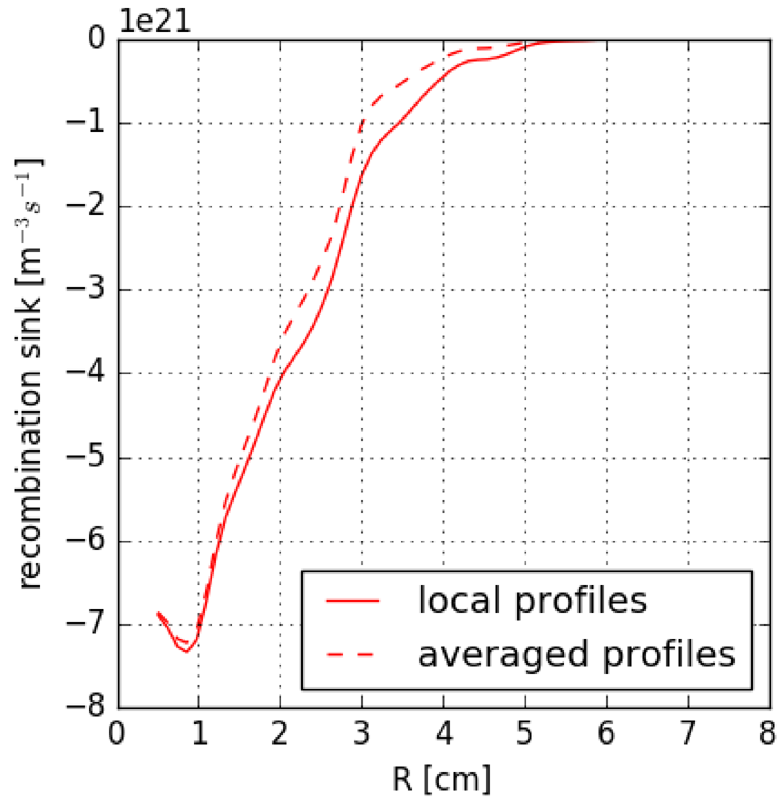
- Note: cold ion model, so electron temperature used for atomic processes



J.Leddy et al. (2016) In Press *Journal of Nuclear Materials & Energy*

Effect of fluctuations



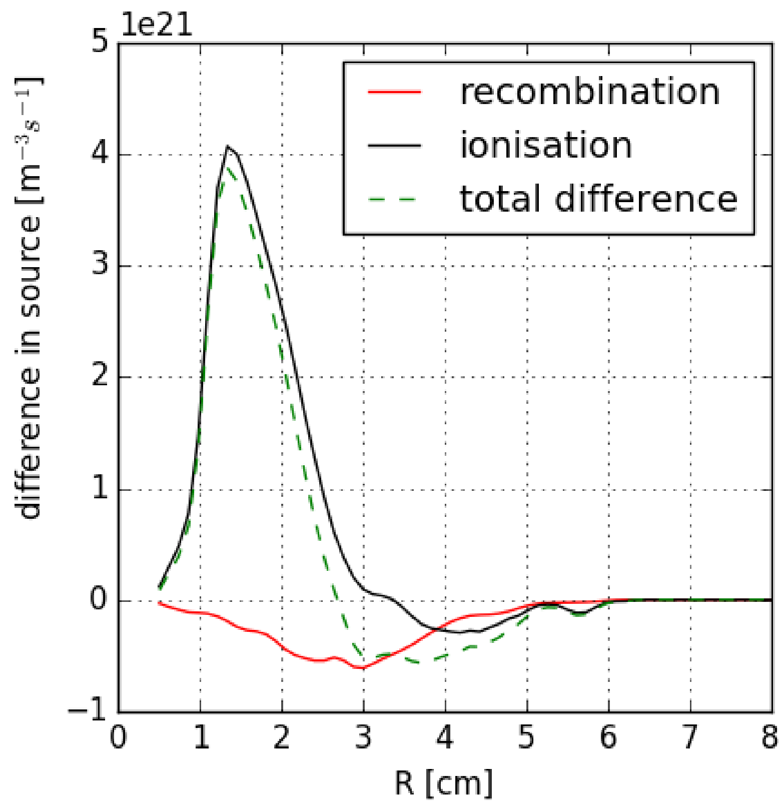


Local profiles

$$n_e n_n \langle \sigma v \rangle (T_e)$$

Axially-averaged profiles

$$\bar{n}_e \bar{n}_n \langle \sigma v \rangle (\bar{T}_e)$$



- Averaged over 8000 ω_{ci}^{-1} (~0.17ms)
- Consistently higher neutral source with turbulence than without
- Difference in source/sinks peaks off-axis
- ~10% max difference in ionisation, ~50% max difference in recombination

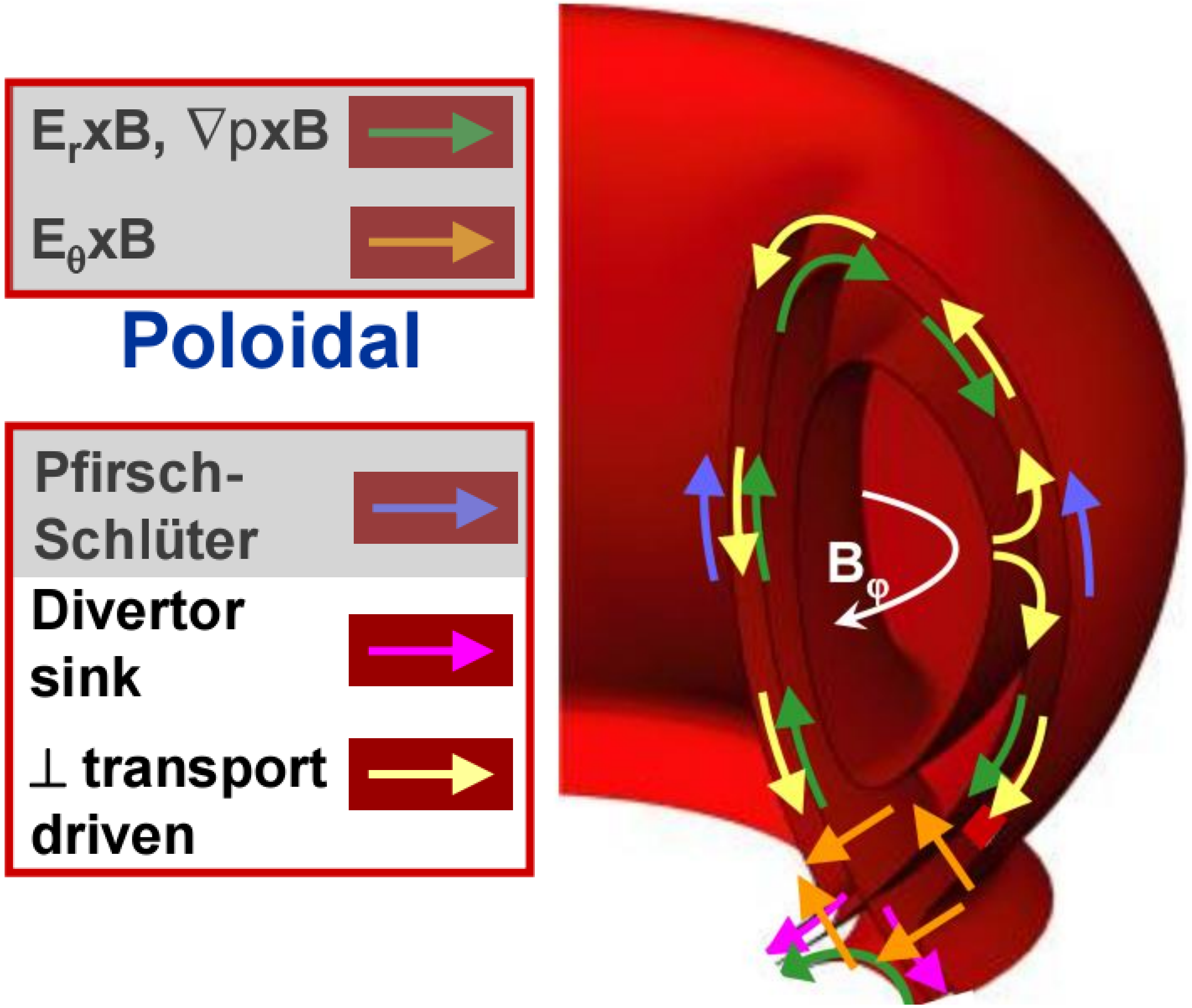
J.Leddy et al. (2016) In Press *Journal of Nuclear Materials & Energy*

Including drifts is challenging

- Balance of diamagnetic, parallel and polarisation currents
- Sheath currents at divertor
- Electric fields modify flows, edge asymmetries

Introduces rapid timescales: Alfven waves, electron parallel dynamics

- Typically reduces timestep by factor of ~10
- Can lead to numerical instabilities

