Diffusion of Mass Through Tera Gauss Fields on the Surface of Neutron Stars in HXRB's

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ps2 THE SITUATION

$$\phi = 10^{20} \text{ ergs/gram} = .1Mc^2$$

$$L = 10^{37}\text{ergs/second}$$

$$\dot{M}_a = 10^{17}\text{grams/s} = 10^{-9}M_S\text{per year}$$

$$A = (100\text{meters})^2 = 3 \times 10^8\text{cm}^2$$

$$\dot{m}_a = 3 \times 10^8\text{grams/cm }^2 seconds > \dot{h} = .3\text{cm/s}$$

$$\Delta\beta = 2.5h \text{ at } 10^4 cm$$

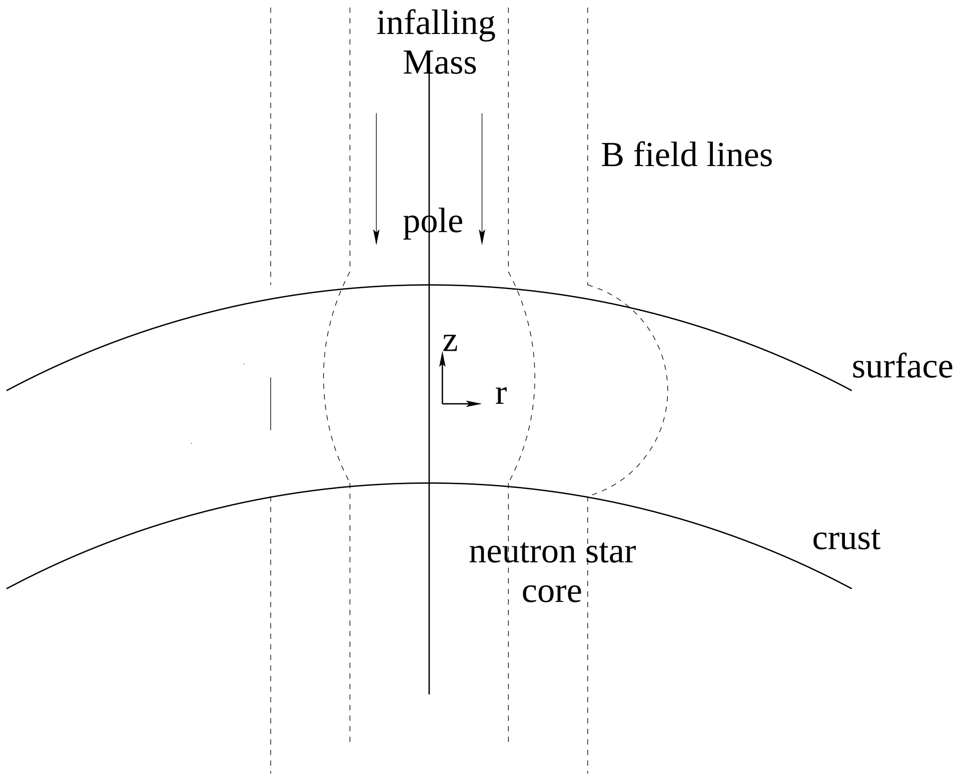

Figure 1: The mound equilibrium

The mound equilibrium

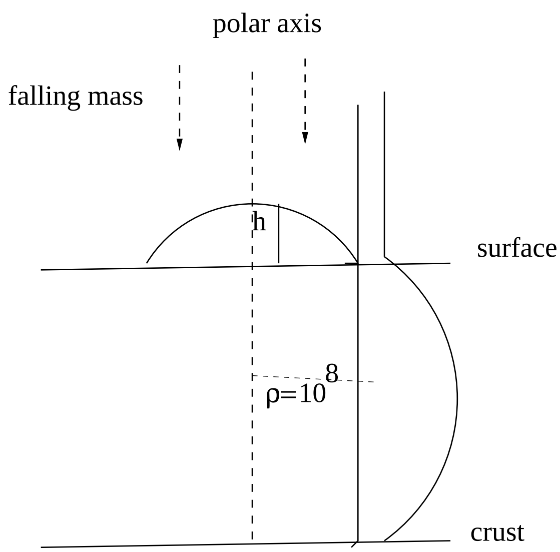

Figure 2: The infalling mass mound equilibrium

Now, for a magnetostatic equilibrium we have

$$\mathbf{j} \times \mathbf{B} = \rho \mathbf{g} = \nabla p. \tag{1}$$

The radial component is

$$j_\theta B_z = -\frac{1}{4\pi r^2} \Delta^* \psi \frac{\partial \psi}{\partial r} = \left(\frac{\partial p}{\partial r} \right)_z ,$$

