Integrating Recent Innovations in Single-Stage Stellarator Optimization for the Columbia Stellarator eXperiment

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PPPL Theory seminar

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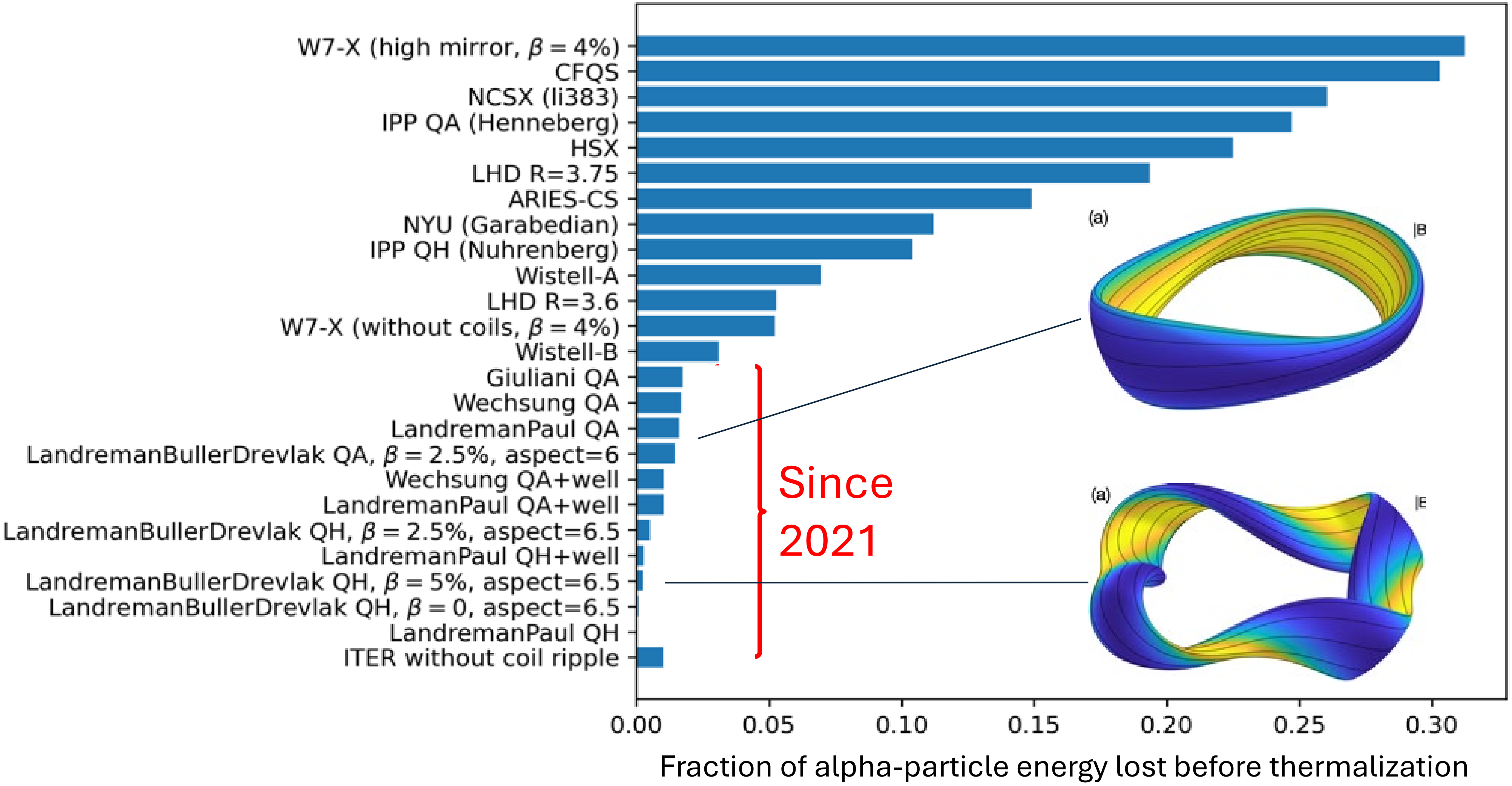
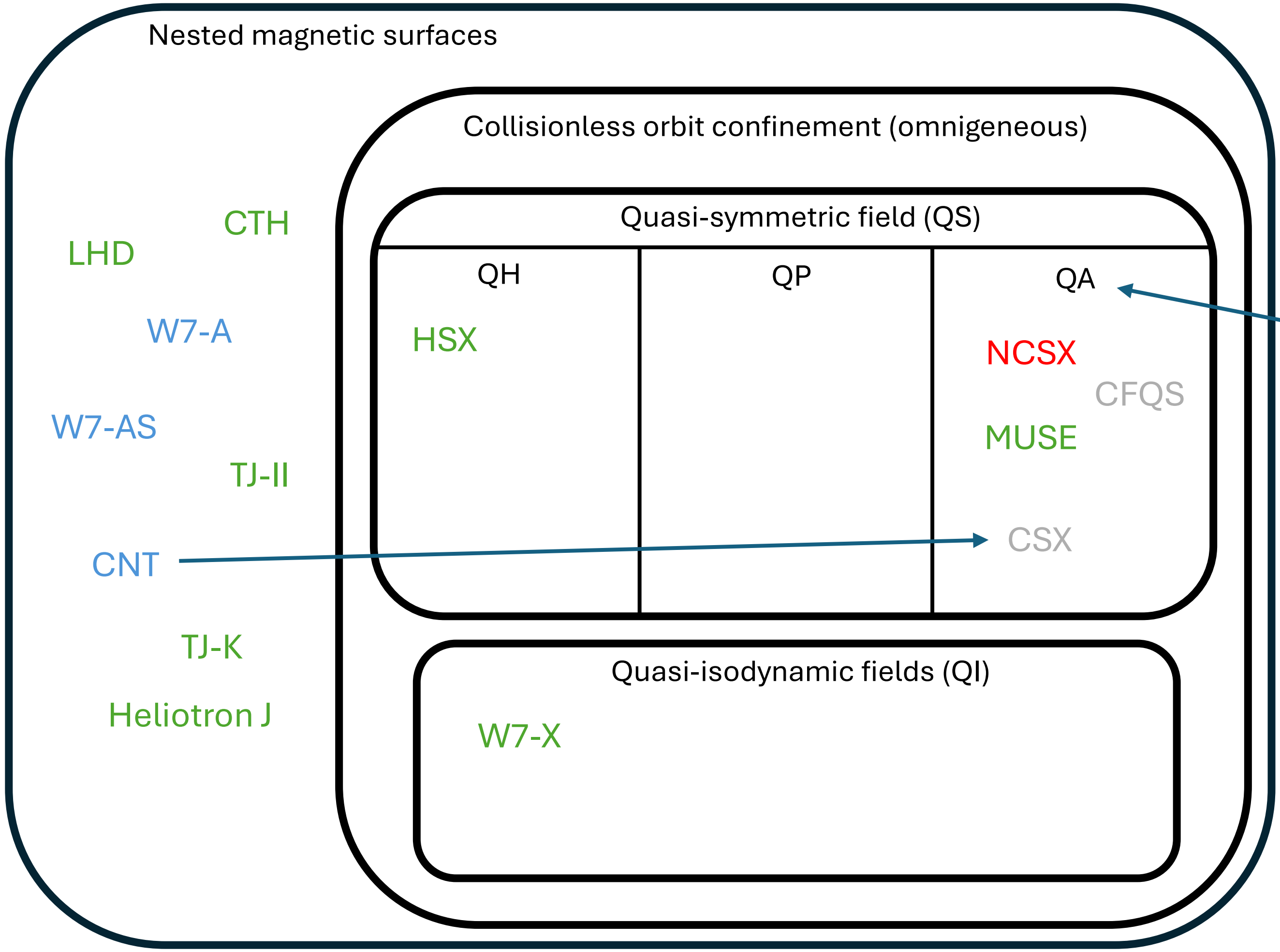
Stellarators require careful optimization



Landreman & Paul PRL (2022),
Wechsung et al PNAS (2022),
Giuliani et al arXiv (2022),
Landreman, Buller & Drevlak arXiv (2022)

Quasisymmetry is one possible path to optimize neoclassical transport in stellarators



Nested magnetic surfaces

CTH

LHD

W7-A

W7-AS

TJ-II

CNT

TJ-K

Heliotron J

Collisionless orbit confinement (omnigeneous)

Quasi-symmetric field (QS)

QH

HSX

QP

QA

NCSX

CFQS

MUSE

CSX

Quasi-isodynamic fields (QI)

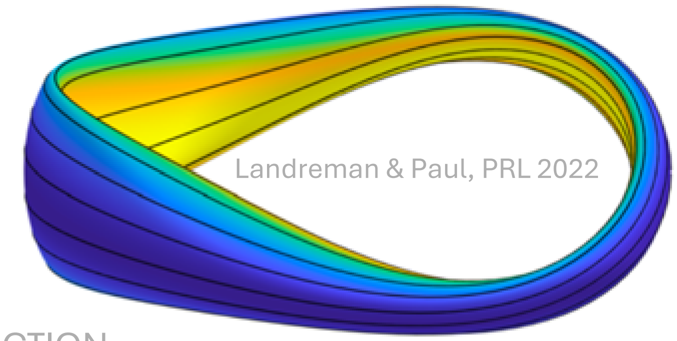
W7-X

OPERATIONAL

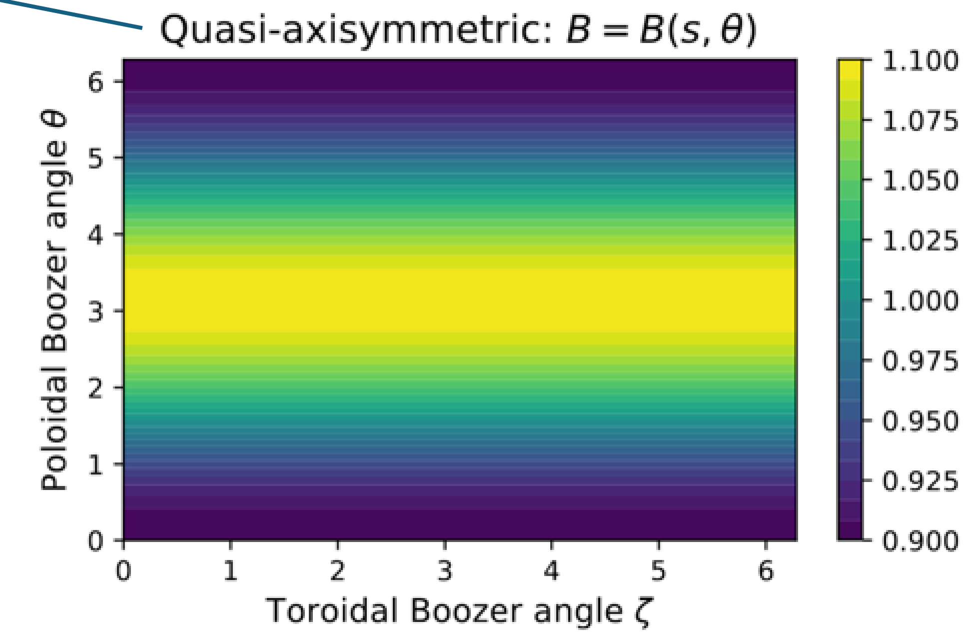
SHUT DOWN

CANCELLED

UNDER CONSTRUCTION

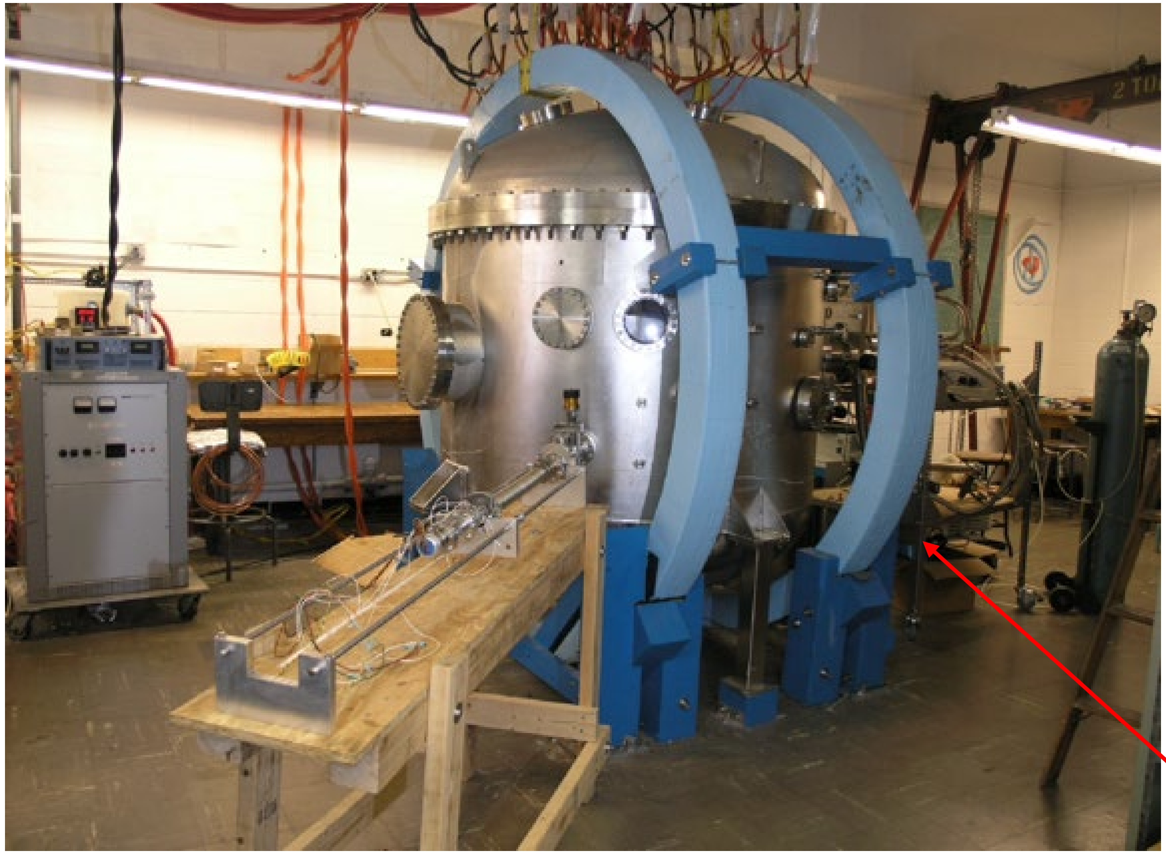


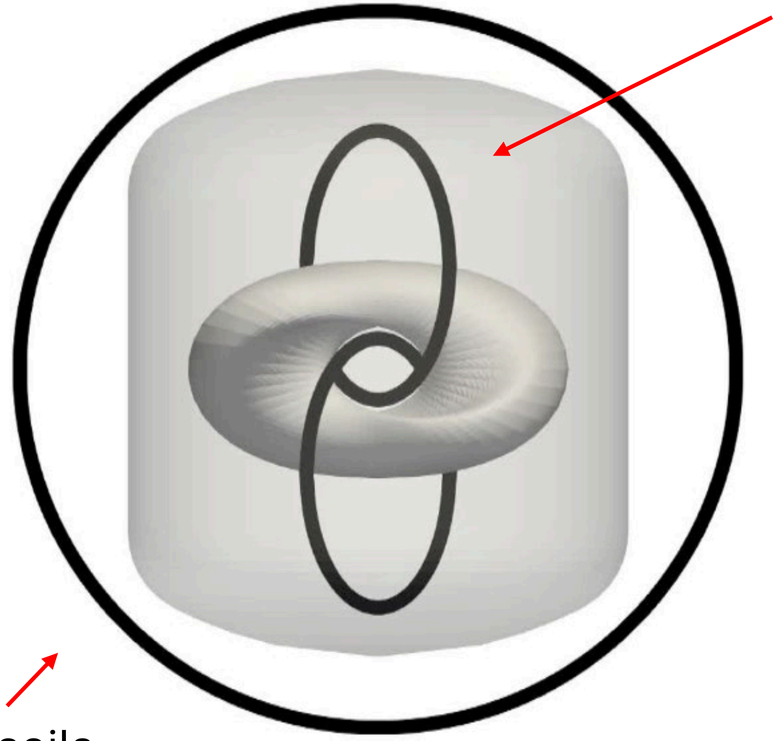
Landreman & Paul, PRL 2022



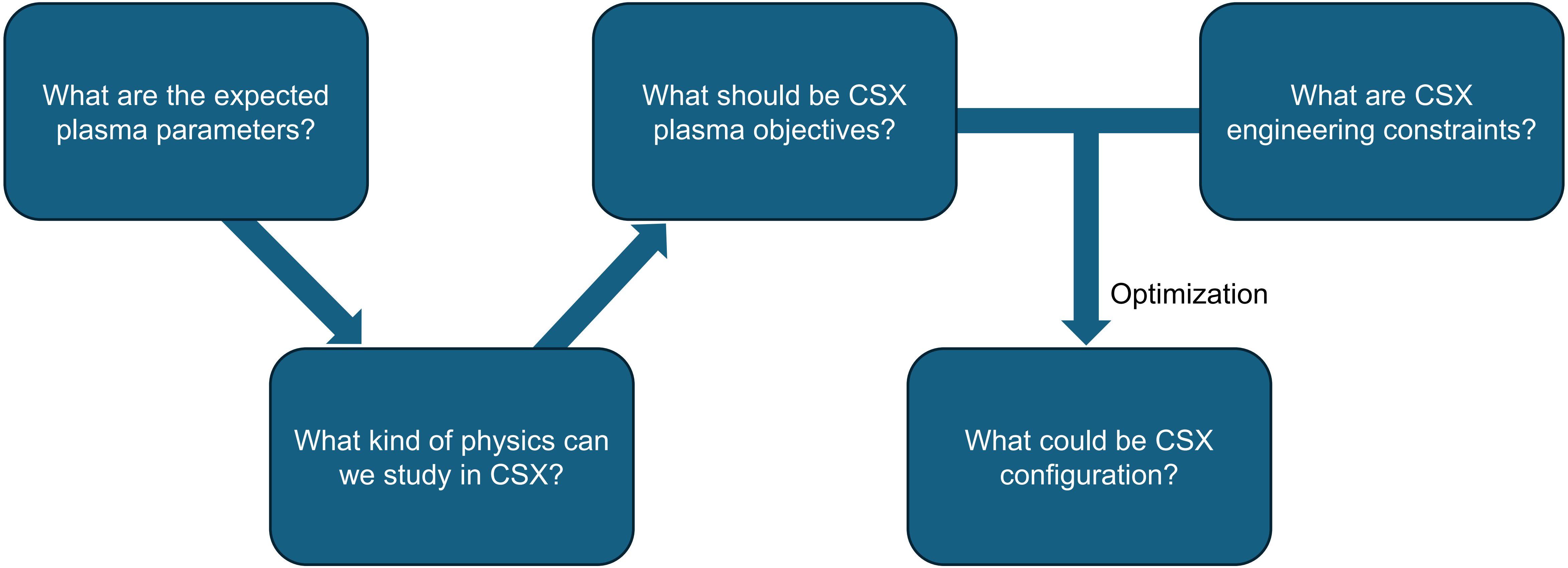
Quasi-axisymmetric: $B = B(s, \theta)$

CNT device at Columbia University can be refurnished to study quasi-axisymmetry and NI-HTS coils





Physics of interest guides optimization targets



What are the expected plasma parameters?

What kind of physics can we study in CSX?

What should be CSX plasma objectives?

What are CSX engineering constraints?

Optimization

What could be CSX configuration?