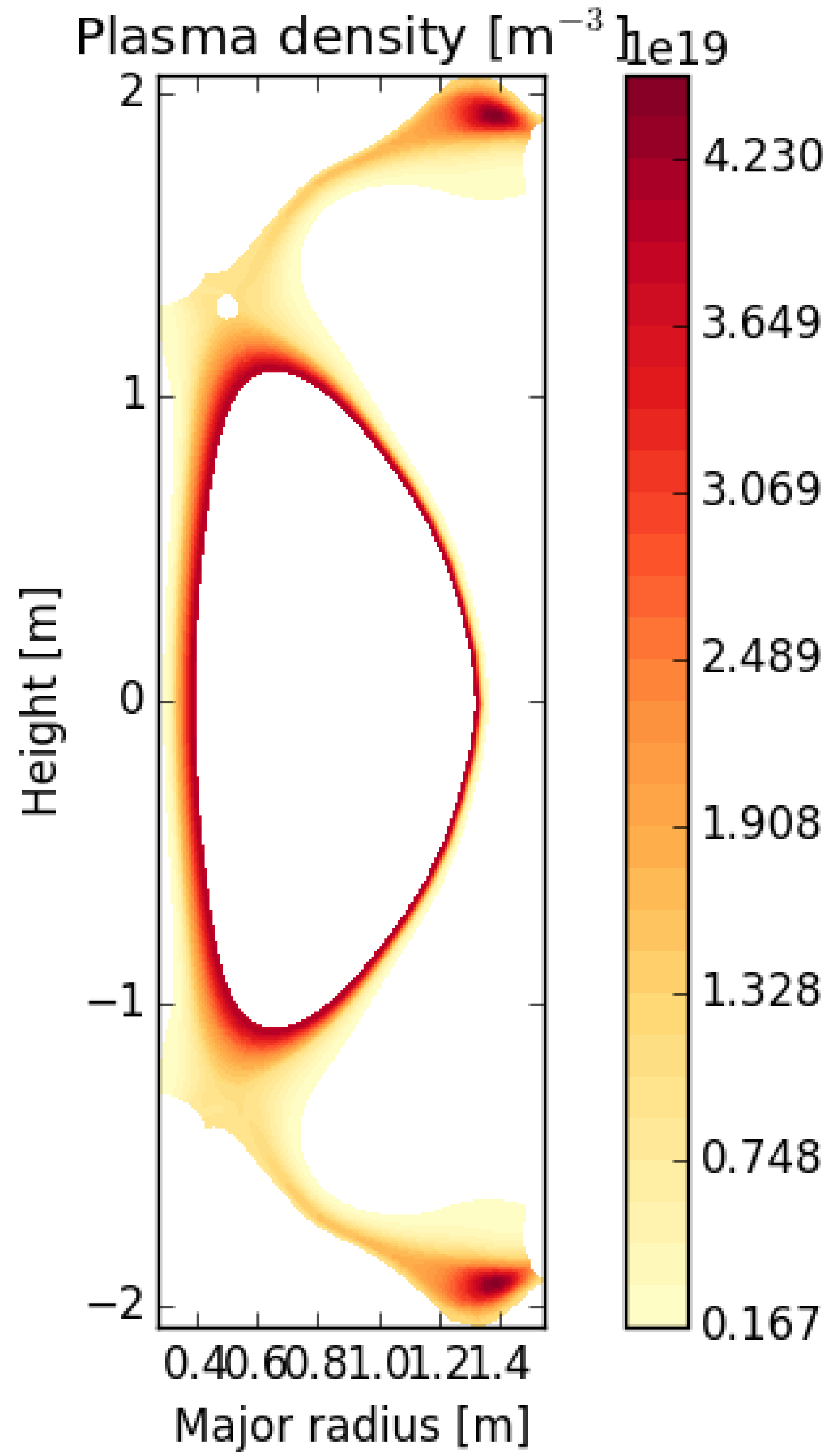


R.Pitts (2015) IAEA TM on Divertor Concepts

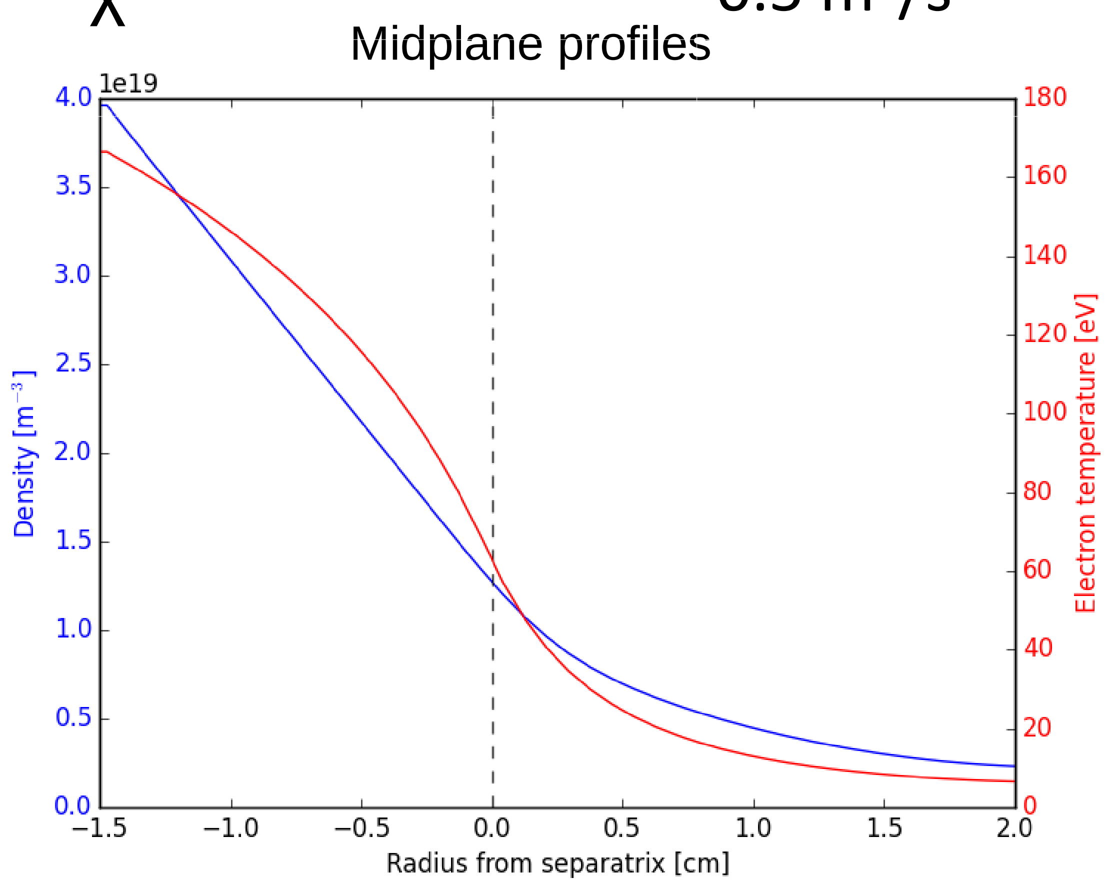
T D Rognlien, et al. (1999) *Physics of Plasmas*

MAST-Upgrade simulations

Axisymmetric fluid simulation: No electric fields, no turbulence



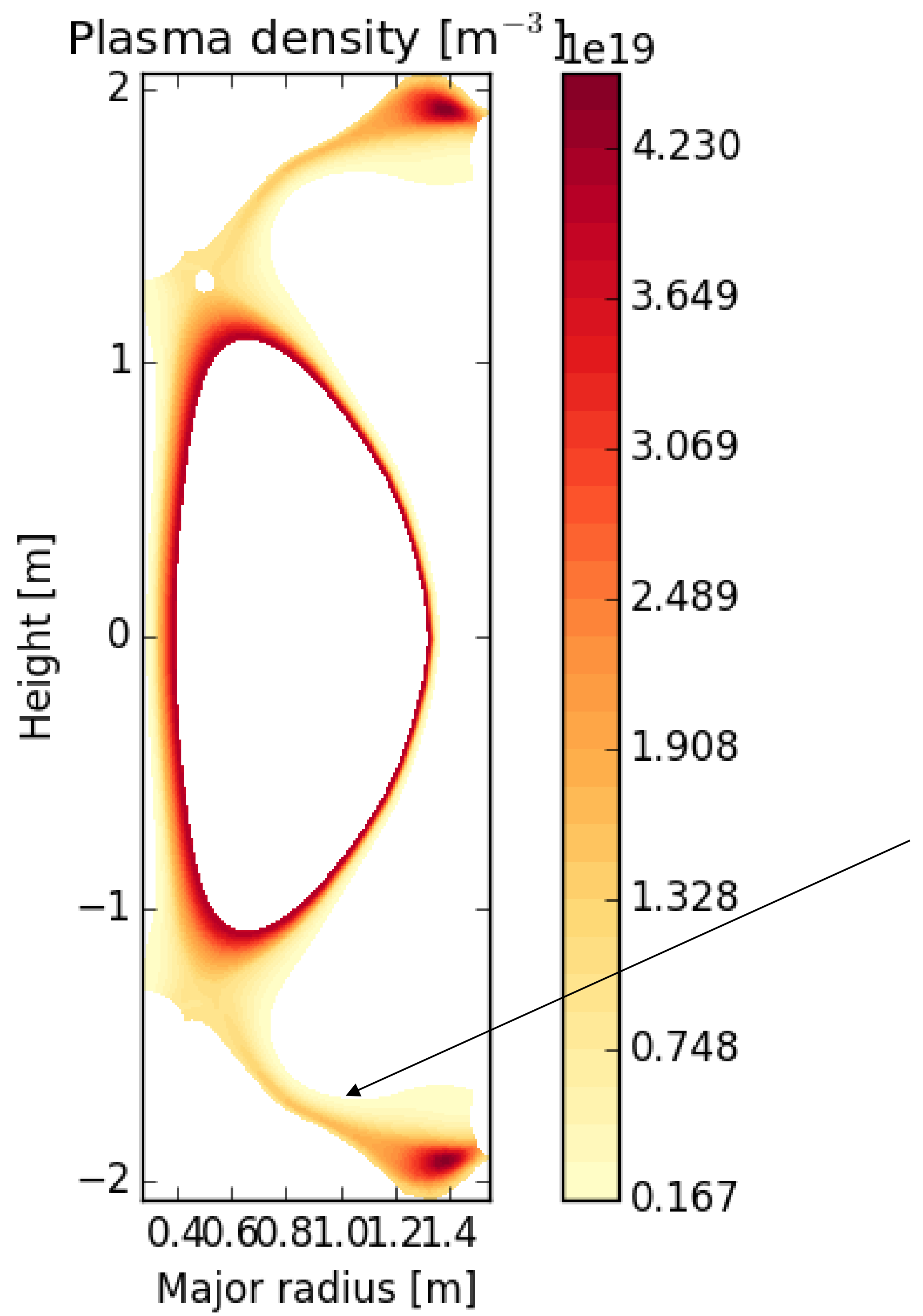
Recycling fraction	80%
Carbon fraction	1%
$n_{e,sep}$	1.3 x 10^{19} m^{-3}
$T_{e,sep}$	63 eV
D	0.2 m^2/s
X	0.5 m^2/s