The vertical component is from hydrostatic equilibrium

$$\left(\frac{\partial p}{\partial z}\right)_{\psi} = -\rho g.$$

$$p = 1.5 \times 10^{13} \rho^{5/3}$$

$$\rho(\psi, z) = 12(z + h(\psi))^{3/2} \text{g cm}^{-3}, \qquad (2)$$

$$p(z, h) = 6.5 \times 10^{14} (z + h(\psi))^{5/2} \text{ergs cm}^{-3}$$

$$-\frac{\Delta^* \psi}{4\pi r^2} = 1.62 \times 10^{15} (z + h(\psi))^{3/2} \frac{dh}{d\psi}.$$

$h(\psi)$ is given on each line to get a unique solution

It is related to the mass by

$$m(\psi) = \rho_0(z_{crust}) h(\psi) = 5.4 \times 10^8 \times h(\psi) \text{ g / cm}^2 \quad (3)$$

How is $h(\psi)$ found?

People who have solved these equations

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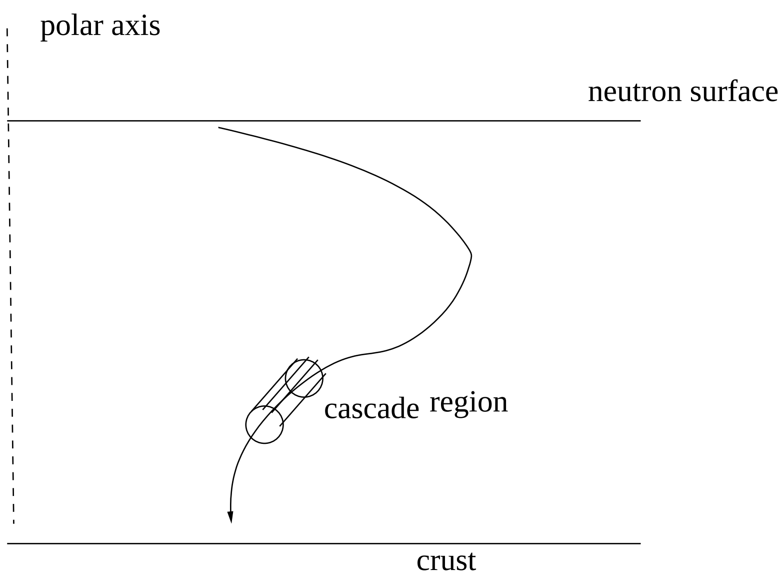