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Key plasma parameters are estimated from past CNT data

	Low power	**High beta**	**Low collisionality**
Density	$10^{17} m^{-3}$	$5 \cdot 10^{18} m^{-3}$	$10^{17} m^{-3}$
Heating system	10kW, 2.45GHz ECH	40-100kW	40-100kW
Electron temperature	$5eV$	$30eV$	$30eV$
Magnetic field	$0.1T$	$0.08T$	$0.5T$
Rotational transform	0.27	0.27	0.27
Minor radius	$13cm$	$13cm$	$13cm$
Volume	$0.1m^3$	$0.1m^3$	$0.1m^3$
Plasma beta	0.002%	$\mathbf{0.94}\%$	0.0005%
Electron collisionality ν^*	0.25	0.37	$\mathbf{0.009}$
Ion collisionality ν^*	0.17	0.25	$\mathbf{0.006}$
Neoclassical regime	Plateau	Plateau	Low collisionality

K. Hammond 2017

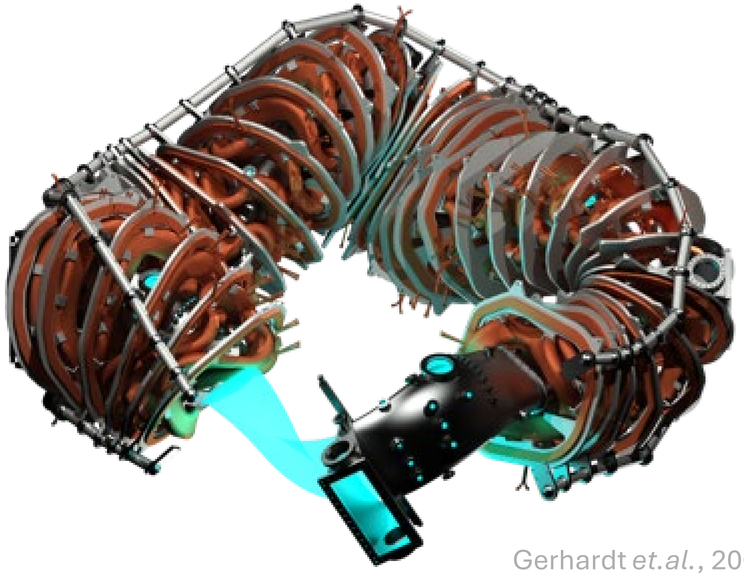
Plasma flow damping is reduced in QS fields

- Capability to sustain strong flows and quasisymmetry are equivalent

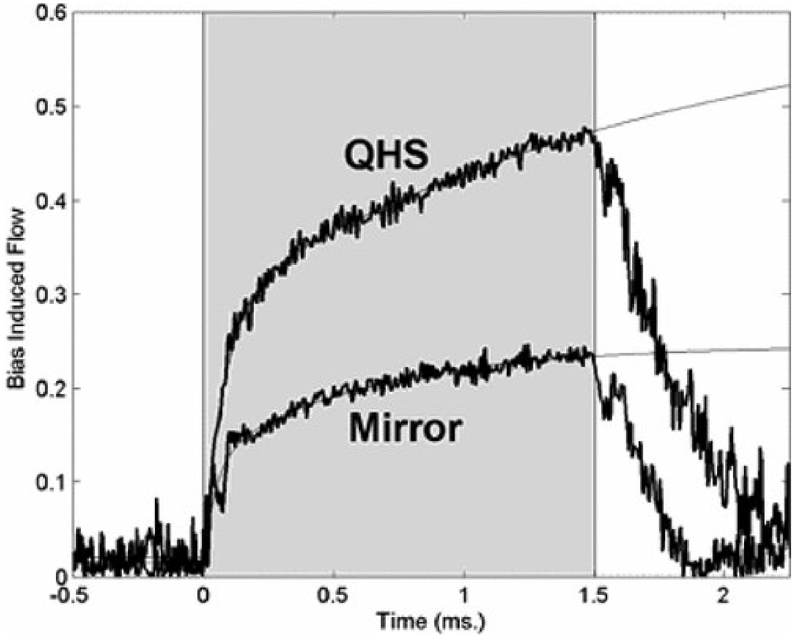
Helander *et.al.*, 2008

- **Flow is possible in symmetry direction independently of collisionality regime**
  - Minor radius 0.13m
  - Magnetic field on axis 0.5T
  - 50kW heating power

- HSX observed reduced flow damping with QS



Gerhardt *et.al.*, 2005



Weak flow damping condition dictates required QA precision

If $\boldsymbol{B} = \boldsymbol{B_{QS}} + \alpha \boldsymbol{B_{non-QS}}$, strong flows are possible if

$$|\alpha\partial_\theta B_{non-QS}| \sim \alpha|\partial_\theta B_{QS}|$$

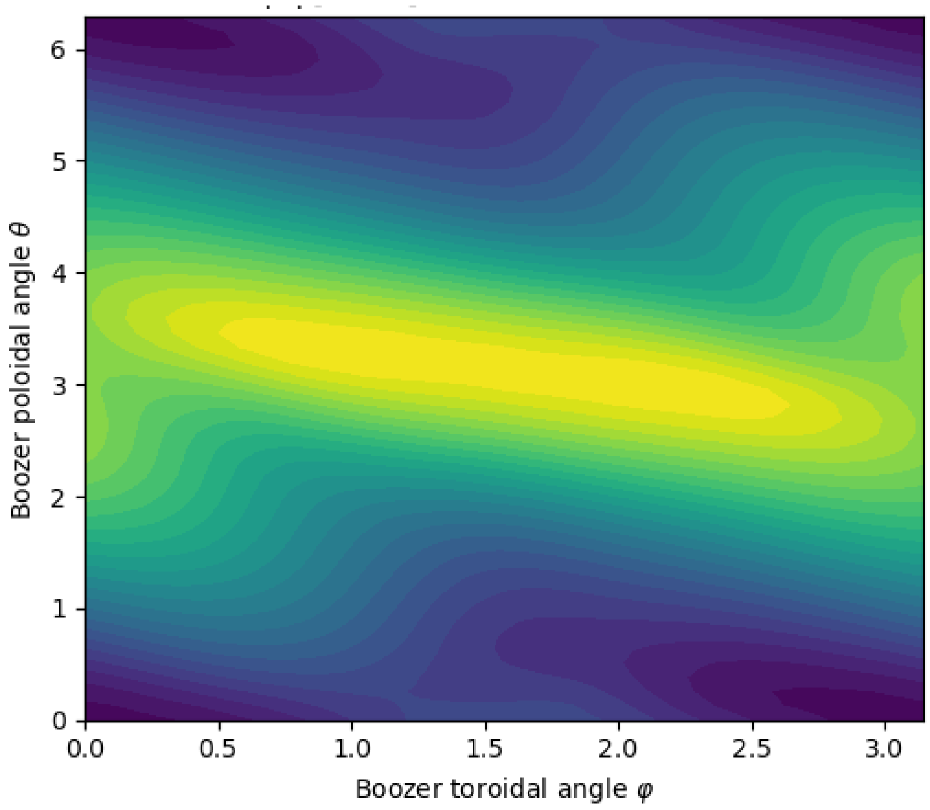
$$|\alpha\partial_\zeta B_{non-QS}| \sim \alpha|\partial_\zeta B_{QS}|$$

Calvo *et.al.*, 2013

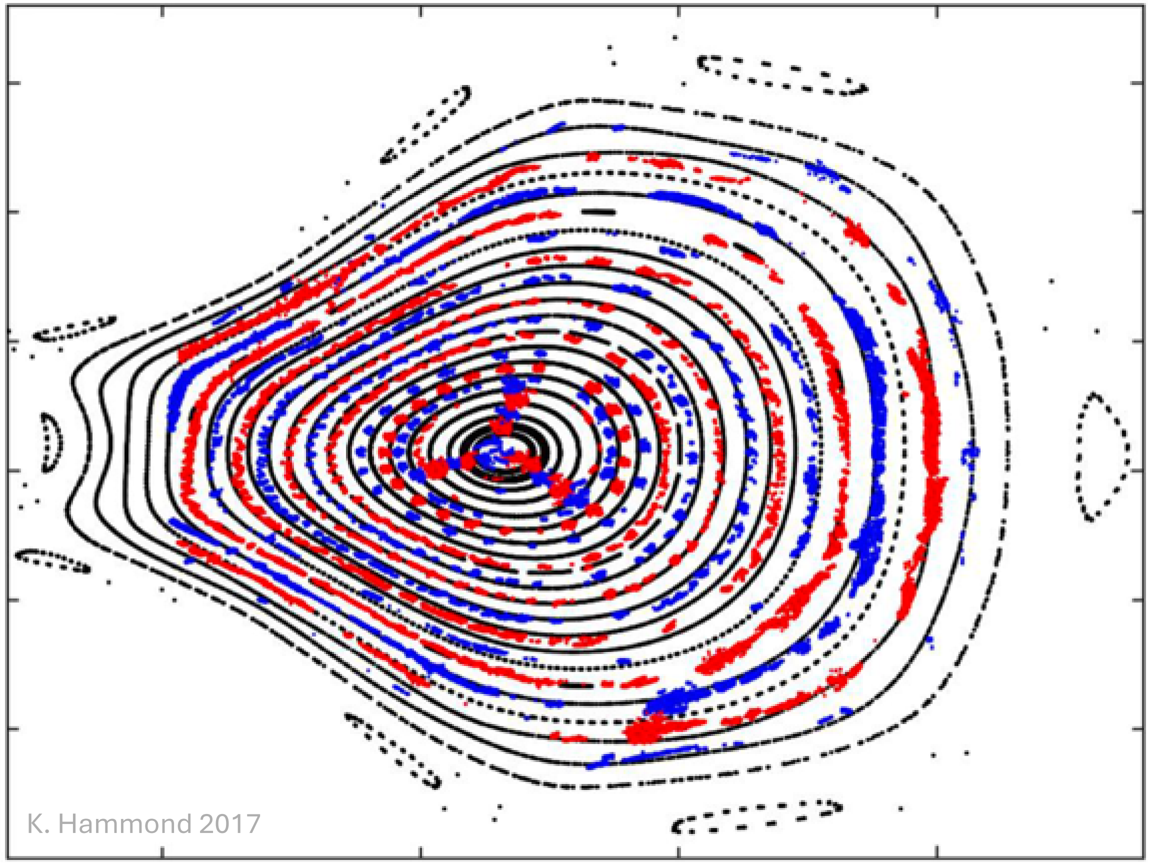
Calvo *et.al.*, 2015

Then flows close to sonic speed are obtained as long as

$$\alpha < \sqrt{\frac{\rho_i}{|\nabla \ln B_0|^{-1}}} \sim \sqrt{\frac{\rho_i}{a}} \sim 10\%$$

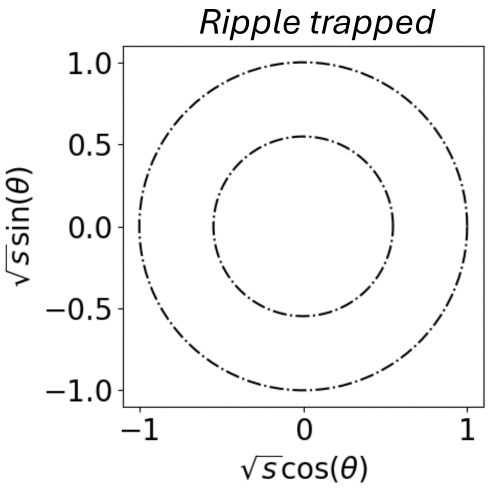


Electron beam mapping can be used to study electron trajectories

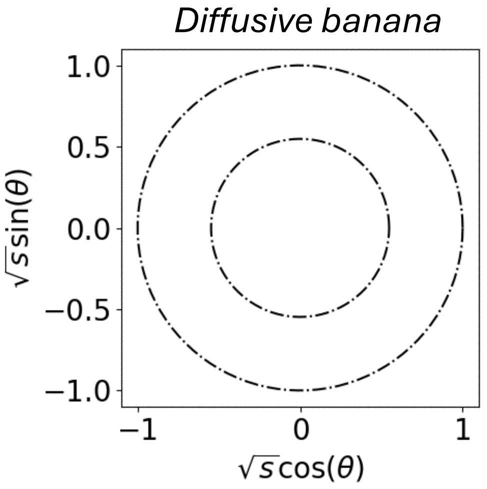


K. Hammond 2017

- **Validate** magnetic field
- Design of error field **correction coils**
- Study electron loss channels



Ripple trapped



Diffusive banana

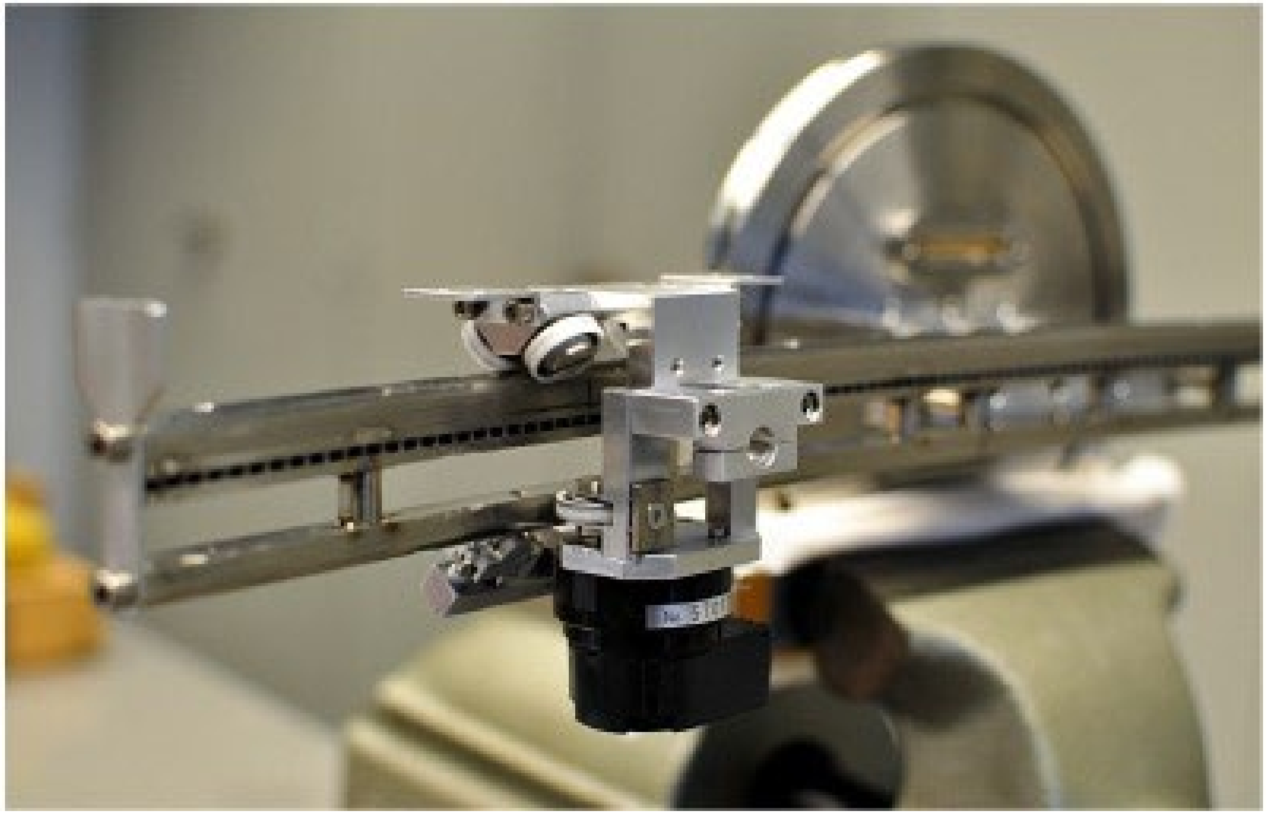
“Fast” ions can be generated with a source in the plasma

Trapped bulk ions: need small banana width for confinement
→ Need sufficient rotational transform

$$\Delta r \sim \frac{\rho}{\pi \iota}\sqrt{\frac{2}{\epsilon(1+\epsilon)}} \quad \Longrightarrow \quad \iota > \iota_c = 0.21$$

Trapped “fast” ions: not confined. Rotational transform too low.

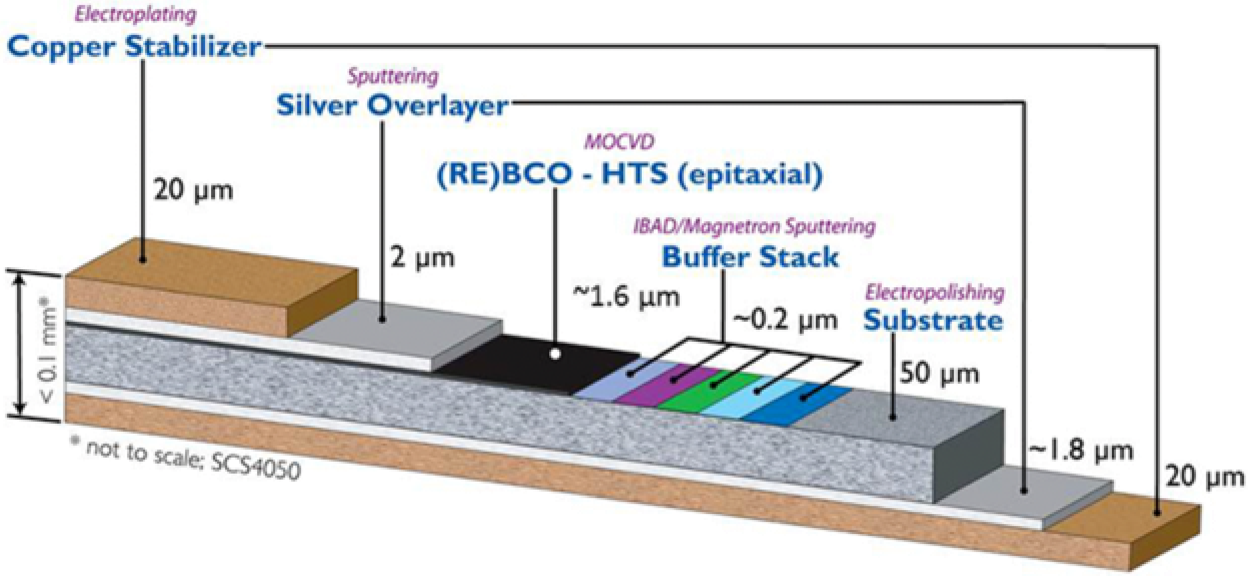
Passing ions:
Study interaction with MHD activity

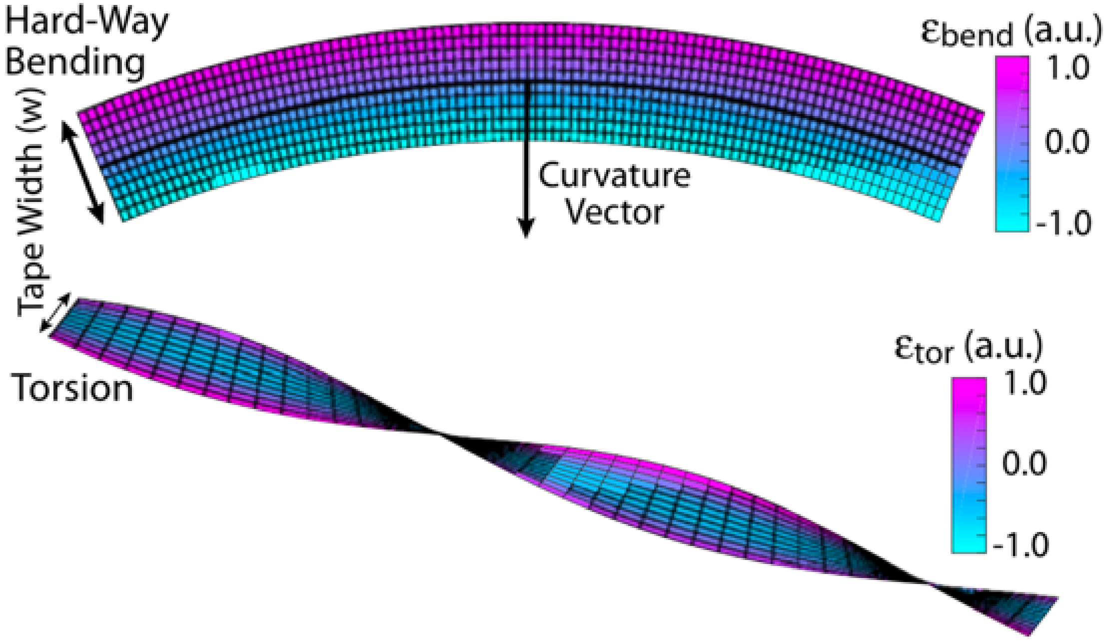


Fast ion source in TORPEX

Bovet *et.al.*, 2012

IL coils are made with High-Temperature Superconducting (HTS) tape





$$\epsilon_{tot} = \max(\epsilon_{bend}(x) + \epsilon_{tor}(x)) < 0.004$$

Paz-Soldan, 2020

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- Expected parameters & experimental possibilities
- Objectives and constraints
- Optimization
- Coil manufacturing
- Future plans and conclusions