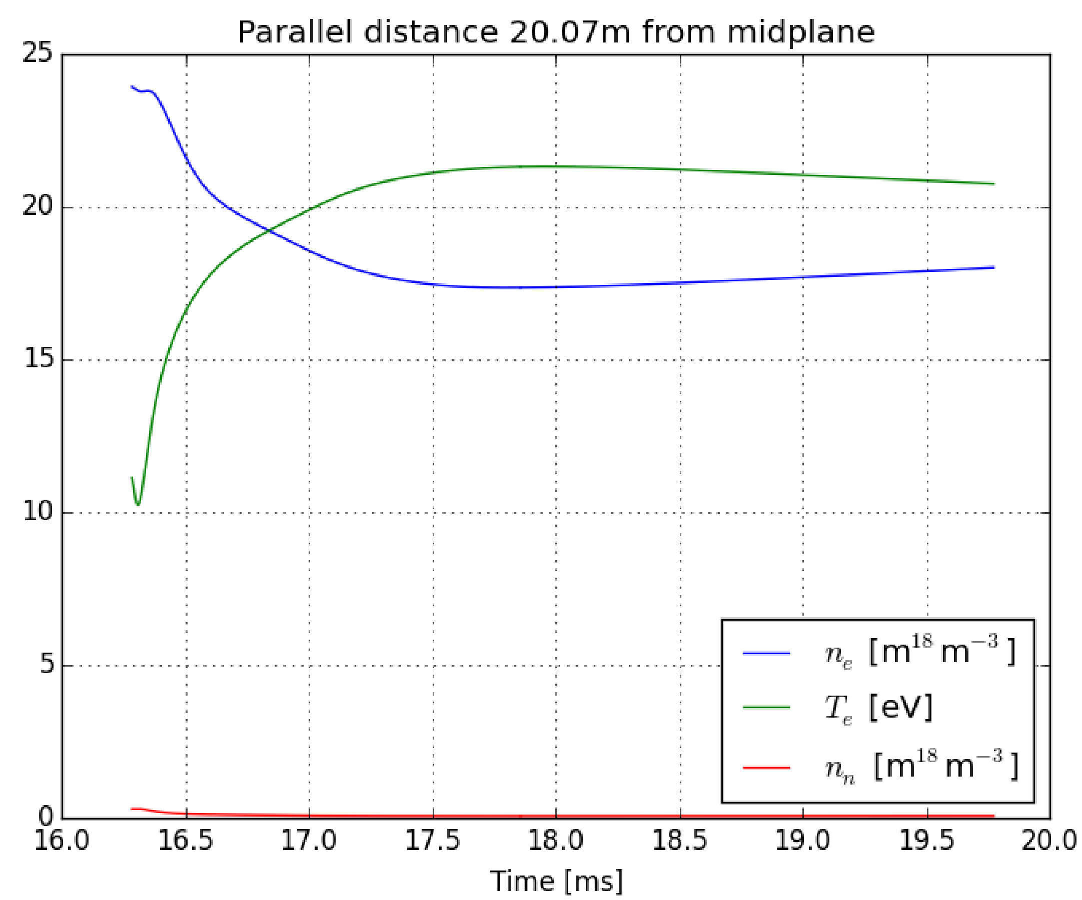
E.Havlicova, et al. (2014) *Contributions to Plasma Physics*

MAST-Upgrade simulations

Axisymmetric fluid simulation: No electric fields, no turbulence



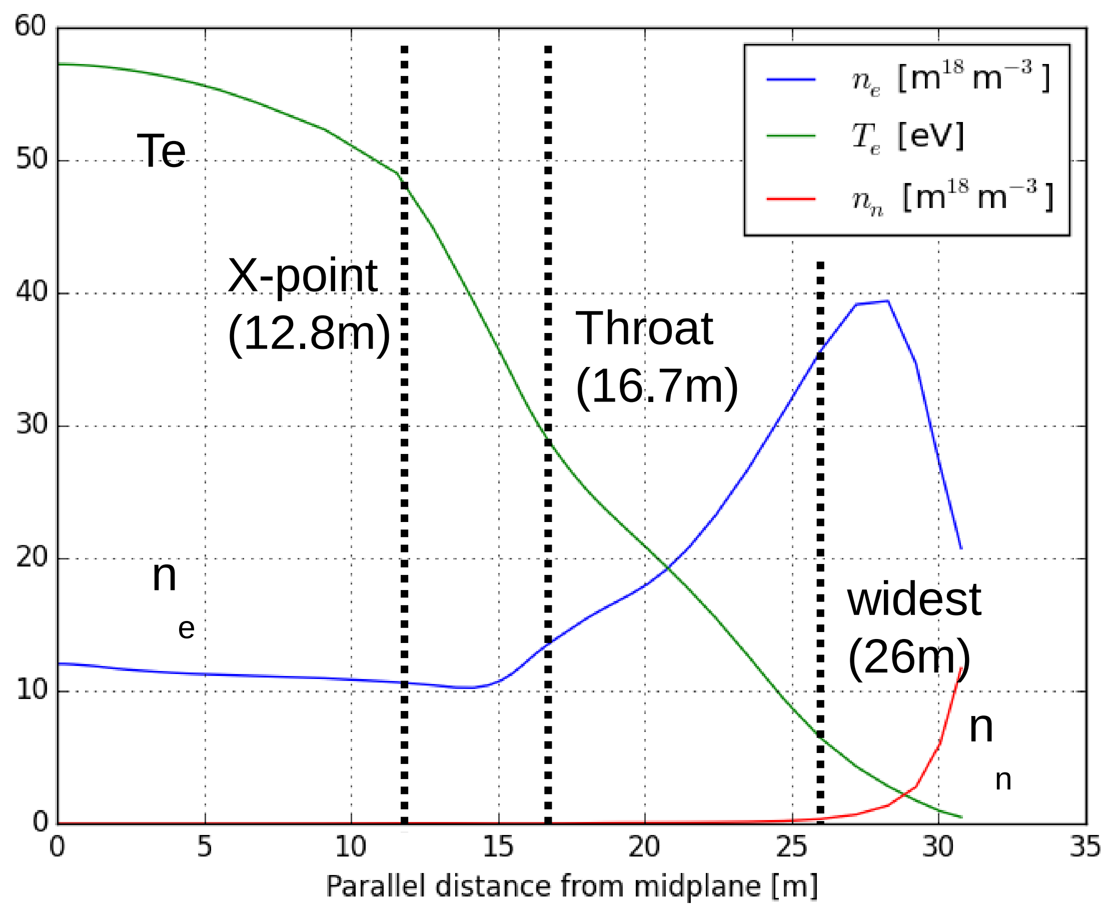
Recycling fraction	80%
Carbon fraction	1%
$n_{e,sep}$	1.3 x 10^{19} m^{-3}
$T_{e,sep}$	63 eV
D	0.2 m^2/s
X	0.5 m^2/s

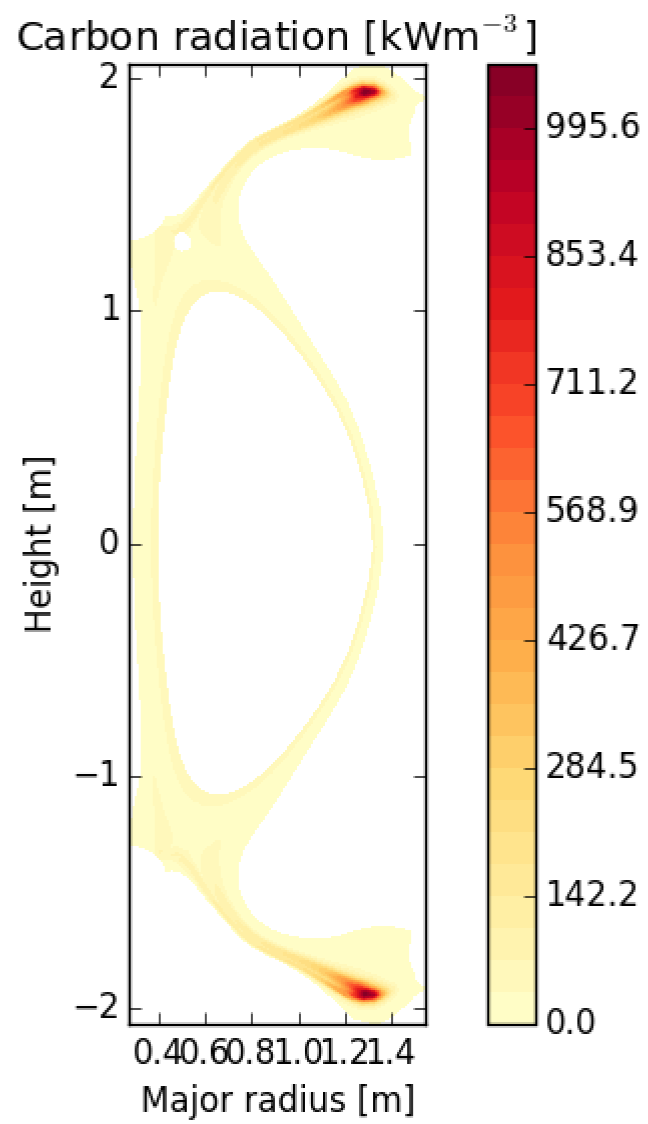


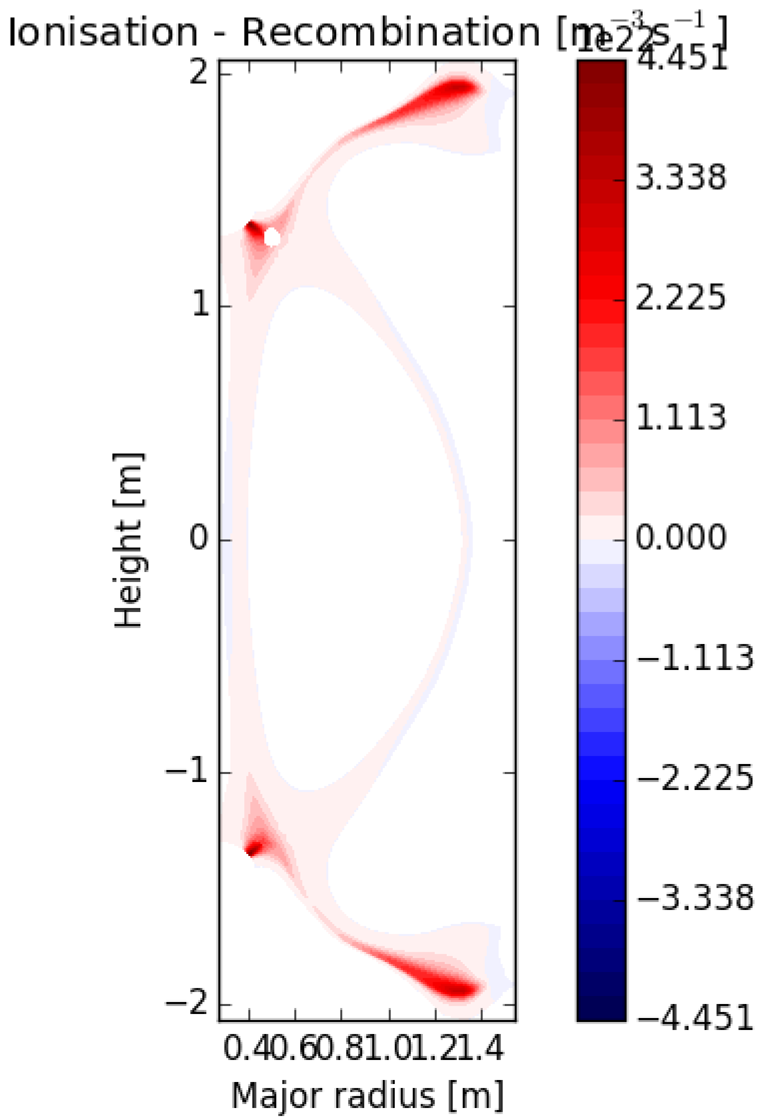
MAST-Upgrade simulations

- Obtained stable solutions in Super-X geometry
- Net volume recombination near target plates