Figure 3: The place occupied by the cascade

THE INSTABILITY

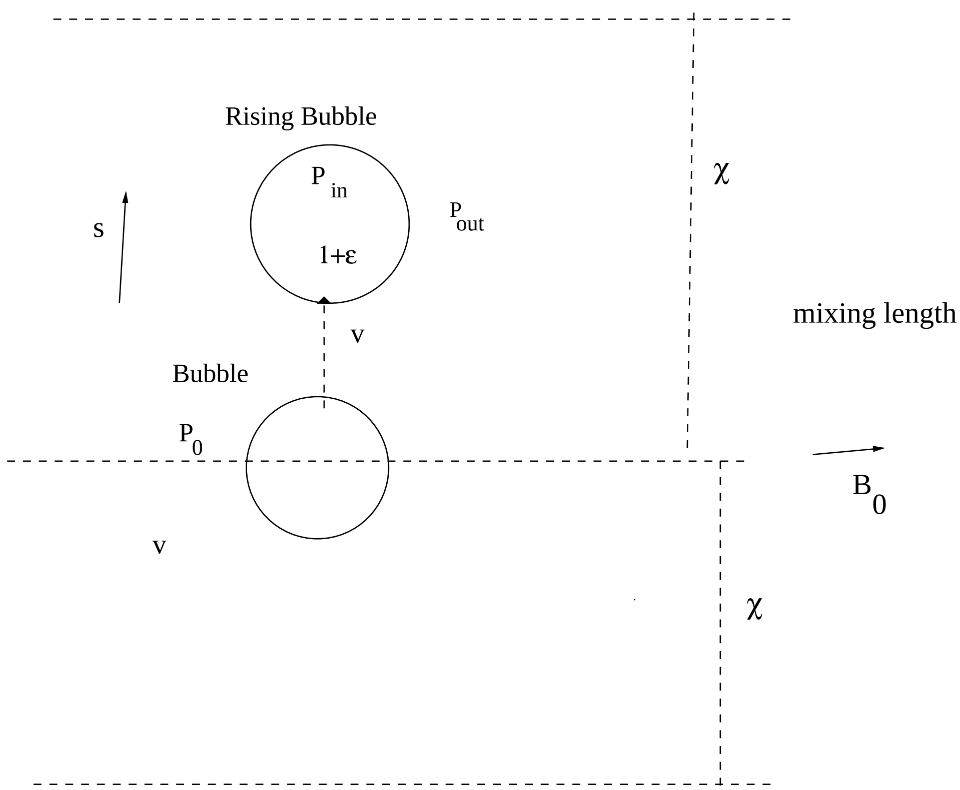

Figure 4: The buoyancy of a rising bubble

Solar convection zone (Schwarzchild instability)

$$\nabla ln(p/\rho^{\gamma}) = 0$$

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Neutron star (Our instability)

$$P_{in} = \left(p + \frac{B^2}{8\pi}\right) = p_0(1-\epsilon)^{\gamma_A} + \frac{B^2}{8\pi}(1-\epsilon)^2$$

$$= p_0 + \frac{B^2}{8\pi} - \left(\gamma_A p_0 + 2\frac{B^2}{8\pi}\right)\epsilon, \quad (4)$$

$$P_{out} = \left(p + \frac{B^2}{8\pi}\right) + \left(p_0' + \frac{BB'}{4\pi}\right)\delta z,$$

$$\epsilon = \frac{-P'}{\Gamma P}.$$

The upward force on the bubble is

$$F_g = -g(\rho_{in} - \rho_{out}) = -g(-\epsilon\rho - \delta z \rho') = -g\rho\left(\frac{P'}{\Gamma P} - \frac{\rho'}{\rho}\right)\delta z.$$

The upward force on the bubble reduces to

$$F_g == \frac{g\rho}{\gamma_A}\frac{\Delta}{(1+\gamma_A\beta/2)}\delta z = g\rho\frac{\Delta}{C},$$

where

$$\Delta = \frac{p'}{p} - \gamma_A\frac{B'}{B} = \frac{d}{dz}\ln\frac{p}{B^{\gamma_A}},$$

and

$$C = \gamma_A(1+\gamma_A\beta/2).$$

For instability Δ must be positive.

$$\nabla ln(p/B^{\gamma_A}) > 0$$

The force acts over a mixing length ξ and the resulting non linear velocities are essentially harmonic vibrations between $s = -\xi$ and $s = \xi$.

They tend not to be damped by ideal motions. There are many such modes and they lead to a statistically steady state.

$$< \mathbf{v}^*_{\mathbf{k}',\omega'} \mathbf{v}_{\mathbf{k}\omega} >= J(k,\omega)\delta_{\mathbf{k}'\mathbf{k}}\delta(\omega' - \omega)(\mathbf{I} - \hat{\mathbf{k}}\hat{\mathbf{k}}).$$

$$v_0 = \gamma_0 \sqrt{\Delta}$$

where $\gamma_0 = \sqrt{g'}$

The TURBULENT CASCADE

$$|\mathbf{B}| = B_0 + b$$

b is evolved passively by the velocities

$$\frac{\partial b}{\partial t} = \nabla \cdot (\mathbf{v} b),$$

or in Fourier space

$$\frac{\partial b_{\mathbf{k}}}{\partial t} = \Sigma_{\mathbf{k}''} \mathbf{k} \cdot (\mathbf{v}_{\mathbf{k}''} b_{\mathbf{k}'}),$$