Designing CSX requires balancing objectives with engineering constraints

OBJECTIVES

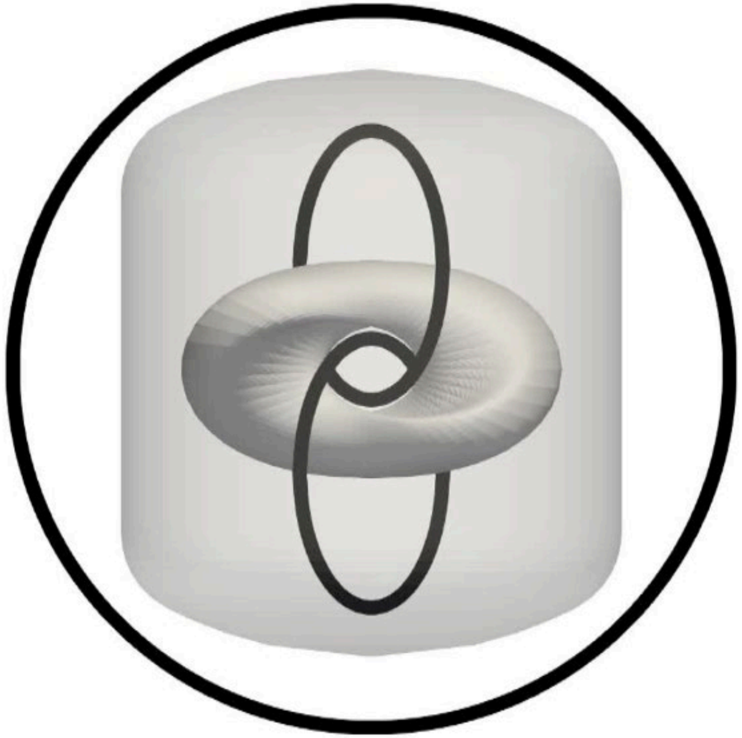
- Flow damping studies -> good quasi-symmetry
- Bulk ions confinement-> minimum rotational transform
- Large volume
- Filled with magnetic surfaces

CONSTRAINTS

- Use external PF coils
- Use only two IL coils
- Fits within the existing vacuum vessel
- Satisfies HTS constraints
- Robust to manufacturing errors

Limited parameter space

Strong engineering constraints



Coils degrees of freedom are limited

- PF coils:
  - Fixed geometry
  - Current is free

1 dof

- IL coils:
  - Current is fixed
  - Geometry is free, stellarator symmetry enforced

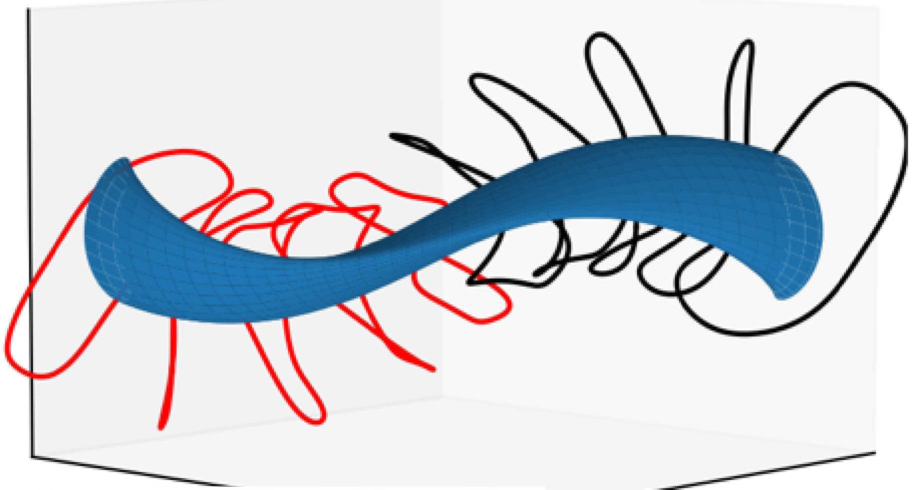
$$x = x_0 + \sum_{n=1}^{7} x_n \cos(2\pi n l)$$

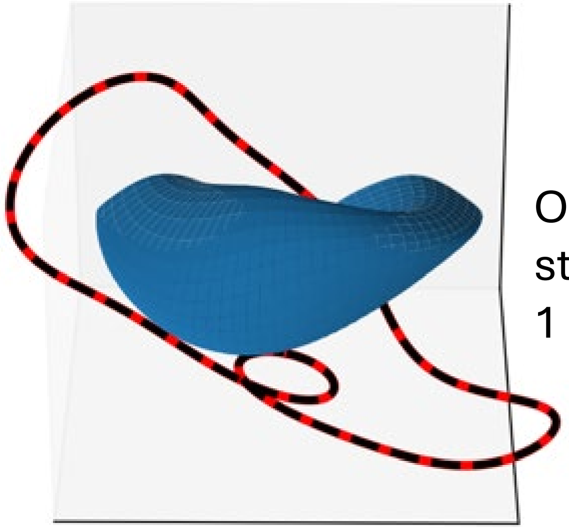
$$y = \sum_{n=1}^{7} y_n \, sin(2\pi n l)$$

$$z = \sum_{n=1}^{7} z_n \, sin(2\pi n l)$$

22 dofs

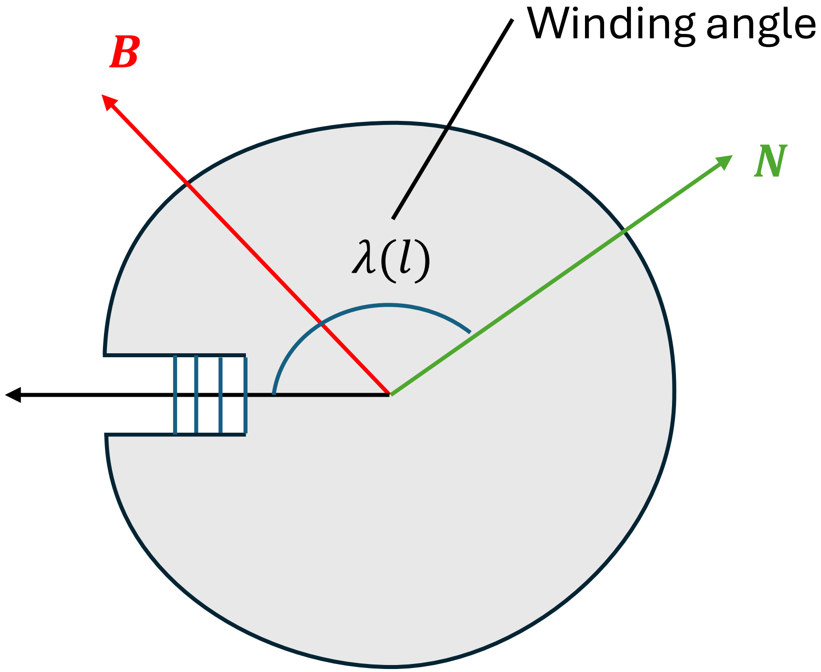
One field period, stellarator symmetric, 10 coils per field period



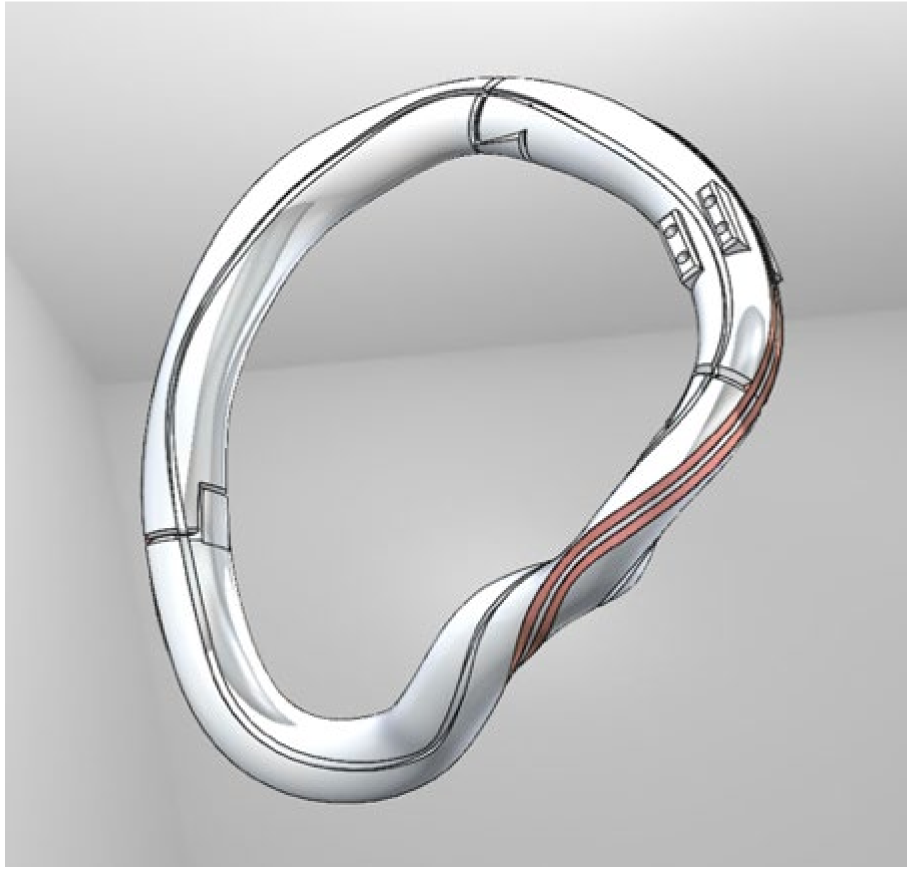


One field period, stellarator symmetric, 1 coils per field period

Winding angle freedom is used to minimize HTS strain



$(\boldsymbol{T}, \boldsymbol{N}, \boldsymbol{B})$ is the curve centroid frame

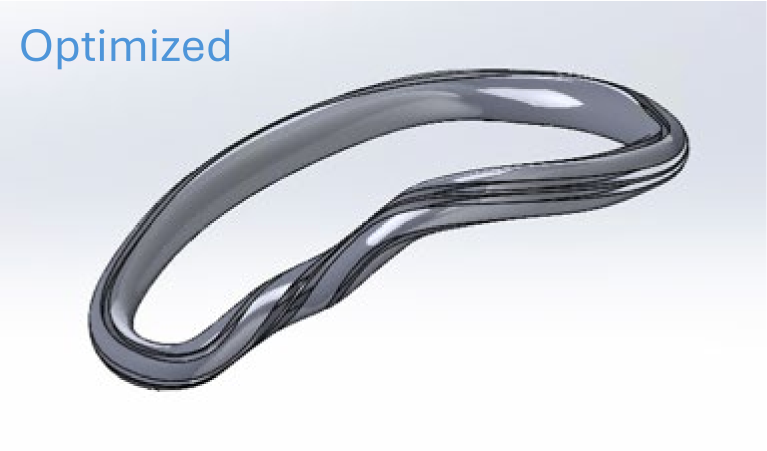


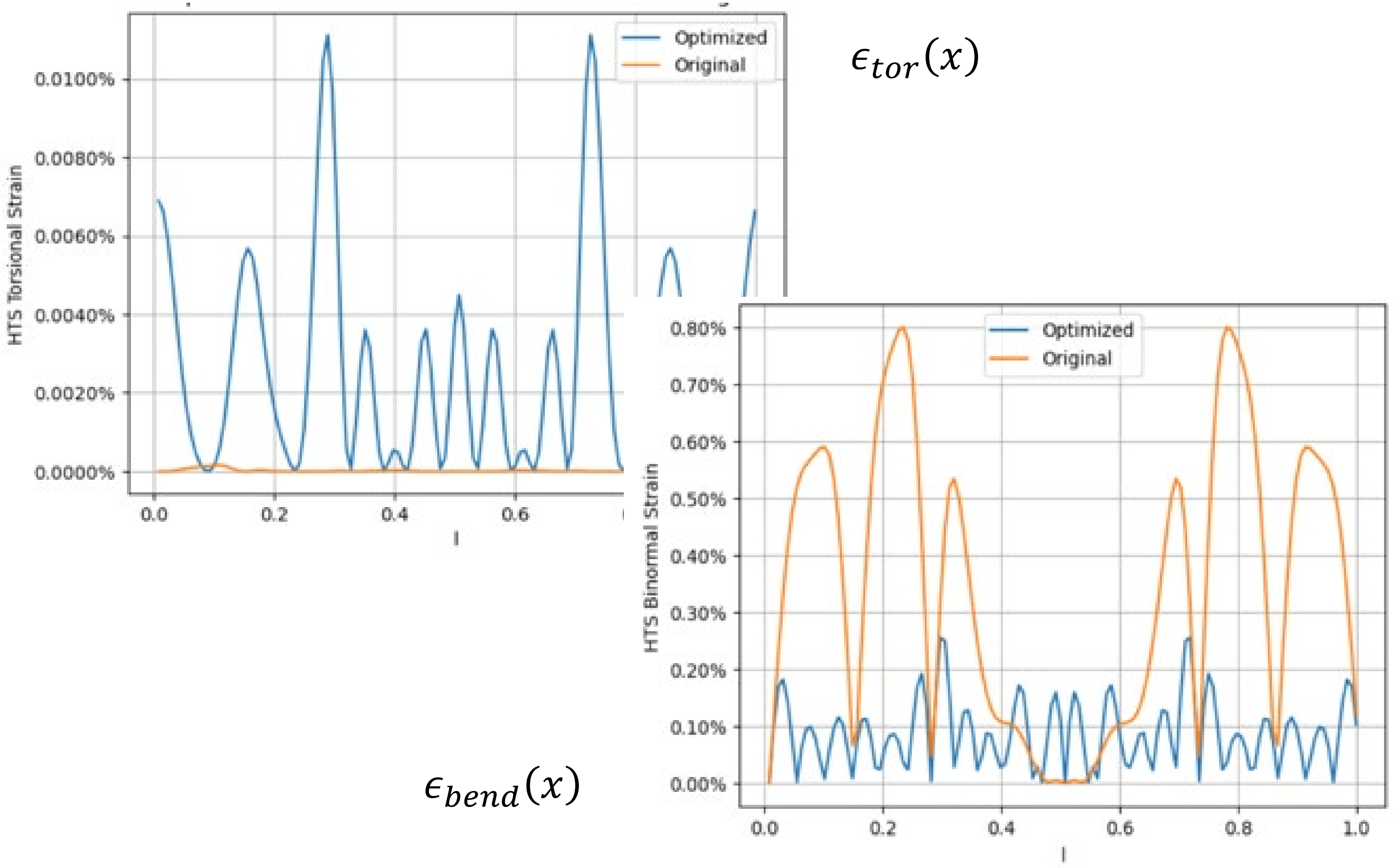
$$\lambda(l) = \lambda_0 + \sum_{n=1}^{10} \lambda_{cn} \cos(2\pi n l) + \sum_{n=1}^{10} \lambda_{sn} \sin(2\pi n l)$$

➡ 21 dofs $\{\lambda_0, \lambda_{cn}, \lambda_{sn}\}$

$\boldsymbol{d} = \{\lambda_0, \lambda_{cn}, \lambda_{sn}\}$

Binormal strain is traded off with torsional strain by optimizing the winding angle





$\epsilon_{tor}(x)$

$\epsilon_{bend}(x)$

Designing CSX requires balancing objectives with engineering constraints

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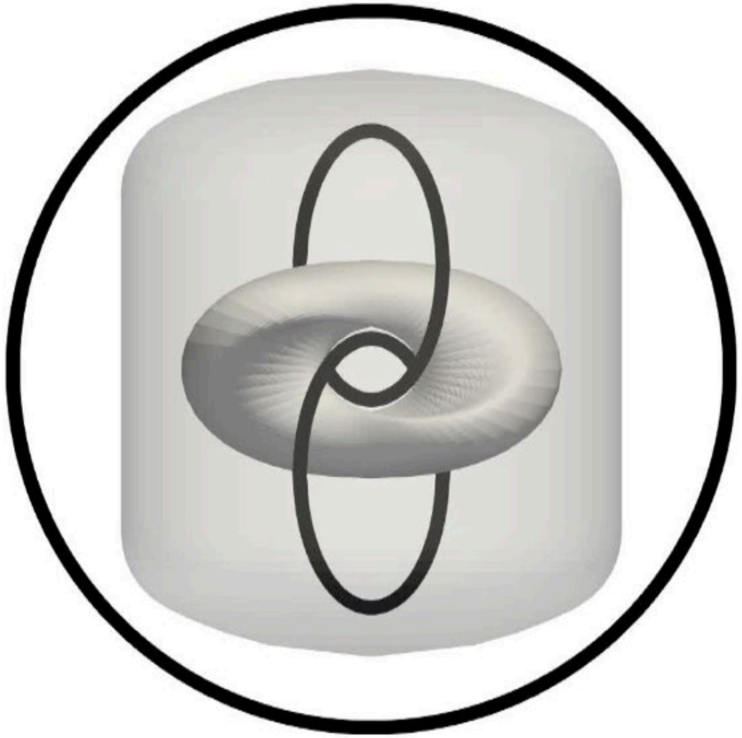
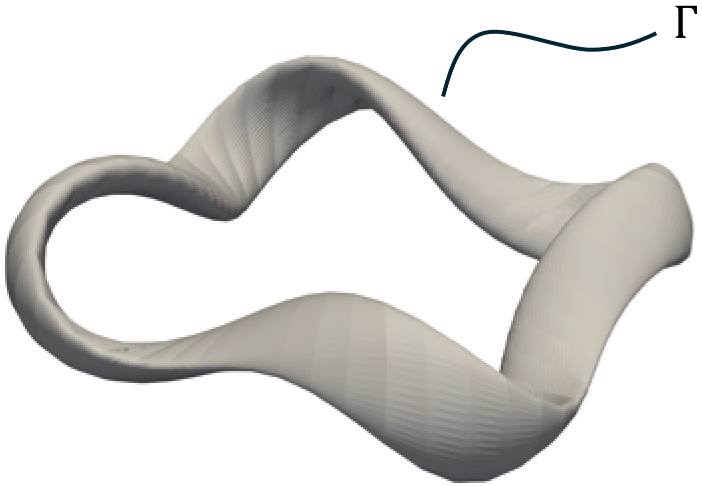
Strong engineering constraints