Input power	497 kW (thermal)
	510 kW (total)
Input particles	5.7 x 10^{21}/s
Carbon radiation	340 kW (67%)
Volumetric loss	56 kW (11 %)





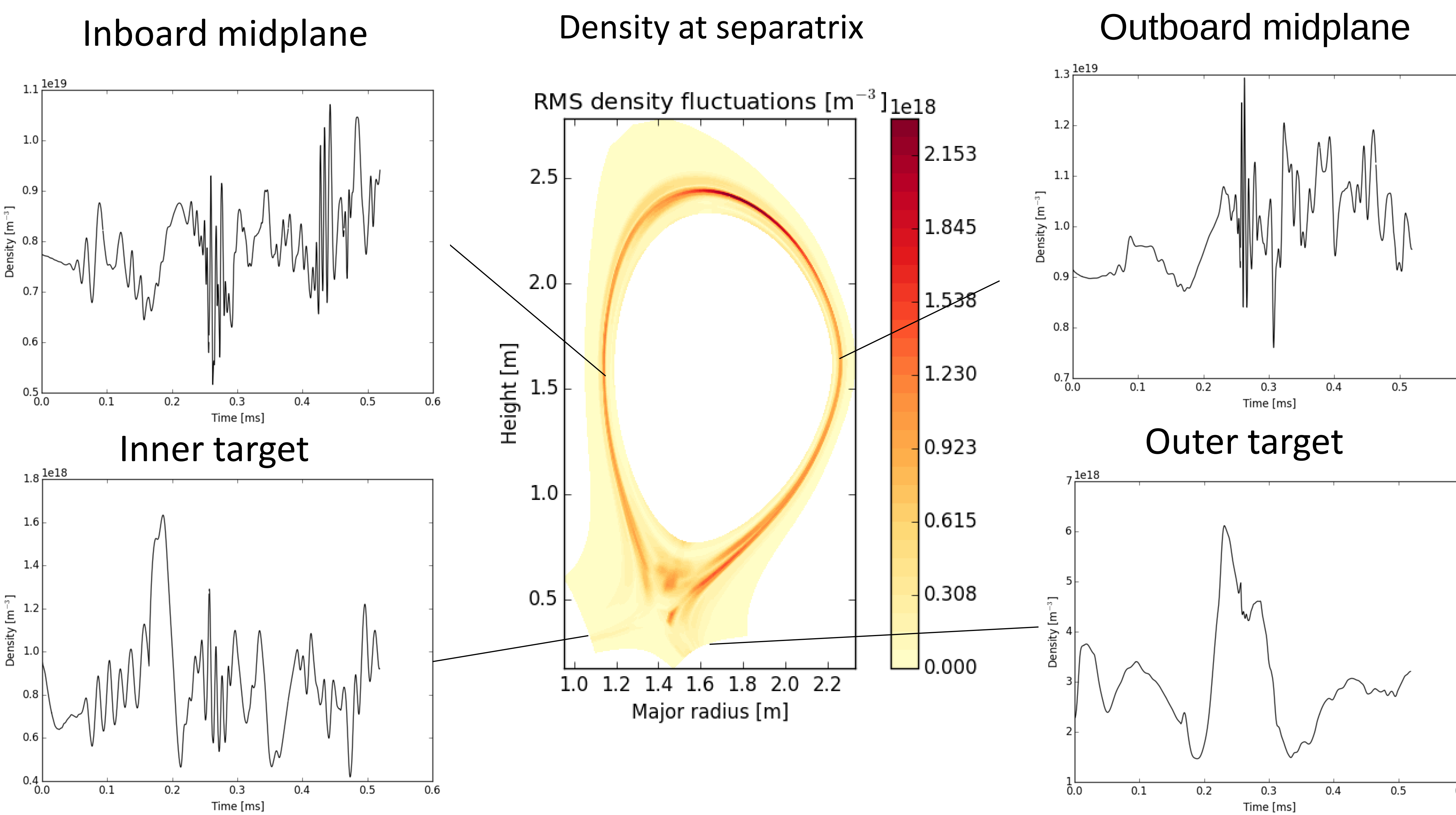


Future work

- Evolving axisymmetric electric field
- Simulate turbulent transport in Super-X geometry

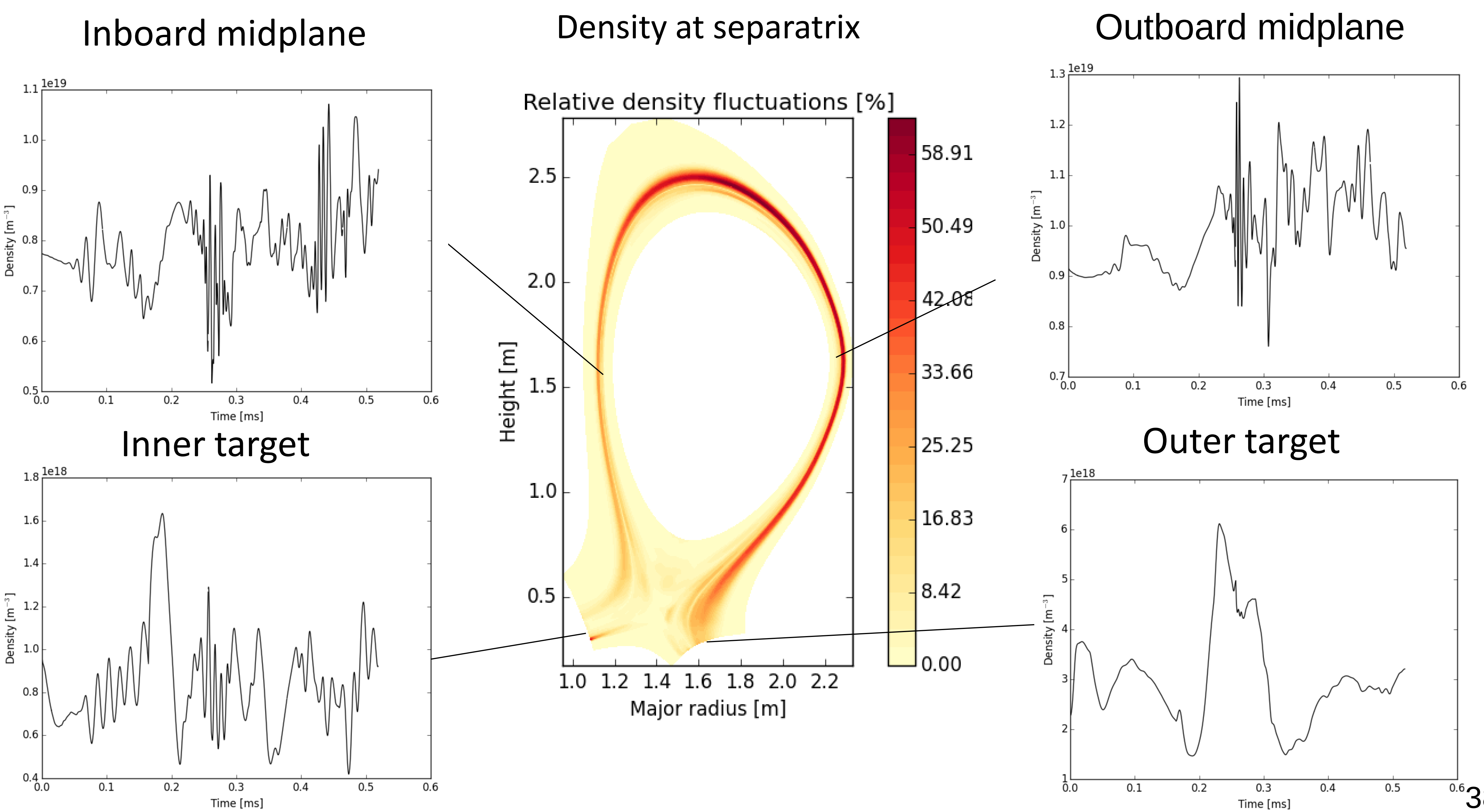
Turbulence in X-point geometry

Extending the mesh in the toroidal direction, turn off anomalous cross-field transport, and add random noise to vorticity