$$\mathbf{k} = \mathbf{k}' + \mathbf{k}''.$$

Essentially this equation was solved by Kulsrud and Anderson 1992

I copy their procedure with the proper modifications

The spectrum of b is given by $M(k)$

$$\int M(k)dk = \int |b|^2$$

The equation for M is in our case is

$$\frac{\partial M}{\partial t} = \frac{\gamma}{16}\left(k^2\frac{\partial^2 M}{\partial k^2} + k\frac{\partial M}{\partial k} - M\right)$$

$$\gamma = k_0 v_0 \quad k_0 = 2\pi/\xi$$

A Green's function solution of this equation is

$$M(k,t) = \frac{8}{\sqrt{\pi}}\frac{\gamma}{(\gamma t)^{3/2}}e^{-\gamma t/16}$$
$$. \quad e^{-4(\ln(k/k_0)^2/\gamma t}\ln(k/k_0) \quad (5)$$

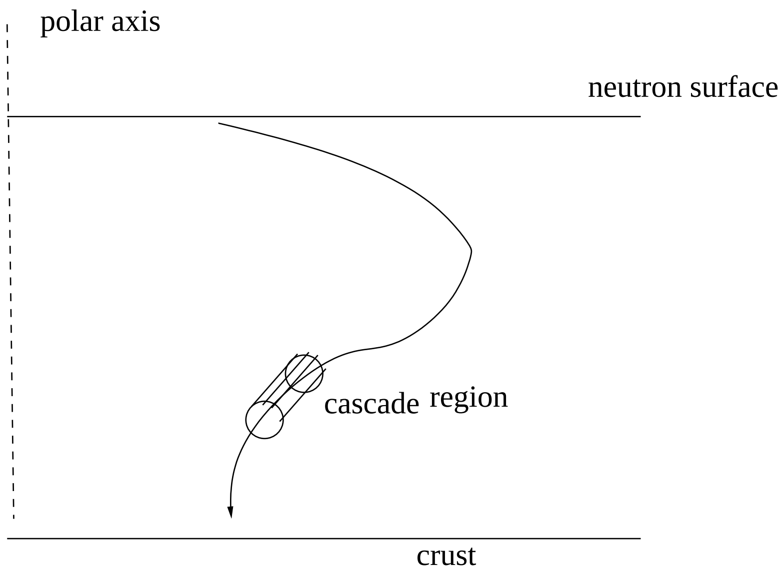


Figure 5: The place occupied by the cascade

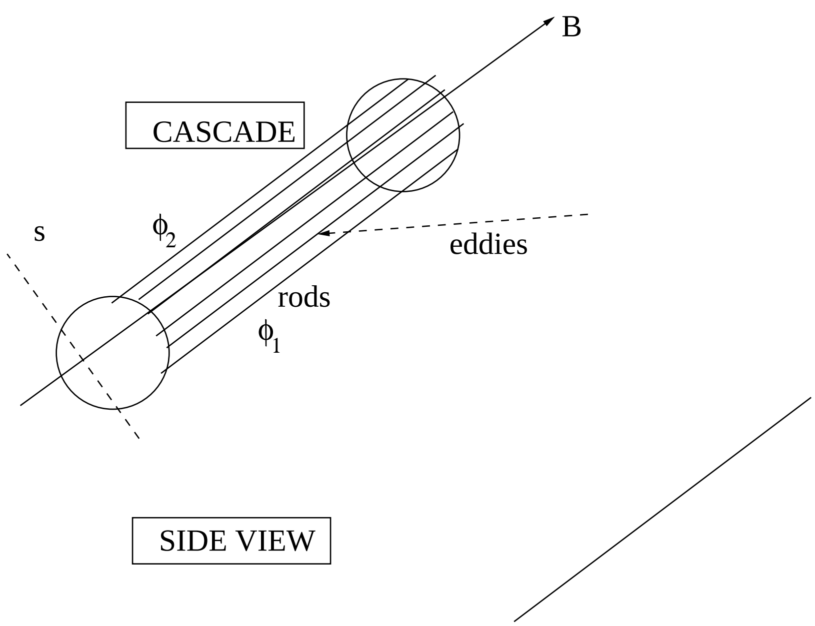