Traditional stellarator optimization uses the “two-stage” approach

STAGE I

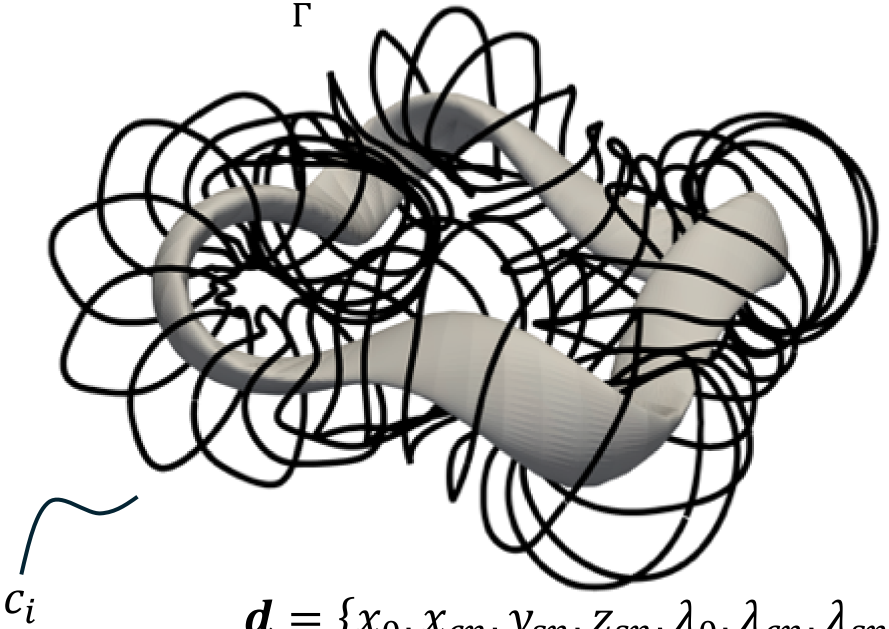
$$f_I(\Gamma) = f_{QS}(\Gamma) + f_\iota(\Gamma) + f_A(\Gamma) + f_{turb}(\Gamma) + \ \dots$$



$$\boldsymbol{d} = \{\Gamma_n\}$$

STAGE II

$$f_{II}(\{\mathrm{c_i}, \lambda_i\}) = \int_\Gamma (\boldsymbol{B}(\{c_i\}) \cdot \hat{n})^2 \, dS + f_{reg}(\{c_i, \lambda_i\})$$



$$\boldsymbol{d} = \{x_0, x_{cn}, y_{sn}, z_{sn}, \lambda_0, \lambda_{cn}, \lambda_{sn}, I\}$$

Strong engineering constraints calls for combined coil-plasma optimization approaches

Different algorithms exist:

Henneberg *et.al.*, 2021

- Using fixed-boundary MHD equilibrium code

  Jorge *et.al.*, 2023

- Using free-boundary MHD equilibrium codes

  Smiet *et.al. 2024*

- Optimizing vacuum field, by directly constructing plasma boundary from the vacuum field

  Giuliani *et.al.*, 2023

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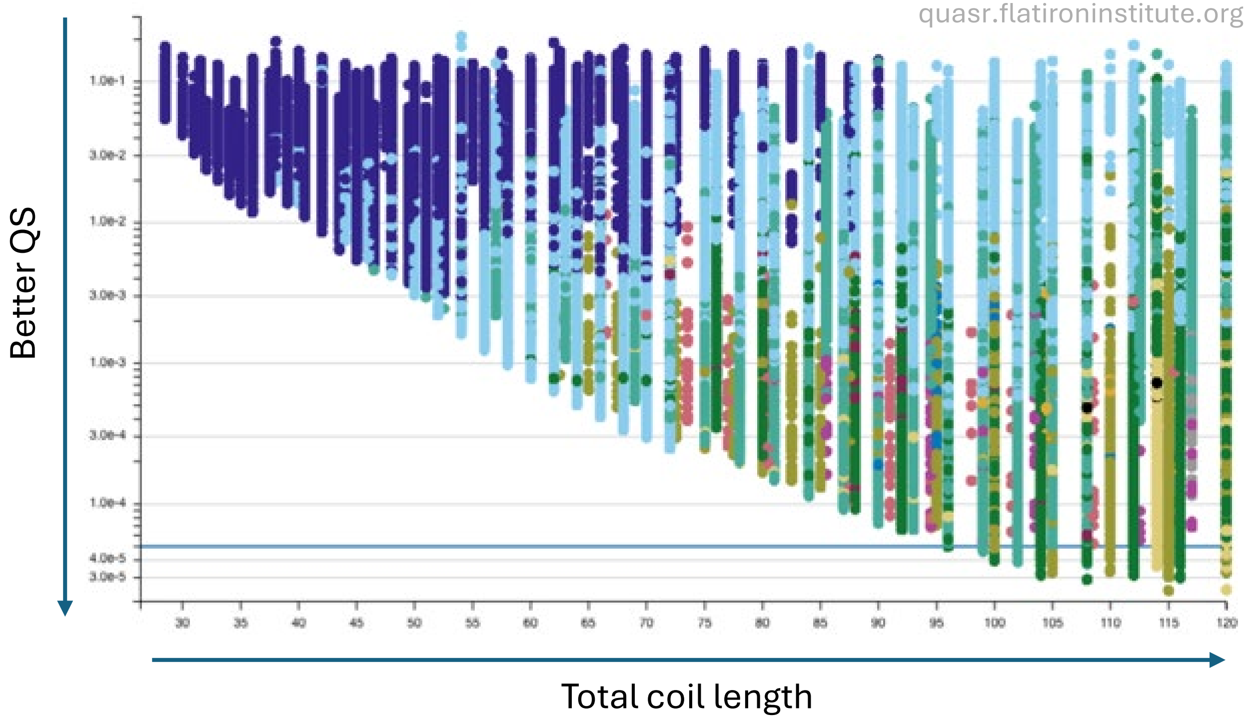

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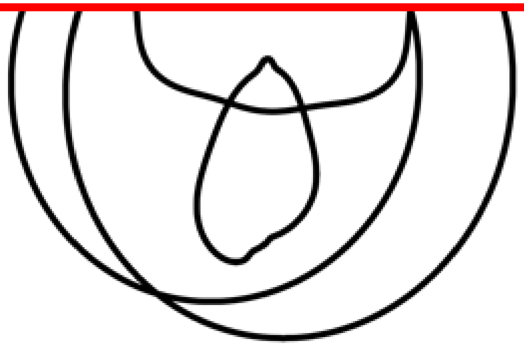
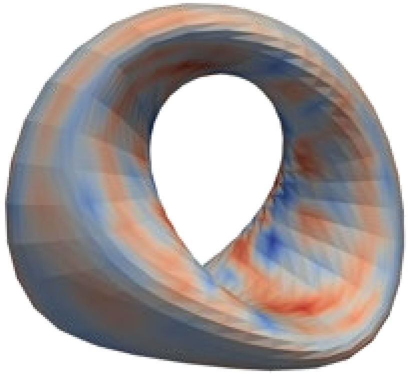
First approach uses fixed-boundary equilibrium solver

$$f(\Gamma,\{c_i,\lambda_i\}) = f_I(\Gamma) + f_{reg}(\{c_i,\lambda_i\}) + \int_\Gamma \left(\frac{\boldsymbol{B}(\{c_i\})\cdot\hat{n}}{|\boldsymbol{B}|}\right)^2 dS$$

$$\boxed{\boldsymbol{d} = \{\Gamma_n, x_0, x_{cn}, y_{sn}, z_{sn}, \lambda_0, \lambda_{cn}, \lambda_{sn}, I_{PF}\}}$$





Targets:
- Quasisymmetry
- Rotational transform
- Volume

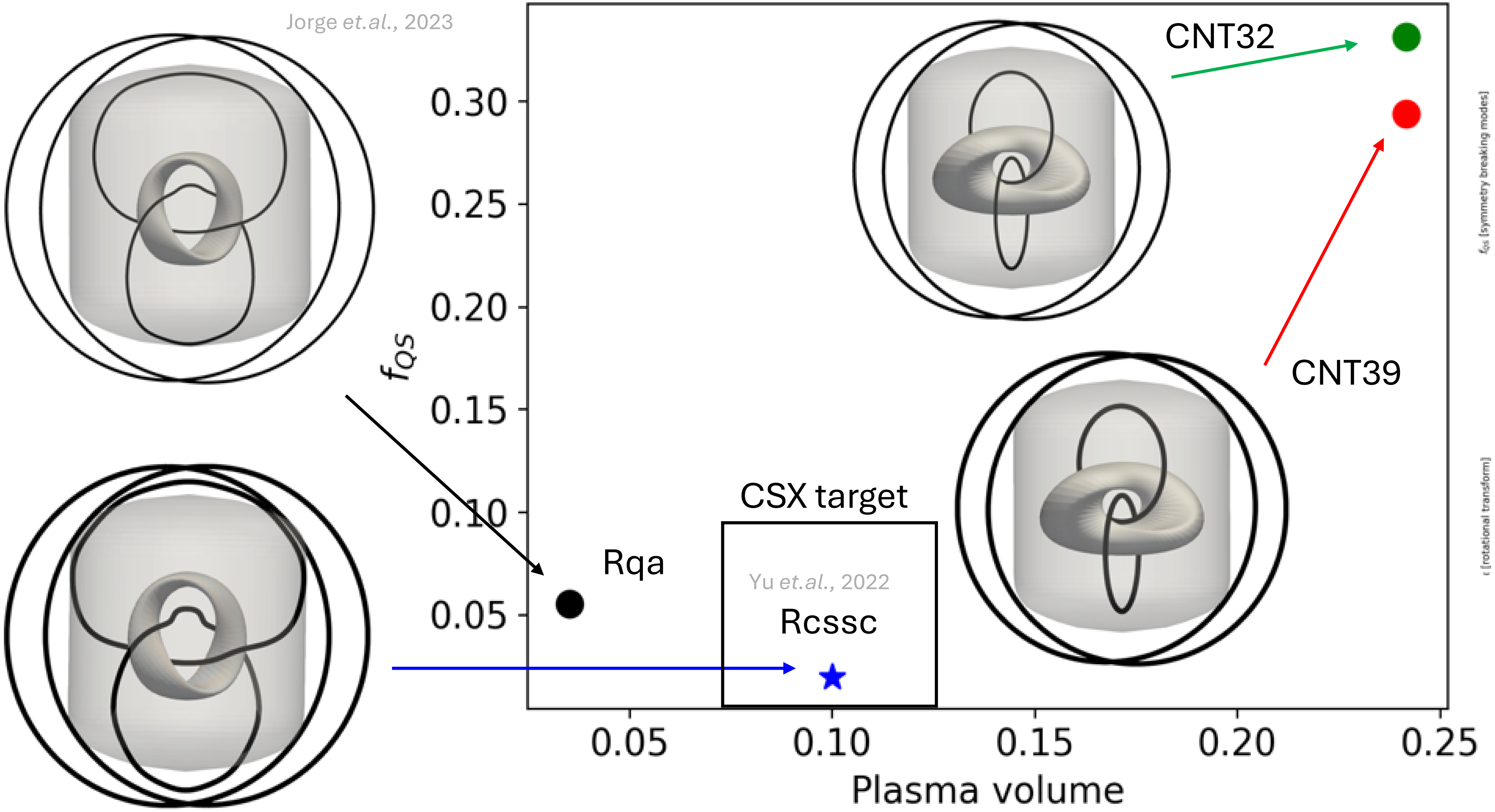
Solver: VMEC
Derivatives: Finite differences

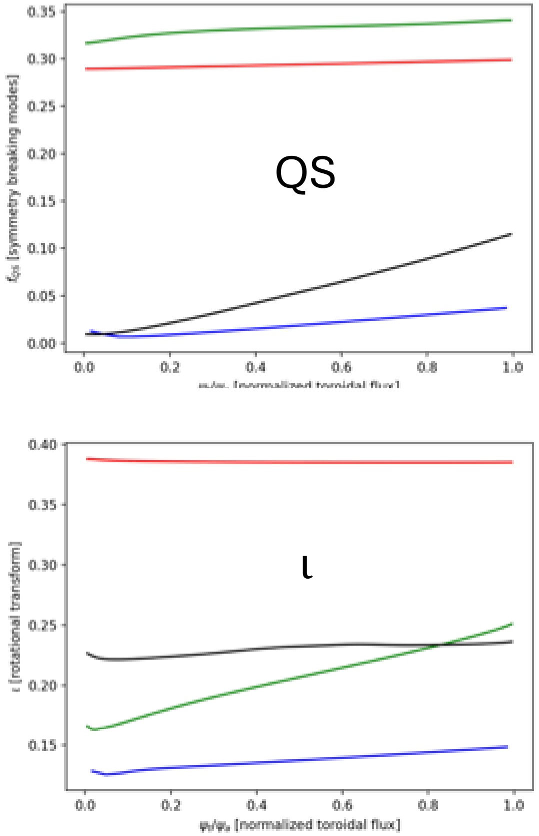
Target:
- Within vacuum vessel
- Max length
- Min coil-coil distance
- Min coil-plasma distance
- Max HTS strain
- Max Arclength variation

Derivatives: Automatic differentiation

Jorge *et.al.*, 2023

We consider various initial guesses

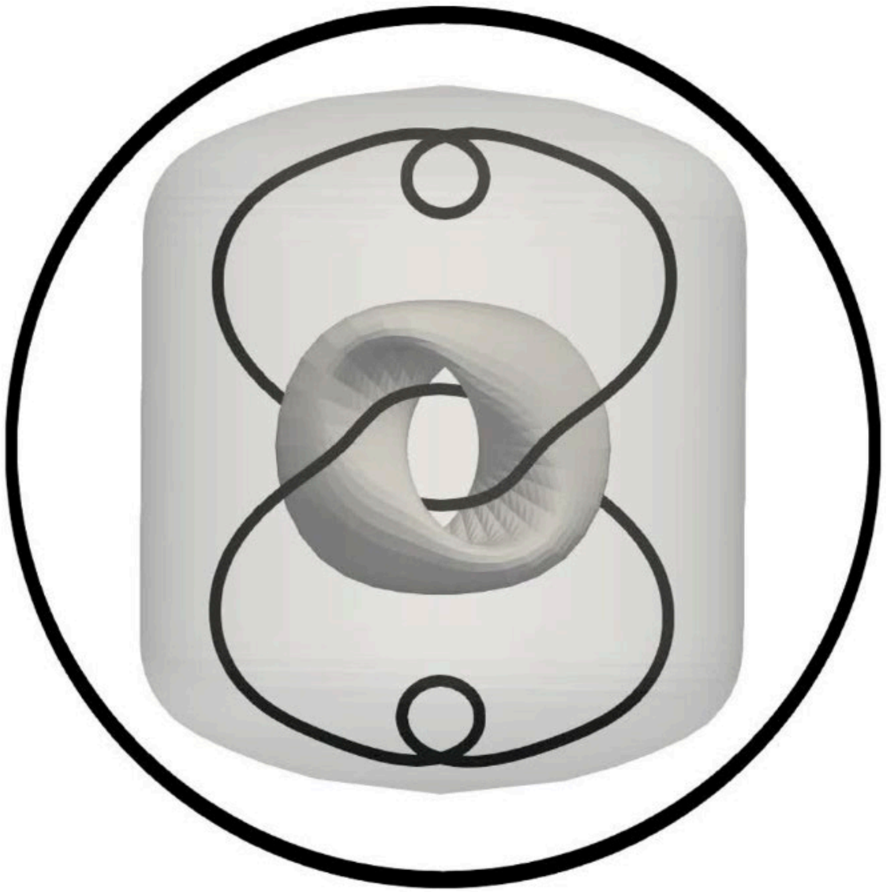


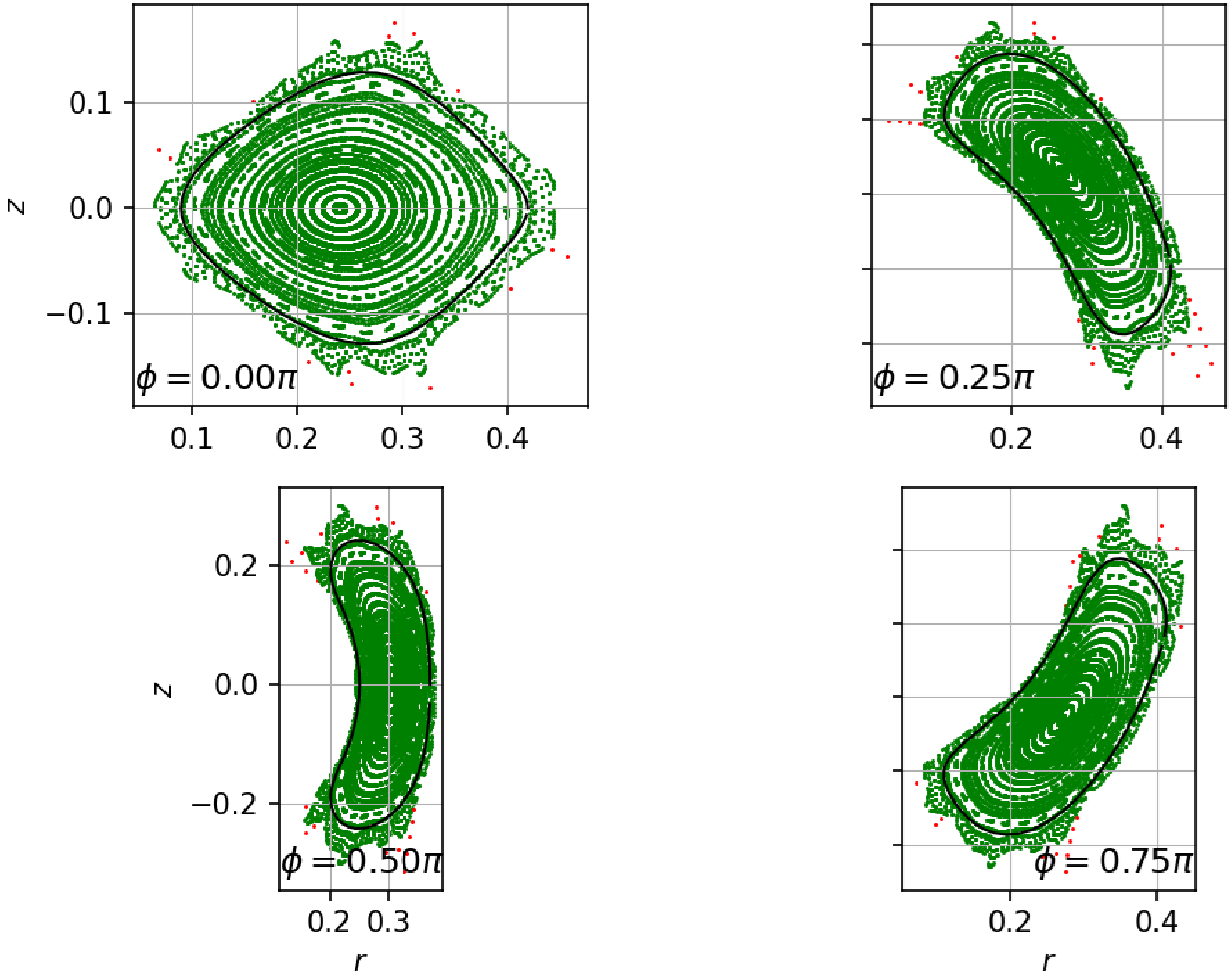


Jorge et.al., 2023

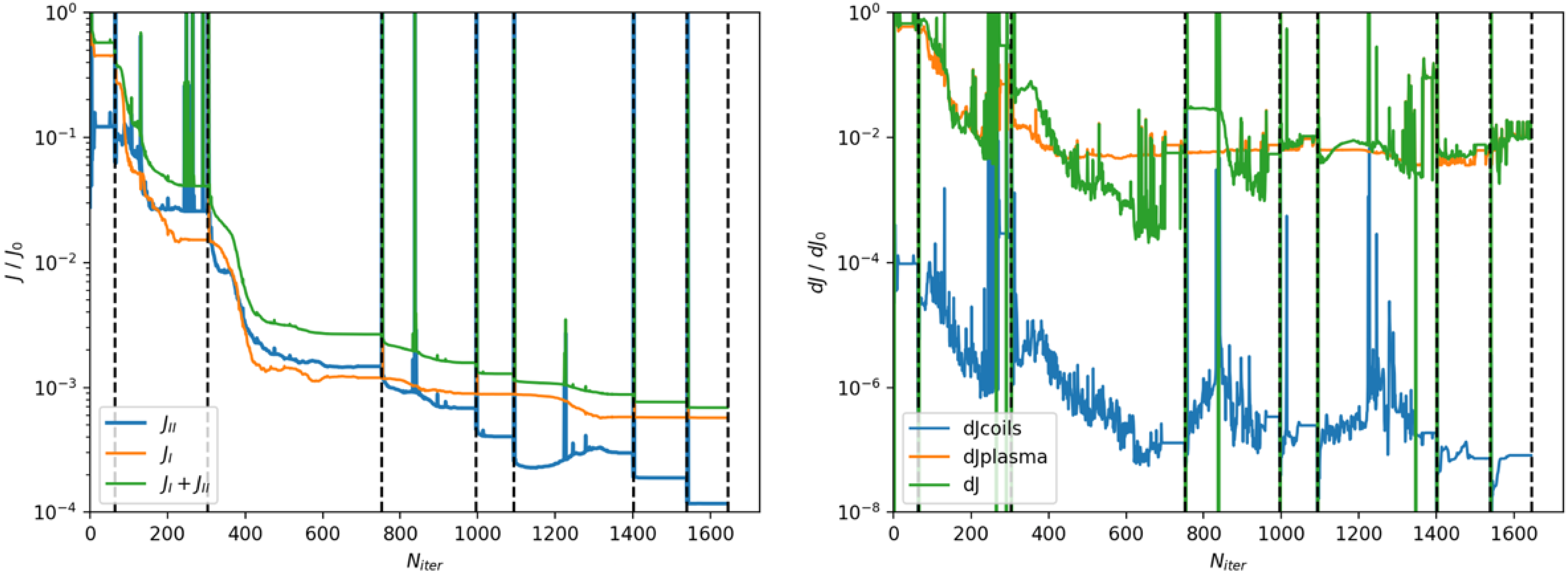
QS can be improved from CNT39

- $\iota = 0.38 \to 0.268$
- Max QS error = $30\% \to 10\%$
- Max HTS Strain = 0.006
- Volume = 0.24 →0.098 m^3





Finite difference errors hinder optimization results



Second approach does not rely on MHD solver

- Parameterize surface $\Gamma(\boldsymbol{r_s})$ in Boozer coordinates, total current G and rotational transform ι obtained from vacuum field.