Turbulence in X-point geometry

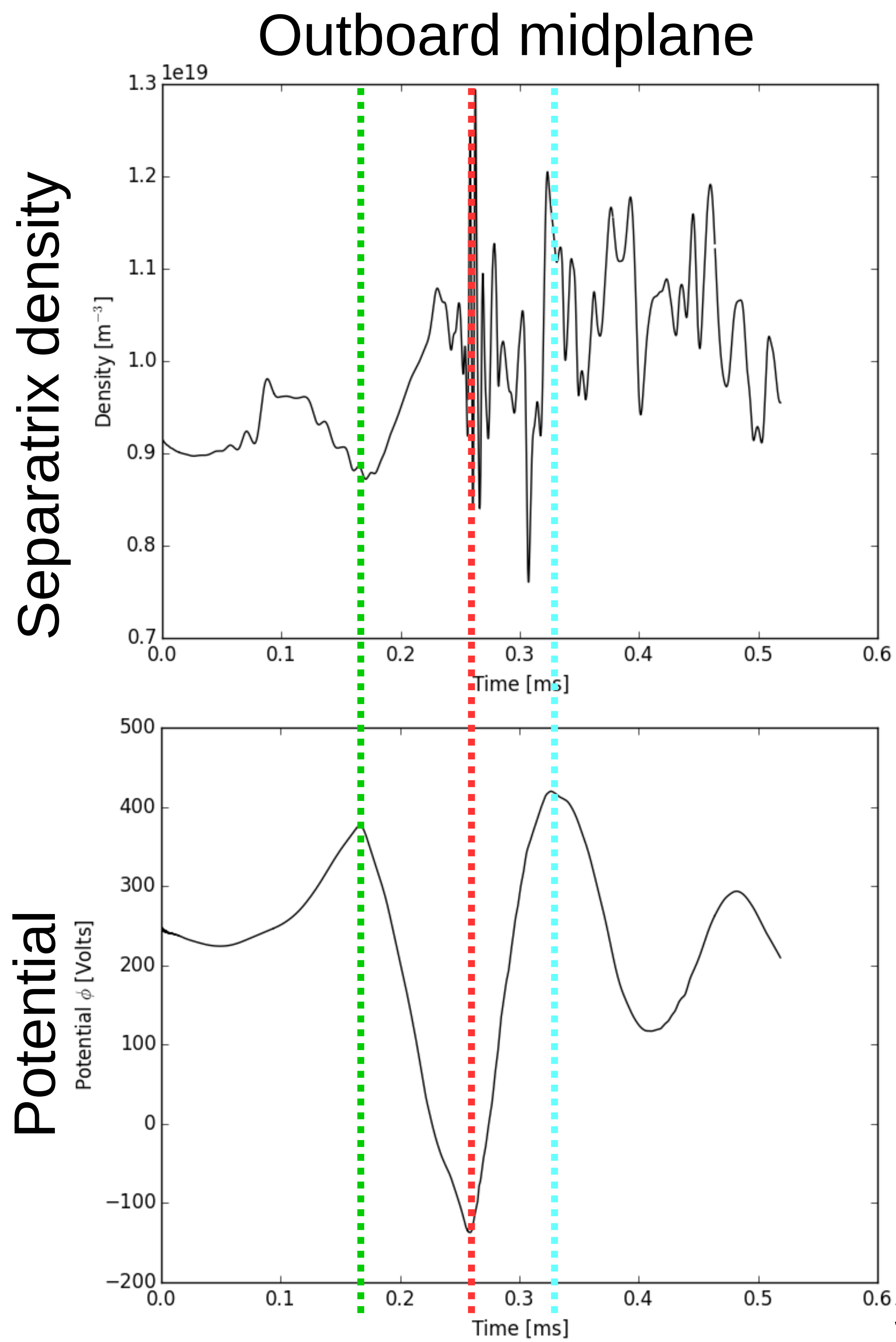
Extending the mesh in the toroidal direction, turn off anomalous cross-field transport, and add random noise to vorticity

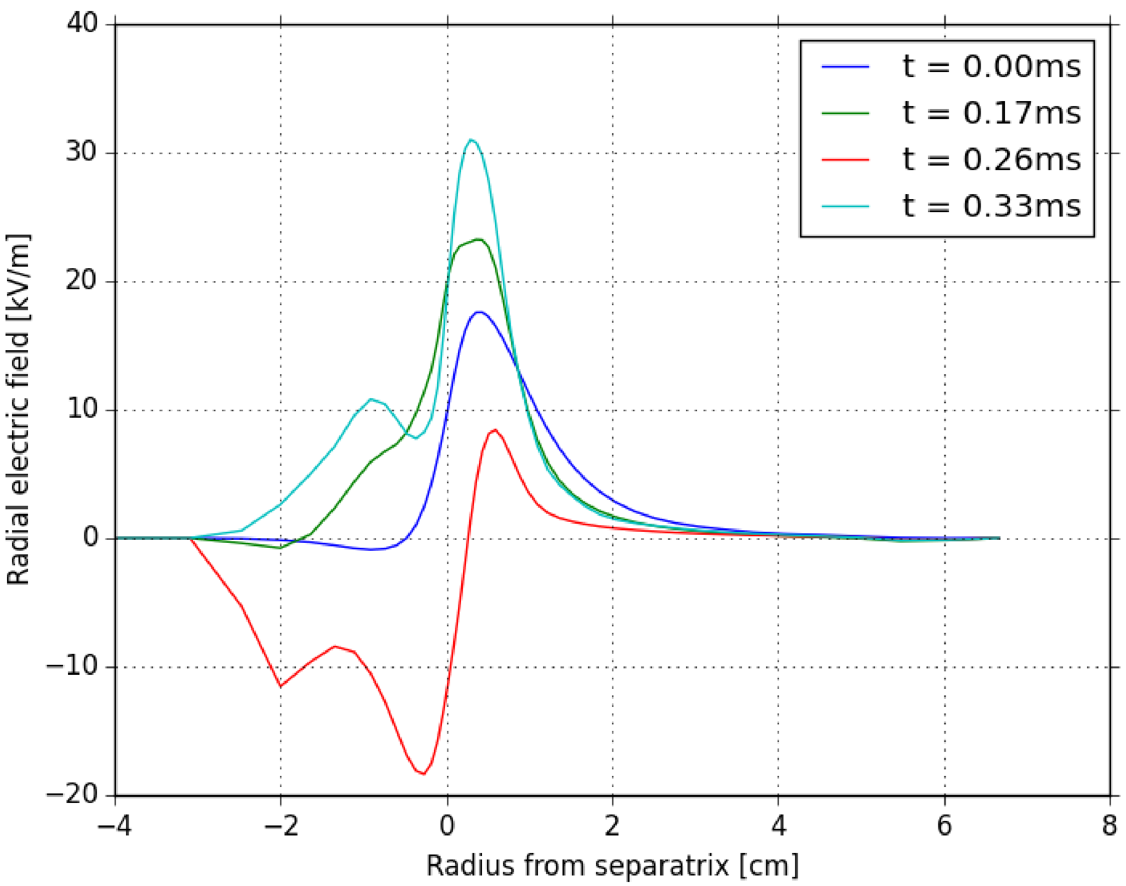


Turbulence in X-point geometry

Extending the mesh in the toroidal direction, turn off anomalous cross-field transport, and add random noise to vorticity

- **Fluctuations extended poloidally**
- **Observed in divertor region, including inner leg PF region**
- **Large n=0 oscillation in potential**