Figure 6: The eddies represented by rigid rods

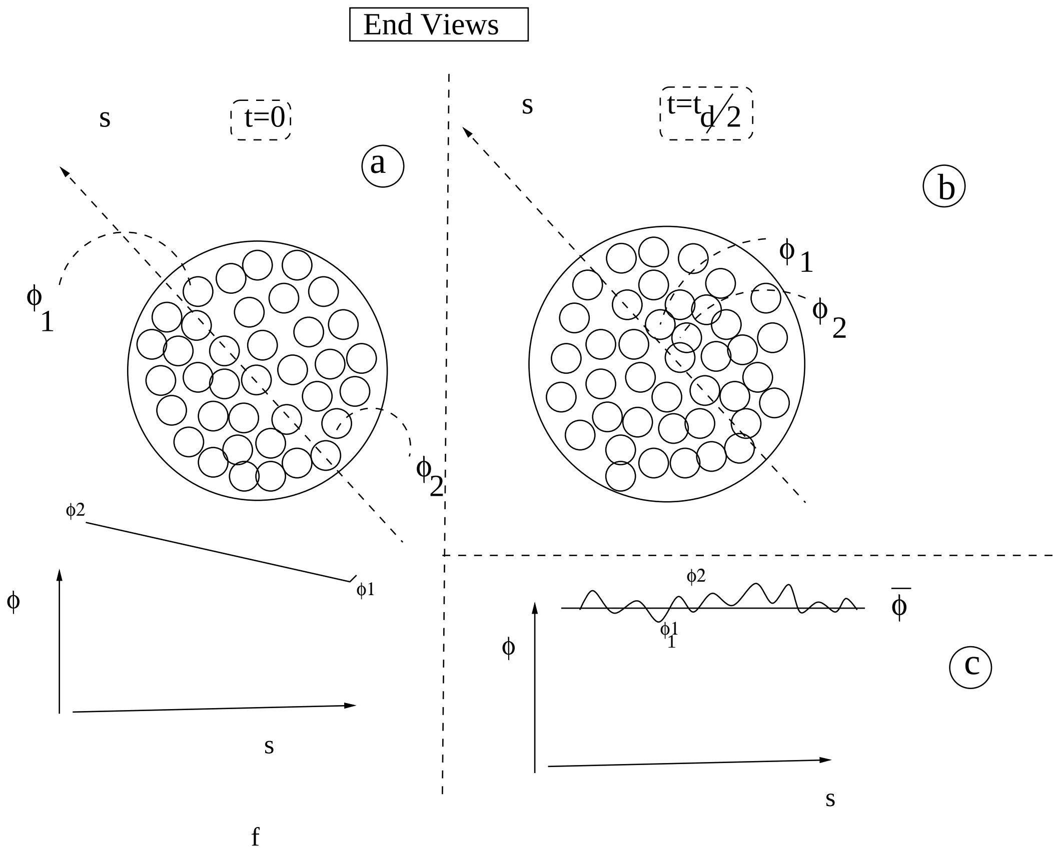


Figure 7: The rate of damping of eddies

When two tubes with different densities and pressures come close together than there is a surface current j and an ηj

There is a critical wave length

$$R(k) = k_\eta^2 \eta' = k_0 v_0$$

Equalizing the densities damps out the fluctuations and the mode.

The accumulative relative damping up to a time t is

$$R_0(t) = \frac{\int_0^t \int_0^{k_\eta} R(k) M(k,t) dk dt}{\int M dk}$$

At a time t_D when $R_0 = 1$ the mode is damped.

$$R_0 = \frac{8}{\sqrt{\pi}} \frac{k_0^2 \eta'}{(1 + 2/\gamma_A \beta)} \int_0^{t_D} dt \frac{e^{-\gamma t/16}}{(\gamma t)^{3/2}} \int_0^{x_\eta} e^{-4x^2/\gamma t + x^3} x dx \tag{6}$$