Giuliani *et.al.*, 2023

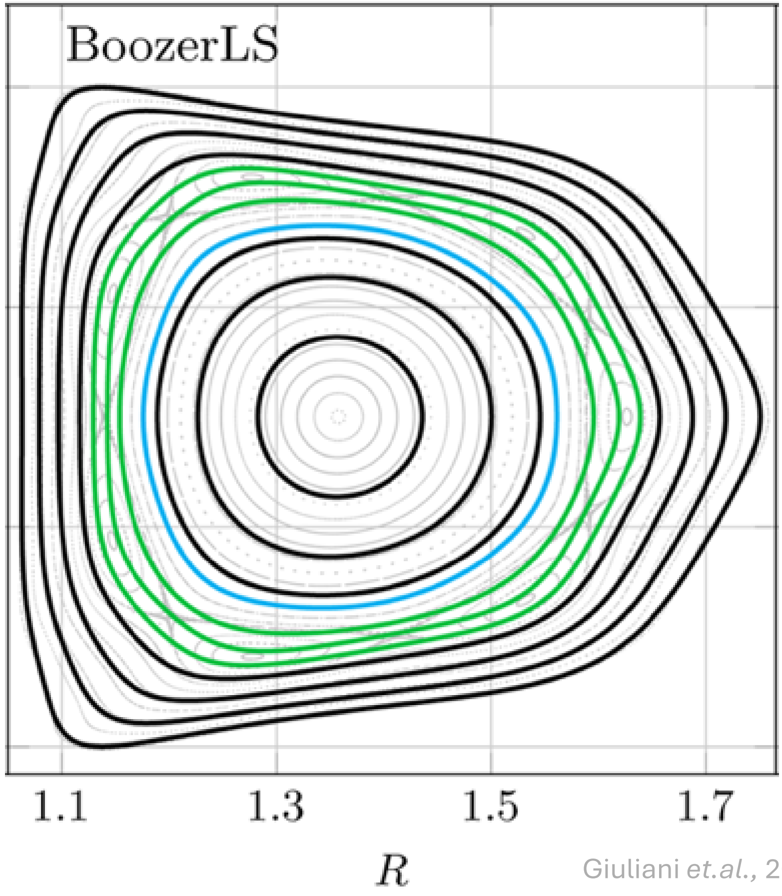
$$\boldsymbol{R} = G\frac{\boldsymbol{B}}{|\boldsymbol{B}|} - |\boldsymbol{B}|\left(\frac{\partial \boldsymbol{r_s}}{\partial \phi} + \iota\frac{\partial \boldsymbol{r_s}}{\partial \theta}\right)$$

- Target function is then

$$f(\{c_i, \lambda_i\}) = f_I(\mathbf{r_s}(\{\mathrm{c_i}\})) + f_{reg}(\{\mathrm{c_i}, \lambda_i\}) + |\boldsymbol{R}|^2$$

- (+) **Derivatives are available**
    - Speed
    - Robustness

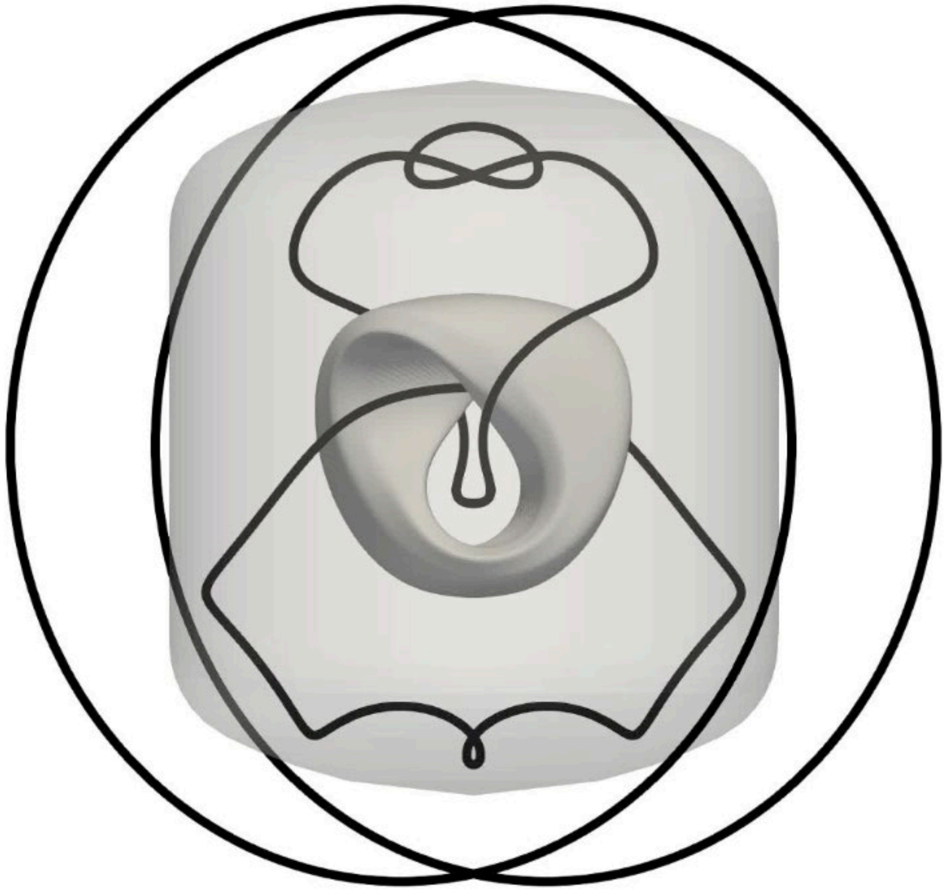
- (-) Only for vacuum fields

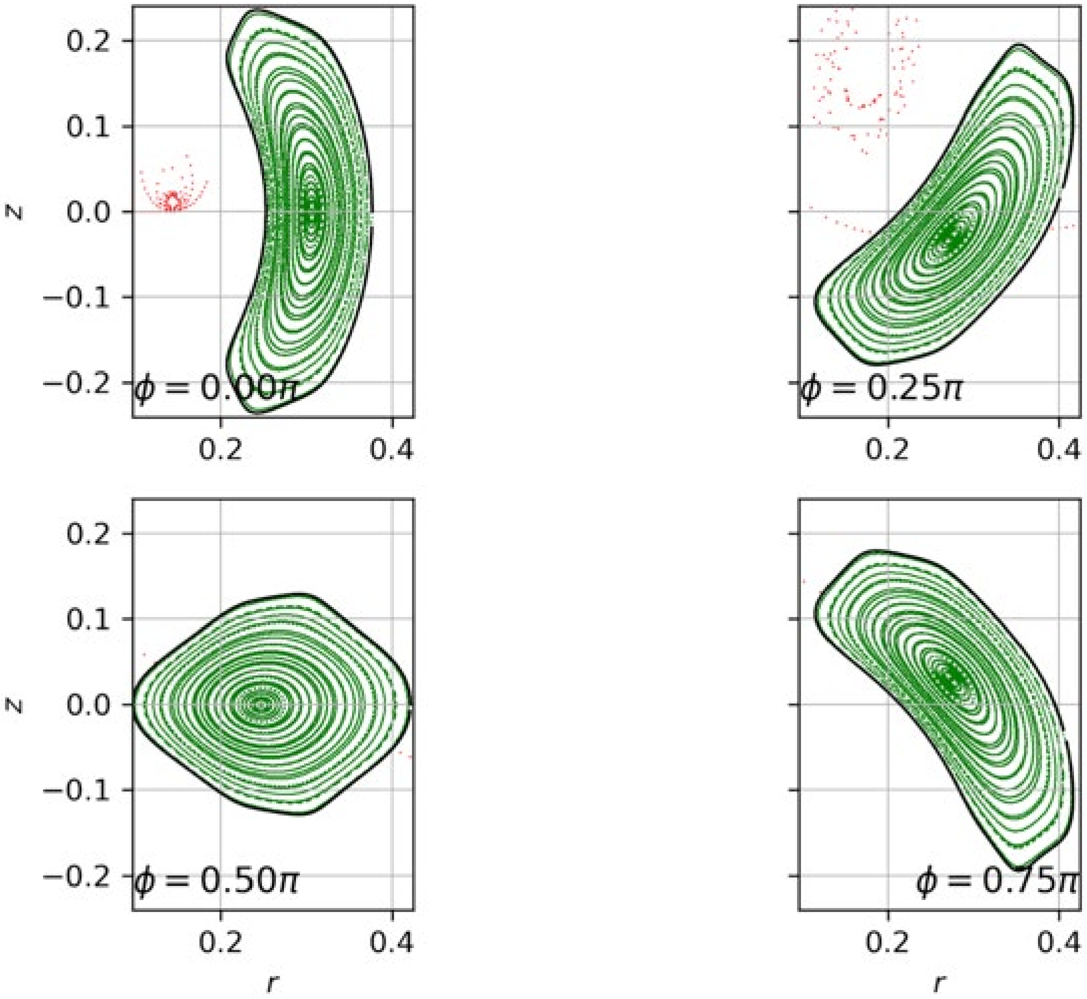


Giuliani *et.al.*, 2022

Better QA is achieved with the Boozer surface approach

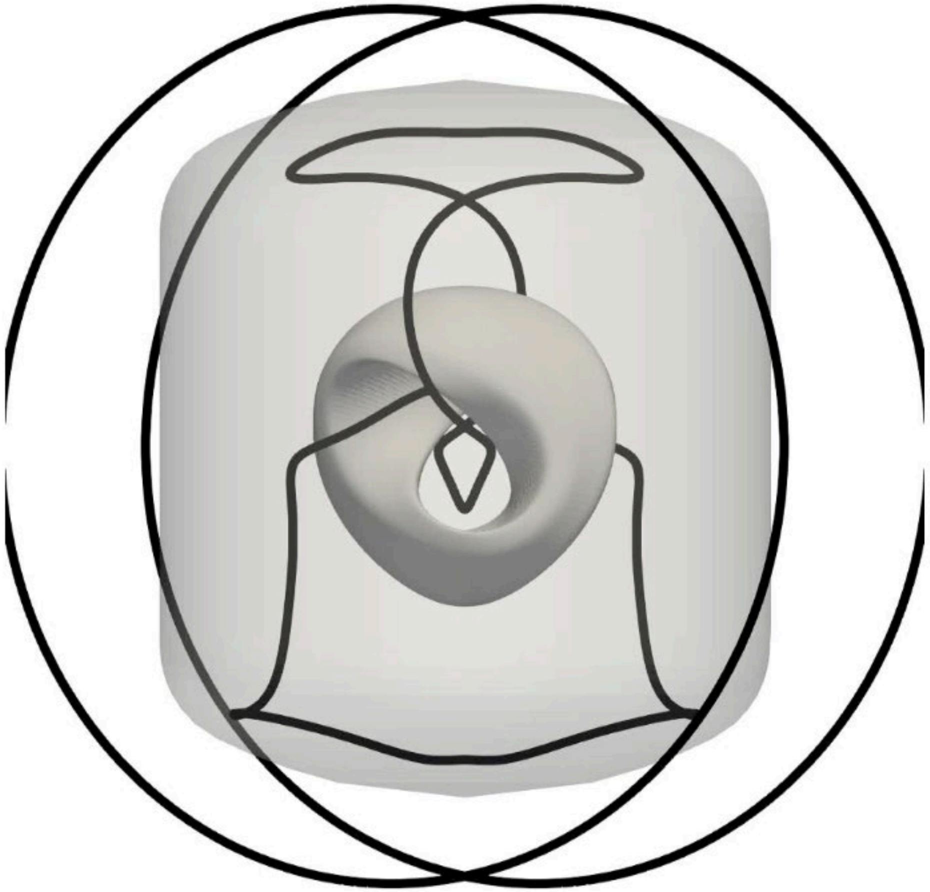
- $\iota = 0.27$
- Max QS error = 5.9%
- Max HTS Strain = 0.003
- Volume = 0.1m^3

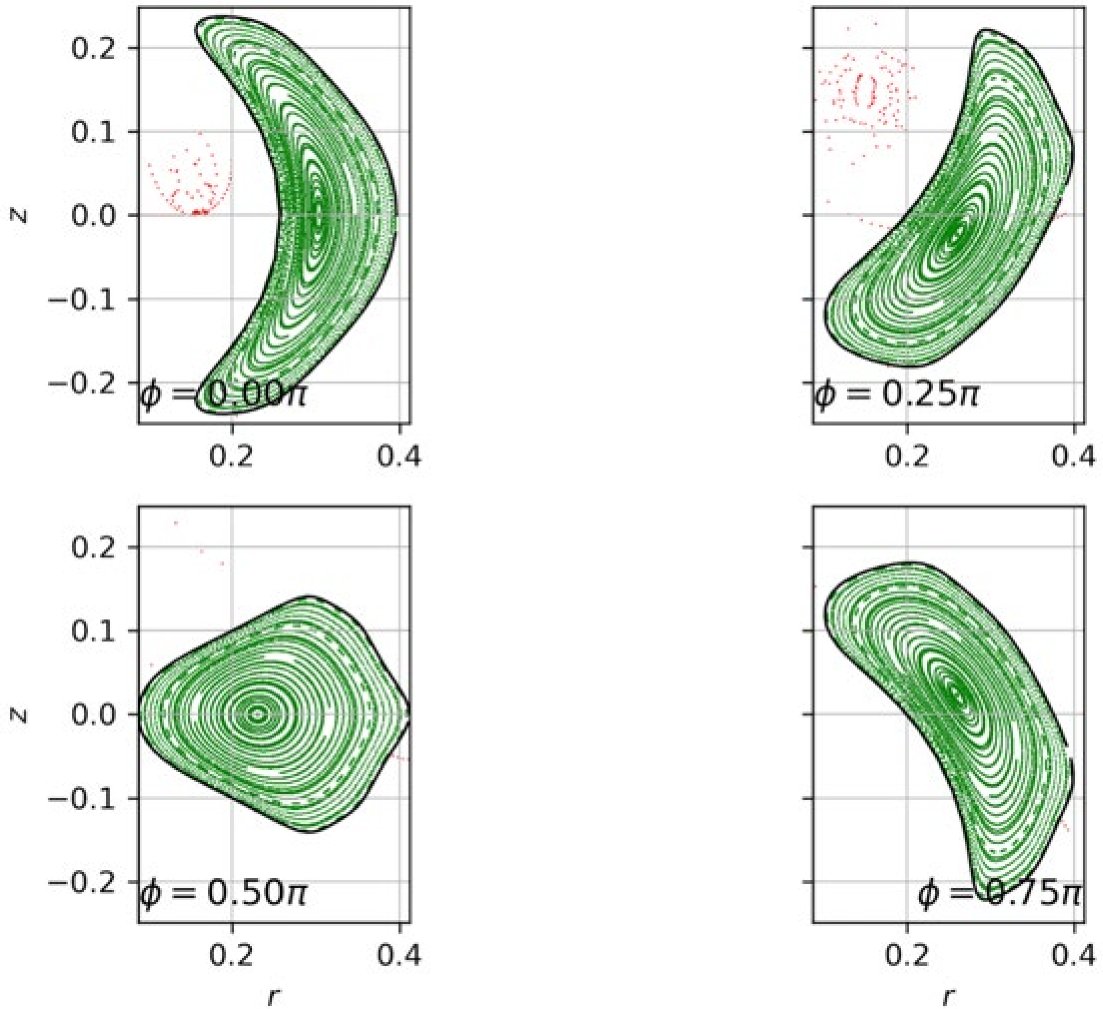




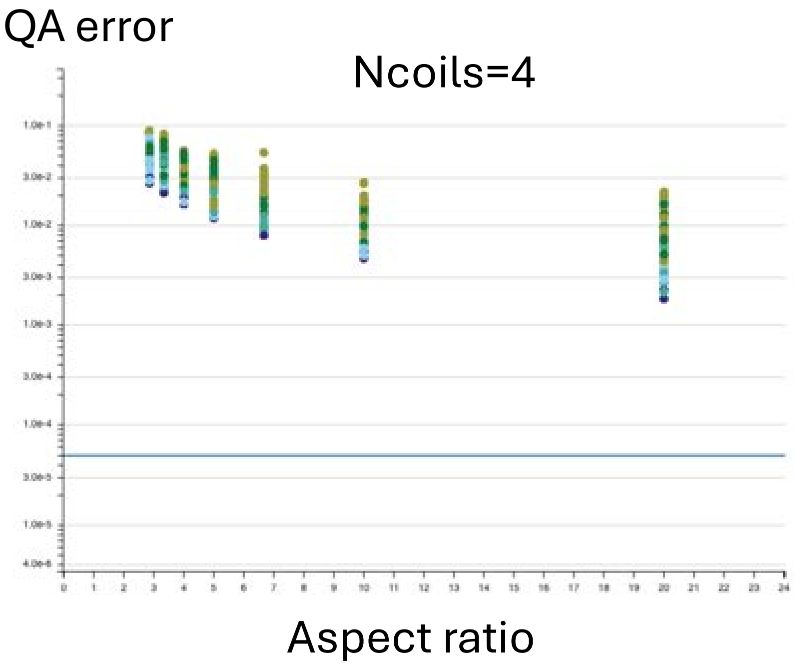
Rsccs initial guess leads to better configurations