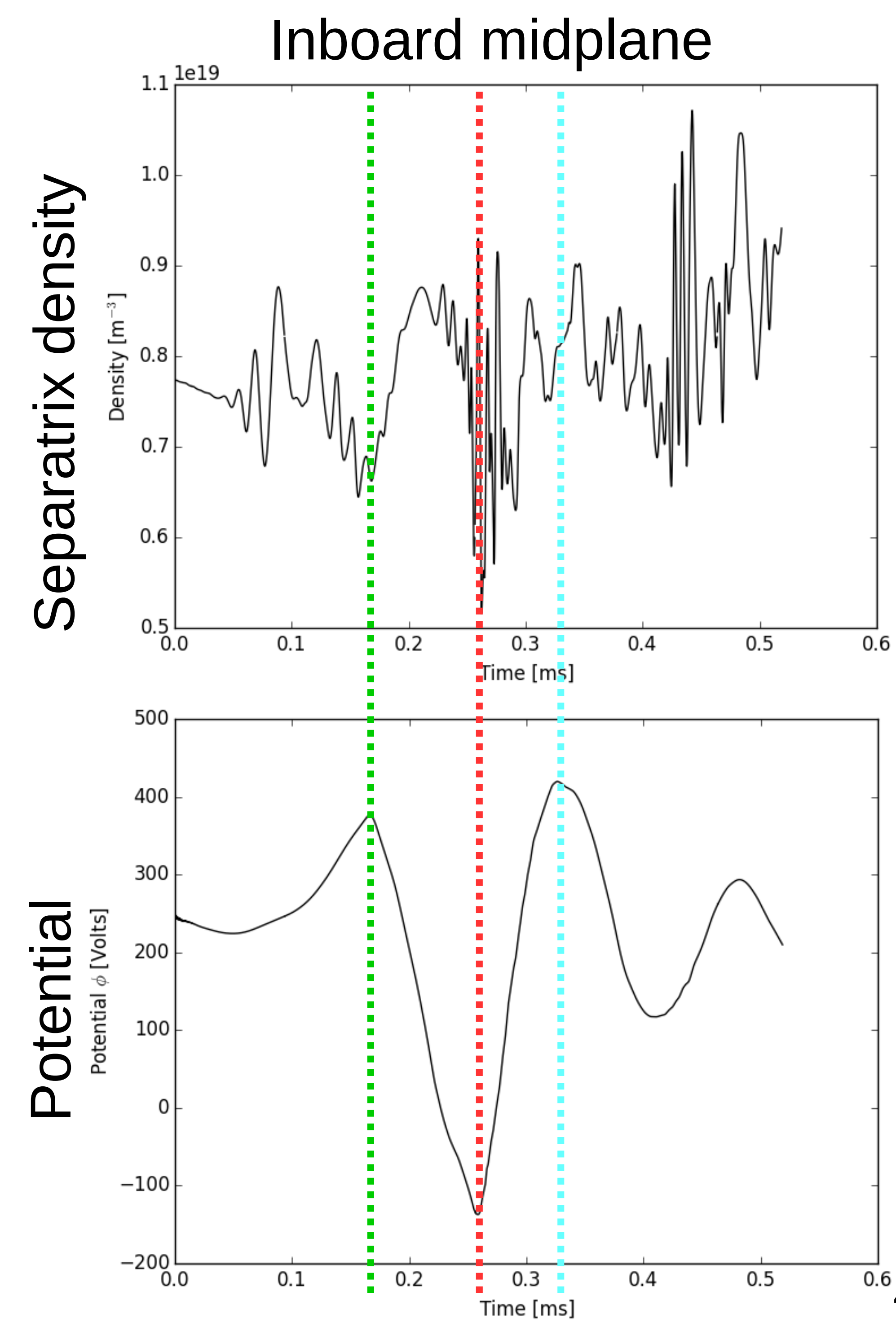


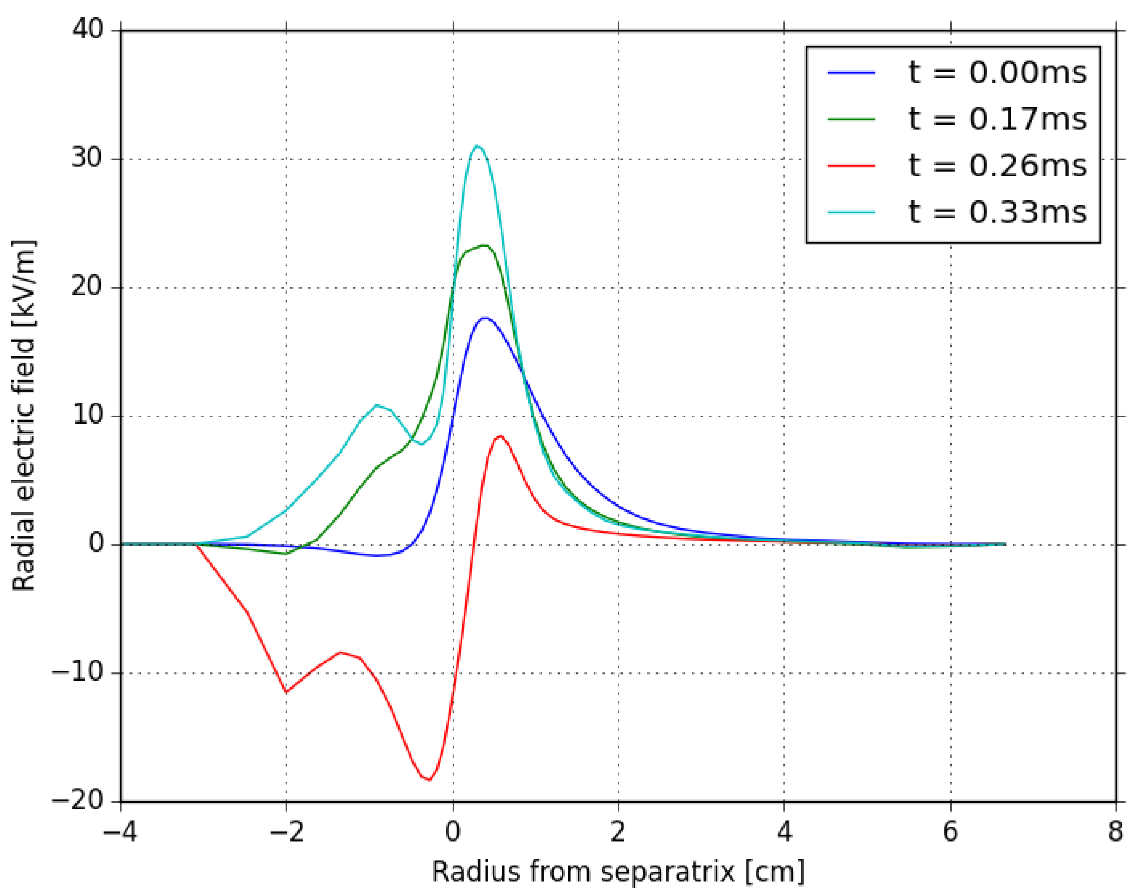


Turbulence in X-point geometry

Extending the mesh in the toroidal direction, turn off anomalous cross-field transport, and add random noise to vorticity

- **Fluctuations extended poloidally**
- **Observed in divertor region, including inner leg PF region**
- **Large n=0 oscillation in potential**





Conclusions

- Numerical methods improved for tokamak and non-axisymmetric geometries
- Hermes model being developed (using BOUT++) to study the interaction of transport and turbulence
- Improvements made to model equations and numerical methods allow stable evolution of n=0 electric fields and currents in X-point geometry for the first time in BOUT++
- Fluid neutral model allows study of high recycling regimes. Simulations in linear device demonstrate interaction between plasma turbulence and neutral gas

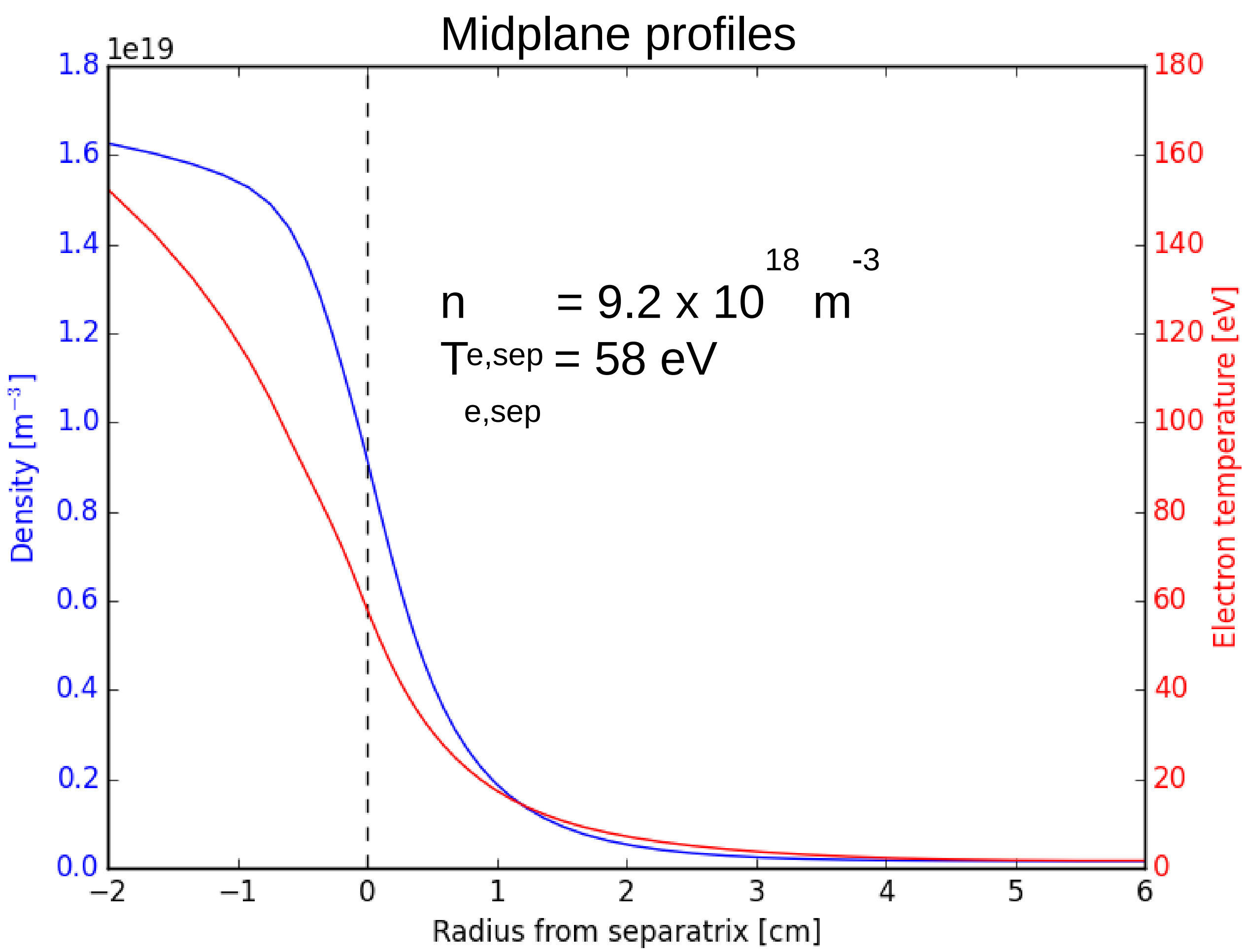
This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

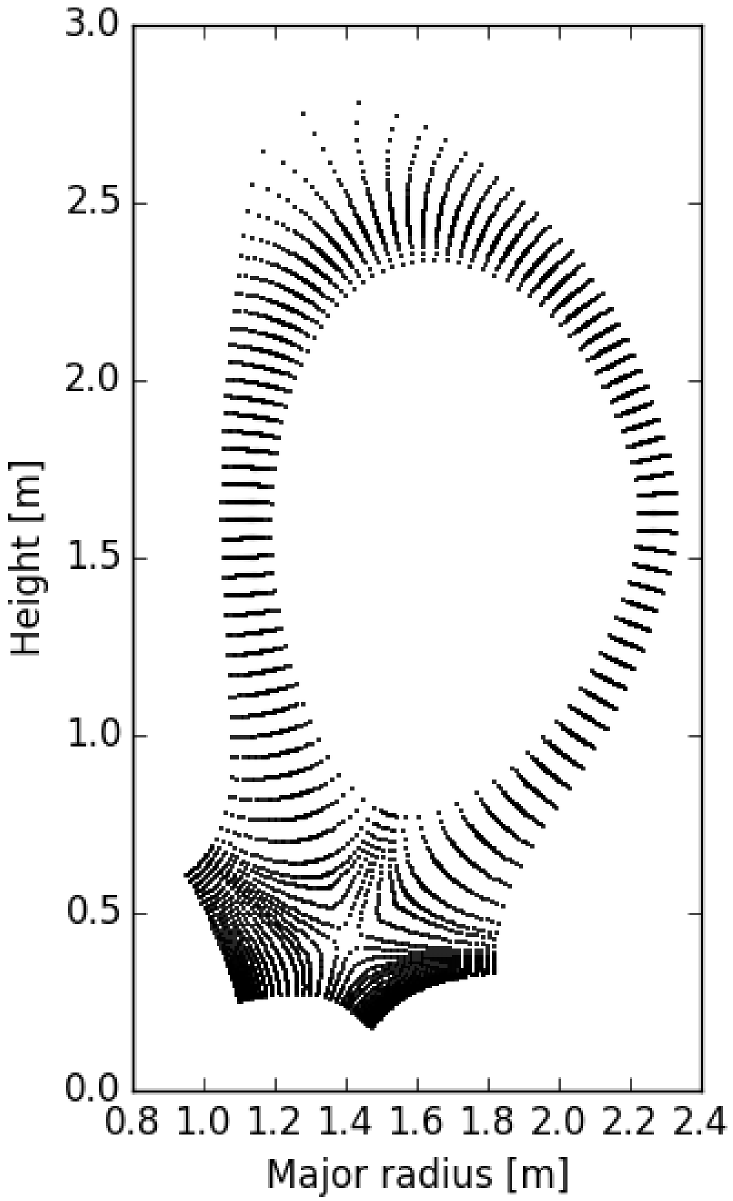
Extra slides

Example equilibrium (DIII-D like)

Hermes can be run as an axisymmetric transport code (e.g. SOLPS, EDGE2D, UEDGE, ...)