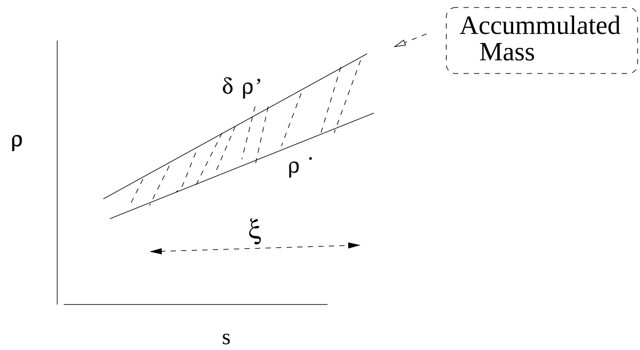
where

$$\begin{aligned} x &= \ln\left(\frac{k}{k_0}\right) \\ x_\eta &= \ln\left(\frac{k_\eta}{k_0}\right) \end{aligned} \tag{7}$$

$$R_0 \approx constant e^{x_\eta^3} e^{\gamma t}$$

so

$$t_D = 16 x_{\eta'}^3 / \gamma$$



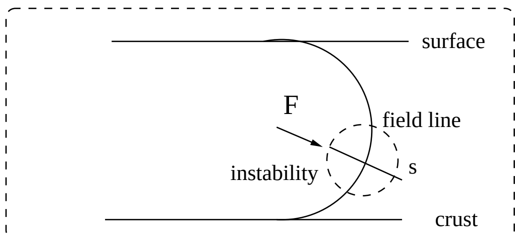


Figure 8: The relation between the flow rate F and Δ'

What is Δ?

Introduce the flow

The local mass flow F at the site of the instability is,

$$F = \frac{\Phi}{2\pi r \ell \epsilon} = \frac{10^7}{\epsilon} \text{ g/s cm}^{-2} = \rho_0 v_F$$

$$v_F = 10 \text{ cm/s}$$

F changes ρ' and therefore

$$\frac{1}{2}\frac{\delta(\rho')}{\rho}\xi^2 = v_F t_D$$

$$\Delta = 2v_F \frac{10^4 C^{3/2}}{\xi^2}\frac{16x_\eta^3}{\gamma_0 \Delta^{1/2}}$$

or

$$\Delta = \left(\frac{32v_F}{\gamma_0 \xi^2}\right)^{2/3} C$$

$$\Delta = 3 \times 10^{-6}$$

SUMMARY

Our goal is to determine the equilibrium for different values of M_a

The standard approach is to solve the Grad Shafranov equation

$$-\frac{\Delta^*\psi}{4\pi r^2} = 1.62 \times 10^{15}(z + h(\psi))^{3/2}\frac{dh}{d\psi}.$$

The problem is to determine $h(\psi)$ Instead of invoking flux freezing we propose that one finds the solution in which δ is very slightly positive on a region of every line and negative outside this region.

The stability condition is roughly

$$-2\pi\frac{dh}{d\psi} > 1$$

With this prescription there is only one parameter, the total accumulated mass M_a.

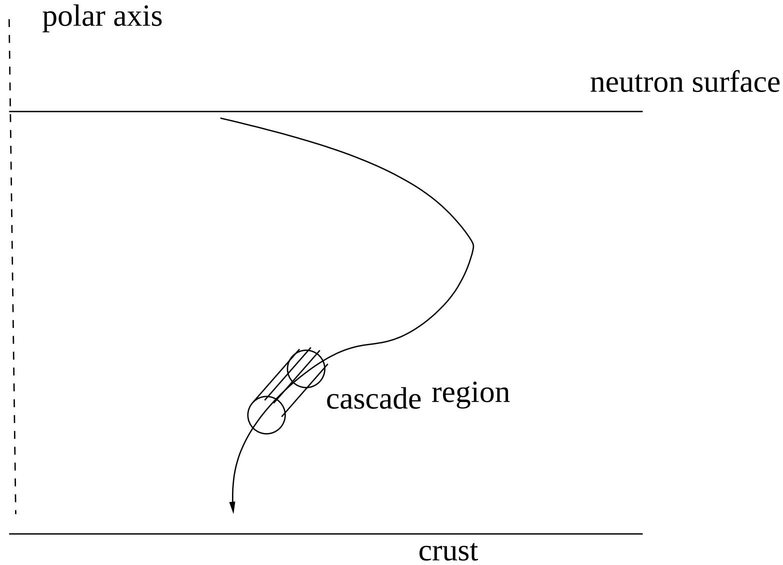


Figure 9: The piece occupied by the cascade