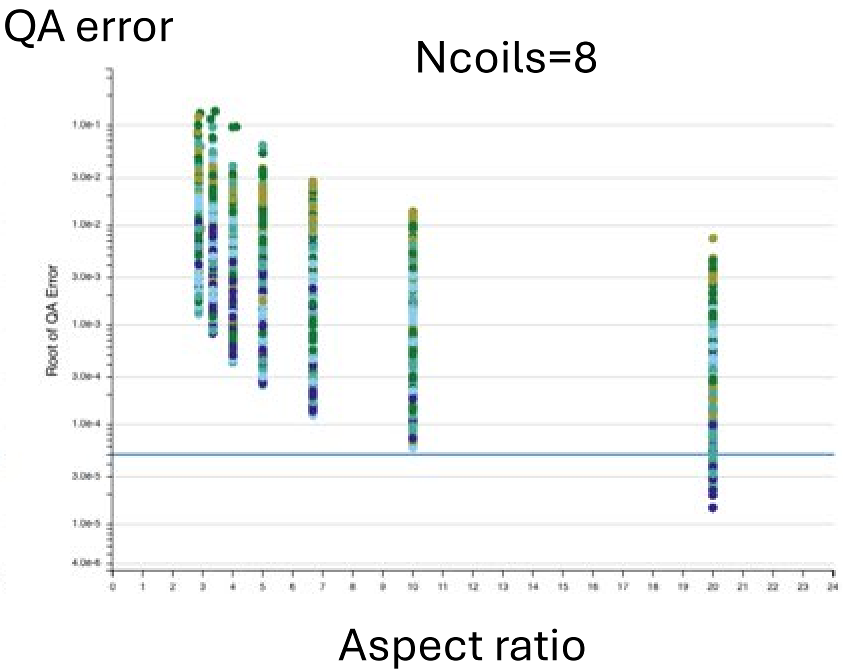
- $\iota = 0.27$
- Max QS error = 5%
- Max HTS Strain = 0.0028
- Volume = $0.1m^3$

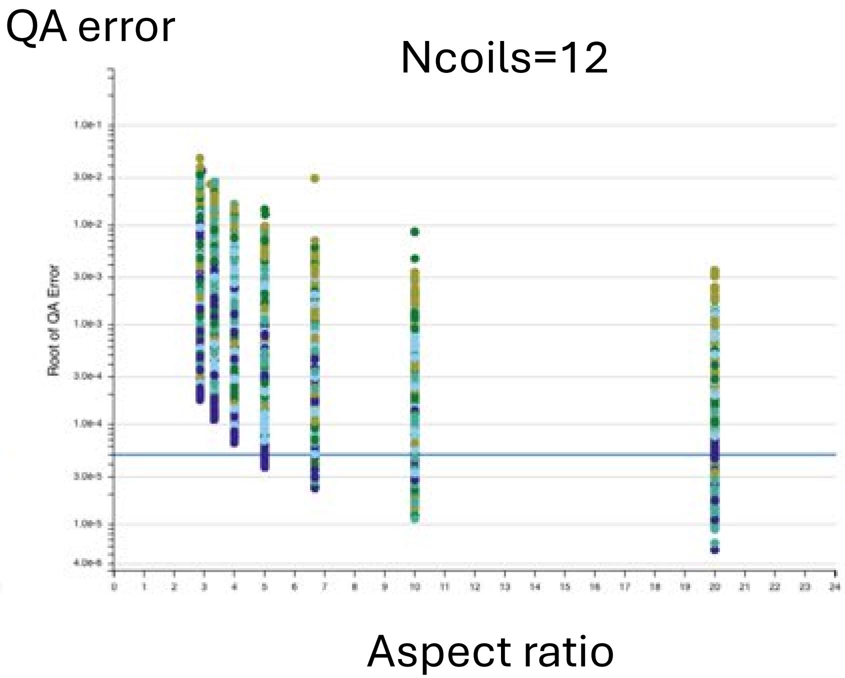




Tight aspect ratio competes with QA







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■ Mean Iota = 0.1 ■ Mean Iota = 0.2 ■ Mean Iota = 0.3 ■ Mean Iota = 0.4 ■ Mean Iota = 0.5

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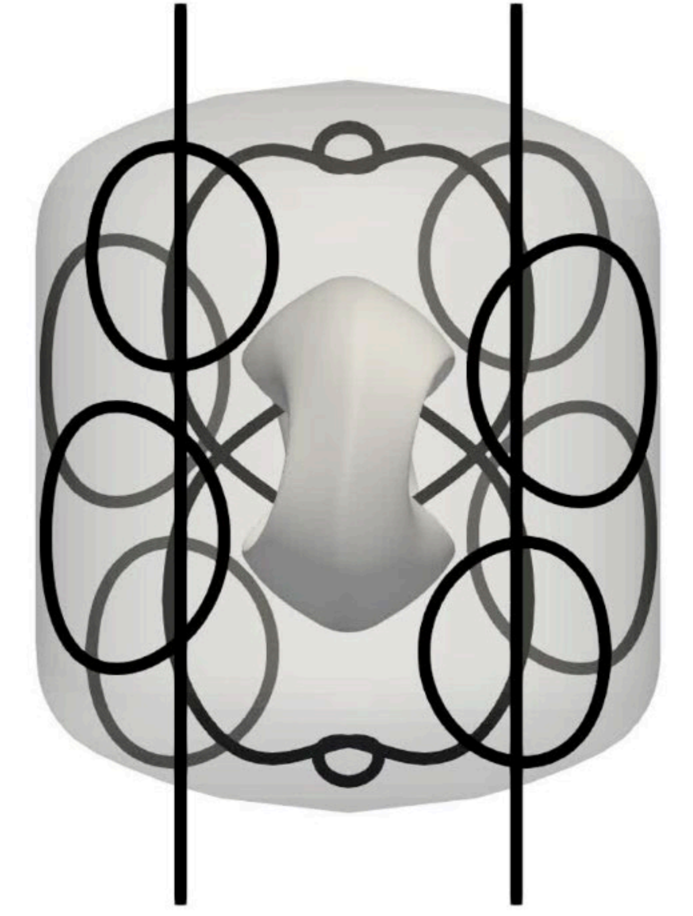
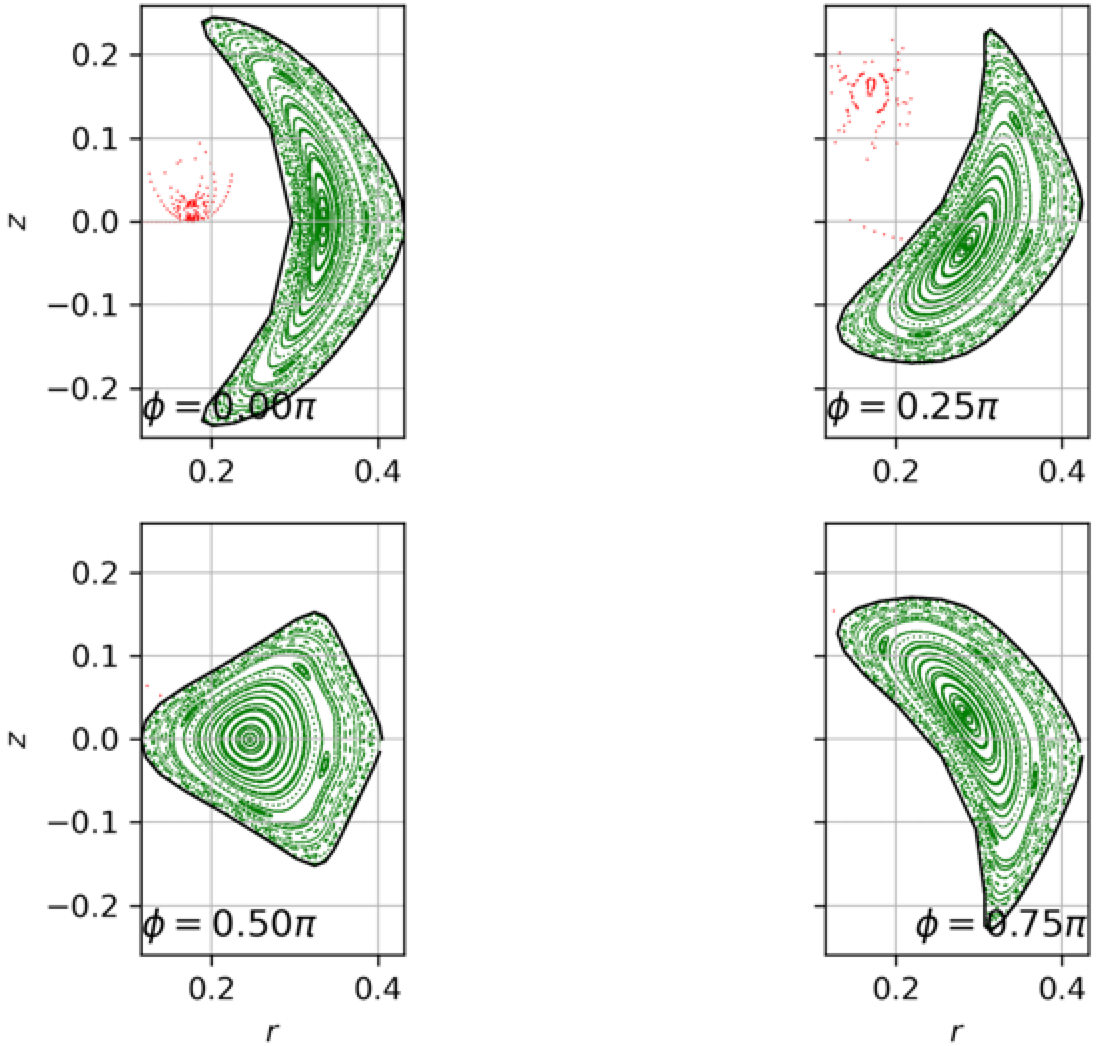
Better QA is obtained by including windowpane coils

- $\iota = 0.27$
- Max QS error = 2%
- Max HTS Strain = 0.0032
- Volume = 0.1m^3





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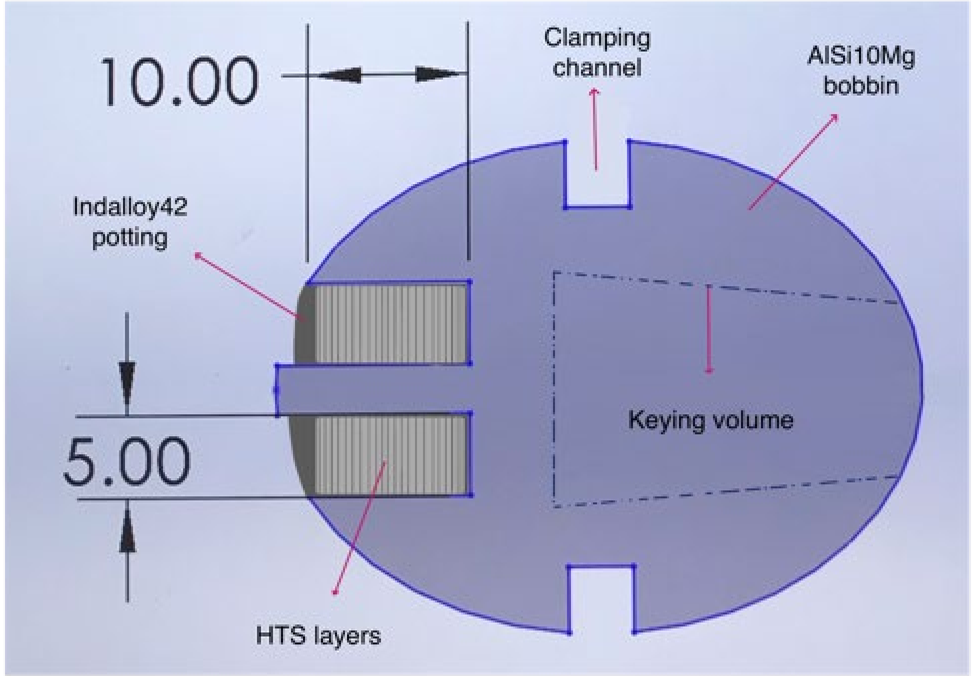
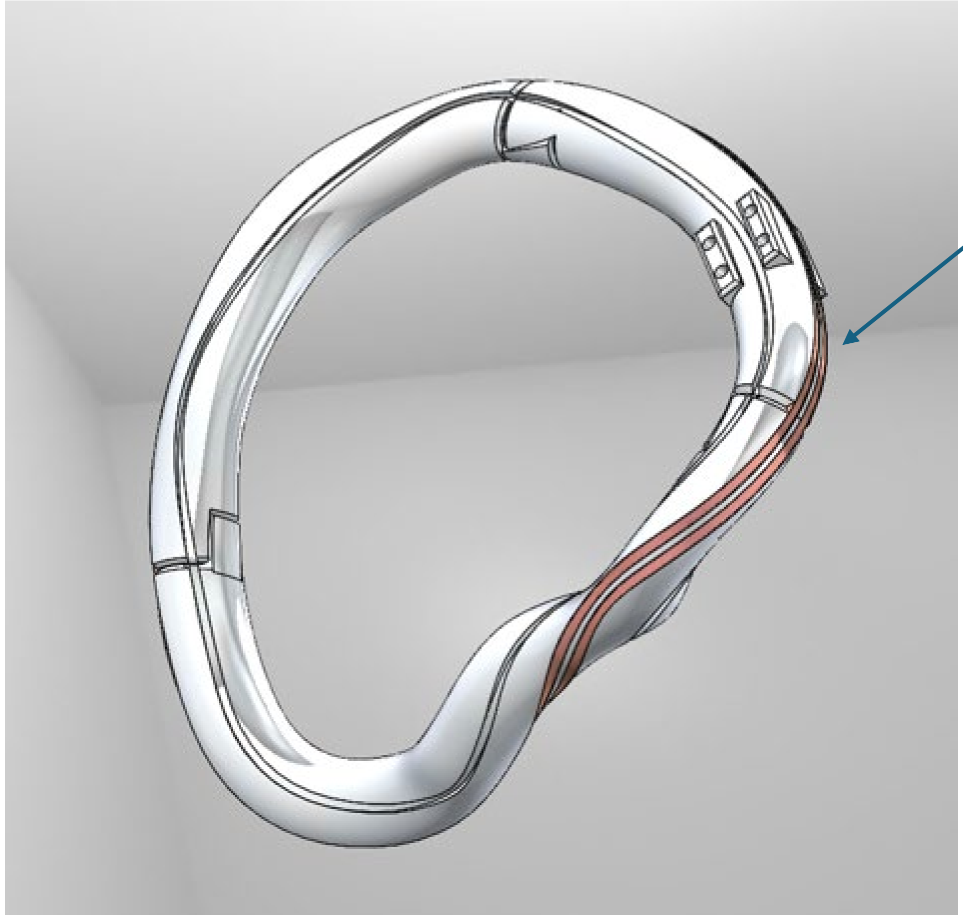
- Future plans and conclusions

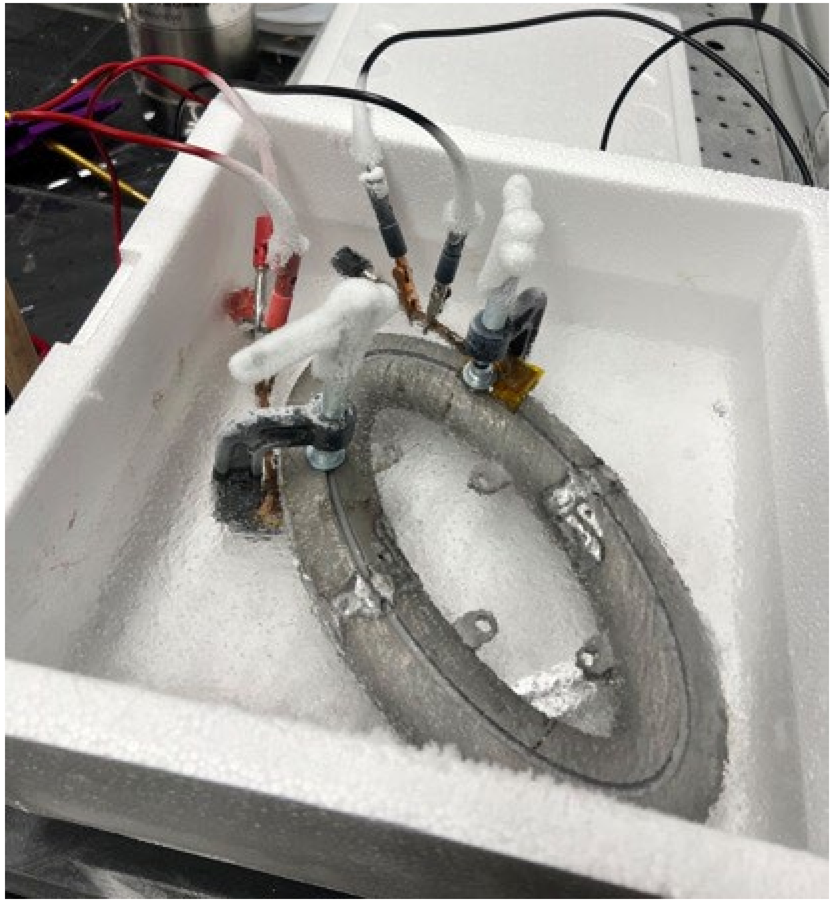
IL coils are made with non-insulated High-Temperature Superconducting (HTS) tape