- **Specify anomalous diffusion coefficients for cross-field transport**
- **Includes (optional) flux limiters as used in SOLPS**
- **Start a simulation without electric fields or drifts**



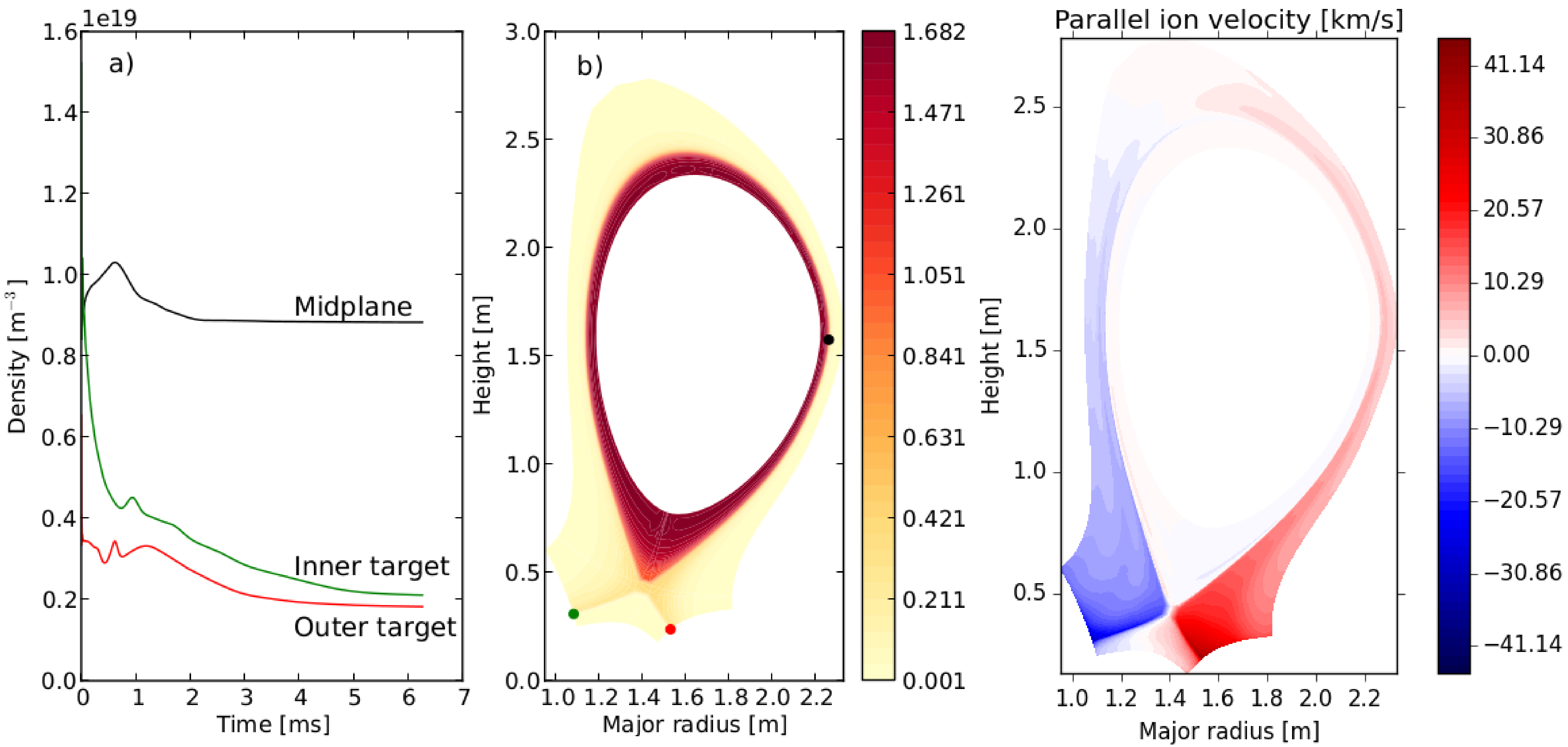


Resolution: 48 x 128 (x 12

Evolving axisymmetric profiles

Hermes can be run as an axisymmetric transport code (e.g. SOLPS, EDGE2D, UEDGE, …)

- **Specify anomalous diffusion coefficients for cross-field transport**
- **Includes (optional) flux limiters as used in SOLPS**
- **Start a simulation without electric fields or drifts**



Evolving axisymmetric potential

Initial Alfvenic oscillations f~500 kHz damp on ~20 μs timescale

Followed by slower oscillation with f ~ 6.7 kHz

Shear Alfven wave

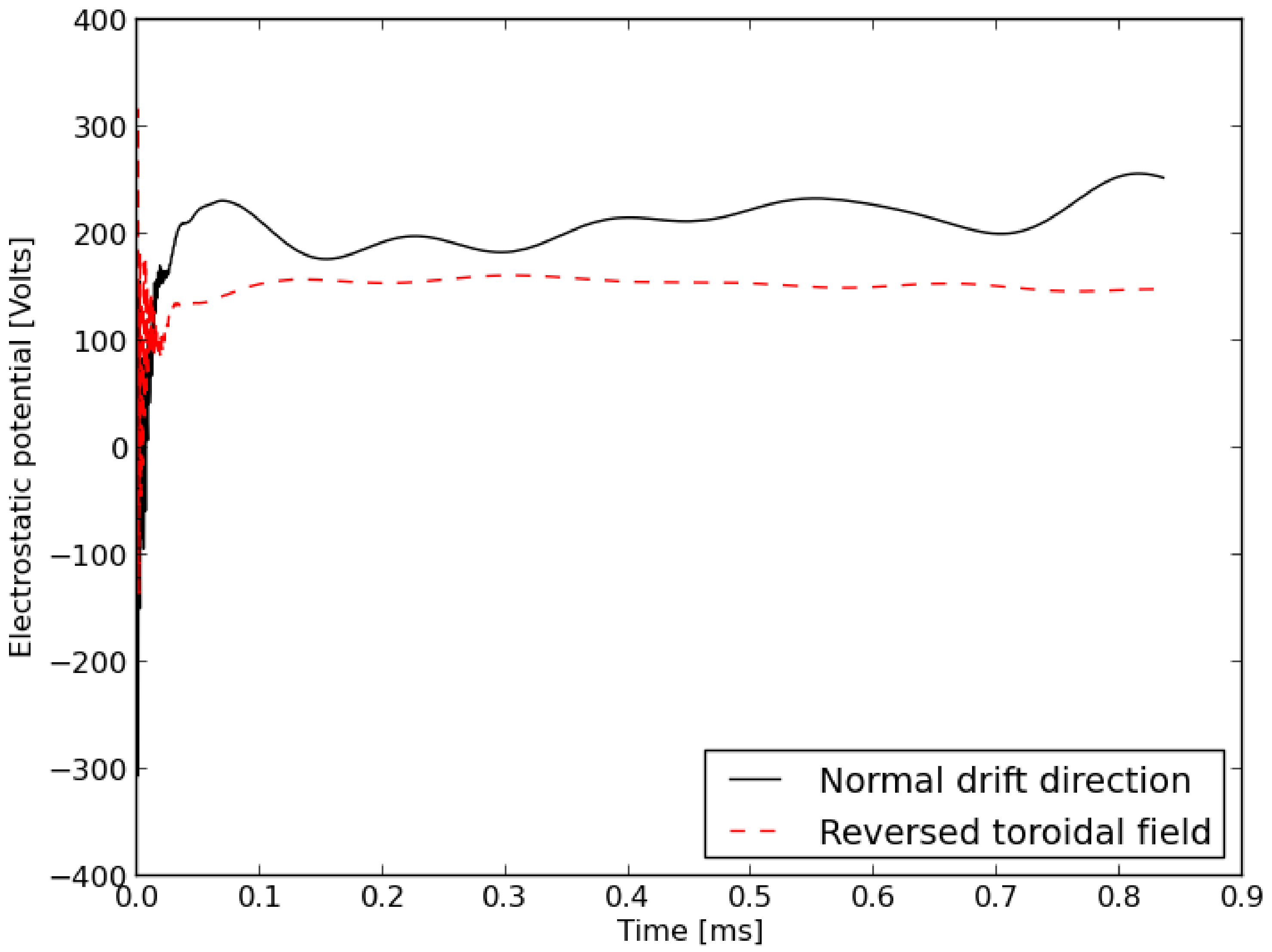
$$f_A = v_A / (2\pi R q) \simeq 550 - 1100\text{kHz}$$

Geodesic Acoustic Mode

$$f_{GAM} = \frac{c_s}{2\pi R}\sqrt{2 + 1/q^2} \simeq 3 - 11\text{kHz}$$

Parallel sound wave

$$f_s = c_s / (2\pi R q) \simeq 0.5 - 2.3\text{kHz}$$



Model includes Alfven waves

Initial Alfvenic oscillations f~500 kHz damp on ~20 μs timescale

Shear Alfven wave

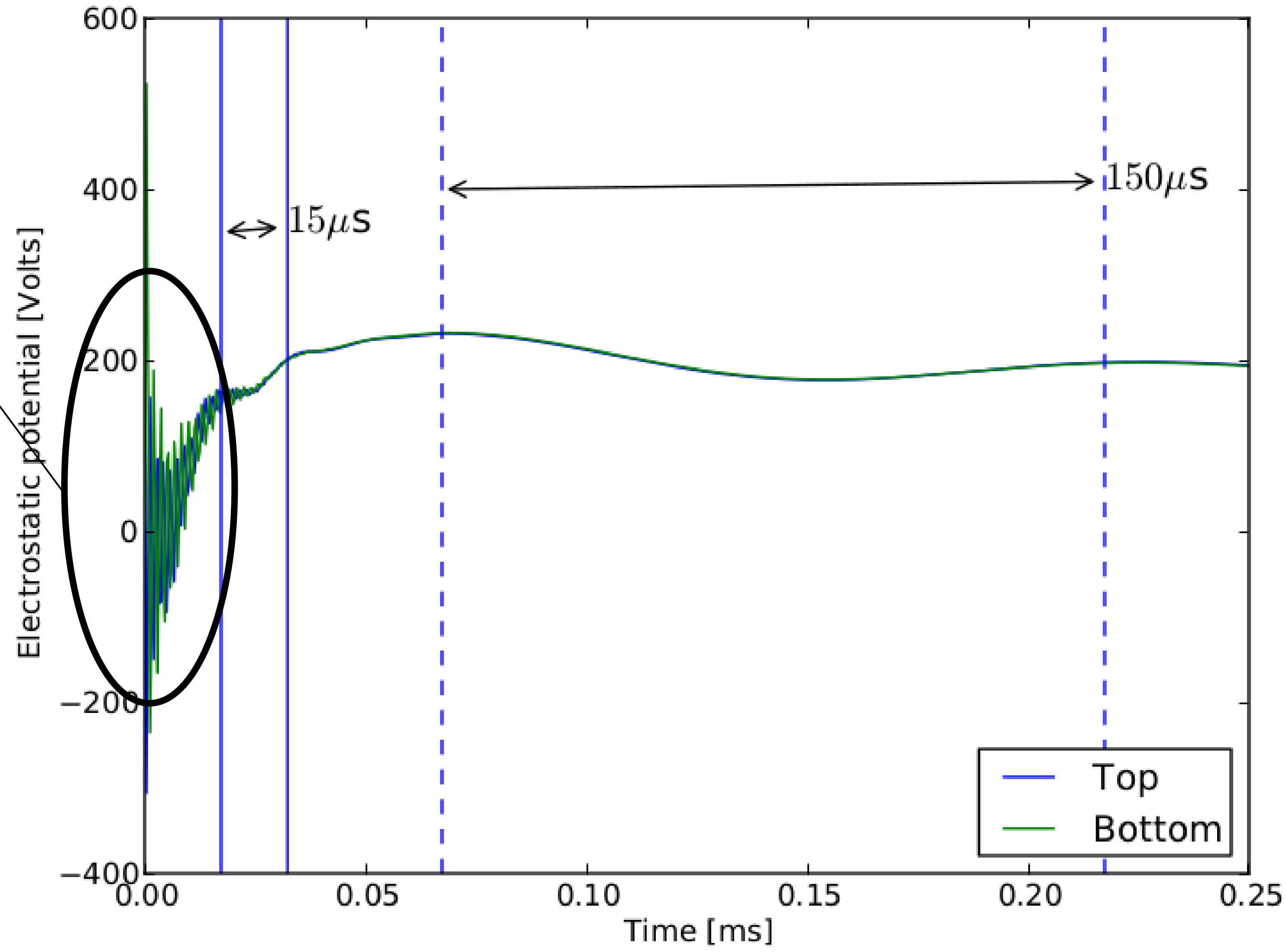
$$f_A = v_A / (2\pi R q) \simeq 550 - 1100\text{kHz}$$