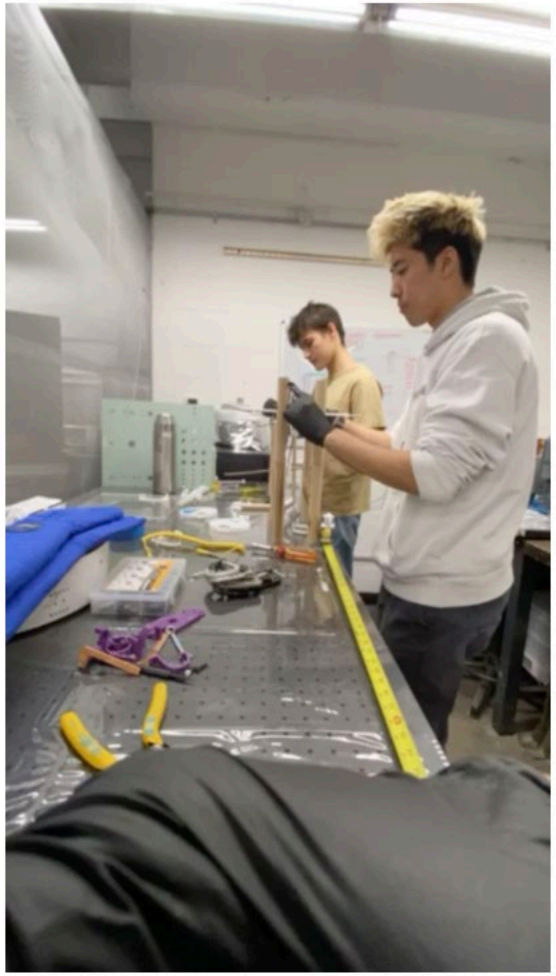




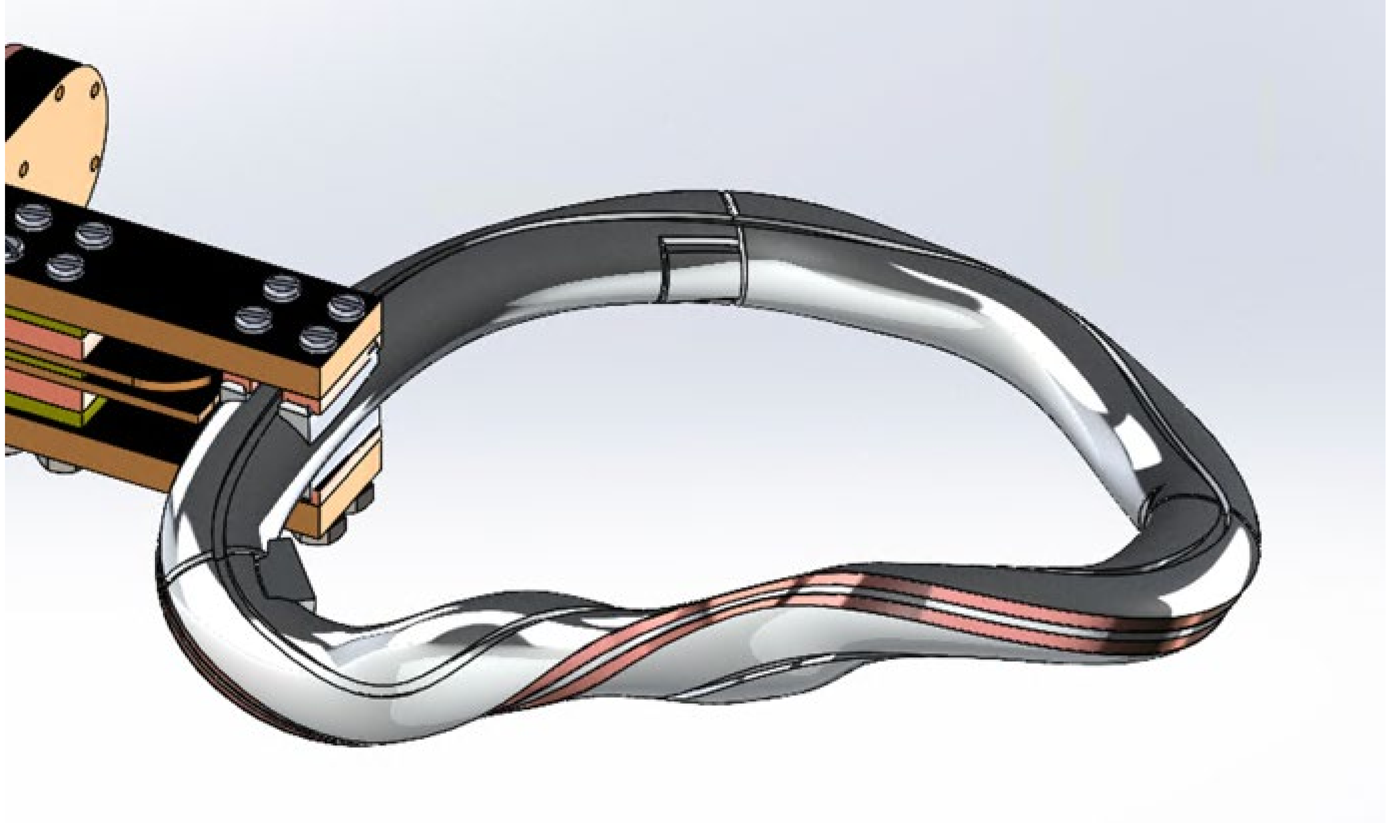
Optimized winding track

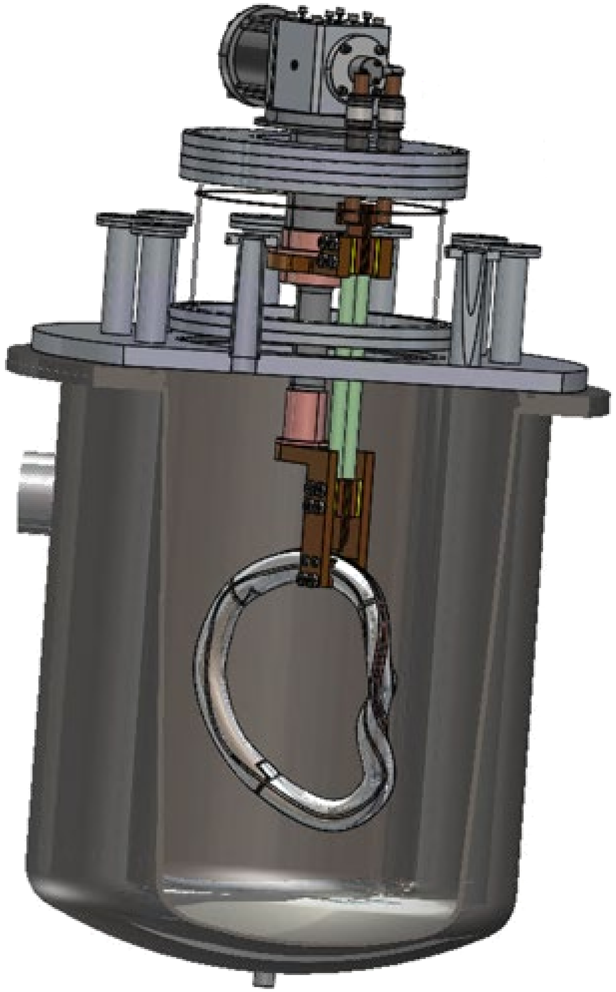
First prototype was planar





Second prototype is non-planar and will be tested this summer





Second prototype was wound recently

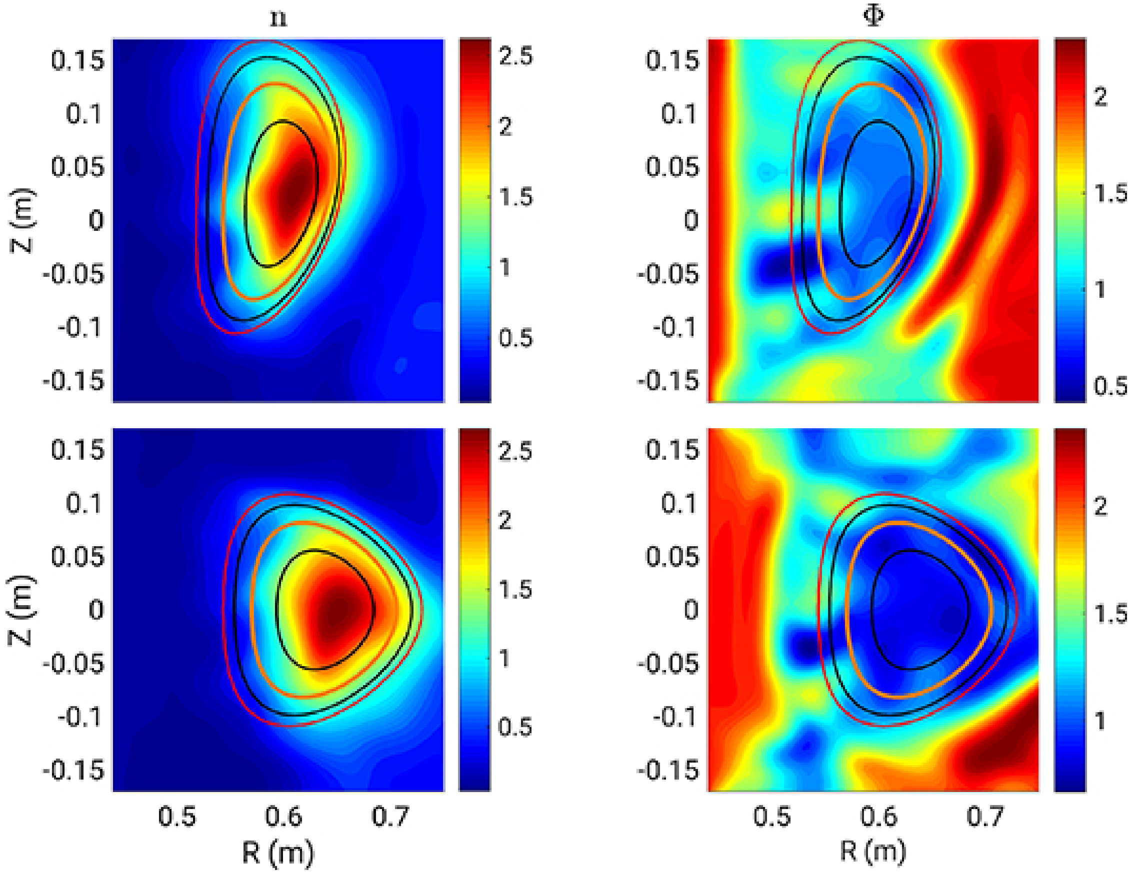
Table of content

Small scale devices are useful to validate fluid codes

- GBS, BSTING are fluid codes, useful to study plasma turbulence in the stellarator boundary
- Relevant for **collision-dominated plasma**
  - Plasma edge
  - Small scale devices
- Important differences with tokamak results call for **validation in stellarator geometries**

Coehlo *et.al.*, 2022

GBS validation in TJ-K



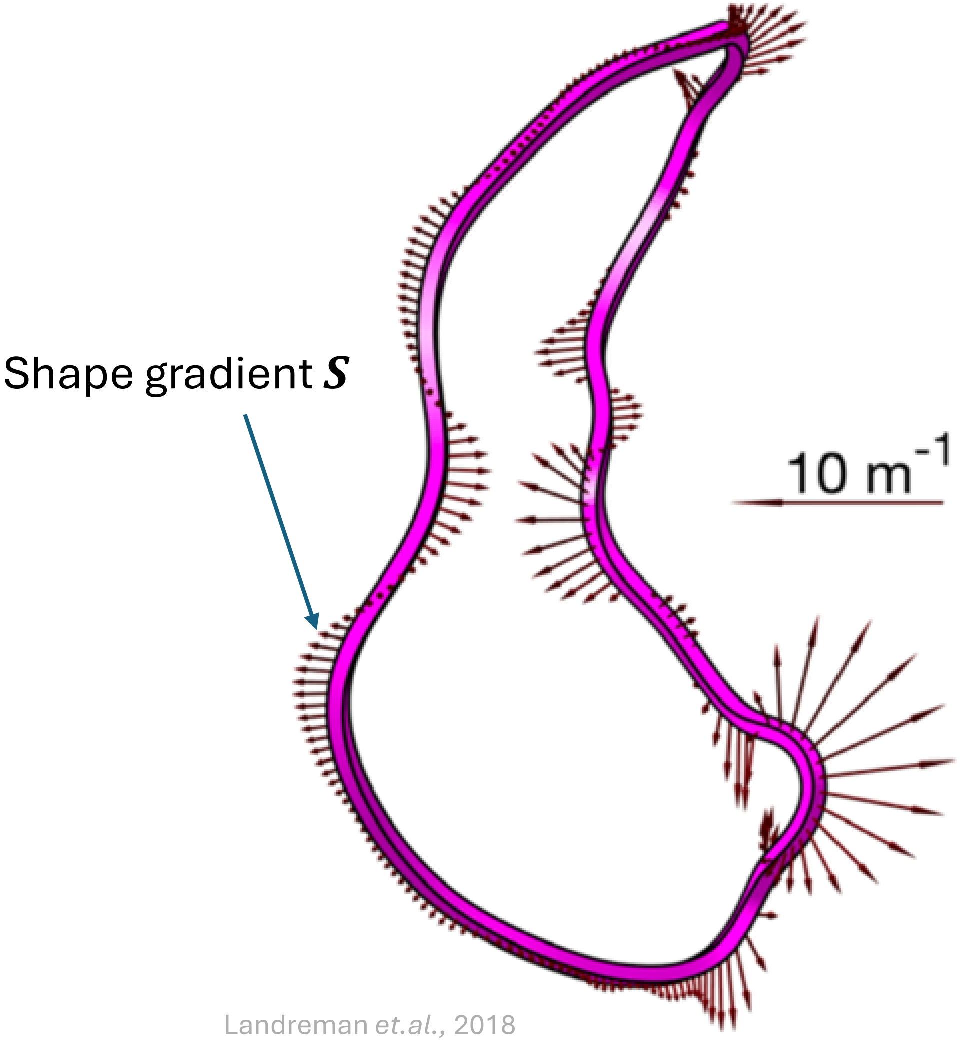
Coehlo *et.al.*, 2023

Shape gradients provide information on coil sensitivity

Goals: Evaluate the shape gradient of coils

$$\delta f = \sum_k \int dl\, \boldsymbol{S}_k \cdot \delta \boldsymbol{r}$$

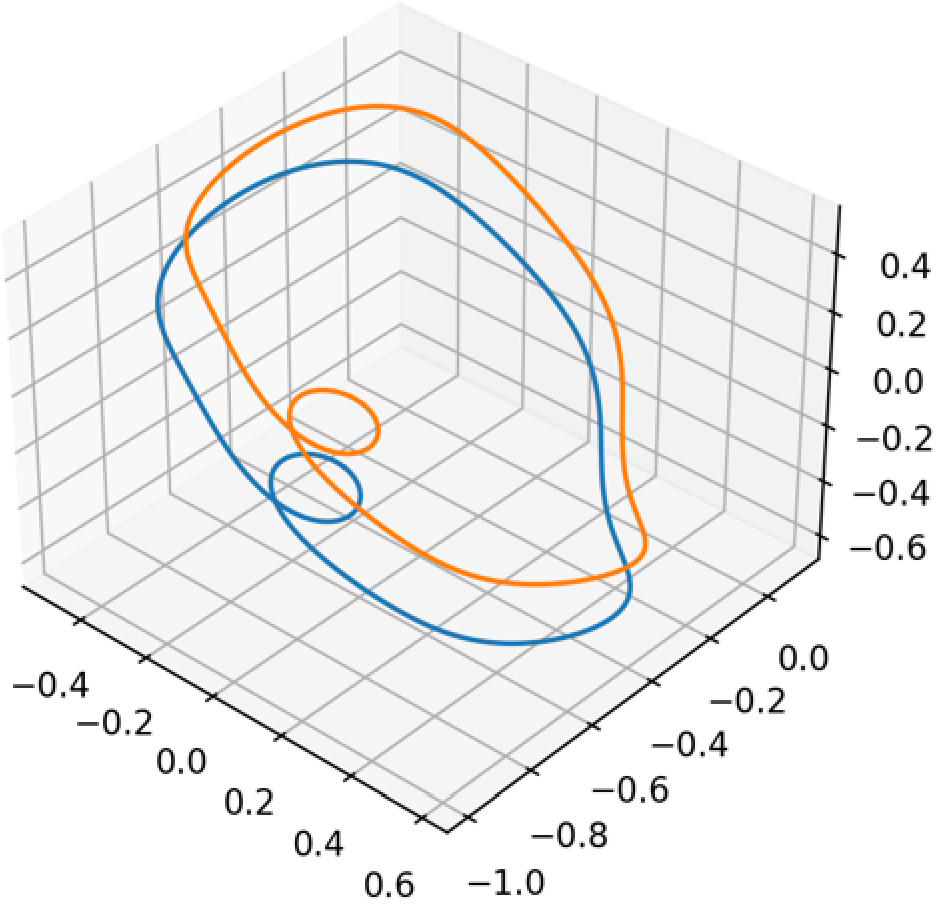
- δf can be a perturbation to QA, mean rotational transform, island width, …
- Estimate required precision from engineers
- Design **control coils** to decrease sensitivity



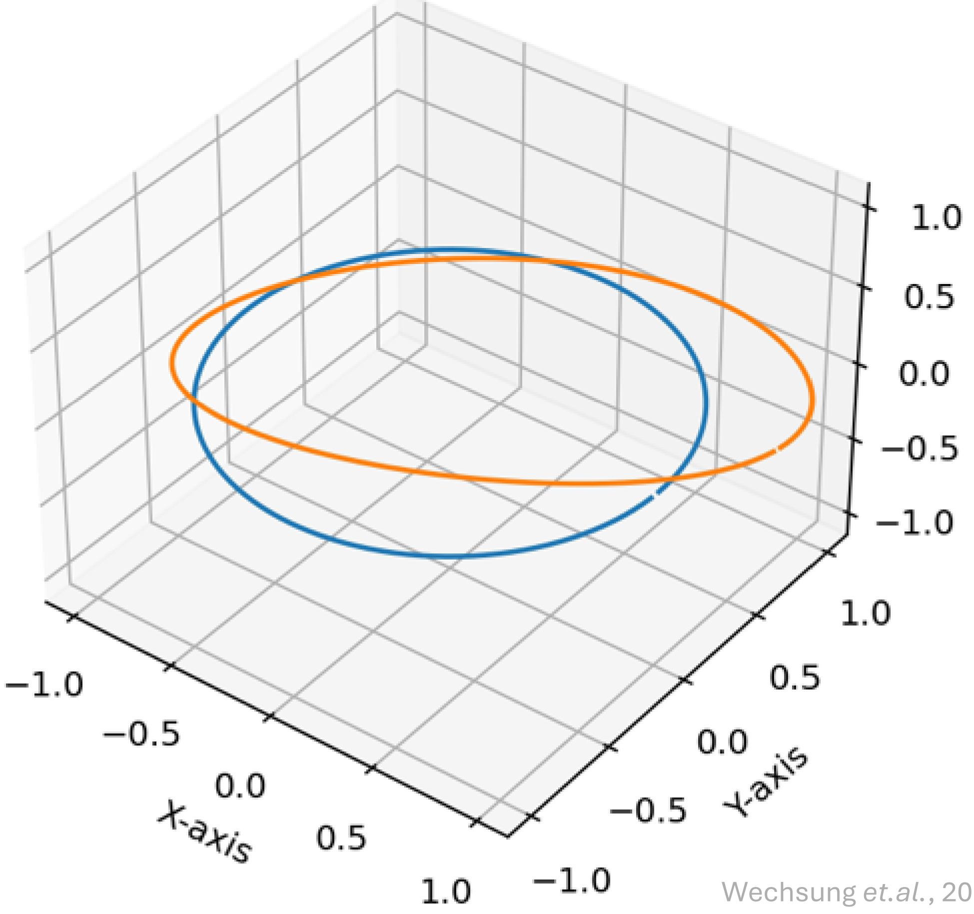
Landreman *et.al.*, 2018

Stochastic optimization

Rigid perturbations – positioning errors

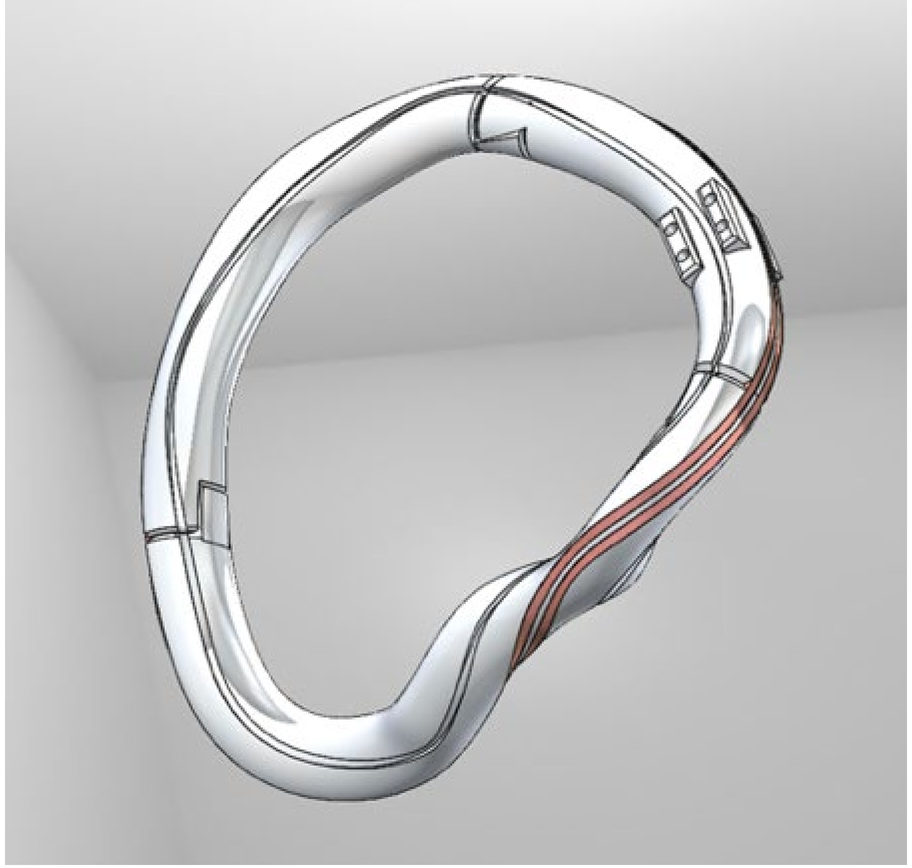


Gaussian perturbation - manufacturing errors



Wechsung *et.al.*, 2022

Effect of coils with finite width



Magnetic field now depends on

- Number of HTS tracks
- Number of HTS turns
- Coil section width
- Winding angle

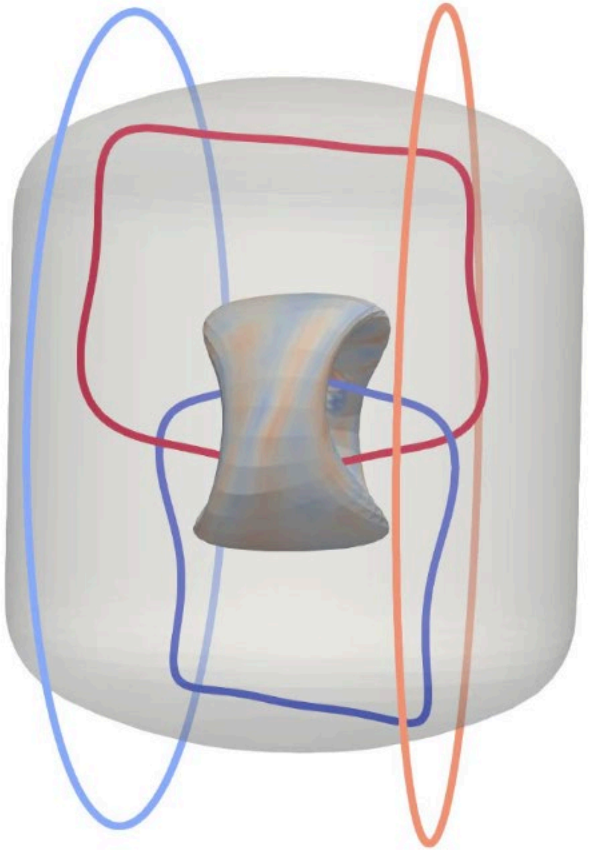
$$\boldsymbol{B} = \boldsymbol{B}(x_0, x_{cn}, y_{sn}, z_{sn}, \boxed{\lambda_0, \lambda_{cn}, \lambda_{sn},} I_{PF})$$

Conclusions

- **The CSX experiment is currently begin designed** at Columbia University
  - Quasisymmetric stellarator
  - Goal is to study flow damping, validate fluid codes, verify fast ion loss channels, and MHD mode saturation
  - We are open to any additional idea!