Geodesic Acoustic Mode

$$f_{GAM} = \frac{c_s}{2\pi R}\sqrt{2 + 1/q^2} \simeq 3 - 11\text{kHz}$$

Parallel sound wave

$$f_s = c_s / (2\pi R q) \simeq 0.5 - 2.3\text{kHz}$$



Model includes GAM oscillations

Initial Alfvenic oscillations f~500 kHz damp on ~20 µs timescale

Followed by slower oscillation with f ~ 6.7 kHz

Shear Alfven wave

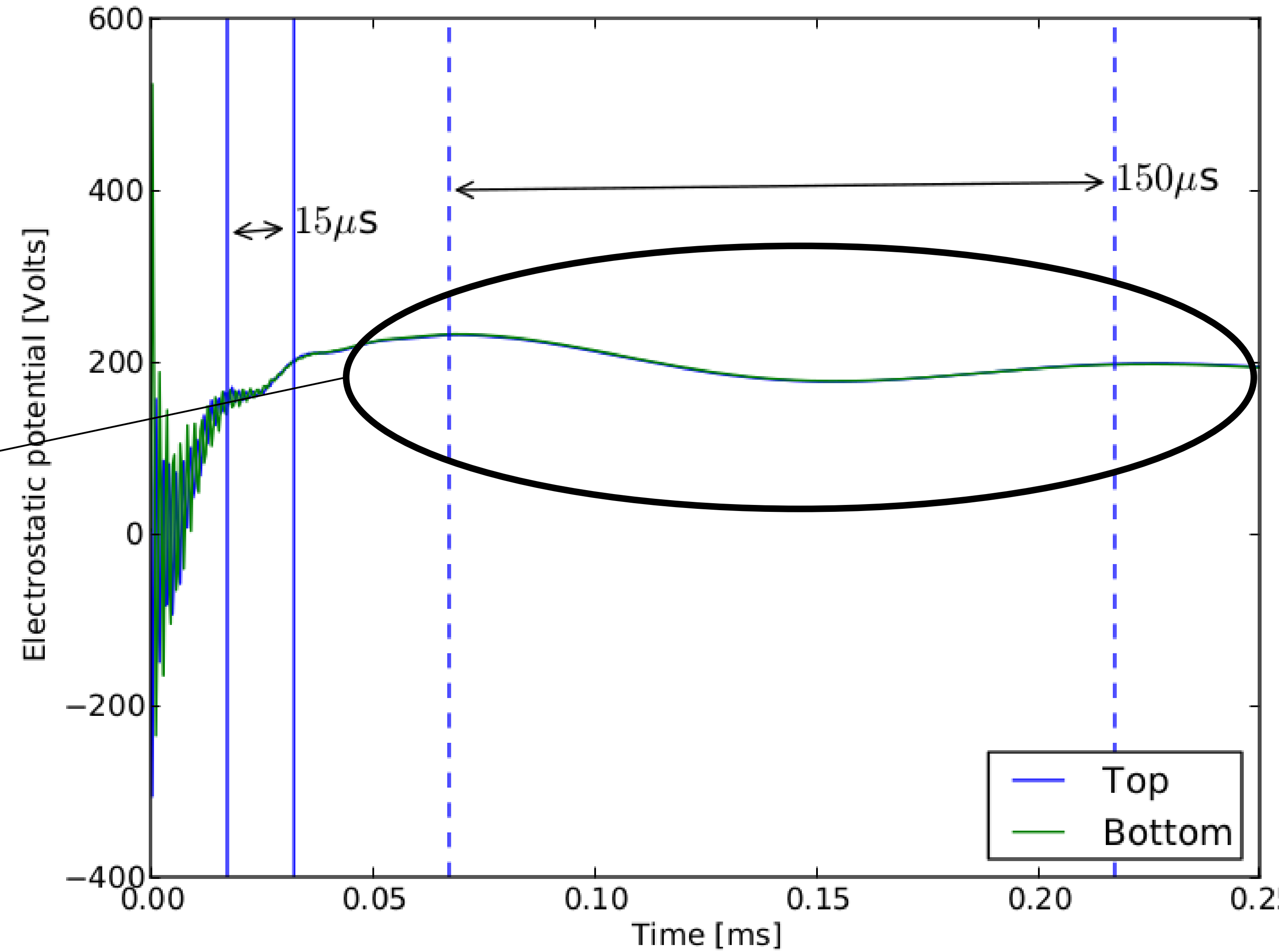
$$f_A = v_A / (2\pi R q) \simeq 550 - 1100\mathrm{kHz}$$

Geodesic Acoustic Mode

$$f_{GAM} = \frac{c_s}{2\pi R}\sqrt{2 + 1/q^2} \simeq 3 - 11\mathrm{kHz}$$

Parallel sound wave

$$f_s = c_s / (2\pi R q) \simeq 0.5 - 2.3\mathrm{kHz}$$



Poloidal flows

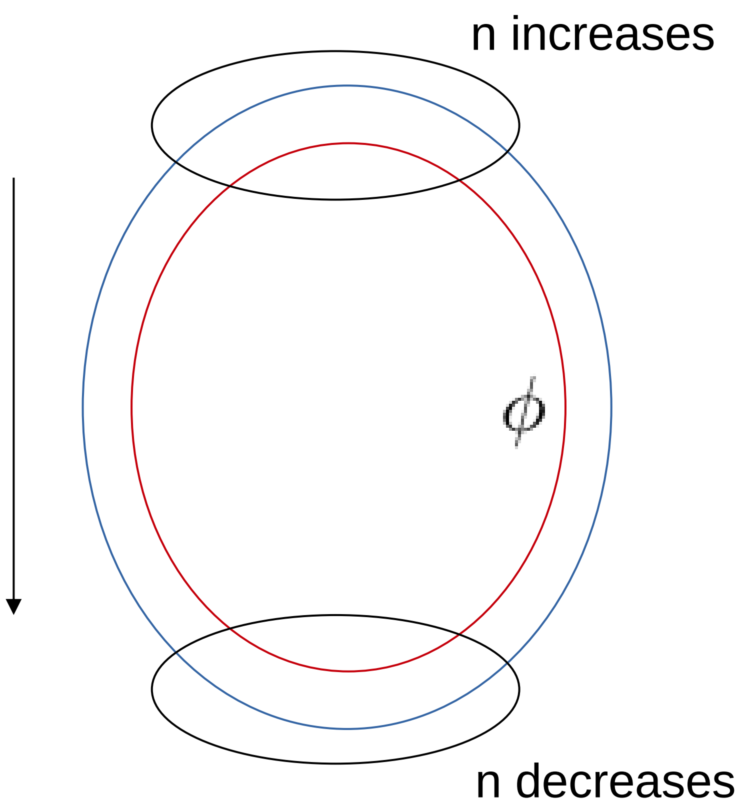
A common way to represent the ExB flow is

$$\nabla\cdot\left(n\frac{\mathbf{b}\times\nabla\phi}{B}\right) = \frac{\mathbf{b}\times\nabla\phi}{B}\cdot\nabla n + n\left[\nabla\times\left(\frac{\mathbf{b}}{B}\right)\right]\cdot\nabla\phi$$

Particles added to some cells, removed from others

- **In general does not conserve particle number**
- **Geometry (curvature) need to be restricted**

$$\nabla\times\left(\frac{\mathbf{b}}{B}\right)$$



Instead, poloidal flows treated in divergence form
$\rightarrow$ Ensures conservation of particles

Drift-plane motion

$$\begin{aligned}\nabla\cdot\left(n\frac{\mathbf{b}\times\nabla\phi}{B}\right) &= \frac{1}{J}\frac{\partial}{\partial\psi}\left(Jn\frac{\partial\phi}{\partial z}\right) - \frac{1}{J}\frac{\partial}{\partial z}\left(Jn\frac{\partial\phi}{\partial\psi}\right)\\ &+ \frac{1}{J}\frac{\partial}{\partial\psi}\left(Jn\frac{g^{\psi\psi}g^{yz}}{B^2}\frac{\partial\phi}{\partial y}\right) - \frac{1}{J}\frac{\partial}{\partial y}\left(Jn\frac{g^{\psi\psi}g^{yz}}{B^2}\frac{\partial\phi}{\partial\psi}\right)\end{aligned}$$

Radial flow due to poloidal electric fields

Poloidal flow due to radial electric fields

Tokamaks : Particle conservation

- Conservation of particle number is important in high recycling regimes
- Since total density, pressure is evolved, numerical sources/sinks could affect fidelity of small-scale fluctuations