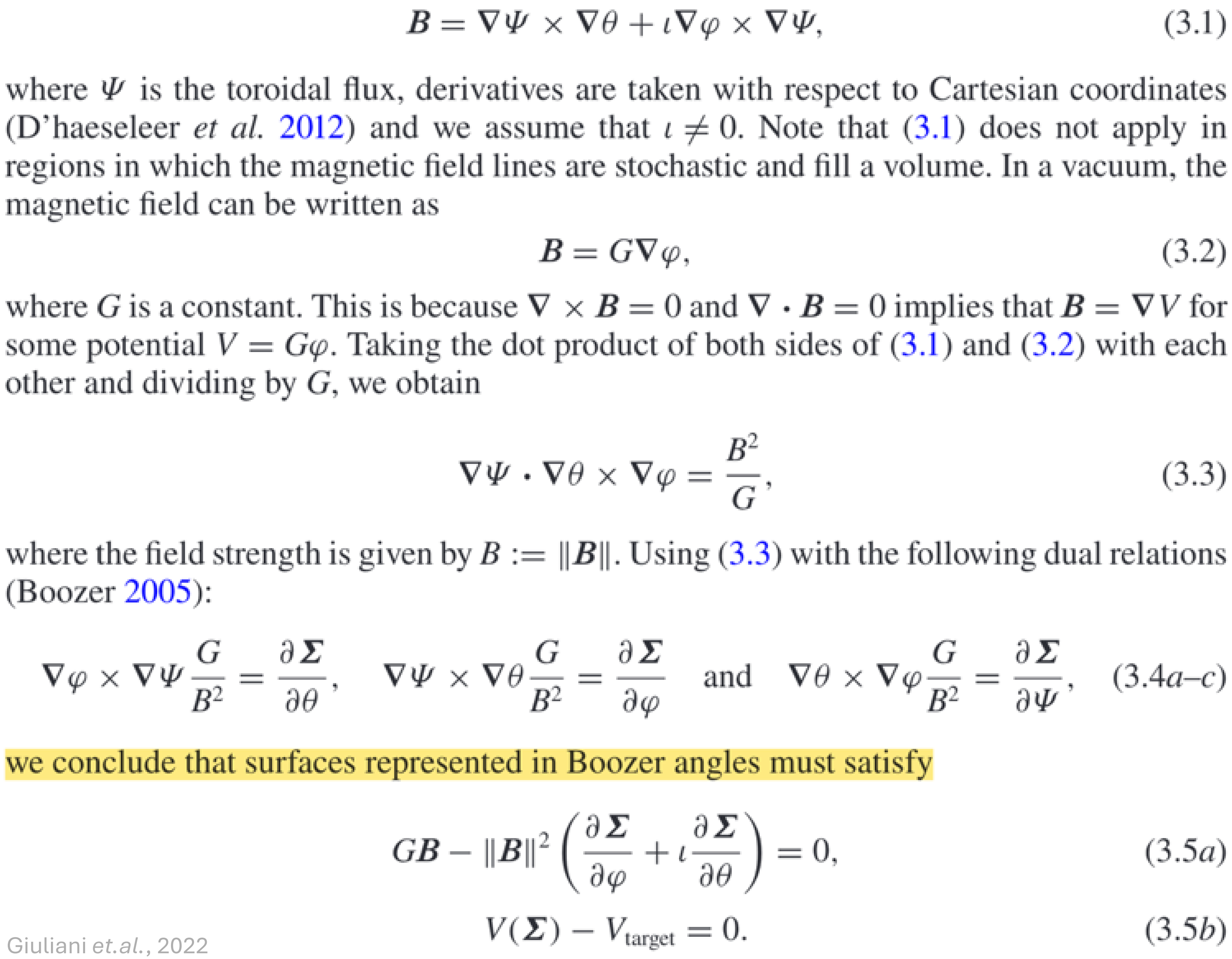
- **Tight engineering constraints** call for the use of combined plasma-coil optimization techniques
  - Refurnishing of CNT vacuum vessel and PF coils
  - Satisfactory QS levels are obtained for reasonable coils

- Coils are **manufactured at Columbia University**
  - Non-insulated HTS technology
  - HTS strain minimization is included in the optimization



Backup slides

Boozer Surface Construction



$$\boldsymbol{B} = \nabla\Psi \times \nabla\theta + \iota\nabla\varphi \times \nabla\Psi, \tag{3.1}$$

where Ψ is the toroidal flux, derivatives are taken with respect to Cartesian coordinates (D'haeseleer *et al.* 2012) and we assume that $\iota \neq 0$. Note that (3.1) does not apply in regions in which the magnetic field lines are stochastic and fill a volume. In a vacuum, the magnetic field can be written as

$$\boldsymbol{B} = G\nabla\varphi, \tag{3.2}$$

where G is a constant. This is because $\nabla \times \boldsymbol{B} = 0$ and $\nabla \cdot \boldsymbol{B} = 0$ implies that $\boldsymbol{B} = \nabla V$ for some potential $V = G\varphi$. Taking the dot product of both sides of (3.1) and (3.2) with each other and dividing by G, we obtain

$$\nabla\Psi \cdot \nabla\theta \times \nabla\varphi = \frac{B^2}{G}, \tag{3.3}$$

where the field strength is given by $B := \|\boldsymbol{B}\|$. Using (3.3) with the following dual relations (Boozer 2005):

$$\nabla\varphi \times \nabla\Psi \frac{G}{B^2} = \frac{\partial \boldsymbol{\Sigma}}{\partial \theta}, \quad \nabla\Psi \times \nabla\theta \frac{G}{B^2} = \frac{\partial \boldsymbol{\Sigma}}{\partial \varphi} \quad \text{and} \quad \nabla\theta \times \nabla\varphi \frac{G}{B^2} = \frac{\partial \boldsymbol{\Sigma}}{\partial \Psi}, \tag{3.4a–c}$$

we conclude that surfaces represented in Boozer angles must satisfy

$$G\boldsymbol{B} - \|\boldsymbol{B}\|^2 \left(\frac{\partial \boldsymbol{\Sigma}}{\partial \varphi} + \iota \frac{\partial \boldsymbol{\Sigma}}{\partial \theta}\right) = 0, \tag{3.5a}$$

$$V(\boldsymbol{\Sigma}) - V_{\text{target}} = 0. \tag{3.5b}$$

Giuliani *et.al.*, 2022

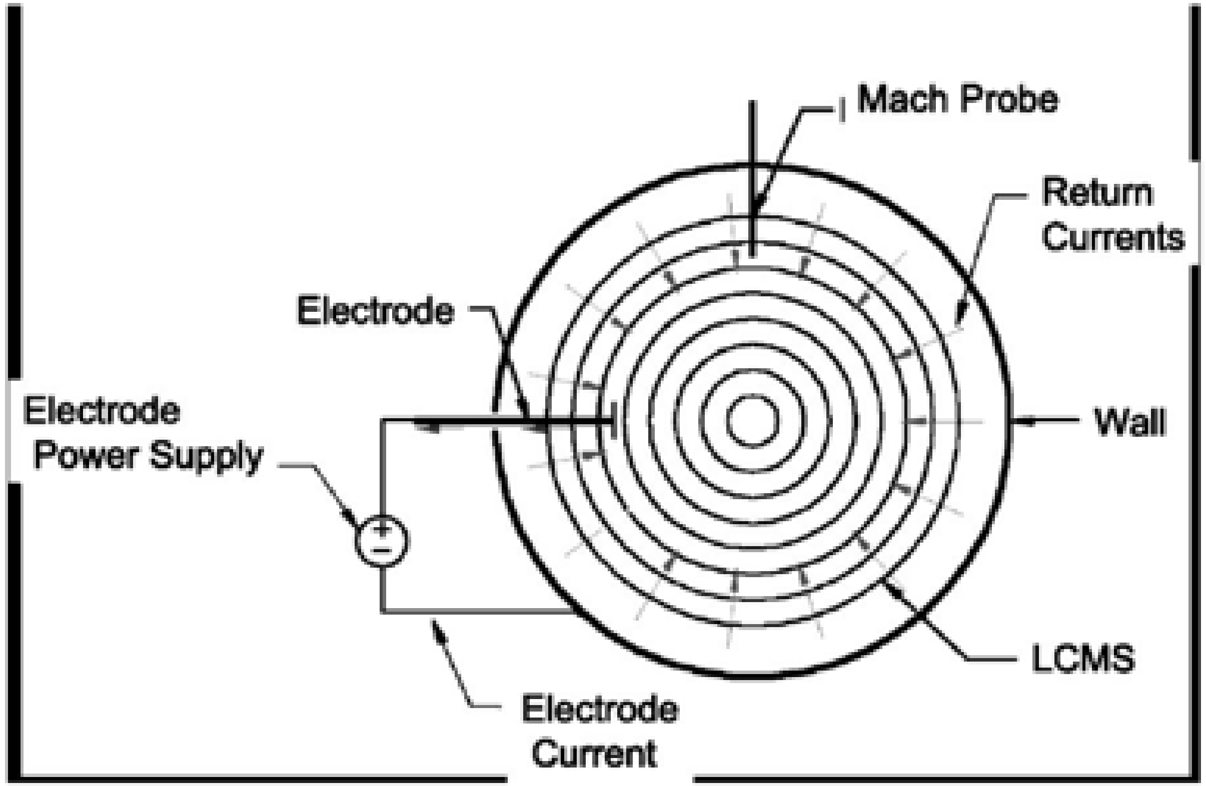
Stellarator symmetry

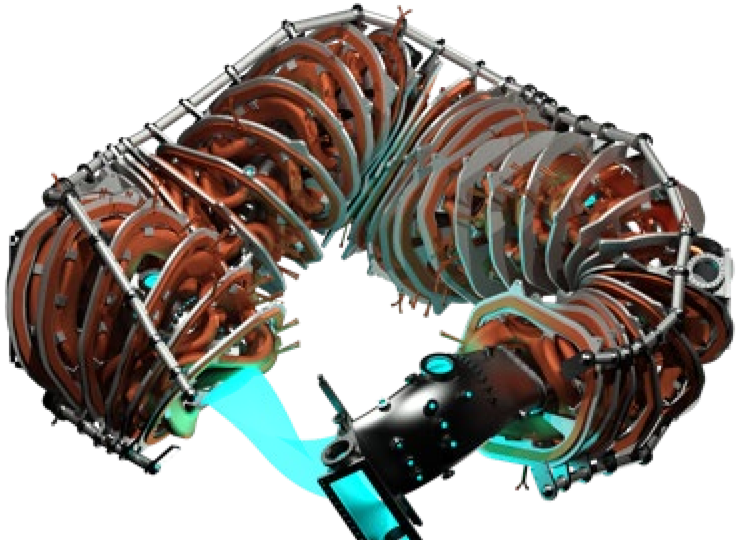
$$I_0 f(\rho, \phi, z) \equiv f(\rho, -\phi, -z)$$

$$I_0[F_\rho, F_\phi, F_z] = [-F_\rho, F_\phi, F_z],$$

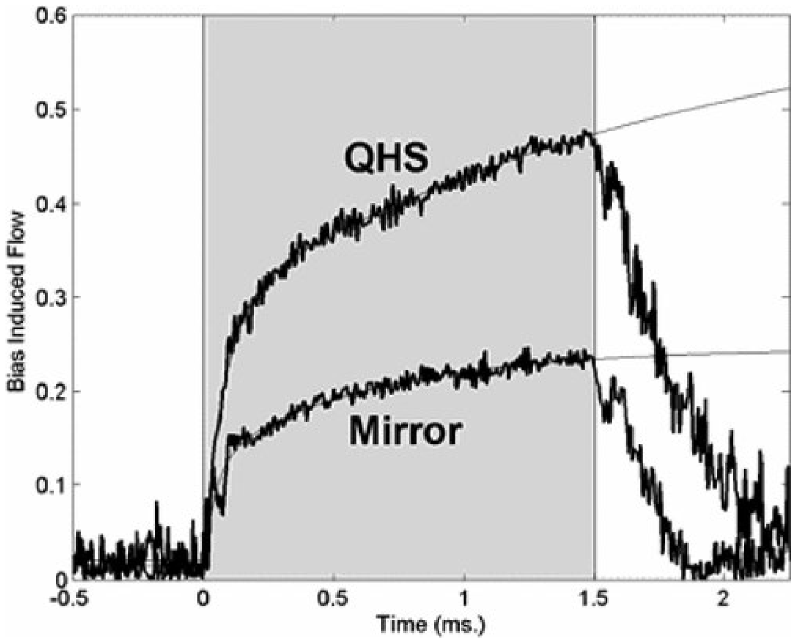
Dewar and Hudson, 1998

Flow damping can be measured in a QS field

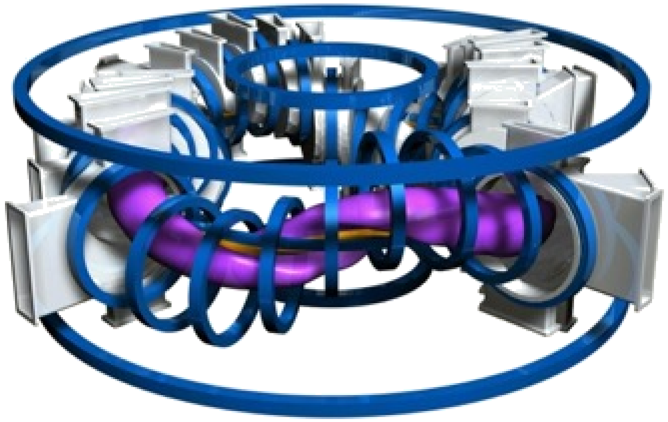




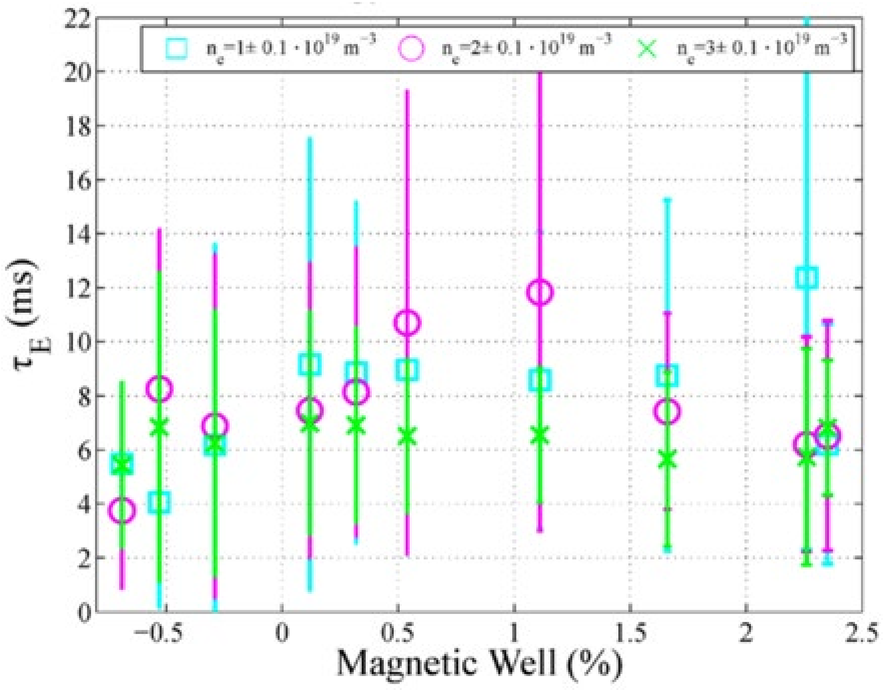
Gerhardt *et.al.*, 2005

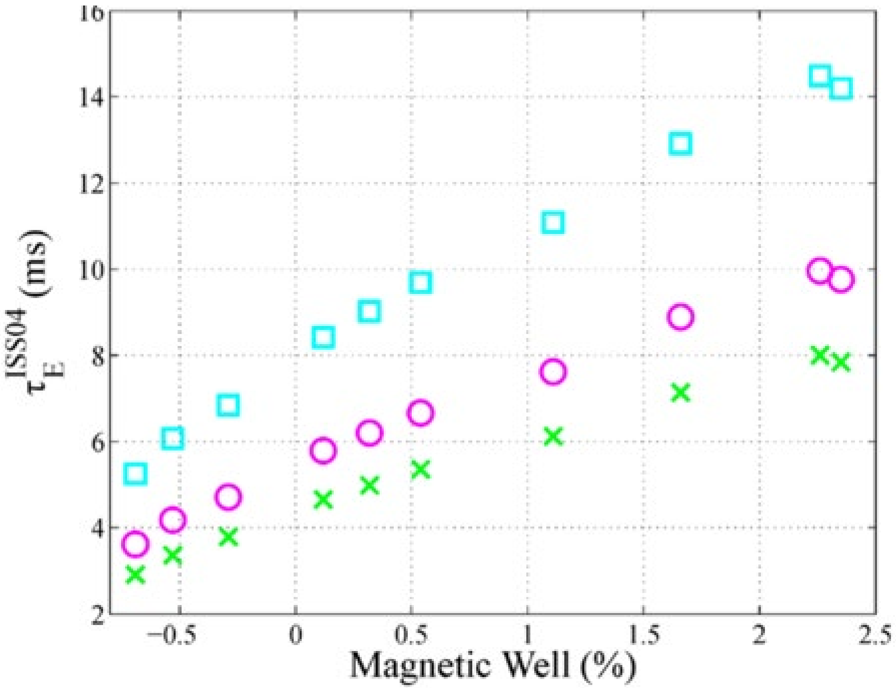


MHD could be studied in CSX



- Experimental evidence of interchange instability saturation at low amplitude
- Small impact on transport
- Is the magnetic well over-constraining our optimizations?





De Aguilera *et.al.*, 2015